# MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE NATIONAL AVIATION UNIVERSITY

Faculty of Transport, Management and Logistics

Higher Mathematics Department

#### METHODICAL GUIDANCE TO THE STUDENTS' SELF-STUDY

#### on «Higher and Applied Mathematics»

Field of study: 07 "Management and administration" Specialty: 073 "Management" Educational Professional programs: "Management of foreign economic activity" "Logistics" "Aviation logistics" "Management of airlines and airports"

> Developed by: Assosiate Professor I. Klyus

Methodical guidance to the students' self-study

was considered and approved
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Department,
Minutes № \_\_\_\_ of \_\_\_\_2021

Head of Department\_\_\_\_\_I. Lastivka

#### METHODICAL SUPPORT OF ARRANGEMENT OF STUDENTS' SELF-STUDY

1. Higher mathematics. Part 1: Manual/ Denisiuk V.P., Grishina L.I., Karupu O. W., Oleshko T.A., Pakhnenko V.V., Repeta V.K. – Kyiv: NAU, 2006.

2. Higher mathematics. Part 2: Manual/ Denisiuk V.P., Demydko V.G., Repeta V.K. – Kyiv: NAU, 2009.

3. Higher mathematics. Part 3: Manual/ Denisiuk V.P., Grishina L.I., Karupu O. W., Oleshko T.A., Pakhnenko V.V., Repeta V.K. – Kyiv: NAU, 2006.

4. Mathematical analysis: Manual / V. P. Denisiuk, V. G. Demydko., O. V. Karupu, T. A. Oleshko, V. V. Pakhnenko, V. K. Repeta. – Kyiv: NAU, 2013. – 396 p.

5.Higher mathematics. Linear algebra. Algebra of vectors. Elements of analytic geometry: Method Guide / compilers: A.O.Antonova, I. S. Klyus, I. O. Lastivka, V. I. Trofymenko. – K. : NAU, 2018. – 60 p

6. Higher mathematics. Introduction to mathematical analysis: Method Guide / compilers: I. S. Klyus, I. O. Lastivka. – K. : NAU, 2019. – 48 p.

7. Higher mathematics. Differential calculus of one variable: Method Guide / compilers: I. S. Klyus, I. O. Lastivka. - K. : NAU, 2021. - 48 p.

8. Higher mathematics. Integral calculus: Method Guide / compilers: I. S. Klyus, I. O. Lastivka. – K. : NAU, 2021. – 72 p.

# Module №1. «Linear algebra. Elements of algebra of vectors and analytic geometry. Precalculus. Calculus»

#### **Topic 1.1. Elements of Linear and Vector Algebra**

#### 1. Concepts, definitions, formulations:

- 1. Determinants of the 2nd, the 3rd and the n-th orders.
- 2. Matrices. Linear operations with matrices. Multiplication of matrices.
- 3. Inverse matrix
- 4. Definite, indefinite, consistent, inconsistent SLAE.
- 5. Matrix form of SLAE.
- 6. Gauss' method of SLAE solution.
- 7. Kronecker-Capelli theorem usage in SLAE investigation.

8. Geometrical vector. Vector addition and subtraction operations, multiplication by scalar.

- 9. Linear dependence and independence of vectors.
- 10. Cartesian coordinate system (CCS).
- 11. Dot product of two vectors.
- 12. Cross product of two vectors.

#### 13. Triple product

### 2. Proofs and conclusions

- 1. Properties of determinants (2nd and 3rd orders).
- 2. Matrix addition and multiplication properties.
- 3. Existence of an inverse matrix.
- 4. Inverse matrix method of SLAE solution.
- 5. Cramer's Theorem.
- 6. Kronecker-Capelli Theorem.
- 7. Projection of vector on axis.
- 8. Representation of a vector in terms of base vectors.
- 9. Properties of a dot product; calculation by coordinates.
- 10. Properties of a cross product; calculation by coordinates.

11. Properties of a triple product; calculation by coordinates

#### 3. Assignments

1. Calculate the determinants of order 2, 3 and n, to be able to lay out a determinant by the elements of any row or column, to reduce determinant to the triangle form.

- 2, Find the matrix sum, difference and product.
- 3. Find the matrix rank.
- 4. Find an inverse matrix.
- 5. Solve the square systems by Cramer's method, through inverse matrix.
- 6. Solve the square systems by Cramer's method, through inverse matrix.
- 7. Solve the arbitrary SLAE by Gauss' method.

8. Analyse SLAE on the consistence (compatibility) according to Kronecker-Capelli Theorem.

9. Analyse SLAE on the consistence (compatibility) according to Kronecker-Capelli Theorem.

- 10. Find the eigenvalues and eigenvectors of matrix.
- 11. Find the vector coordinates, it's length, unit vector. Find the angle between vectors.
- 12. Find the vector sum, difference, dot and cross products.
- 13. Calculate the area of the triangle, volume of pyramid.
- 14. Be able to represent the vector in terms of base vectors.
- 15. Be able to use the condition of two vectors perpendicularity

## **Topic1.2 Introduction to Mathematical Analysis**

1. Sets. Classification of numerical sets. Operations on sets. The modules of a real number.

2. A sequence.

3. A function. Classification of functions. The elementary functions. An inverse function. A composite function.

4. The Limit of a numerical sequence. The Limit of a function. Infinitesimals.

5. Continuity. Continuity of a function at a point and on an interval. Properties of continuous functions. Points of discontinuity and its classification.

#### 2. Proofs and conclusions

- 1. Theorems about limits.
- 2. The first and the second honorable limits.
- 3. Theorems about equivalent infinitesimals.

#### 3. Assignments

- 1. Evaluate the limits.
- 2. Evaluate the limits using the equivalent infinitesimals.
- 3. Investigate functions for continuity.

## **Topic 1.3. Differential Calculus of the Function of One Variable**

#### 1. Concepts, definitions, formulations:

- 1. Definition of a derivative. Geometrical and physical interpretation.
- 2. A table of derivatives. Rules of differentiation.
- 3. A connection between continuity and differentiability.
- 4. A differential. Geometrical interpretation of a differential.
- 5. The usage of the differentials.
- 6. Evaluation of the first and higher order derivatives.
- 7. Leibniz's formula.
- 8. Lagrange's formula.

9. L'Hospital's rule for expansion of indeterminate forms  $\begin{bmatrix} 0\\ 0 \end{bmatrix}$  or  $\begin{bmatrix} \infty\\ \infty \end{bmatrix}$ .

#### 10. Taylor's formula.

- 11. Maclaurin's formula.
- 12. Investigation for function increase and decrease on the given interval.
- 13. Investigation of a function for extremum.
- 14. Minimum and maximum values on the interval.
- 15.Concavity intervals. Inflection points.
- 16. Asymptotes.
- 17. Plan of graph construction.

## 2. Proofs and conclusions

- 1. The derivatives of elementary functions.
- 2. The first order and higher order derivatives of the parametric functions.
- 3. Theorem about continuity of differentiable functions.
- 4. Geometrical interpretation of the first order differential.
- 5. Equation of a tangent line and a normal to the curve.
- 6. Lagrange's and Fermat's theorems.
- 7. L'Hospital's rule for expanding of indeterminate form  $\begin{bmatrix} 0\\ 0 \end{bmatrix}$ .
- 8. The necessary monotony conditions.
- 9. The necessary and sufficient extremum conditions.
- 10.Curve asymptotes seeking rule.

#### 3. Assignments

1. Find the derivatives of functions.

2. Find the derivatives of composite functions, implicit functions and parametric functions.

3. Find the differentials of functions.

4. Find the derivatives and the differentials of higher order.

5. Solve tasks for geometrical and physical interpretation of a derivative.

6. Investigate elementary functions.

7. Sketch the graphs of elementary functions.

8. Find different limits with the help of L'Hospital's rule.

9. Find intervals of function increase and decrease, local extremum.

- 10. Find concavity intervals.
- 11. Find graph asymptotes.
- 12. Construct the graph.

# Module №2 «Integral calculus. Differential equations. Introduction to operations research»

#### **Topic 2. 1. Integral Calculus of the Function of One Variable**

#### 1. Concepts, definitions, formulations:

- 1. Antiderivative. Indefinite integrals. Table of integrals. Evaluating techniques.
- 2. Polynomial functions. Rational functions.
- 3. Integrating of rational functions by partial fractions.
- 4. Integrals involving powers of trigonometric functions.
- 5. Integrating of irrational functions.
- 6. Definite integrals. Newton-Leibniz fundamental theorem.
- 7. Properties of definite integrals. Evaluating techniques.
- 8. Improper integrals. Convergence of improper integrals.
- 9. Application of the definite integrals

## 2. Proofs and conclusions

1. Concepts of antiderivative and the indefinite integral. The table of the integrals.

2. The substitution technique.

- 3. Integration by parts.
- 4. Integrating of partial fractions. Integrating of rational functions.
- 5. Integrals involving powers of trigonometric functions.
- 6. Integrating of irrational functions.
- 7. Definite integrals. Newton-Leibniz fundamental theorem.
- 8. Properties of definite integrals.
- 9. Improper integrals. Convergence and evaluating.

10. Application of the definite integrals in geometry and mechanics.

## 3. Assignments

- 1. Find indefinite integrals applying table of integrals.
- 2. Find indefinite integrals applying substitution technique.

3. Find indefinite integrals applying integration by parts.

4. Find integrals of rational functions by partial fractions.

5. Find integrals involving powers of trigonometric functions.

6. Find integrals of irrational functions.

7. Find definite integrals applying Newton-Leibniz formula..

8. Find definite integrals applying evaluating techniques.

9. Investigate improper integrals for convergence. Find improper integrals.

10. Apply definite integrals for solving geometric and mechanical problems.

## **Topic 2. 2. Differential Equations**

## 1. Concepts, definitions, formulations:

1. Differential equations of the first order. General definitions. Integral curve. Cauchy problem.

2. Differential equations of the first order: separable equation, homogeneous differential equation, linear differential equations of the first order, Bernoulli equation, exact differential equations.

3. Differential equations of higher order. Basic concepts and definitions.

4. Differential equations which allow reduction of order.

5. Linear differential equations of order n.

6. Linear homogeneous differential equations with constant coefficients.

7. Linear non-homogeneous equations. Method of undetermined coefficients.

8. Systems of the differential equations. Normal system of differential equations.

The method of elimination and integration combinations of solutions of systems of differential equations in normal form.

9. System of differential equations with constant coefficients.

# 2. Proofs and conclusions

1. Differential equations of the first order. General and particular solutions of differential equation. Cauchy problem.

2. Separable equation.

- 3. Homogeneous differential equation.
- 4. Linear differential equations of the first order.
- 5. Bernoulli equation.
- 6. Exact differential equations.

7. Differential equations of higher order.

8. Linear differential equations of order n. Fundamental system of solutions. Structure of the general solution of the homogeneous linear differential equation of order n.

9. Method of variation of constants.

10. Linear homogeneous differential equations with constant coefficients. The structure of the general solution of a linear homogeneous equations.

11. Linear non-homogeneous equations. The structure of the general solution of a linear non-homogeneous equations.

12. Method of undetermined coefficients.

13. Systems of the differential equations. Normal system of differential equations. The method of elimination and integration combinations of solutions of systems of differential equations in normal form.

#### 3. Assignments

1. Finding general solutions of the differential equations of the first order: separable equation, homogeneous differential equation, linear differential equations of the first order, Bernoulli equation, exact differential equations.

2. Finding the particular solution of the differential equation through the given point. Linear differential equations with constant coefficients.

3. Finding the most general solution of the simultaneous equations.

4. Some applications of differential equations.

#### Literature

1.Denisiuk V. P. Higher mathematics. Part 2: Manual / V. P. Denisiuk, V. G. Demydko, V. K. Repeta. — K. : NAU, 2009. — 248 p.

2.Ластівка І.О., Коновалюк В.С., Ковтонюк І.Ю., Паламарчук Ю.А., Петрусенко В.П., Чуб Л.О. Вища математика. Модуль 3. Невизначений та визначений інтеграли: Навч. посібник– К.:Книжкове вид-во НАУ, 2007. – 208 с.

#### Guidelines

- 1. Elaboration of lecture material
- 2. Preparation for practical classes
- 3. Doing homework for practical classes
- 4. Completion of individual homework
- 5. Elaboration of literature.