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STUDY OF THE DYNAMICS OF HEAVY QUADCOPTER AUTOMATIC CONTROL LOOPS

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Abstract—*Considered the issues of a heavy quadcopter control automation. A mathematical model of the dynamics of motion of a quadcopter is proposed, taking into account the effect of inertia of the change in the speed of rotation of the lifting rotors. The model reveals the dependences of aerodynamic forces and moments acting on a quadcopter. For the purpose of simplification, it is proposed to use mathematical model of only an isolated pre-linearized longitudinal motion of a quadcopter for research. The use for automatic control of a quadrocopter is substantiated not by PID, but by PD-regulation. Variants of control laws are proposed that exclude the influence of the inertia effect on the dynamics of control loops.*

Index Terms—Quadcopter; lifting rotors; the rotor rotation speed; control inertance; control forces and moments; control laws.

I. INTRODUCTION

A quadcopter (QC) is an unmanned aerial vehicle with four lifting rotors. Unlike a helicopter, which uses a swashplate for control, which changes direction of the lifting rotor thrust, and the total pitch and adequately the magnitude of the rotor thrust, in QC the thrust direction relative to the body and the pitch of the rotor blades are unchanged.

To change the flight altitude, the QC synchronously changes the rotation speed and, accordingly, the magnitude of the thrust of all four rotors. And for horizontal movement, QC changes its angular orientation (roll and pitch angles), while the horizontal components of the rotor thrust change, and the velocity vector QC rotates in the horizontal plane. To change the roll and pitch angles, the corresponding control moments are created by changing the speeds of rotation of different rotors. To compensate for the reactive moments of the rotors, the opposite direction of rotation of individual pairs of rotors is used. Changes in the speeds of rotation of these pairs distort the condition of compensation and create a control yaw moment. Thus, the control of QC is reduced to the control of the rotor rotation speed, which greatly simplifies the design of the control system.

At present, QC are used quite widely and variedly, but this use is limited mainly by modes of "manual" remote control from the operator's console. However, the task of developing and researching an automatic control system that makes it possible to implement an autonomous flight of QC is now becoming very urgent.

When solving such problems in the dynamics of movement of miniature QC, the change persistence in the rotational speed of the rotor is usually ignored. However, there is also "heavy" QC [2], the total weight of which is about 15 ... 20 kg and which have massive lifting rotor blades with a large span (length). For example, the Airborg H8 drone has a propeller span of almost two meters. Naturally, the inertance of change in the speed of rotation of such rotors (inertia of control) can significantly affect the quality of control processes QC.

II. PROBLEM STATEMENT

The purpose of this work is to develop a mathematical model of QC and to study the effect of the time delay of the change in the lifting rotational speed of the rotor (inertia of control) on the dynamics of automatic control loops QC.

The mathematical model of QC should be simple enough for research; therefore, when obtaining it, the following assumptions are made:

- the quadcopter is a rigid body with constant inertial-mass characteristics;
- the axes of the bound coordinate system (CS) of the quadcopter coaxial with the principal axes of inertia;
- the gyroscopic moments of the engines and lifting rotors are not keep in mind;
- the Earth is flat, does not rotate or move in inertial space;
- the wind is considered as external disturbance.

III. Problem Solution

The equations of motion of QC can be obtained from the basic laws of classical mechanics, which in vector form have the form [4]:

$$m\frac{d\vec{V}}{dt}=\vec{F}, \tag{1}$$

$$\frac{d\vec{K}}{dt}=\vec{M}, \tag{2}$$

here m is the mass of QC; $\vec{V}$ is the airspeed vector; $\vec{F}$ is the vector of forces acting on QC; $\vec{K}$ is the angular momentum vector; $\vec{M}$ is moments acting on QC.

By projecting vector equations (1) and (2) on the axis of the bound CS, according to the rule of differentiation of a vector given in a moving CS (Boer's rule), we obtain a differential equation system describing the spatial motion of QC as a rigid body with six degrees of freedom.

$$\begin{aligned} &m\left(\dot{V}_x+\omega_y V_z-\omega_z V_y\right)=F_x,\\ &m\left(\dot{V}_y+\omega_z V_x-\omega_x V_z\right)=F_y,\\ &m\left(\dot{V}_z+\omega_x V_y-\omega_y V_x\right)=F_z,\\ &\dot{K}_x+\omega_y K_z-\omega_z K_y=M_x,\\ &\dot{K}_y+\omega_z K_x-\omega_x K_z=M_y,\\ &\dot{K}_z+\omega_x K_y-\omega_y K_x=M_z, \end{aligned} \tag{3}$$

In the bound CS, the equation of moments is simplified, since in that case, the centrifugal moments of inertia I_{xy}, I_{yz}, I_{zx} are equal to zero, and the projections of the angular momentum on the axis of the bound CS take on a simple form:

$$\begin{aligned} K_x&=I_x\omega_x,\\ K_y&=I_y\omega_y,\\ K_z&=I_z\omega_z, \end{aligned} \tag{4}$$

where I_x, I_y, I_z – axial moments of inertia; ω_x, ω_y, ω_z – the projection of the angular velocity vector of QC rotation on the axis of the bound CS.

Taking into account (4), the equation of moments in (3) will have the following form:

$$\begin{aligned} &I_x\dot{\omega}_x+\left(I_z-I_y\right)\omega_z\omega_y=M_x,\\ &I_y\dot{\omega}_y+\left(I_x-I_z\right)\omega_x\omega_z=M_y,\\ &I_z\dot{\omega}_z+\left(I_y-I_x\right)\omega_y\omega_x=M_z. \end{aligned} \tag{5}$$

But it is more expedient to write the equations of forces in a nonrotating normal CS $OX_gY_gZ_g$. In normal CS, the OY_g axis is directed upward along the local vertical, and the direction of the OX_g and OZ_g axes is selected in accordance with the task. In the absence of rotation, the equations of forces in (3) take the form:

$$\begin{aligned} m\dot{V}_{x_g}&=F_{x_g},\\ m\dot{V}_{y_g}&=F_{y_g},\\ m\dot{V}_{z_g}&=F_{z_g}. \end{aligned}$$

Let us supplement this system with equations describing changes in the speeds of rotation $\Omega_{1\text{-}4}$ and, accordingly, changes in the magnitudes of the thrust $T_{1\text{-}4}$ of four main rotors:

$$\left.\begin{aligned} J_\omega\dot{\Omega}_i&=M_{\mathrm{sh}_i},\\ T_i&=c_T\Omega_i^2, \end{aligned}\right.\qquad i=1...4$$

where J_ω are moments of inertia (equal for all rotors); $\left(M_{\mathrm{sh}_i}=M_{\mathrm{cont}_i}-M_{\mathrm{load}_i}\right)$ are moments on shafts of lifting rotors; $\left[M_{\mathrm{cont}_i}(u_i)\right]$ are moments of control u_i created by rotor motors; $\left[M_{\mathrm{load}_i}(\Omega_i,V_{y_g})\right]$ are load moments on shafts of lifting rotors; c_T is the thrust coefficient of each rotor, which depending from its geometric characteristics and from the air density ρ, i.e. from the flight altitude H. With an increase in the speed of climb, the angle of attack of the rotor blades decreases, therefore, the thrust coefficient also decreases. With oblique blowing of the rotors, the thrust coefficient increases. Thus, the thrust coefficient is a function $c_T=f(H,V_{x_g},V_{y_g})$.

Complementing the obtained equations with kinematic and geometric relations, as well as equations describing the trajectory motion of the center of mass, we obtain a closed system of equations describing the spatial motion of QC.

However, some problems of the research of controlled QC movement can be fulfilled using mathematical models of only isolated longitudinal or only isolated lateral movement.

The closed system of equations describing the longitudinal motion of QC can be extracted from the complete system of equations provided that the lateral motion parameters, as well as the roll and yaw control actions, are equal to zero. Then the system of equations describing the isolated longitudinal motion of QC is reduced to the form:

$$m\dot{V}_{x_g} = F_{x_g},$$
$$m\dot{V}_{y_g} = F_{y_g},$$
$$I_z\dot{\omega}_z = M_z,$$
$$\dot{\vartheta} = \omega_z,$$
$$\dot{Y}_g = \dot{H} = V\sin\Theta,$$
$$\dot{X}_g = V\cos\Theta,$$
$$J_\omega\dot{\Omega}_i = M_{\text{tor}_i}, \qquad i = 1...4$$
$$T_i = c_T\Omega_i^2,$$

Expanding the right components of the equations of forces, we note that the change in the components of the flight speed V_{x_g} and V_{y_g} occurs under the influence of the components of the thrust force $T = \sum_{i=1}^{i=4} T_i$ (control actions), gravity G, as well as the aerodynamic drag force of the fuselage QC and the blade-swept area of the lifting rotor.

Fuselage drag compared to the aerodynamic drag force of the blade-swept area of the lifting rotor is small, it is 3...5% of the thrust, so in the mathematical model it can be neglected.

The drag force vector of the blade-swept area of the rotors R is opposite to the airspeed vector of QC. The value of R is proportional to the swept area of the rotors S, air density ρ and the square of the speed V and depends on the angle of attack of the plane of the rotor α, which is understood as the angle between the plane of rotation of the rotor and the projection of the airspeed vector on the plane of symmetry. Since the QC moves in any direction in the air, the angle of attack of the rotors can vary range ±180°.

Thus, the projections of the force vector on the axes of normal CS have the form:

$$F_{x_g} = T\left(H, V_{x_g}, V_{y_g}\right)\sin\vartheta - R(H,V,\alpha)\sin\Theta,$$
$$F_{y_g} = T\left(H, V_{x_g}, V_{y_g}\right)\cos\vartheta - G - R(H,V,\alpha)\cos\Theta,$$

where Θ is the angle of trajectory obliquity.

Airspeed V and angular parameters Θ and α can be obtained from the relations:

$$V = \sqrt{V_{x_g}^2 + V_{y_g}^2}, \quad \Theta = \text{arctg}\frac{V_{y_g}}{V_{x_g}}, \quad \alpha = \vartheta - \Theta.$$

The components of the moment M_z acting on the QC are:

- the control moment $M_z(u)$, which is created by the difference between the rods of the front T_1 and rear T_2 rotors – $M_z(u) = (T_1 - T_2)l$, (l is the distance from the rotor axis of the lifting rotor to the center of mass QC);
- the moment of own aerodynamic damping $M_z(\omega_z)$.

For the purpose of further simplification, the QC model can be linearized using control effects as inputs and center of mass coordinates and angular coordinates as outputs. As an unperturbed (programmed) motion, it is advisable to choose a rectilinear horizontal flight with a constant speed, assuming that the pitch angle is equal to the angle of attack.

It was on this model that studies were carried out on the influence of the inertance of the QC automatic control loops. Matlab Simulink software environment was used for mathematical modeling.

For automatic control of the QC motion, PD-control based control loops were used. And this is natural, since the P-controller makes the control system oscillatory, and when using the PI-controller, the system can become unstable. When trying to improve the precision characteristics of the system at the expense of PID-control, problems arise when working out the specified values of the control parameters (significant overshoot).

Any synthesis method can be used to determine the coefficients of the PD-controls. In this case, reasoning from the desired transition time, the synthesis method which based of modal control was used [3]. For the synthesis, we used simplified models of the vertical velocity channel and the channel of longitudinal progressive and angular motion. The simplifications ignored counterbracing between the channels, and also did not take into account the inertia of changing the speed of rotation of the lifting rotors, that is, the governing force $T(u_{Vy}) = (T_1 + T_2)$ and moment $M_z(u_\vartheta) = (T_1 - T_2)l$ formed instantly. In expressions for forces and moments u_{Vy}, u_ϑ - control actions formed according to the control laws of the PD-controllers

$$u_{Vy} = K_H(H - H_{\text{set}}) + K_{Vy}V_y,$$
$$u_\vartheta = K_\vartheta(\vartheta - \vartheta_{\text{set}}) + K_{\omega z}\omega_z.$$

Here $K_H, K_{Vy}, K_\vartheta, K_{\omega z}$ are coefficients of the PD-controls; H_{set}, ϑ_{set} are set point values of flight altitude and pitch angle.

Matlab Simulink software environment was used for mathematical modeling. When experimentation from the mathematical model was excluded and then reintroduced the relations describing the change in the rotational speed of the lifting rotors. Simultaneously, the playback dynamics of the set point values of the flight altitude and the pitch angle by synthesized control loops was simulated.

The simulation results (Fig. 1) show that the inertance effect of control is especially pronounced in the pitch angle control loop and manifests itself in an increase of the transient process oscillativity.

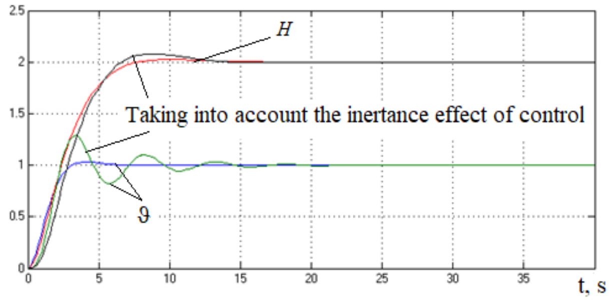

Fig. 1. Results of modeling the inertance effect of the lifting rotors rotation speed change on the dynamics of the automatic control loops of QC

Efforts to eliminate this effect by increasing the portion of the damping signal leads to the fact that an exponential component appears in the transient process, on which the oscillatory component is superimposed (curve 1 in Fig. 2).

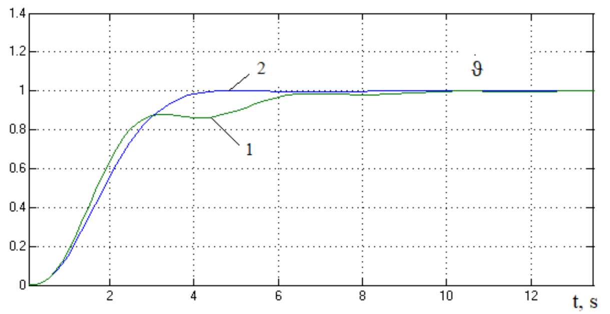

Fig. 2. Simulation results of control law correction options

The paper proposes to eliminate the inertance effect of control by introducing a forcing component $K_{p\omega z}(p+1)/(0.1p+1)$ into the signal circuit of the angular velocity sensor. The result of such a correction of the control law is demonstrated by curve 2 of the transient process in Fig. 2.

IV. Conclusions

The proposed mathematical model of a quadcopter adequately describes the processes of the inertance effect of the lifting rotors rotation speed change. The proposed variants of control laws eliminate the influence of the control inertance effect.

References

[1] I. I. Ogoltsov, N. B. Roznin, and V. V. Sheval, "Development of a mathematical model of space flight quadcopter," Proceedings of MAI. no. 83, 2015, pp. 46–58.

[2] Heavy Duty Quadcopters: https://digbox.ru/reviews/s-bolshoy-gruzopodemnosty/

[3] A.O. Belyavsky and S. I. Tomashevich, "Synthesis of an adaptive control system for a quadcopter by the pacification method," *Management of large systems*. Moscow, no. 63, pp. 155–181, 2015.

[4] V. A. Rogozhin, V. M. Sineglazov, and N. K. Filyashkin, *Automated aircraft control systems*. Kyiv: NAU, 2003, 502 p.

[5] K. Geng and N. A. Chulin, "Stabilization algorithms for automatic control of the trajectory motion of a quadcopter," *Electronic journal "Science and education," Proceedings of MSTU*, no. 83, 2015, pp. 218–235.

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М. К. Філяшкін. Дослідження динаміки контурів автоматичного керування важкого квадрокоптера

Розглянуто питання автоматизації керування важким квадрокоптером. Запропоновано математичну модель динаміки руху квадрокоптера, що враховує ефект інерційності зміни швидкості обертання несних гвинтів. У моделі розкриті залежності аеродинамічних сил і моментів, що діють на квадрокоптер. З метою спрощення досліджень пропонується використовувати для моделювання тільки математичну модель ізольованого, попередньо лінеаризованого поздовжнього руху квадрокоптера. Обгрунтовано використання для автоматичного керування квадрокоптера не ПІД-, а ПД-регулювання. Запропоновано варіанти законів керування, які виключають вплив ефекту інерційності на динаміку контурів автоматичного керування.

Ключові слова: квадрокоптер; несучі гвинти; швидкість обертання гвинта; інерційність керування; керуючі сили і моменти; закони керування.

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Напрям наукової діяльності: комплексна обробка інформації в пілотажно-навігаційних комплексах, автоматизація та оптимізація керування повітряними суднами на різних етапах польоту.
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Н. К. Филяшкин. Исследование динамики контуров автоматического управления тяжелого квадрокоптера

Рассмотрены вопросы автоматизации управления тяжелым квадрокоптером. Предложена математическая модель динамики движения квадрокоптера, учитывающая эффект инерционости изменения скорости вращения несущих винтов. В моделе раскрыты зависимости аэродинамических сил и моментов, действующих на квадрокоптер. С целью упрощения исследований предлагается использовать для моделирования только математическую модель изолированного, предварительно линеаризованного продольного движения квадрокоптера. Обосновано использование для автоматического управления квадрокоптером не ПИД-, а ПД-регулирования. Предложены варианты законов управления, исключающие влияние эффекта инерционости на динамику контуров автоматического управления.

Ключевые слова: квадрокоптер; несущие винты; скорость вращения винта; инерционность управления; управляющие силы и моменты; законы управления.

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