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(151 Комп'ютеризовані системи управління та автоматика)

ВИПУСКНИКА ОСВІТНЬОГО СТУПЕНЯ
«БАКАЛАВР»

Тема: «Система орієнтації супутника з використанням магнітометра та датчика Землі»

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MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
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QUALIFICATION PAPER

(151 COMPUTERIZED CONTROL SYSTEMS AND AUTOMATION)

FOR THE ACADEMIC DEGREE OF BACHELOR

Title: “ A satellite orientation system using a magnetometer and a Earth sensor”

Submitted by: student of group CS-404 __________________Oleksangr RYZHKOV

Supervisor: PhD, associate professor ____________________Lev RYZHKOV

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НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ

Факультет аеронавігації, електроніки та телекомунікацій
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ЗАВДАННЯ

на виконання кваліфікаційної роботи

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7	Оформлення роботи	25.05.2024	
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“_____”______________2024

Qualification Paper Assignment for Graduate Student

Ryzhkov Oleksandr Oleksiyovich

1. The qualification paper title “A satellite orientation system using a magnetometer and a Earth sensor ” was approved by the Rector’s order of “ 01 ” April 2024 № 511/ст.

2. The paper to be completed between: 13.05.2024 and 16.06.2024

3. Initial data for the paper:

3.1 Composition of the Earth sensor system, magnetometer.

3.2 The error of determining angles is no more than 0.5.

4. The content of the explanatory note:

4.1 Management systems and automation

5. The list of mandatory illustrations:

6. Timetable

№	Assignment	Dates of completion	Completion mark
1	Collection of information	01.02.2024	
2	Theory writing	01.03.2024	
3	Preparation for practice	05.04.2024	
4	Model development	06.04.2024	
5	Writing code	25.04.2024	
6	Collection of all information	01.05.2024	
7	Designing the work	25.05.2024	
8	Coordination with the manager	02.06.2024	
9	Preliminary work protection	04.06.2024	
10	Work protection	14.06.2024	

7. Assignment issue date: “13” May 2024

Qualification paper supervisor ____________________ Lev RYZHKOV

(the supervisor’s signature)

Issued task accepted ____________________ Oleksandr RYZHKOV

(the graduate student’s signature)

Content

LIST OF TERMS, ABBREVIATIONS AND TERMS

SC is a coordinate system

KA is a space vehicle

OSK is an orbital coordinate system

ZSK is a connected coordinate system

CM is the center of mass

INTRODUCTION

Small spacecraft (SC) are becoming more and more common nowadays. In particular, nanosatellites are used to develop the latest technologies, methods and software and hardware solutions, as well as for educational programs, remote sensing of the Earth and space observations. Due to their small dimensions, weight and cost, as well as a wide range of applications, they have become an integral part of the scientific and space world.

Conducting most scientific and applied research in space involves ensuring a certain orientation of the angular position of the nanosatellite in space. To ensure the necessary orientation of the nanosatellite, an orientation system is created, which consists of an algorithm for determining angular values and a regulator that creates a control moment. In this paper, only the algorithm for determining the orientation due to information from the magnetometer and the Earth sensor is considered. Thanks to the use of this pair of sensors, the creation of an orientation system becomes simpler and more reliable. That allows you to be sure that the assigned mission has been solved.

Despite the large volume of research, in the created algorithms for determining the orientation of a nanosatellite, as a rule, they are built on the basis of the use of two-vector methods [1, 2], since they are easy to establish and sufficiently reliable. Among such algorithms, the TRIAD algorithm [1], which simultaneously determines three orientation angles, has become more common. But when using the data of the meters, it is not optimal, since the Earth sensor provides information about two angles, and the angle that is in the plane perpendicular to the orbital plane remains unknown. Based on this, there is redundant information. It should also be taken into account that the magnetometer is a less accurate meter than the Earth sensor, which also affects the accuracy of the orientation determination by the TRIAD algorithm. Solving the problem of eliminating redundant information, as well as reducing the influence of a less accurate sensor on the determination of the three orientation angles of a nanosatellite, is an actual direction of research. Solving this issue makes it possible to reduce the load on the on-board computer, as well as to eliminate the cross-influence of the gauges on the orientation accuracy. Which, in turn, will reduce the cost of the finished product due to the use of a weaker calculator, and increase the overall accuracy of determining the angular position.

The orientation system of the nanosatellite consists of three main elements, these are the meters of certain physical quantities (orientation sensors), the processing of information sent to the on-board computer (in which the orientation determination algorithm and the control signal generation algorithm are embedded) and the regulator that creates the control moment. The basic quality of determining the angular position of the spacecraft depends on the accuracy of the installed sensors, as well as the orientation algorithm. Determining the orientation of small spacecraft is often accomplished with instruments such as sun sensors and magnetometers. However, these sensors have various disadvantages. For example, solar sensors lose their functionality during periods of solar eclipse in orbit. Magnetometers cannot achieve high accuracy in determining the projection of the intensity of the Earth's magnetic field, due to its constant change. The sensors of the Earth's horizon appeared as an effective and relatively inexpensive meter to ensure accurate determination of the orientation of small spacecraft during low-orbital motion, their accuracy can reach 〖0.1〗^°.

Due to the low cost and acceptable accuracy of determining the orientation, the choice was made to use a magnetometer and an Earth sensor as part of the orientation system. We will analyze the existing orientation systems built on the basis of the Earth sensor and magnetometer.

1.1. Solving the problem of orientation determination using the Earth sensor and magnetometer

The magnetometer determines the intensity vector of the Earth's magnetic field. The accuracy of the magnetometric sensor depends on the quality and is 0.5-5 degrees. For correct operation, the sensor must be isolated from electromagnets physically or by switching magnets in the CA. They are not as accurate as star or horizon sensors, but the low accuracy is compensated by the simplicity, reliability, lightness and low cost of this sensor. Magnetometers weigh approximately 0.3 to 1.2 kg and consume less than 1 W of electricity.

Earth sensors (local vertical sensor) detect the Earth's electromagnetic radiation in the infrared spectrum, caused by the absorption and re-reflection of solar radiation by the Earth's surface and atmosphere, and also work in the optical spectrum and determine orientation through photo and video images. Earth sensors determine the nadir direction to the Earth or roll and pitch angles. The accuracy is from 0.1 to 0.25 degrees.

Orientation systems for small spacecraft, which are based on the use of a magnetometer and an Earth sensor, have long been known and used on real objects. Consider the following known solutions when creating an orientation system:

• The orientation system is based on a horizon sensor and a pair of biaxial magnetometers;

The main idea, which is considered in the article [3] is as follows, an autonomous magnetic orientation system is proposed for a small satellite of remote sensing of the Earth. The orientation detection hardware consists of a horizon sensor and a pair of biaxial magnetometers. Three-axis orientation control is carried out using three magnetic coils. The control laws are derived separately for the orientation determination and retention phases.

Disadvantages of the method:

- To stabilize the satellite, a suspended mass is used, which occupies a certain volume on the space carrier, as well as a long stabilization time;

- This scheme has limitations in orientation angles;

- The impact of magnetometer errors on the operation of the orientation algorithm is not considered.

The weak point of this approach is the satellite orientation time and angular position determination.

• Construction of the algorithm for determining the orientation of the spacecraft using the Kalman filter;

This paper [4] considers the development of an orientation system that determines the position of the satellite only by means of magnetic field measurements. This is achieved using a Kalman filter that estimates the spacecraft's orientation, velocity, and perturbation constants. Advantages of the method:

- Relative cheapness;

- Increasing the size of the spacecraft, due to the installation of an additional sensor.

Disadvantages of the method:

- The complexity of developing an orientation determination algorithm;

- The need to use the most accurate magnetometers;

- Accuracy of knowledge of the local magnetic field;

- Uncertainty at small deviation angles.

That is, the use of the Kalman filter makes it possible to accurately determine the orientation of the satellite, but to reproduce such an algorithm, it is necessary to use expensive sensors given the complexity of implementing this method.

• The algorithm for determining the angular position of the spacecraft using the TRIAD and QUEST matrix algorithms.

This article [5] presents two effective algorithms for determining the three-axis position of a spacecraft, using two or more vector observations. The first of them, the TRIAD algorithm [6], provides deterministic (non-optimal) solutions for orientation based on two vector observations. The second, the QUEST algorithm [7], is an optimal algorithm that determines the orientation that achieves the best weighted overlap of an arbitrary number of reference and observation vectors.

Advantages of the above algorithms:

- Relative simplicity in creating an orientation system.

Disadvantages of the above algorithms:

- Influence of sensor errors on determination of orientation.

The use of matrix algorithms when using a magnetometer and an Earth sensor is not optimal, since redundant information from the local vertical sensor is created, and the error of a less accurate meter is also dependent on the general determination of the satellite's angular position.

That is, the use of a magnetometer to determine the intensity vector of the Earth's magnetic field in the TRIAD and QUEST algorithms causes errors not only in the yaw angle, but also in the pitch and roll angles.

To avoid the listed disadvantages of the above algorithms, we will evaluate the proposed orientation determination algorithm [8], which was not implemented on real spacecraft. The work of the algorithm consists in the use of two meters, a magnetometer and a local vertical sensor. Pitch and roll angles (using the Earth sensor) are considered calculated. Projections of the magnetic field intensity vector in satellite-related coordinate systems were measured using a magnetometer. Based on this information, the yaw angle is determined.

1.1.1Conclusions on the section

Therefore, based on the conducted analysis, the development of a nanosatellite orientation system based on the Earth sensor and magnetometer using the proposed algorithm is a relevant topic of research. The above methods do not consider the use of a simple orientation determination algorithm that does not depend on magnetometer errors. This master's thesis presents the nanosatellite orientation system with a solution to this problem, as well as the requirements for the errors of the magnetometric sensor to ensure the necessary accuracy of the angular position.

The novelty of the work consists in the creation of a calculation model of the orientation system of a nanosatellite with an Earth sensor and a magnetometer, which uses an orientation determination algorithm that is simpler to implement and convenient in error analysis, if compared with standard solutions.

The subject of research is the nanosatellite orientation system.

2. Calculation method

When the orientation of the object in space is determined, an important issue is the determination of the reference systems used and the determination of the relationship between them. The appropriate method of calculation includes the determination of transition matrices and transformation formulas.

2.1. Coordinate systems

Different reference systems are used for tasks that describe the angular position of a nanosatellite in space. The correct choice of the frame of reference ensures the simplification of the equations of motion of the object, and also improves the informativeness of determining the position of the body in space.

2.1.1. Absolute geocentric coordinate system

Geocentric equatorial rectangular coordinate system. As shown in fig. 1, the beginning of this SC O_E is located in the center of mass of the Earth, the main plane OX_E Y_E lies in the plane of the equator, the axis OX_E is directed to the point of the vernal equinox Y, the axis OZ_E coincides with the axis of rotation of the Earth and is directed to the North Pole of the Earth, the axis OY_E completes the system to the right .

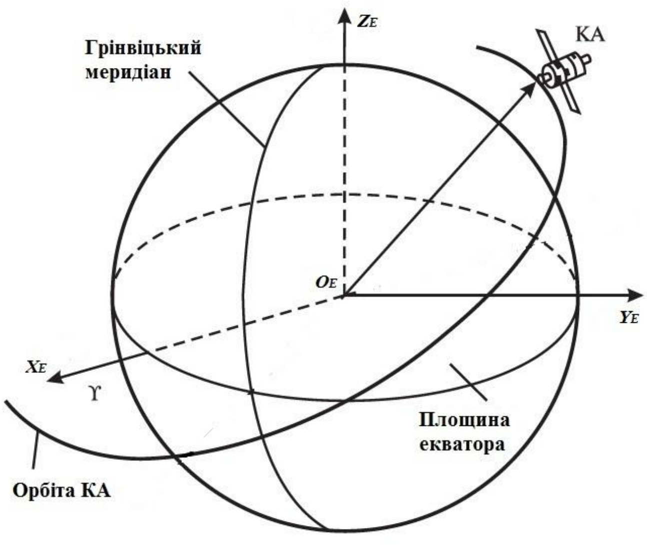

Fig. 1. Absolute geocentric SK

2.1.2. Orbital coordinate system

– orbital coordinate system (OCS), O is the center of mass of the nanosatellite, the axis OX_o lies in the plane of the orbit and is perpendicular to the radius vector of the center of mass of the satellite in the direction of its movement, the axis OZ_o is along the radius vector, the axis OY_o completes the system to the right.

2.1.3. Kepler elements of orbit

Kepler elements of the orbit are used to determine the state vector of the satellite, which fully characterizes its movement and position in space. In fig. 2 shows the spherical Earth and the elliptical orbit of the spacecraft in space. They allow you to determine the values of the coordinates and components of the velocity vector of the center of mass of the satellite at any time.

The main elements of the orbit:

The inclination of the orbit i ($0 \le i \le \pi$) is the angle between the plane of the orbit and the plane of the Earth's equator (counted from the plane of the equator counterclockwise, if you look along the line of nodes from the ascending node in the direction of the descending node);

Longitude of the ascending node of the orbit – the angle in the plane of the equator between the directions to the point of the vernal equinox and to the ascending node of the orbit (it is counted in the plane of the equator from the point of the vernal equinox counterclockwise to the direction to the point of the ascending node, if viewed from the northern end of the Earth's axis);

The perigee argument ω ($0 \le \omega \le 2\pi$) is the angle in the orbital plane between the line of nodes and the line of apsides (it is measured from the ascending node to the perigee of the orbit in the direction of the spacecraft);

The true anomaly ϑ is the angle between the directions from the center of the Earth to the perigee and the location point of the spacecraft;

- *The semi-major axis a is a distance equal to half the length of the apse line;*
- *Eccentricity e is a parameter that determines the shape of the orbit;*

- *The time of passage of the spacecraft through perigee t_П.*

The quantities i and Ω characterize the position of the orbital plane in space, and the quantities a and e – the size and shape of the spacecraft's orbit. The dimension ω represents the angular distance from the ascending node Ω to the perigee of the orbit, that is, it characterizes the state of the ellipse in space.

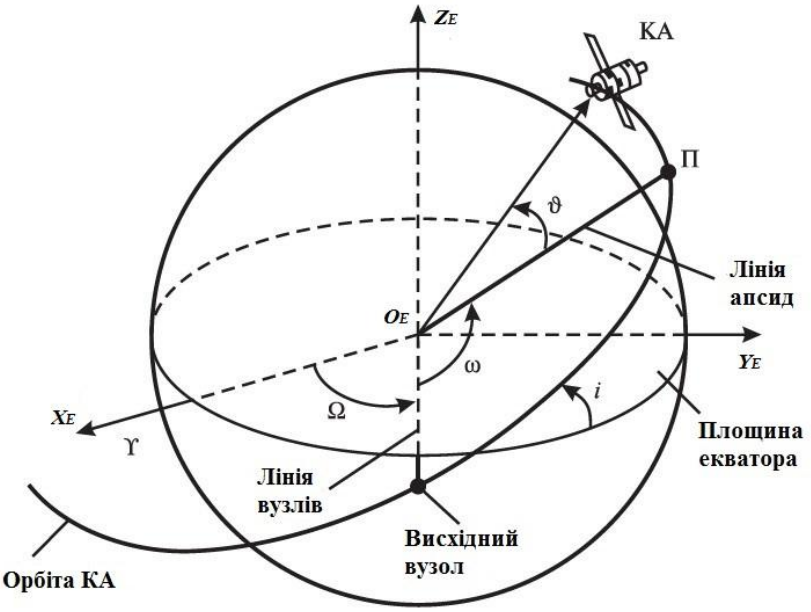

Fig. 2. Spacecraft orbit in space

Knowing all six elements of the orbit i, Ω, ω, a, e, t_П, you can calculate the coordinates of the satellite for any moment in time. This SC is used in descriptions of the true motion of the satellite through the elements of its orbit.

2.1.4. Linked coordinate system

Let's introduce the coordinate system rigidly connected to the satellite in Fig. 3, that is, during the movement of the spacecraft, the coordinates of its points in the system do not change. We place the origin of coordinates O in the center of mass of the KA, the axis OZ_KA is directed along the radius vector r KA, the axis OX_KA is chosen in the plane of the orbit, perpendicular to 〖OZ〗_KA so that when combining Z_KA with the axis OZ_o, the axis OX_KA is combined with the axis OX_o. The OY_KA axis complements the OX_KA Y_KA Z_KA system to the right rectangular coordinate system.

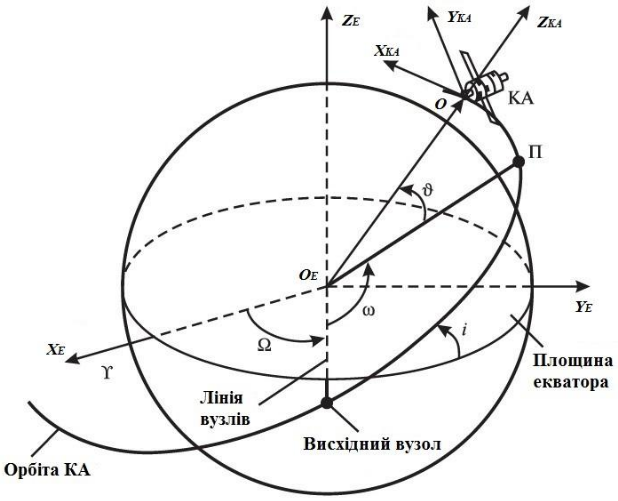

Fig. 3. Connected coordinate system

2.2. Conversion between SC

The position of the axes of the spacecraft OX_ka Y_ka Z_ka relative to the base coordinate system OX_o Y_o Z_o is determined using matrix A of the transition from the orbital to the nanosatellite-linked coordinate system. Since the transition from OSK to ZSK, as can be seen from fig. 4, is performed by successive rotations by the angles ψ, θ and φ, then the matrices of single angular transitions have the form

The first rotation by the angle ψ around the axis OZ_o: ,

The second rotation by the angle θ around the axis

OY_o^': ,

Rotation by an angle φ around the axis :

.

Thus, the resulting transition from orbital to bound SC has the following matrix form

.

.

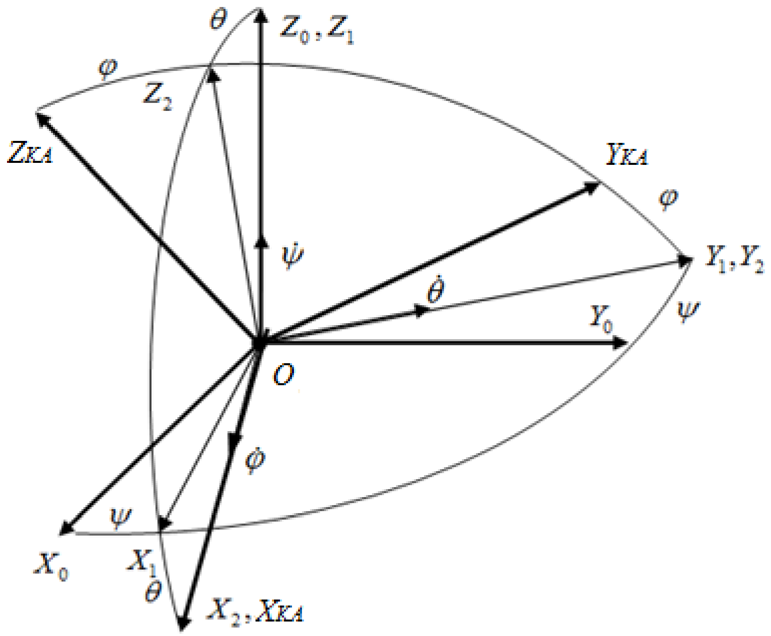

Fig. 4. Transition from USK to ZSK

The position of the spacecraft in orbit in the plane of motion is found using the matrix M of the transition from the geocentric equatorial rectangular coordinate system OX_E Y_E Z_E to the moving coordinate system OX_o Y_o Z_o connected to the center of mass of the nanosatellite. Due to the fact that, the transition from the OX_E Y_E Z_E coordinate system to the OX_o Y_o Z_o coordinate system, as can be seen from fig. 5, occurs due to successive rotations to the angles Ω, i and u=ω+ϑ , then the transition matrix M has the

form .

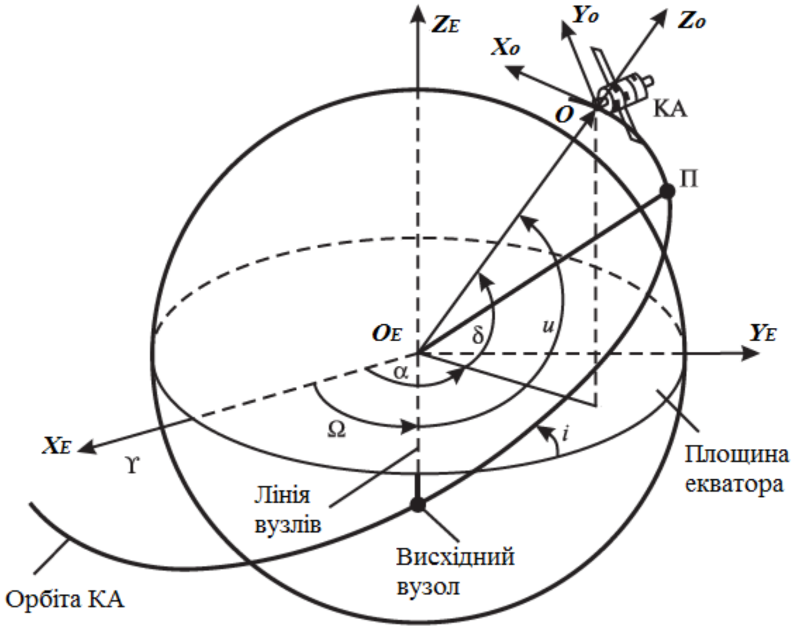

Fig. 5. The position of the moving SC relative to the inertial one

Any vector k specified in the absolute geocentric coordinate system OX_E Y_E Z_E using the transition matrix M can be translated into the moving orbital coordinate system OX_o Y_o Z_o connected to the center of mass of the nanosatellite ,

,

where k _o is the k vector, in the orbital coordinate system, M^(-1) is the inverse matrix M (which is orthogonal).

Conclusions on section

The main coordinate systems have been chosen, which will serve as the basis for creating a nanosatellite orientation system and ensuring a simpler reproduction of the processes taking place. Also in this section, the relationship between coordinate systems is considered and the transition matrices between them are defined. For certainty, the main version of the orientation of the nanosatellite in space will be the orbital coordinate system located at the center of mass of the spacecraft.

3. Development of a model of the system for determining the orientation of nanosatellite

3.1. Earth's magnetic field model

The complexity of the research and the accuracy of describing the angular motion of the nanosatellite in the Earth's magnetic field largely depends on the selected field model. The simpler the model, the more possibilities the developer has in using research methods, but due to this, the reliability of the description of transient movements decreases. To obtain general ideas about the behavior of a nanosatellite, it is necessary to obtain the most universal description without excessive detail, which forces the use of a simpler model. A simple magnetic field model will be used to calculate a specific orientation determination system.

3.1.1. International Geomagnetic Reference Field/World magnetic model

The most complete geomagnetic field is described by the IGRF (International Geomagnetic Reference Field) and WMM (World Magnetic Model) models. Both models use the expansion of the potential of the internal field to the Gaussian series [9]:

– ,

where λ_0 is the longitude of the point where the field induction vector is determined, $\vartheta_0 = 90^\circ - \theta_0$, θ_0 is its latitude, r is the distance from the center of the Earth, R is the average radius of the Earth, g_n^m and h_n^m are coefficients (in nT) from table [10], P_n^m are quasi-normalized Schmidt adjoint Legendre functions. The induction and field strength vectors are determined by the expression

$B = \mu_0 H$,

where $\mu_0 = 4\pi \cdot 10^{-7}$ kg·m·A^{-2}·s^{-2} is the magnetic constant. The values of the coefficients in this series are determined empirically with the help of statistical processing of numerous measurements of the geomagnetic field. Both models differ only in coefficients and have common limitations. They are used for altitudes up to 600 kilometers above the Earth's surface (WGS84) and are defined by a specific year. In the last year of the model, the International Geodetic and Geophysical Union publishes new IGRF coefficients valid for the next five years. The US National Oceanic and

Atmospheric Administration does the same for the WMM model. Such models are usually used on board the satellite to achieve the highest possible accuracy and in the numerical simulation of its motion during the development phase, but are not used in analytical studies.In fig.7 it is worth noting that the IGRF model is usually used to provide satellite orientation.

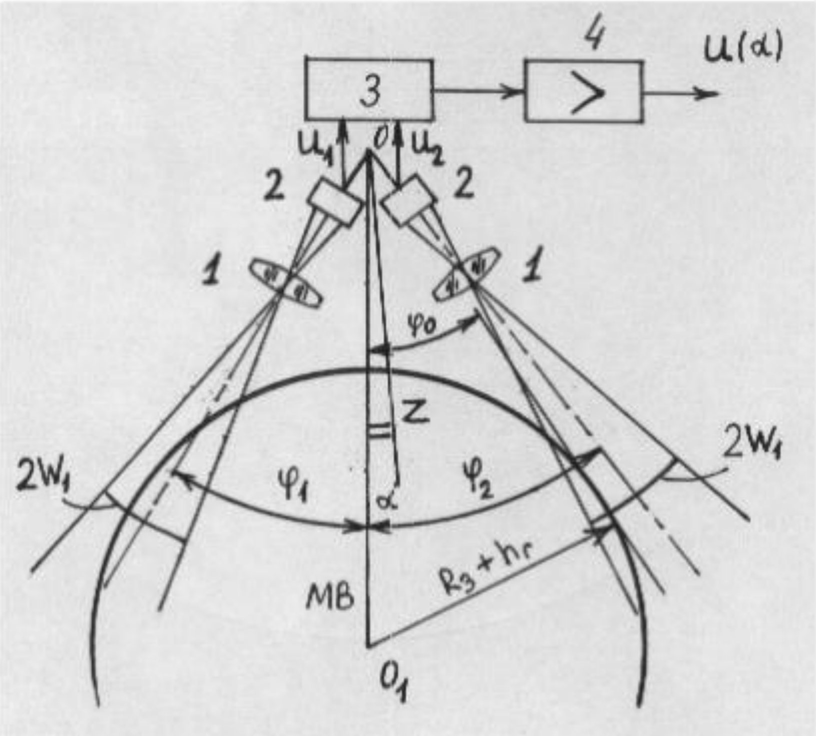

Fig. 6

3.1.2. Direct dipole 26

In the future, a simplified model will be used - the direct dipole model [11]. The geomagnetic field is approximated by the field of a dipole located in the center of the Earth and antiparallel to its axis of rotation. In this case, in the OXYZ system, the dipole direction vector has the form k=(0,0,-1). Accordingly, the expression for the geomagnetic induction vector in the OXYZ system takes the form

$$— \quad \_ \qquad (3.1)$$

where r is the value of the radius vector of the point in which the induction is calculated, $\mu_e=\mu_0 \mu_m/4\pi$ is the value determined by the first three components of the expansion, $\mu_m=7.7245\cdot 10^6$ T·km^2 is the dipole value Earth

The size of the induction vector B changes during the movement of the satellite in orbit and is

— (3.2)

In the orbital coordinate system $OX_o Y_o Z_o$, the geomagnetic field is written in the form

— (3.3)

The use of the direct dipole model will make it possible to take into account the non-uniformity of the rotation of the local magnetic induction vector during the movement of the nanosatellite in orbit. Also, this model will allow to fairly correctly describe the main properties of the Earth's magnetic field for the correct operation of the magnetometric orientation sensor.

3.2. Motion of the nanosatellite relative to the center of mass

To write the equations of motion of the satellite [12,13,14,15] relative to the center of mass, we use the reference coordinate system $OX_o Y_o Z_o$ and the moving $OX_{ka} Y_{ka} Z_{ka}$. We will determine the position of the nanosatellite-related coordinate system relative to the orbital one using the orientation angles ψ, θ, and φ. The transition between systems will be performed due to the orthogonal transition matrix A.

Euler's kinematic equations

Kinematic equations establish a relationship between the orientation angles and the angular velocity of the nanosatellite. The angular velocity of the spacecraft in the reference coordinate system can be represented as ω =ψ +θ +φ .

Let's project the absolute angular velocity vector ω on the OX_ka Y_ka Z_ka axis using the transition matrix A and get the projections ω_x, ω_y, ω_z in the connected coordinate system

(3.4)

Equations (3.4) are called Euler's kinematic equations, they connect the projections of the spacecraft's angular velocity vector with the derivatives of the orientation angles. To fully describe the movement of the spacecraft around the center of mass, the kinematic equations must be supplemented with dynamic equations.

Euler's dynamic equations

Dynamic equations describe the motion of a rigid body near a fixed point, which are derived from the kinetic momentum change theorem, according to which the time derivative of the kinetic moment vector relative to a fixed point is equal to the principal moment of external forces

— (3.5)

Dynamics equation in a general form, the space vehicle in the projection on the axis of the moving coordinate system OX_ka Y_ka Z_ka under the condition that the connected axes are the main axes of inertia

(3.6)

where I_x, I_y, I_z are the axial moments of inertia of the spacecraft, M_x, M_y, M_z are the moments of external forces acting on the spacecraft.

Euler's dynamic equations relate the change in angular velocity of the spacecraft to the action of control and disturbance moments acting on the satellite.

Thus, the motion around the center of mass of the space vehicle as a completely solid body is described by a system of six ordinary nonlinear differential equations of the first order with respect to six unknown functions of time: ψ, θ, φ, ω_x, ω_y, ω_z. The system of equations includes: three dynamics equations and three kinematic equations. These equations make it possible to judge the change in the angular position of the spacecraft under the action of the controls and disturbing moments.

By substituting the kinematic equations (3.5) into the dynamics equation, instead of six first-order equations, three second-order differential equations can be obtained with respect to the three orientation angles ψ, θ, and φ. Such equations establish a direct connection between orientation angles, which determine the angular position of the spacecraft, and external moments acting on the satellite.

Assuming that the orbit is circular (angular velocity ω_0 of the moving coordinate system = const), the linearized equations of motion of the spacecraft under the influence of gravitational and disturbing moments, as well as at small deviation angles, have the following form:

(3.7)

Let's use the Laplace transform to the system of equations (3.7) and express the orientation angles from it

——

——

——

We will consider the resulting system of equations as a simplified model of a nanosatellite, which will make it possible, at the initial stage of design, to simulate the angular motion of the spacecraft under the action of disturbing moments. (3.8)

3.3. Undisturbed movement of the spacecraft in orbit

Forecasting the position of the spacecraft at a given moment requires a high-quality mathematical model of the movement of its center of mass in the Earth's gravitational field. Let's consider the formation of a mathematical model of the Keplerian motion of a nanosatellite, that is, without taking into account the influence of disturbing factors.

3.3.1. The equation of motion of the center of mass of the KA

We consider the undisturbed motion [16,17,18,19] of the spacecraft relative to the Earth, taking into account the following assumptions:

we neglect the action of aerodynamic forces, forces of light pressure, forces of gravity of other celestial bodies;

the center of mass of the central attracting body moves in a straight line and uniformly;

the attracting celestial body (planet Earth) has the shape of a sphere with a spherical density distribution. In this case, the gravitational field is central

$F\ _{rez}=-\mu\ m/r^2\ (e_r)$ (3.9)

where $\mu=398600.5\ km^2 \cdot s^{-2}$ is the gravitational parameter of the Earth, m is the mass of the spacecraft, (e_r) is the radius vector of the spacecraft, r is the distance to the satellite.

the mass of the spacecraft is very small compared to the mass of the attracting body, that is, we neglect the force of gravity from the side of the spacecraft.

We consider the movement of the nanosatellite relative to the inertial coordinate system $OX_E\ Y_E\ Z_E$.

In celestial mechanics, this model is considered as a limited two-body problem. Then the origin of the spacecraft coordinate system is combined with the center of the Earth, the vector differential equation of motion of the nanosatellite looks like

(3.10)

Projecting equation (3.10) on the axis of the inertial coordinate system, we obtain the equation of the undisturbed motion of the spacecraft in the coordinate form:

(3.11)

When integrating this system of differential equations, we get 6 integrals that contain time t and 6 arbitrary constants determined from the initial conditions.

3.3.2. Orbit equation. Determination of the radius vector of the space vehicle and velocity projections through the elements of orbit

The movement of the nanosatellite occurs in a constant plane (Laplace plane). The trajectory of the spacecraft is a flat curve – the orbit of the satellite. To obtain the orbit equation, we use the Laplace vector [17]. First, we find the scalar product of the vectors f on r:

$f \cdot r = (V \times C) \cdot r - \mu r / r \cdot r = C (r \times V) - \mu r = C^2 - \mu r$ (3.12)

where V is the energy integral vector, C is the plane integral vector

By definition of the scalar product

$f \cdot r = f \cdot r \cdot \cos\vartheta$ (3.13)

where ϑ is the angle between the vectors f and r, then we get

$f \cdot r \cdot \cos\vartheta = C^2 - \mu r$ (3.14)

From here we get the equation of the conic section in polar coordinates with the center at the focus of the orbit:

(3.15)

where p is an orbit parameter that determines its linear dimensions in space:

(3.16)

The conic section is symmetric with respect to the Laplace vector, and the polar angle ϑ (true anomaly) determines the rotation of the current radius vector relative to the axis of symmetry. The obtained result reflects Kepler's first law.

The speed of the spacecraft. Depending on the shape and dimensions of the orbit, as well as the position of the satellite in the orbit, its speed can vary in a fairly wide range. Given the distance to the center of attraction (Earth), the speed of the satellite is one of the main parameters of the movement. Let's decompose the velocity vector into two components. Let's assume that one component (V_r) is directed along the radius-vector, and the second (V_n) - along the normal to the radius-vector in the direction of the satellite's movement.

Radial component of velocity:

(3.17)

Transversal component of speed:

(3.18)

The absolute speed of the spacecraft along the trajectory:

(3.19)

It follows from this formula that the absolute speed of the satellite in a given orbit varies within fixed limits. The maximum speed is reached in the pericenter of the orbit.

The coordinates of the unit vectors (e_r) and (e_n) directed along the radius vector r of the spacecraft in orbit and along the normal to it in the plane of motion, respectively, are found using the transition matrix M using the formulas:

(3.20)

(3.21)

The value of the radius vector in the absolute geocentric coordinate system:

(3.22)

The value of the velocity vector in the absolute geocentric coordinate system:

(3.23)

Based on this, the radius vector r (t) of the position of the spacecraft and its velocity vector V (t) at the moment of time t were completely specified through the parameters of the orbit, in the geocentric equatorial coordinate system.

3. Development of a model of the system for determining the orientation of nanosatellite 25

3.3.3. Dependence of parameters of motion along an elliptical orbit. Solving the Kepler equation

To determine the law of the satellite's orbital motion, it is necessary to establish the dependence of the motion parameters on time. The distance of the nanosatellite to the center of mass of the Earth and its speed can be calculated quite simply if the value of the true anomaly ϑ is known. Therefore, it is necessary to associate the truth of the anomaly with the time of movement t.

The equation of the dependence [17] of the true anomalies ϑ in the time interval from t_Π to some arbitrary point on the orbit where the satellite is at the moment of time t:

(3.24)

As we can see, the integral (3.24) depends on the sign of the eccentricity e, that is, on the type of satellite orbit. For an elliptical orbit 0<e<1. To calculate the integral (3.24), it is necessary to switch to a new variable E - the eccentric anomaly.

The relationship between the eccentric anomaly E and the truth ϑ has the following form

(3.25)

If we replace ϑ with E in the integral (3.24) and differentiate, we get

(3.26)

Now let's introduce the concept of average anomaly

(3.27)

where – the average motion of the spacecraft along the orbit.

The Kepler equation can be derived from the obtained ratio

(3.28)

Determining the position of the satellite at a given time requires solving Kepler's transcendental equation (3.28).

To solve Kepler's equation [29,20] we will use an iterative algorithm. The average anomaly, eccentricity and truth at the initial moment of time (t=0) are denoted as M, E and ϑ, at any moment of time (Δt) – as M_1, E_1 and ϑ _1.

1) To obtain the true anomaly ϑ _1 at the time Δt, we use equations (3.25, 3.27, 3.28) as follows:
2) we set the initial value of the eccentric anomaly E_1=M_1, the average anomaly is found by the formula (3.27);
3) we calculate the new value of the eccentric anomaly E_new according to the Kepler equation E_new=M_1+e·sinE_1;
4) the error is equal to the difference of absolute values 〖ε=|E〗_new-E_1 |;

5) then let 〖E_1=E〗_new, now this is the new value of E_1;

6) steps 1–4 will be repeated until the error ε becomes small;

7) true anomaly ϑ _1 is expressed from equation (3.25);

8) end of the algorithm.

Conclusions on the section

The main aspects of the formation of the model of the spacecraft movement relative to the center of mass and motion along the orbit are considered, and the main model of the magnetic field, which depends on the orbital position of the satellite, is also given. Using the presented models of the movement and position of the satellite in orbit, it is possible to consider the tasks of determining the orientation using a magnetometer and an Earth sensor, as the main measures of the angular position relative to the reference coordinate system.

4. Synthesis of the orientation system of nanosatellite

In order to solve the problem of determining the orientation of a nanosatellite, in addition to using the models discussed above, it is also necessary to consider orientation meters (in our case, it is a magnetometer and a vertical sensor of the Earth) and the main orientation algorithms that will be compared (the algorithm for determining the yaw angle and the TRIAD algorithm). The selected sensors have adequate orientation accuracy and are also sufficiently simple and reliable to use. Also, the use of appropriate algorithms will make it possible to assess the influence of the accuracy of the meters, mostly for the magnetometer (as it is a less accurate sensor), on the complete determination of the angular position of the spacecraft.

The general principle of operation is as follows: under the influence of external moments on the satellite during orbit movement, an angular displacement of the SC connected to the satellite relative to the reference one is formed, this angular displacement will be reflected in the change in the position of the magnetic stress vector in the orbital coordinate system, which in its the magnetometer will measure the turn, and the direction vector to the Earth will also change, the change of which is determined by the Earth sensor. Further, the measured data are used in the orientation determination algorithms, and after determining the angular position, an accuracy assessment is made depending on the specified errors of the meters and initial conditions.

In fig. 7. a demonstrative reference coordinate system OX_o Y_o Z_o relative to which the formation of mathematical models of the movement of the spacecraft, as well as the operation of sensors and algorithms for determining the orientation will be carried out.

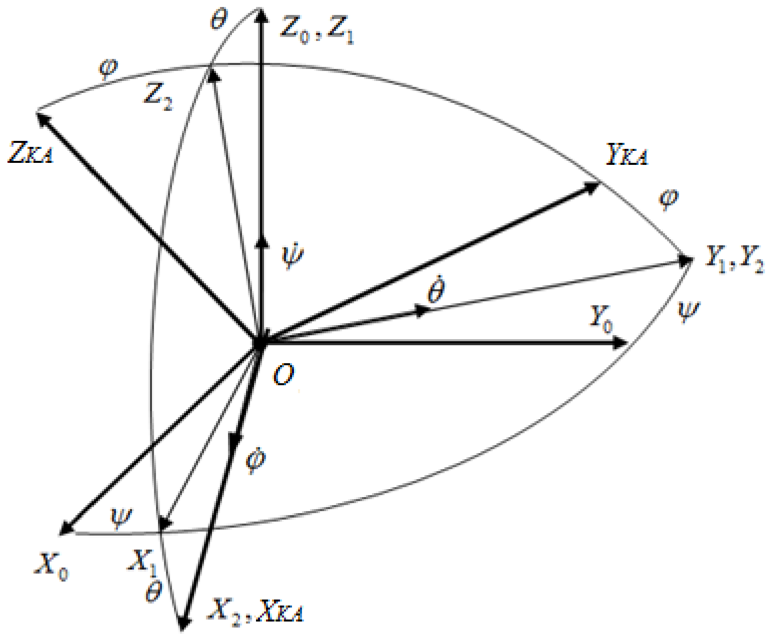

Fig. 7. Reference coordinate system

4.1. Models of measurement sensors

4.1.1. Mathematical model of magnetometer

Using a magnetometer as part of a low-orbit satellite

The mathematical model of magnetometer measurement [21] without taking into account the non-orthogonality of the sensitive elements of the axes has the following form:

. (3.29)

where h is the vector of measurements of the magnetometer, H is the vector of the Earth's magnetic field strength in the coordinate system linked to the nanosatellite, Δ is the error of the scale factor, μ is the zero displacement vector of the magnetometer.

Formula (3.29) can be written in the following form:

. (3.30)

Based on the fact that we know the true angular position of the satellite in space at this moment in time, due to motion simulation and the magnetic field model, we determine the H_o vector in the orbital SC. Then the magnitude of this vector in the connected SC is calculated by the formula:

(3.31)

With the help of the given ratios, the determination of the intensity vector of the Earth's magnetic field by the magnetometer, which is on board the spacecraft, is reproduced. This model of the magnetometer is approximate, it takes into account only the error of the scale factor and the displacement of zero along the corresponding axes.

4.1.2. Mathematical model of the Earth sensor

The line that passes through the center of the Earth O_E and the center of mass of the spacecraft O is called the local vertical [12]. The OZ_o axis of the orbital coordinate system is located on it. The vertical sensor is needed to build an on-board orbital coordinate system. When using the Earth as a reference point, one axis of the satellite (-OZ_{KA}) will be constantly directed towards the Earth and coincide with the local vertical, provided that there is an ideal orientation.

In this work, the Earth sensor is considered in two possible versions, the first as a meter of pitch angles θ and roll φ, which are formed as follows. Let the initial position of the axes of the connected system $OX_{ka} Y_{ka} Z_{ka}$ coincide with the axes of the orbital coordinate system $OX_o Y_o Z_o$. The rotation by the pitch angle θ is performed around the connected axis OY_{ka}. After the first turn, the connected axes of the spacecraft occupy the position $(OX_{ka} Y_{ka} Z_{ka})^{'}$. The turn to the roll angle φ is performed around the connected axis $O X_{ka}^{'}$. Information from which is necessary for the operation of the yaw angle finding algorithm.

The second option is a direct meter of the direction vector to the center of mass of the Earth. Information from the Earth sensor in this form is used for the TRIAD vector algorithm.

The measured values from the Earth sensor will be considered under the influence of the added zero displacement error for two angles θ, φ. The mathematical model has the following form:

(3.32)

are the pitch and roll angles measured by the Earth sensor, respectively, θ, φ are the true values of the orientation angles, $\Delta\theta$, $\Delta\varphi$ are the zero offset of the orientation angles.

Mathematical model of local vertical vector measurement with scale factor error and zero offset:

.

where e_x, e_y, e_z are the measured projections of the local vertical vector on the connected axes of the spacecraft, E_x, E_y, E_z are the true values of the local vertical vector in the connected SC, Δ_x, Δ_y, Δ_z are the errors of the scale factor on the corresponding axes, σ_x, σ_y, σ_z – displacement of sensor zeros.

(3.33)

The Earth sensor is one of the most accurate meters [22] that are installed on small spacecraft during low-orbit flight, but it has a significant drawback, which is the limitation of the maximum deviation at which the sensor can be oriented to the Earth. In the future, the dependence of the maximum possible deviation angles of the spacecraft, which do not affect the accuracy of determining the orientation, will be considered.

4.2. Algorithms for determining the orientation of a nanosatellite

4.2.1. The TRIAD algorithm

The algorithm for determining the angular position of the spacecraft relative to the reference coordinate system is TRIAD [22,23]. The result of determining the orientation is formed by observing at least two non-parallel vectors located in two different SCs. The operation of this algorithm requires information about two vectors located in the coordinate system connected to the nanosatellite. This information is provided by selected meters (magnetometer, Earth sensor), due to the selection of these sensors, the orientation system will be reliable and simple to implement.

We consider the orientation of the object in the coordinate systems OX_o Y_o Z_o and OX_ka Y_ka Z_ka, the relative position is shown in fig. 6. Assume that two vectors e and s are known in the given coordinate systems. In the reference coordinate system OX_o Y_o Z_o vectors are denoted as (e_o) =E_o=[E_Xo,E_Yo,E_Zo]^T and (s_o) =S_o=[S_Xo,S_Yo,S_Zo]^T. In the connected coordinate system OX_ka Y_ka Z_ka we denote the vectors by (e_KA) =E_KA=[E_X,E_Y,E_Z]^T and (s_KA) =S_KA=[S_X,S_Y,S_Z]^T. Next, the normalized (unit) vectors of the reduced vectors are constructed: —,

—,

——,

——.

We construct a normalized vector that is perpendicular to the plane formed by the two vectors e _no and s _no. This vector, denoted by m _no, is found by the following equation: .

Having obtained the vector m _no, we determine the unit vector n _no=e _no×m _no. Using the previously generated vectors, we will get an orthogonal triplet of vectors e_{no}, m_{no}, n_{no} (triad), which forms a basis built on the vectors (e_o) and (s_o) .

Similarly, we construct a basis for a connected coordinate system from the vectors e_{nKA}, m_{nKA}, n_{nKA}, and write the normalized vectors in the matrix of the following form:

, (3.34)

, (3.35)

Based on the concept of the TRIAD algorithm, the matrix of direction cosines of the transition from the reference coordinate system to the connected one has the form

. (3.36)

where M_o^T is the transposed matrix M_o.

To determine the angles from the matrix M, you need to use the following expressions:

(3.37)

(3.38)

(3.39)

It should also be noted that the TRIAD algorithm is sensitive to the instrumental errors of the meters. If we consider the error caused by the inaccuracy of the magnetometer h_y along the OY_{KA} axis, denote it as Δh_y, then the error in determining the yaw angle will have the following form:

.

From the given equation, we can see that the error of the magnetometer causes a change in the yaw angle, i.e., even if the nanosatellite is perfectly oriented in space, the orientation determination system will receive information about the angular deviation, which in turn causes excessive movement of the spacecraft.

4.2.2. Algorithm for determining the yaw angle

Consider the algorithm [8] that determines the orientation of the spacecraft relative to the axis OZ_ka – the yaw angle ψ. For its operation, it is necessary to know the angles of pitch θ and roll φ relative to the reference coordinate system OX_o Y_o Z_o, which are determined using the Earth sensor. The obtained values are found in the form (3.32). It is also necessary to know the stress vector of the Earth's magnetic field measured by a magnetometer (3.30). The task is to determine the yaw angle ψ, based on the relevant information.

The movement of the nanosatellite is considered in the reference coordinate system OX_o Y_o Z_o, the position of the connected axes is shown in fig. 6. We consider the projections of the intensity vector of the Earth's magnetic field a_x0, a_y0, a_z0 in the orbital coordinate system and in the connected SC a_x, a_y, a_z to be known. The pitch and roll angles measured by the Earth sensor must also be known.

From fig. 6, we determine the dependence of the projection of the tension vector H in the orbital SC relative to the connected one by the given relation

$$\begin{aligned} a_x &= a_{x0}\cos\psi\cos\theta + a_{y0}\sin\psi\cos\theta - a_{z0}\sin\theta; \\ a_y &= a_{x0}(-\sin\psi\cos\varphi + \cos\psi\sin\varphi\sin\theta) + a_{y0}(\cos\psi\cos\varphi + \sin\psi\sin\varphi\sin\theta) + \\ &+ a_{z0}\cos\theta\sin\varphi; \\ a_z &= a_{x0}(\sin\psi\sin\varphi + \cos\psi\cos\varphi\sin\theta) + a_{y0}(-\cos\psi\sin\varphi + \sin\psi\cos\varphi\sin\theta) + \\ &+ a_{z0}\cos\theta\cos\varphi. \end{aligned} \tag{3.40}$$

We denote μ_1 and μ_2 by expressing from the second and third equations (3.40):

$$\begin{cases} a_{x0}\cos\theta\cos\psi + a_{y0}\cos\theta\sin\psi = \mu_1; \\ -a_{y0}\cos\psi + a_{x0}\sin\psi = \mu_2, \end{cases} \tag{3.41}$$

де , .

From the system of equations (3.41), the expression by which the yaw angle is calculated is:

$$\sin\psi = \frac{\mu_2 a_{x0}\cos\theta + \mu_1 a_{y0}}{\cos\theta(a_{x0}^2 + a_{y0}^2)} \tag{3.42}$$

Expression (3.42) is inserted into the on-board computer to find the yaw angle ψ. It should also be noted that the accuracy of the angle determination by this algorithm, when the value of the latitude argument u=90^° and u=270^° decreases significantly, this follows from the following ratio:

This problem is also observed in the TRIAD algorithm, this is due to the fact that the vectors at the corresponding values of the width argument become parallel, that is, the system becomes one-vector.

Conclusions on the section

In this section, the main mathematical models of sensors for orientation determination (magnetometer and Earth sensor) with scale factor errors and zero offset are considered. The main algorithms for determining the orientation of a nanosatellite (the algorithm for determining the yaw angle and the TRIAD algorithm), their dependence on the parameters of the orbital motion, and their mathematical implementation are also given.

5. Modeling the system in the Matlab Simulink software package

We use the Matlab/Simulink program package to test the proposed methods and solutions, as well as to create a simulation model that reflects the complex creation of an orientation system. The simulated model of the orientation system of a nanosatellite with a magnetometer and a vertical sensor of the Earth, built on the basis of the use of the mathematical and shown in fig. 8.

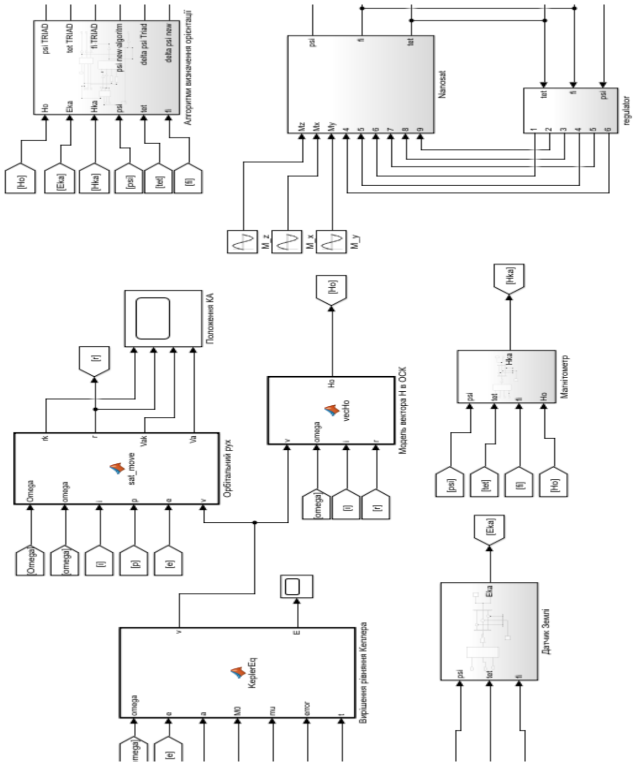

Fig. 8. Complete simulation model of the nanosatellite orientation system

The model consists of the following blocks and subsystems:

- "Initial conditions of the orbit", a subsystem in which the parameters of the orbit are set;
- "Solution of Kepler's equation", function block in which Kepler's equation is solved;
- "Orbital motion", a block for forming the nanosatellite's state vector during orbital motion;
- "H vector model in OSK", in this block the magnetic field intensity vector is formed in the orbital coordinate system;
- "Magnetometer", a subsystem in which the mathematical model of the magnetometer is implemented;
- "Earth sensor", a subsystem in which the mathematical model of the Earth sensor is implemented;
- "Orientation determination algorithms", a subsystem in which the yaw angle determination algorithm and the TRIAD algorithm are embedded;

- "regulator", a subsystem that implements damping of the angular motion of the spacecraft;
- "nanosat", a subsystem that reproduces the mathematical model of a nanosatellite.
- When modeling the given scheme, we will get a display of the satellite's orbital movement, the angular movement around the center of mass of the spacecraft, and the determined values of the nanosatellite's angular position relative to the reference coordinate system.
- The set parameters of the orbit:
- The longitude of the ascending node of the orbit Ω=0^°;
- The argument of the perigee of the orbit ω=0^°;
- Orbit inclination i= 〖98〗 ^°;
- Semi-major axis a=6971 km;
- Eccentricity e=0.01 km;
- Average anomaly M=0^°;
- Gravitational parameter μ=398600.5 km^2·s^(-2);
- The value of the error of solving the Kepler equation ε=1e-10;
- Simulation time t=5854.8 s.

5.1. Model of nanosatellite movement relative to CM

5.2. Model of orbital motion of nanosatellite

5.3. Block of formation of the Earth's magnetic field

The results of the spacecraft's orbital motion, obtained during the simulation of the nanosatellite calculation system for one period T, are shown in Fig. 9 - 12.

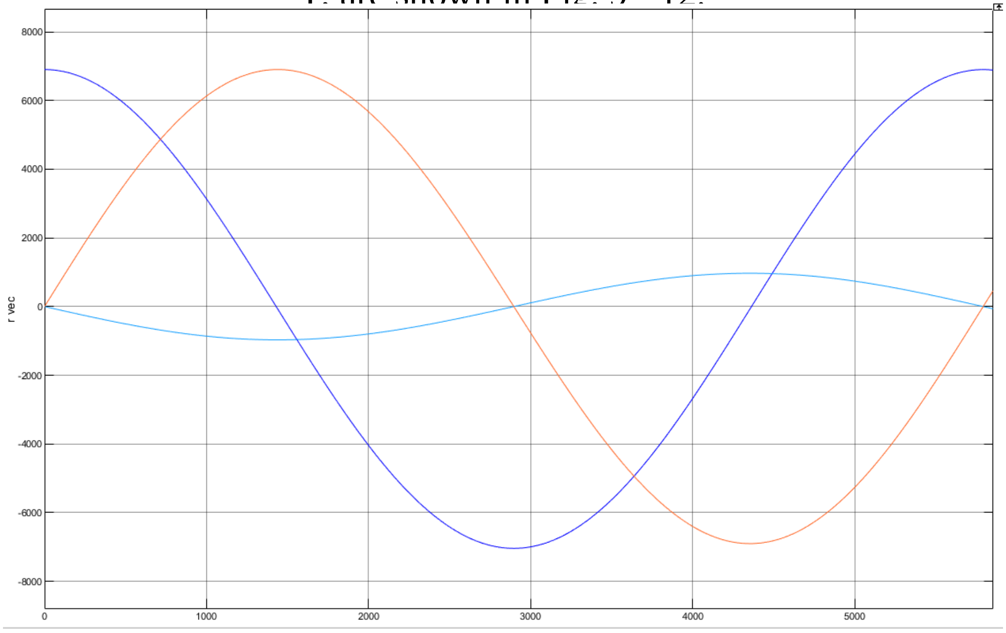

Fig. 9. The value of the radius vector in the geocentric coordinate system

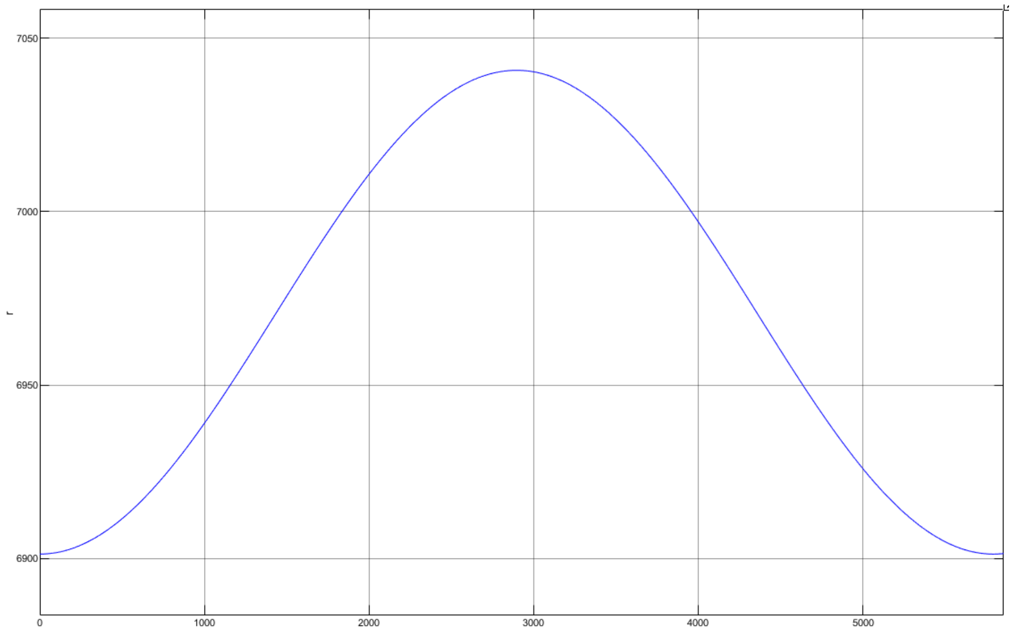

Fig. 10. The trajectory of the spacecraft's orbital motion

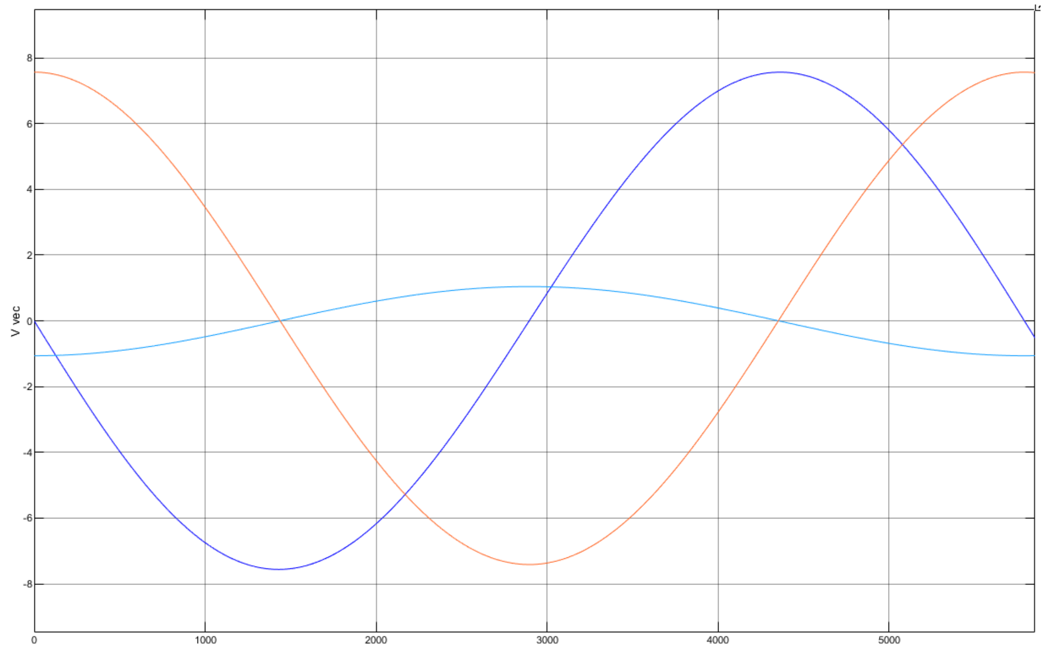

Fig. 11. The value of the velocity vector in the geocentric coordinate system

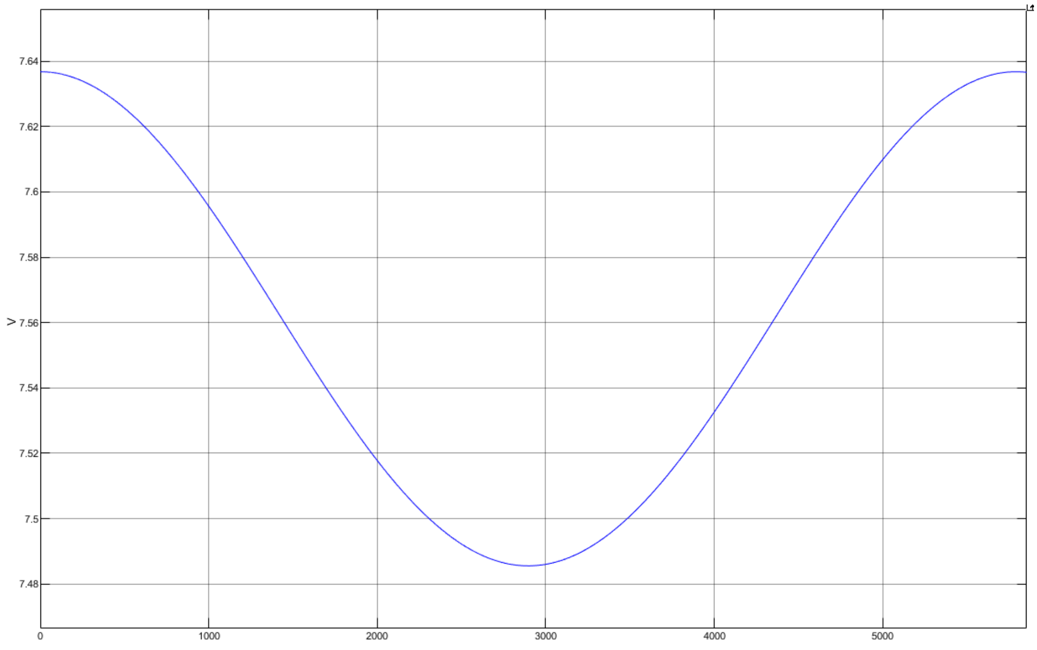

Fig. 12. Absolute speed of a nanosatellite

The following graphs show the actual movement of the center of mass of the nanosatellite along the orbit depending on the set parameters of the orbit. The corresponding parameters were chosen according to the principle of optimality of the orbit to ensure the maximum time of illumination of the spacecraft by the Sun's rays.

A model of nanosatellite motion relative to the CM

In the "nanosat" subsystem (Fig. 13), a system of differential equations (3.8) is embedded, which reflects the movement of the nanosatellite relative to the center of mass. With the help of the corresponding equations, the angular motion is modeled when gravitational restoring and disturbing moments act on the satellite, as well as small deviation angles.

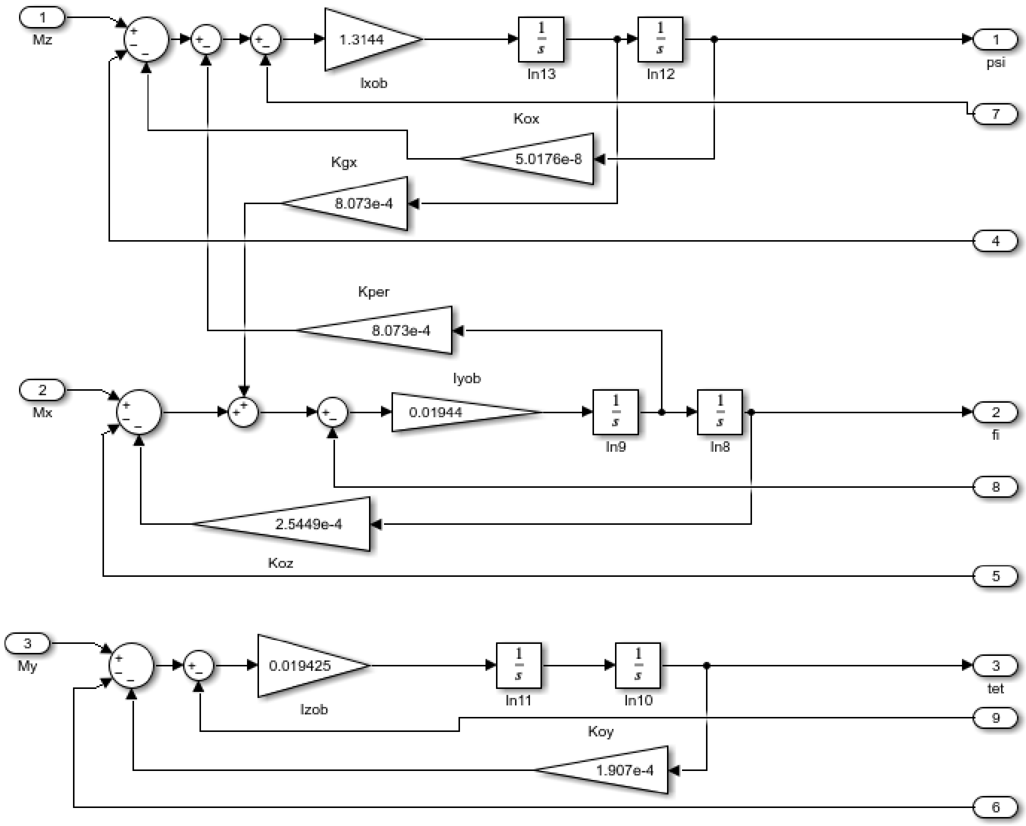

Fig. 13. Structural diagram of the "nanosat" subsystem

where

— — —

The initial conditions for deviation of yaw, pitch and roll angles are 0.1 radians, which we set in every second integrator of the corresponding channel.

Let's set the optimal values of the coefficients for small spacecraft. In equation (3.8), which characterizes the simulation of the angular motion of the satellite:

— — —

where ω_0 is the angular speed of rotation of the spacecraft in orbit.

In this simulation model, there is no damping of natural oscillations relative to the connected axes. Let's solve this problem by introducing the "regulator" subsystem into the system (Fig. 14), which will perform the task of damping links of the formed angular movements.

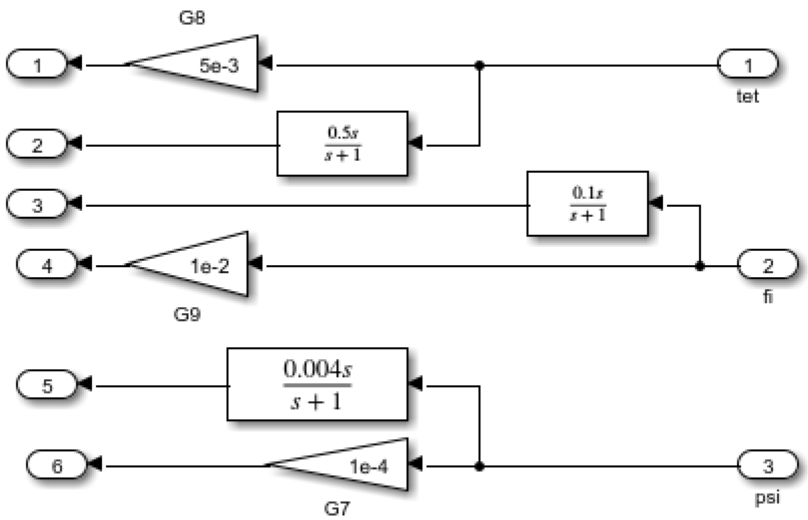

Fig. 14 Structural diagram of the "regulator" subsystem

We will use the optimal damping coefficients:

We will obtain the results of modeling the angular motion of the nanosatellite using the given coefficients, the graphs of the formed deflection angles are shown in fig. 15 - 17.

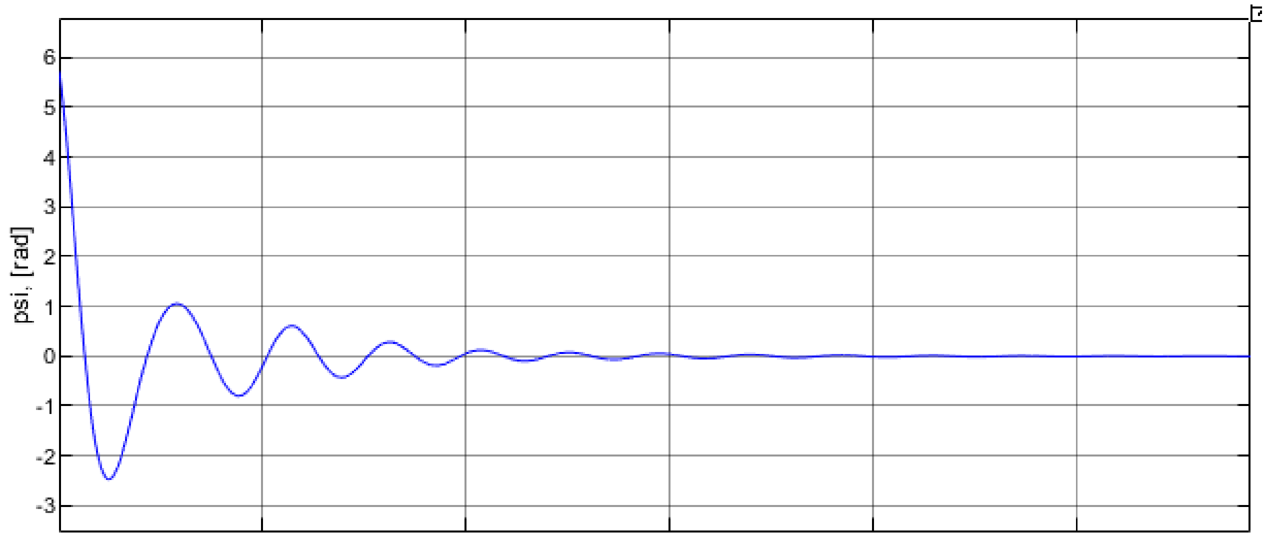

Fig. 15. Graph of deviation of the nanosatellite along the course angle

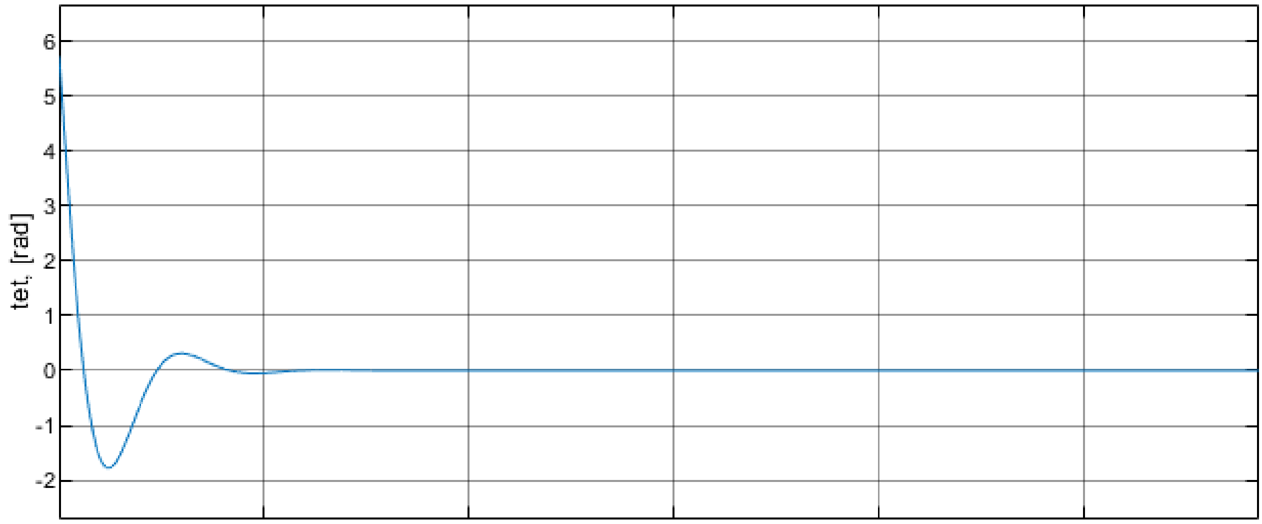

Fig. 16. Graph of nanosatellite deflection by pitch angle

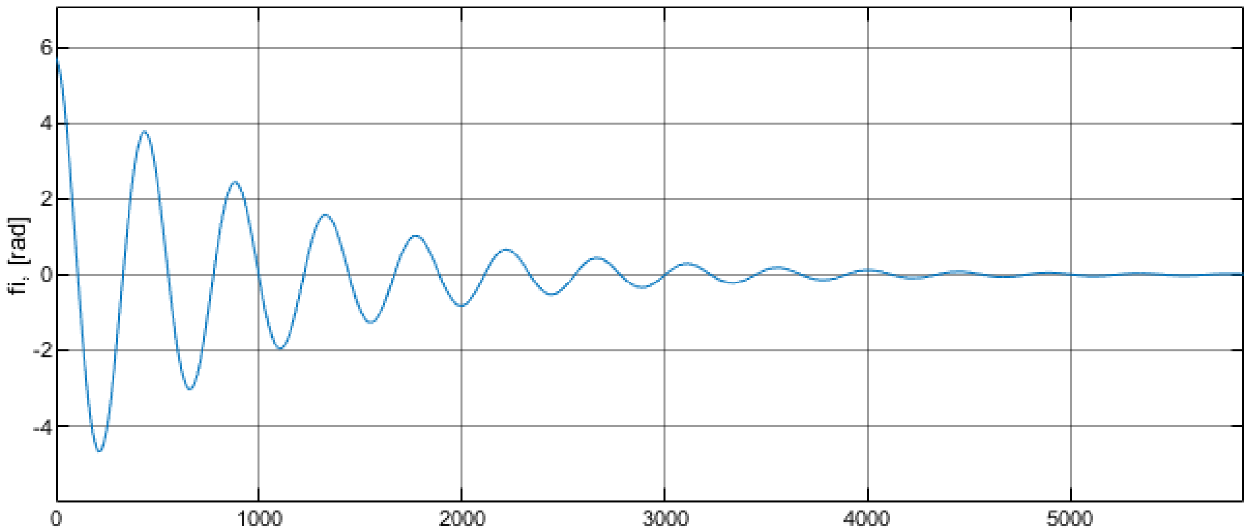

Fig. 17. Graph of deviation of the nanosatellite by roll angle

The obtained graphs demonstrate the behavior of the nanosatellite when external moments act on it, the output angles must then be measured with the help of a magnetometer and the Earth sensor and processed in the orientation algorithm.

A model of the orbital motion of a nanosatellite

The structural diagram of the nanosatellite motion simulation (Fig. 18) consists of the "Solving the Kepler equation" and "Orbital motion" blocks.

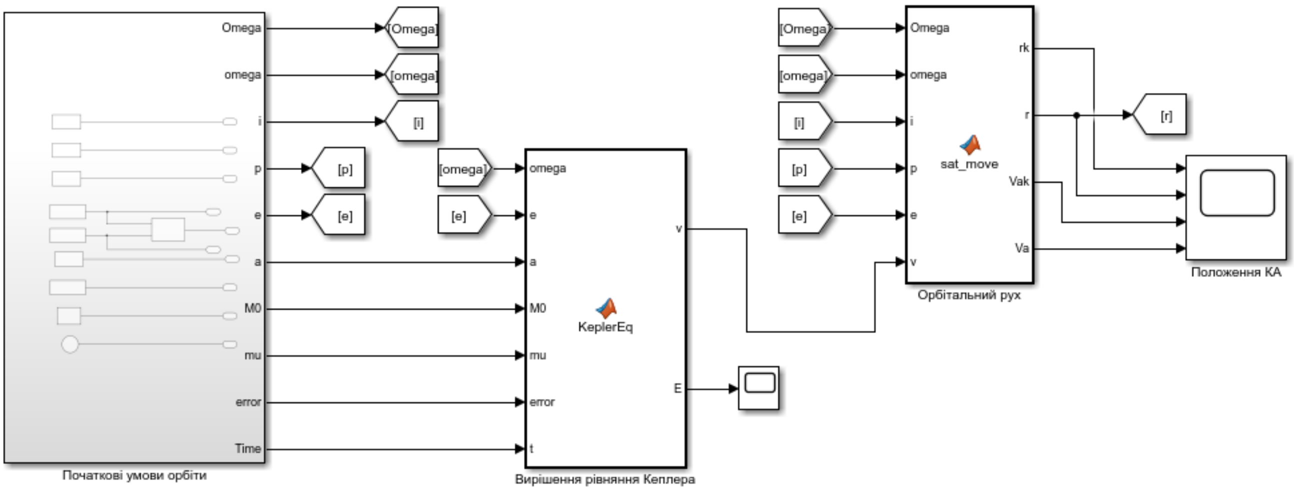

Fig. 18. Structural modeling of the orbital motion of a nanosatellite

The iterative algorithm for solving the transcendental equation described in clause (3.3.3) is embedded in the "solving the Kepler equation" block, given the initial conditions and orbit parameters. The MATLAB script of the corresponding algorithm is given in Appendix A.1.

Let's consider the "Orbital motion" block, in this block the programmed equations were set forth in the description of the mathematical model of item (3.3.2). The input data are the previously set parameters of the orbit, as well as the truth anomaly ϑ calculated with the help of the block "solving the Kepler equation", which characterizes the rotation of the radius vector of the spacecraft along the orbit. The output values are the velocity and position vectors in the absolute geocentric coordinate system, as well as the absolute values of the velocity and position of the satellite in orbit. The implementation is outlined in Appendix A.2.

Block of formation of the Earth's magnetic field

The use of the "Vector H model in OSK" block (Fig. 19) is necessary to simulate the Earth's magnetic field, which in turn will be measured by the magnetometer installed on board the spacecraft.

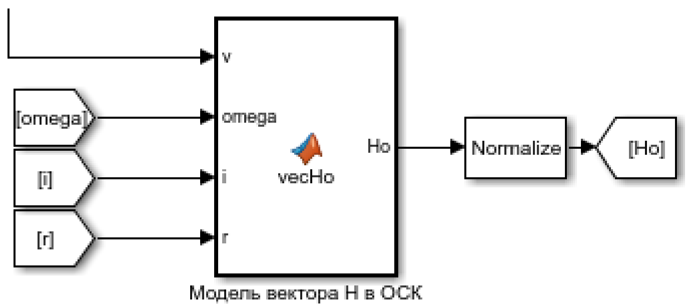

Fig. 19 Block for simulating the Earth's magnetic field

The values of the perigee argument, the inclination of the orbit and the distance of the satellite to the center of the Earth are given to the input of the block. The formulas contained in this block are considered in the section on the direct dipole model (3.1.2). The implementation is outlined in Appendix A.3.

5.4. Block of orientation detection sensors 53

In the "Magnetometer" subsystem, the output signal of the magnetometer is simulated, the type of signal is the projection of the magnetic field intensity vector in the bound coordinate system. The input of the subsystem is supplied with the vector of the magnetic field strength in the orbital SC and the angles of deviation of the spacecraft to simulate the simulation of work. The magnetometer, according to the mathematical model considered in point (4.1.1), has scale coefficient errors and zero offset error. The structure of the subsystem is shown in fig. 20.

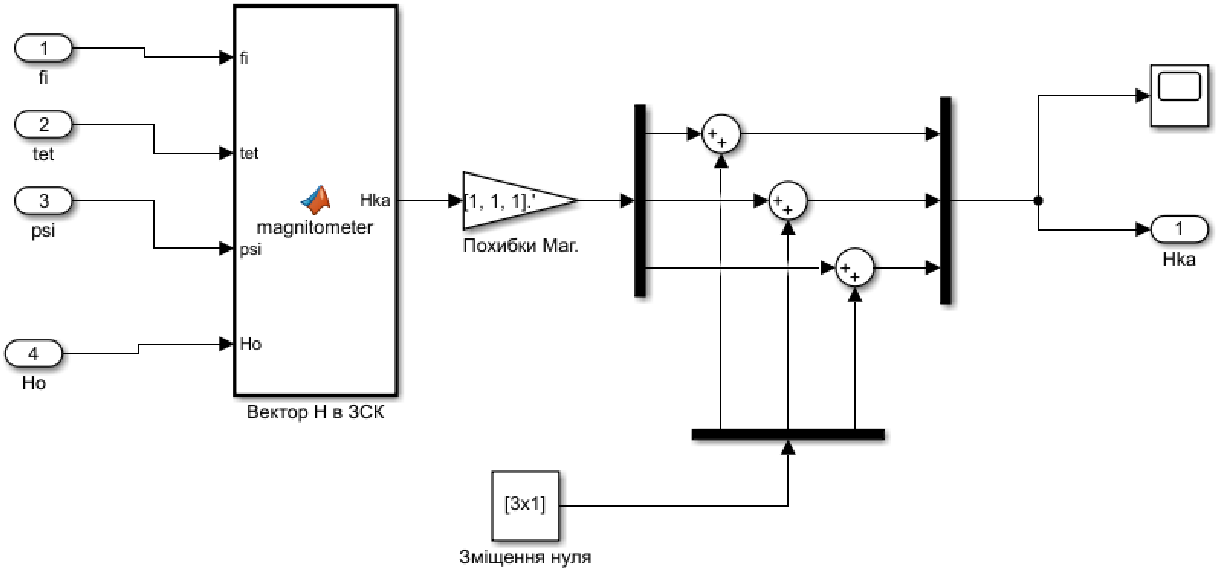

Fig. 20. Structural diagram of magnetometer simulation

The conversion of H_o to H_KA is implemented using the script file provided in Appendix B.1.

Similarly, a model for simulating the operation of the Earth sensor is created, the vector of the direction to the ground in the reference coordinate system and the satellite deflection angles are input. Also, according to the mathematical model from point (4.1.2), a structural one with measurement errors is created. The structure of the "Earth sensor" subsystem is shown in fig. 21.

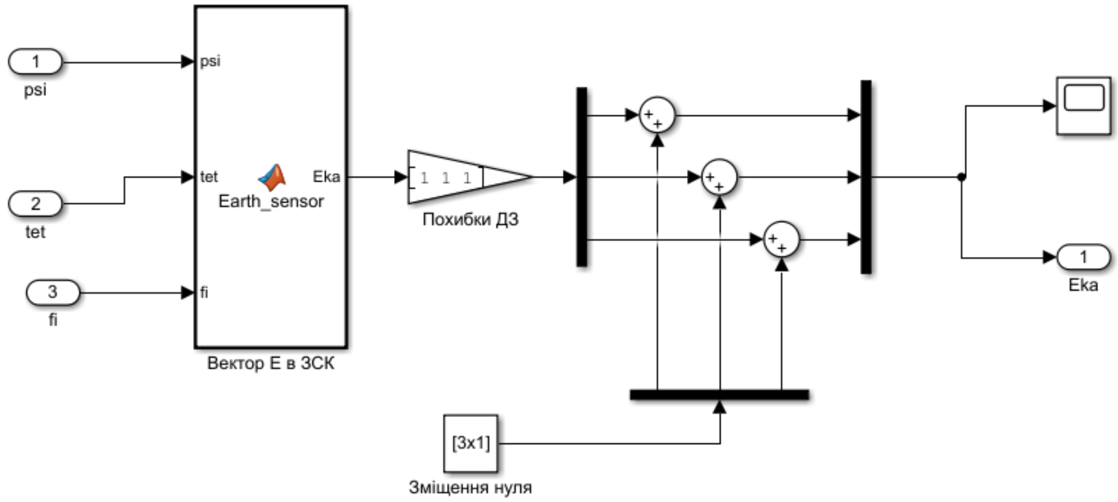

Fig. 21. Structural diagram of the simulation of the operation of the Earth sensor

The script file for the transformation of the local vertical vector from the reference coordinate system to the connected SC is given in Appendix B.2.

5.5. Block of orientation determination algorithms

Let's consider the structure of the "Orientation determination algorithms" subsystem, in which two orientation algorithms are located - the TRIAD algorithm and the proposed algorithm. The structure of the subsystem is created on the basis of a mathematical explanation of the operation of the corresponding algorithms, considered in clauses (4.2.1 - 4.2.2). The algorithm simulation scheme is shown in fig. 22.

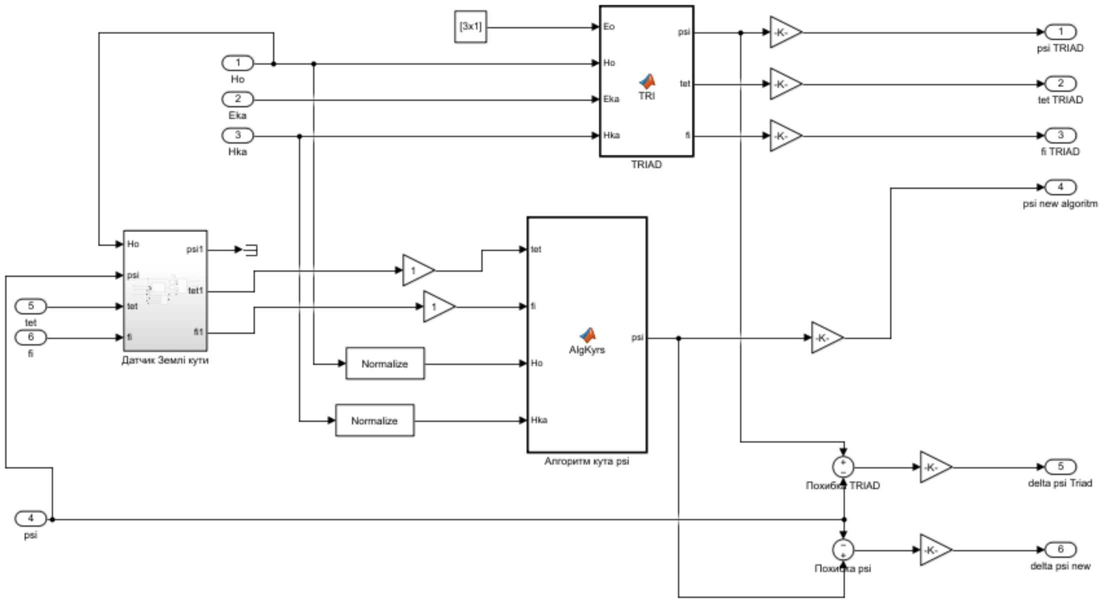

Fig. 22. Structural diagram of orientation algorithms

The implementation of the proposed algorithm and the TRIAD algorithm using a Matlab script is given in Appendix B.1. and B.2. in accordance.

Next, we will perform a comparative analysis of the accuracy of the given algorithms with respect to the reference values of the orientation angles and consider the influence of meter errors on the accuracy of the TRIAD algorithm and the proposed algorithm.

Conclusions on the section

In this section, the structure of the simulation model of the calculation system of the orientation of a nanosatellite with an Earth sensor and a magnetometer in the Matlab/Simulink software package is implemented. The specified mathematical models of measurement sensors, the Earth's magnetic field, the orbital motion of the spacecraft, the angular motion of the satellite relative to the center of mass and the corresponding algorithms are reproduced. In fig. 23 is MatLab code and Graph.

```
clc
clear
i=98*pi/180;
k=1.01;
for p=1:1:2*pi*10
u=(p-1)/10;
axo=cos(u)*sin(i);
ayo=cos(i);
azo=-2*sin(u)*sin(i);
psi_gr=5;  tet_gr=3;  fi_gr=4;
psi=psi_gr*pi/180;  tet=tet_gr*pi/180;   fi=fi_gr*pi/180;
cpsi=cos(psi); spsi=sin(psi); ctet=cos(tet); stet=sin(tet); cfi=cos(fi); sfi=sin(fi);
Eb_x=axo*cpsi*ctet+ayo*spsi*ctet-azo*stet;
Eb_y=k*(axo*(-spsi*cfi+cpsi*sfi*stet)+ayo*(cpsi*cfi+spsi*sfi*stet)+azo*ctet*sfi);
Eb_z=axo*(spsi*sfi+cpsi*cfi*stet)+ayo*(-cpsi*sfi+spsi*cfi*stet)+azo*ctet*cfi;
v(p)=u;
mu1=Eb_x+azo*sin(tet);
mu2=Eb_z*sin(fi)-Eb_y*cos(fi);
delt = cos(tet)*(axo^2+ayo^2);
delts = mu2*axo*cos(tet)+mu1*ayo;
sinpsi = delts/delt;
```

```
psi_1 = asin(sinpsi)*180/pi;
del_psi_1(p)=psi_1-psi_gr;
end
plot(v/2/pi,del_psi_1,'k','linewidth',1),grid
xlabel('ВЮЯ, НПАЁРХ')
ylabel('ОНУХАЙХ, ЦПЮД')
```

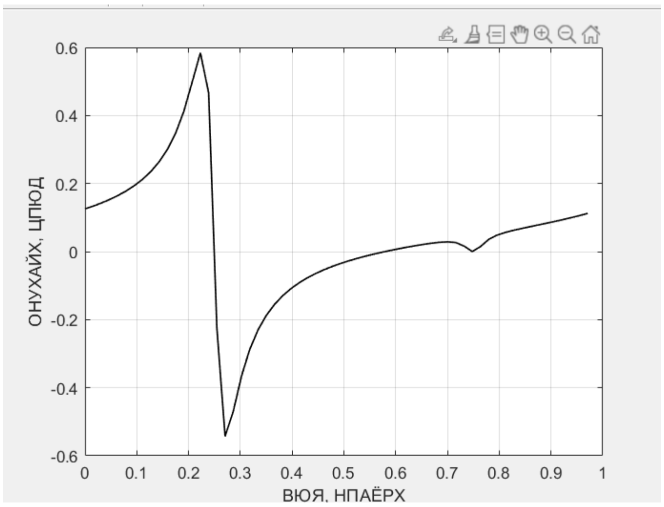

Fig.23.

CONCLUSIONS

1. A nanosatellite orientation system was developed based on the Earth sensor and magnetometer to determine the angular position of the spacecraft;
2. Basic mathematical models of measurement and movement sensors are described nanosatellite;
3. It is proposed to use the yaw angle determination algorithm in the composition orientation systems based on the Earth sensor and magnetometer;
4. The structure of the nanosatellite orientation system was developed and implemented model in the Simulink program;

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APPENDIX A

A.1. SOLVING THE KEPLER EQUATION

```
function [v,E]  = KeplerEq(omega,e,a,M0,mu,error,t) %Функція вирішення
% рівняння Кеплера
n = sqrt(mu/a^3); % середнє переміщення
M = M0 + (n*t); % середня аномалія

Enew  = M;
    Enew1 = Enew - (Enew-e*sin(Enew)- M)/(1 - e*cos(Enew));
    while ( abs(Enew1-Enew) > error )
        Enew = Enew1;
        Enew1 = Enew - (Enew - e*sin(Enew) - M)/(1 - e*cos(Enew));
    end;
    E = Enew1;

 v=2*atan(tan(E/2)*((1+e)/(1-e))^0.5); % істина аномалія

 u = omega + v; % аргумент широти
```

A.2. Orbital motion of the spacecraft

```
r = p/(1+e*cos(v)); % рівняння орбіти

Vr = sqrt(mu/p)*e*sin(v); %радіальна швидкість
Vn = sqrt(mu/p)*(1+e*cos(v)); %трансверсальна швидкість
Va = sqrt(Vr^2 + Vn^2); %абсолютна швидкість

Mu = [cos(u) sin(u) 0; -sin(u) cos(u) 0; 0 0 1];
Mi = [1 0 0; 0 cos(i) sin(i); 0 -sin(i) cos(i)];
MO = [cos(Omega) sin(Omega) 0; -sin(Omega) cos(Omega) 0; 0 0 1];

M = Mu*Mi*MO; %матриця переходу M

er = M.'*[1,0,0].'; %орта по X
en = M.'*[0,1,0].'; %орта по Y
rk = r*er; %радіус-вектор супутника

Vak = Vr*er + Vn*en; %вектор абсолютної швидкості
```

A.3. Model of Earth's magnetic field

```
function Ho  = vecHo(v,omega,i,r)

M = 8.3*10^16; % магнітний момент Землі
lambda = M/r^3;

u = omega + v; % аргумент широти
ax=cos(u)*sin(i);
ay=cos(i);         % проекції вектора напруженості  на ОСК
az=-2*sin(u)*sin(i);

Ho = [ax, ay, az].'*lambda; % вектор напруженості H
```

APPENDIX B

B.1. THE MAGNETIC FIELD INTENSITY VECTOR IN THE ZSK

```
function Hka = magnitometer(psi,tet,fi,Ho)

Afi = [1 0 0; 0 cos(fi) sin(fi); 0 -sin(fi) cos(fi)];
Atet = [cos(tet) 0 -sin(tet); 0 1 0; sin(tet) 0 cos(tet)];
Apsi = [cos(psi) sin(psi) 0; -sin(psi) cos(psi) 0; 0 0 1];

A = Afi*Atet*Apsi; %Матриця переходу від АГ до З СК

Hka = A*Ho; % визначений вектор Н в СК КА
```

B.2. The local vertical vector in the ZSK

```
function Eka = Earth_sensor(psi,tet,fi)
Eo = [0 0 -1].'; %вектор місцевої вертикалі в ОСК

Afi = [1 0 0; 0 cos(fi) sin(fi); 0 -sin(fi) cos(fi)];
Atet = [cos(tet) 0 -sin(tet); 0 1 0; sin(tet) 0 cos(tet)];
Apsi = [cos(psi) sin(psi) 0; -sin(psi) cos(psi) 0; 0 0 1];

A = Afi*Atet*Apsi; %Матриця переходу від орбітальної до З СК

Eka = A*Eo; %вектор місцевої вертикалі в ЗСК
```

APPENDIX C

C.1. THE PROPOSED ALGORITHM

```
function psi = AlgKyrs(tet,fi,Ho,Hka)

axo = Ho(1);
ayo = Ho(2); % проекції вектора H в ОСК
azo = Ho(3);

ax = Hka(1);
ay = Hka(2); % проекції вектора H в ЗСК
az = Hka(3);


mu1=ax + azo*sin(tet);
mu2=az*sin(fi) - ay*cos(fi);

delts = mu2*axo*cos(tet) + mu1*ayo;
delt = cos(tet)*(axo^2 + ayo^2);

sinpsi = delts/delt;

psi = asin(sinpsi); %розрахунок кута psi
```

C.2. TRIAD algorithm

```
function [psi,tet,fi] = TRI(Eo,Ho,Eka,Hka)

% усунення складання двох векторів
if Eo==Ho
    Eo(1)=Ho(1)+0.0000000000000001;
end
if Eka==Hka
    Eka(1)=Hka(1)+0.0000000000000001;
end

Eon=Eo./norm(Eo);
Son=Ho./norm(Ho);
hon=cross(Eon,Son)./norm(cross(Eon,Son)); % формування одиничних
von=cross(Eon,hon)./norm(cross(Eon,hon)); % ортогональних векторів в ОСК

Mo=[Eon hon von]; % матриця векторів в ОСК

Ebn=Eka./norm(Eka);
Sbn=Hka./norm(Hka);                       % формування одиничних
hbn=cross(Ebn,Sbn)./norm(cross(Ebn,Sbn)); % ортогональних векторів в ОСК
vbn=cross(Ebn,hbn)./norm(cross(Ebn,hbn));

Mb=[Ebn hbn vbn]; % матриця векторів в ЗСК

M = Mb*Mo';  %M - обчислюється матриця направляючих косинусів

psi=atan(M(1,2)/M(1,1));
tet=asin(-M(1,3));      % визначаються кути орієнтації
fi=atan(M(2,3)/M(3,3));
```