MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL AVIATION UNIVERSITY

Faculty of Transport, Management and Logistics

Department of Higher mathematics

METHODICAL GUIDANCE TO THE STUDENTS' SELF-STUDY

on
«Higher Mathematics»

Field of study: 27 « Transport Services »
Specialty: 272 « Aviation Transport »
Educational and Professional Program: «Maintenance and Repair of Aircraft and Aircraft Engines »

Developed by:
Senior lecturer V. Kravchenko

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by the meeting of the Higher Mathematics Department,
Minutes № ___ of _____2021

Head of Department___________I. Lastivka

METHODICAL SUPPORT OF ARRANGEMENT OF STUDENTS' SELF-STUDY

1. Higher mathematics. Part 1: Manual/ Denisiuk V.P., Grishina L.I., Karupu O. V., Oleshko T.A., Pakhnenko V.V., Repeta V.K. – Kyiv: NAU, 2006. – 272 p.
2. Higher mathematics. Part 2: Manual/ Denisiuk V.P., Demidko V.G., Repeta V.K. – Kyiv: NAU, 2009. – 248 p.
3. Higher mathematics. Part 3: Manual/ Denisiuk V.P., Grishina L.I., Karupu O. W., Oleshko T.A., Pakhnenko V.V., Repeta V.K. – Kyiv: NAU, 2006. – 232 p.
4. Higher mathematics. Part 4: Manual/ Denisiuk V.P., Bobkov V.M., Grishina L.I., Demidko V.G., Karupu O. V., Oleshko T.A., Pakhnenko V.V., Pogrebetska T.O., Repeta V.K. – Kyiv: NAU, 2006. – 248 p.

Module №1 „ Elements of Linear Algebra,Vector Algebra and Analytical Geometry"

Topic 1. 1. Elements of Linear and Vector Algebra

1. Concepts, definitions, formulations:

1. Determinants of the 2nd, the 3rd and the *n– th* orders.
2. Matrices. Linear operations with matrices. Multiplication of matrices.
3. Inverse matrix
4. Definite, indefinite, consistent, inconsistent SLAE.
5. Matrix form of SLAE.
6. Gauss' method of SLAE solution.
7. Kronecker-Capelli theorem usage in SLAE investigation.
8. Geometrical vector. Vector addition and subtraction operations, multiplication by scalar.
9. Linear dependence and independence of vectors.
10. Cartesian coordinate system (CCS).
11. Dot product of two vectors.
12. Cross product of two vectors.
13. Triple product

2. Proofs and conclusions

1. Properties of determinants (2nd and 3rd orders).
2. Matrix addition and multiplication properties.
3. Existence of an inverse matrix.
4. Inverse matrix method of SLAE solution.
5. Cramer's Theorem.
6. Kronecker-Capelli Theorem.

7. Projection of vector on axis.
8. Representation of a vector in terms of base vectors.
9. Properties of a dot product; calculation by coordinates.
10. Properties of a cross product; calculation by coordinates.
11. Properties of a triple product; calculation by coordinates

3. Assignments

1. Calculate the determinants of order 2, 3 and *n*, to be able to lay out a determinant by the elements of any row or column, to reduce determinant to the triangle form.
2, Find the matrix sum, difference and product.
3. Find the matrix rank.
4. Find an inverse matrix.
5. Solve the square systems by Cramer's method, through inverse matrix.
6. Solve the square systems by Cramer's method, through inverse matrix.
7. Solve the arbitrary SLAE by Gauss' method.
8. Analyse SLAE on the consistence (compatibility) according to Kronecker-Capelli Theorem.
9. Analyse SLAE on the consistence (compatibility) according to Kronecker-Capelli Theorem.
10. Find the eigenvalues and eigenvectors of matrix.
11. Find the vector coordinates, it's length, unit vector. Find the angle between vectors.
12. Find the vector sum, difference, dot and cross products.
13. Calculate the area of the triangle, volume of pyramid.
14. Be able to represent the vector in terms of base vectors.
15. Be able to use the condition of two vectors perpendicularity

Topic 1. 2. Analytical Geometry

1. Concepts, definitions, formulations:

1. Different equations of a straight line (typical problems of finding equations of a straight line).

2. Curves of the second order: a circle, an ellipse, a hyperbola, a parabola (their standard equations).

3. A plane. Different equations of a plane (typical problems on finding of equations of a plane).

4. Cylindrical, conic surfaces.

5. Surfaces of revolution.

6. Method of sections.

2. Proofs and conclusions

1. Different forms of the equation of a straight line on a plane (general, symmetric, parametric, passing through two points, in slope — intercept form, in «segments», normal).
2. Mutual location of two straight lines. An angle between two straight lines. Conditions of parallelism and perpendicularity.
3. Distance from a point to a straight line.
4. Equation of a plane passing through a point perpendicularly to a given vector.
5. Equation of a plane passing through three given points.
6. Equation of a circle, an ellipse, a parabola.
7. Symmetric equations of a straight line in space.

3. Assignments

1. To work out the equation of a straight line passing through two points, through one point in the given direction.
2. To work out equations of a plane passing through a point perpendicularly to a vector, through three points.
3. To find angles between straight lines and planes.
4. To find an intersection point of a straight line and a plane.
5. To reduce equations of the second order to the standard form and to sketch their graphs.

Module №2 „ Introduction to Mathematical Analysis. Differential calculus of function of one variable. Differential calculus of function of several variables ”

Topic 2. 1. Limits

1. Concepts, definitions, formulations:

1. Sets. Classification of numerical sets. Operations on sets. The modules of a real number.
2. A sequence.
3. A function. Classification of functions. The elementary functions. An inverse function. A composite function.
4. The Limit of a numerical sequence. The Limit of a function. Infinitesimals.
5. Continuity. Continuity of a function at a point and on an interval. Properties of continuous functions. Points of discontinuity and its classification.

2. Proofs and conclusions

1. Theorems about limits.
2. The first and the second honorable limits.
3. Theorems about equivalent infinitesimals.

3. Assignments

1. Evaluate the limits.
2. Evaluate the limits using the equivalent infinitesimals.
3. Investigate functions for continuity.

Topic 2. 2. Differential Calculus of the Function of One Variable

1. Concepts, definitions, formulations:

1. Definition of a derivative. Geometrical and physical interpretation.
2. A table of derivatives. Rules of differentiation.
3. A connection between continuity and differentiability.
4. A differential. Geometrical interpretation of a differential.
5. The usage of the differentials.
6. Evaluation of the first and higher order derivatives.
7. Leibniz's formula.
8. Lagrange's formula.
9. L'Hospital's rule for expansion of indeterminate forms $\left[\frac{0}{0}\right]$ or $\left[\frac{\infty}{\infty}\right]$.
10. Taylor's formula.
11. Maclaurin's formula.
12. Investigation for function increase and decrease on the given interval.
13. Investigation of a function for extremum.
14. Minimum and maximum values on the interval.
15. Concavity intervals. Inflection points.
16. Asymptotes.
17. Plan of graph construction.

2. Proofs and conclusions

1. The derivatives of elementary functions.
2. The first order and higher order derivatives of the parametric functions.
3. Theorem about continuity of differentiable functions.
4. Geometrical interpretation of the first order differential.
5. Equation of a tangent line and a normal to the curve.
6. Lagrange's and Fermat's theorems.
7. L'Hospital's rule for expanding of indeterminate form $\left[\frac{0}{0}\right]$.
8. The necessary monotony conditions.
9. The necessary and sufficient extremum conditions.
10. Curve asymptotes seeking rule.

3. Assignments

1. Find the derivatives of functions.
2. Find the derivatives of composite functions, implicit functions and parametric functions.
3. Find the differentials of functions.
4. Find the derivatives and the differentials of higher order.
5. Solve tasks for geometrical and physical interpretation of a derivative.
6. Investigate elementary functions.
7. Sketch the graphs of elementary functions.
8. Find different limits with the help of L'Hospital's rule.

9. Find intervals of function increase and decrease, local extremum.
10. Find concavity intervals.
11. Find graph asymptotes.
12. Construct the graph.

Topic 2. 3. Functions of several Variables

1. Concepts, definitions, formulations:

1. Functions of several variables.
2. Limit, and continuity of the functions of several variables.
3. Partial derivatives.
4. Differential.
5. Relative extrema.
6. Tangent plane and normal to a surface.
7. Gradient.
8. Extrema on a polygon.

.2. Proofs and conclusions

1. Functions of several variables. Domain of a function of several variables.
2. Properties of continuous of the functions of several variables.
3. Formulas for calculation of partial derivatives.
4. Differential. Properties and calculation.
5. Relative extrema. Necessary and sufficient conditions.
6. Tangent plane and normal to a surface.
7. Gradient. Properties and calculation.
8. Eextrema on a polygon.

3. Assignments

1. Finding domain of a function of several variables.
2. Finding the first and higher order partial derivatives and the differentials.
3. Finding partial derivatives of the composite functions.
4. Implicit partial differentiation.
5. Finding equations of tangent plane and normal to a surface.
6. Finding gradient.
7. Finding relative extrema and extrema on a polygon.

Module №3 „Integral calculus of functions of one variable. Differential equations ”

Topic 3. 1. Complex numbers

1. Concepts, definitions, formulations:

1. Complex numbers.
2. Forms of the complex numbers.

3. Operations with them
4. Sets. Classification of numerical sets. Operations on sets. The modules of a real number.
5. A sequence.
6. A function. Classification of functions. The elementary functions. An inverse function. A composite function.
7. The Limit of a numerical sequence. The Limit of a function. Infinitesimals.
8. Continuity. Continuity of a function at a point and on an interval. Properties of continuous functions. Points of discontinuity and its classification.

2. Proofs and conclusions

1. Complex number.
2. Module and argument of a complex number
3. Theorems about limits.
4. The first and the second honorable limits.
5. Theorems about equivalent infinitesimals.

3. Assignments

1. Operate with complex numbers
2. Evaluate the limits.
3. Evaluate the limits using the equivalent infinitesimals.
4. Investigate functions for continuity.

Topic 3. 2. Integral Calculus of the Function of One Variable

1. Concepts, definitions, formulations:

1. Antiderivative. Indefinite integrals. Table of integrals. Evaluating techniques.
2. Polynomial functions. Rational functions.
3. Integrating of rational functions by partial fractions.
4. Integrals involving powers of trigonometric functions.
5. Integrating of irrational functions.
6. Definite integrals. Newton-Leibniz fundamental theorem.
7. Properties of definite integrals. Evaluating techniques.
8. Improper integrals. Convergence of improper integrals.
9. Application of the definite integrals

2. Proofs and conclusions

1. Concepts of antiderivative and the indefinite integral. The table of the integrals.
2. The substitution technique.
3. Integration by parts.
4. Integrating of partial fractions. Integrating of rational functions.
5. Integrals involving powers of trigonometric functions.
6. Integrating of irrational functions.
7. Definite integrals. Newton-Leibniz fundamental theorem.
8. Properties of definite integrals.

9. Improper integrals. Convergence and evaluating.
10. Application of the definite integrals in geometry and mechanics.

3. Assignments

1. Find indefinite integrals applying table of integrals.
2. Find indefinite integrals applying substitution technique.

3. Find indefinite integrals applying integration by parts.

4. Find integrals of rational functions by partial fractions.
5. Find integrals involving powers of trigonometric functions.
6. Find integrals of irrational functions.
7. Find definite integrals applying Newton-Leibniz formula..
8. Find definite integrals applying evaluating techniques.
9. Investigate improper integrals for convergence. Find improper integrals.
10. Apply definite integrals for solving geometric and mechanical problems.

Topic 3. 3. Differential Equations

1. Concepts, definitions, formulations:

1. Differential equations of the first order. General definitions. Integral curve. Cauchy problem.
2. Differential equations of the first order: separable equation, homogeneous differential equation, linear differential equations of the first order, Bernoulli equation, exact differential equations.
3. Differential equations of higher order. Basic concepts and definitions.
4. Differential equations which allow reduction of order.
5. Linear differential equations of order *n*.
6. Linear homogeneous differential equations with constant coefficients.
7. Linear non-homogeneous equations. Method of undetermined coefficients.
8. Systems of the differential equations. Normal system of differential equations. The method of elimination and integration combinations of solutions of systems of differential equations in normal form.
9. System of differential equations with constant coefficients.

2. Proofs and conclusions

1. Differential equations of the first order. General and particular solutions of differential equation. Cauchy problem.
2. Separable equation.
3. Homogeneous differential equation.
4. Linear differential equations of the first order.
5. Bernoulli equation.

6. Exact differential equations.
7. Differential equations of higher order.
8. Linear differential equations of order *n*. Fundamental system of solutions. Structure of the general solution of the homogeneous linear differential equation of order *n*.
9. Method of variation of constants.
10. Linear homogeneous differential equations with constant coefficients. The structure of the general solution of a linear homogeneous equations.
11. Linear non-homogeneous equations. The structure of the general solution of a linear non-homogeneous equations.
12. Method of undetermined coefficients.
13. Systems of the differential equations. Normal system of differential equations. The method of elimination and integration combinations of solutions of systems of differential equations in normal form.

3. Assignments

1. Finding general solutions of the differential equations of the first order: separable equation, homogeneous differential equation, linear differential equations of the first order, Bernoulli equation, exact differential equations.
2. Finding the particular solution of the differential equation through the given point. Linear differential equations with constant coefficients.
3. Finding the most general solution of the simultaneous equations.
4. Some applications of differential equations.

Module №4 „Elements of Theory of Probability and Mathematical Statistics ”

Topic 4.1: Random events

Tasks for independent work

1. Sets and operations on them.
2. Elements of combinatorics.
3. Events. Types of events. Algebra of events.
4. Classical, geometric, statistical definition of probability.
5. Theorems of addition and multiplication of probabilities.
6. Full probability. Bayesian formulas.
7. Repeated independent tests. Bernoulli's formula. The most likely number of successes for an event to occur n Bernoulli trials.
8. Local Muavra-Laplace theorem. Basic properties of the Gaussian function.
9. Muavra-Laplace integral theorem. Basic properties of the Laplace function.
10. Poisson's theorem.

Guidelines

1. Elaboration of lecture material
2. Preparation for practical classes
3. Doing homework for practical classes
4. Completion of individual homework
5. Elaboration of literature.

Questions for self-control

1. Formulate the basic principles of combinatorics (sum and product).
2. What compounds are called placements with ″ items by k $(k \le n)$ Give a formula for calculating their number and give examples of placements.
3. What compounds are called permutations? Give the formula for their number″ elements and give examples of permutations.
4. What compounds are called combinations (combinations) with n items by k $(k \le n)$. Give the formula for their number and give examples of combinations.
5. What is a random event? reliable? impossible? space of elementary events? Give definitions and give examples.
6. What events are called incompatible? compatible? opposite? Give examples.
7. What events form a complete group? Give examples.
8. What is the sum of events? product? Name the properties and give a geometric interpretation of operations on events.
9. Give the definition of the probability of the event: a) classical; b) geometric; c) statistical. Name the properties of probability and give examples of probability calculation.
10. Formulate the addition theorem for incompatible events and its consequences.
11. What events are called independent? dependent? Give examples.
12. Give the definition of conditional probability. Give examples of its calculation.
13. Formulate the probability theorem of dependent events; independent. Give the consequences of these theorems.
14. Formulate an addition theorem for compatible events. What does it look like for independent events? for addicts?
15. Formulate and prove the formula of complete probability.
16. What events are called hypotheses? Why is the sum of the probabilities of the hypotheses?
17. Formulate Bayesian formulas for the probabilities of hypotheses.
18. What tests are called independent? Bernoulli's trials?
19. Name Bernoulli's formula for the probability of occurrence k times of some event in n Bernoulli trials. What is the consequence of Bernoulli's theorem?
20. Give the formula for the most probable number of occurrence of an event in n Bernoulli trials.

21. Formulate Poisson's theorem on the probability of occurrence of an event k times in n Bernoulli trials.
22. Formulate the local Muavra-Laplace theorem.
23. What function is called the Gaussian function? Name its main properties and give a geometric interpretation.
24. Formulate the Muavra-Laplace integral theorem.
25. What function is called the Laplace function? Name its main properties and give a geometric interpretation.
26. Formulate Bernoulli's theorem on the probability of deviation of a relative frequency from a constant probability.

Topic 4.2: Random variables

Tasks for independent work

1. Definitions and types of random variables.
2. A series of distribution of a discrete random variable.
3. The distribution function of a random variable and its properties.
4. The density of the distribution of a random variable and its properties.
5. Numerical characteristics of random variables and their properties.
6. Probabilistic generating function and its application to find the numerical characteristics of discrete distributions.
7. Distributions of an integer discrete random variable.
8. Distributions of continuous random variables.
9. The law of large numbers.
10. The concept of a system of two random variables. Distribution matrix of a system of discrete random variables and series of distribution of its components.
11. The distribution function of a system of two random variables and its properties.
12. The density of the distribution of the system of two continuous random variables and its properties.
13. Conditions of independence of random components of the system.
14. The main numerical characteristics of the components of the system.
15. Conditional laws of distribution of system components.
16. Correlation moment and correlation coefficient of a system of random variables.

Guidelines

1. Elaboration of lecture material
2. Preparation for practical classes
3. Doing homework for practical classes
4. Completion of individual homework.
5. Elaboration of literature.

Questions for self-control

1. What is a random variable?
2. What value is called discrete?
3. What is a distribution series? distribution polygon?
4. Why is the sum of the probabilities in the distribution series?
5. What is the integral distribution function.
6. What is the graph of the distribution function for discrete random variables?
7. What is the law of distribution of a discrete random variable? Name the ways to specify the distribution law. Give examples.
8. What is the binomial law of distribution of DVV? Give an example.
9. Explain what is the Poisson distribution of DVV? Give an example.
10. What are the geometric and hypergeometric distributions of DVV? Give examples.
11. What is the mathematical expectation of DVV? What is the probabilistic meaning of mathematical expectation?
12. Name the main properties of the mathematical expectation of DVV.
13. What is DVV dispersion?
14. Name and prove the main properties of DVV dispersion. How to calculate the variance?
15. What is the deviation of DVV from its mathematical expectation? Why is his mathematical expectation equal?
16. What is the standard deviation of DVV?
17. How to calculate the numerical characteristics of equally distributed independent random variables?
18. What function is called the integral distribution function of random variables? What is the graph of the DVV distribution function?
19. By what formula do we find the differential distribution function? Name its properties.
20. What is the geometric meaning of the property of the density of distribution $\int_{-\infty}^{\infty} f(x)\mathrm{dx} = 1$?
21. For which random variables is there an integral distribution function?
22. For which random variables is there a differential distribution function?
23. What value is called random? discrete random? a continuous random variable?
24. Name the main properties of the distribution function of a random variable. What is the graph of the distribution function of a continuous random variable (HBR)?
25. What is called the differential distribution function (distribution density)? What is the relationship between integral and differential distribution functions?
26. Name the main properties of the distribution density.
27. How are the numerical characteristics of the HBV calculated?
28. Define a system of two continuous random variables.
29. What is the integral probability distribution function of two-dimensional NVV, its geometric meaning?
30. What is called the density of the combined probability distribution of two-dimensional HBV.
31. How, knowing the density, can we find the distribution function?

32. Numerical characteristics of two-dimensional random variables.
33. Why is the correlation moment of two independent NVVs equal?