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MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL AVIATION UNIVERSITY
Faculty of Transport, Management and Logistics
Higher Mathematics Department

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«___»_____2021 p.

APPROVED
Vice-Rector for Academics

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«___»_____2021 p.

Quality Management System

COURSE TRAINING PROGRAM
on
"Higher Mathematics"

Educational Professional Programs:	«Computer-aided Control Systems and Automatics» «Computer-Integrated Technological Processes and Production» «Information Technologies and Aviation Computer Systems Engineering»
Field of study:	15 «Automation and Instrumantation»
Specialty:	151 «Automation and Computer Integrated Technologies»

Training Form	Semester	Total (hours/credits ECTS)	Lectures	Practicals	Self-Study	HW/CGP	TP/CP	Semester Grade
Full-time:	1-4	585/19,5	119	170	296	1 HW – 1s. 1 HW – 2s. 1 HW – 3s. 1 HW – 4s.	-	Graded Test 1,2,3s Exam 4s

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INTRODUCTION

The Course Training Program on «Higher Mathematics» is developed based on the "Methodical guidance for the subject course training program", approved by the order № 249/роз. of 29.04.2021 p., and correspondent normative documents.

1. EXPLANATORY NOTES

1.1 Place, objectives, tasks of the subject

Place: This subject is considered as a theoretical and practical basis of knowledge and skills that form the expert's profile in the field of transport technologies.

The subject **target** is to teach students to master the mathematical apparatus that is sufficient to develop mathematical models related to the further practical activities of specialists.

Objectives of the subject are:

- mastering the logical and algorithmic thinking of students;
- mastering the necessary theoretical knowledge and the main directions of their application in the system of disciplines by specialty;
- to instill primary skills of mathematical research of applied problems;
- mastering the ability to use independently the necessary methods and special literature in solving problems.

1.2. Learning outcomes the subject makes it possible to achieve

As a result of this subject mastering a student should acquire such **learning outcomes**:

- know linear and vector algebra, differential and integral calculus, functions of many variables, functional series, differential equations for functions of one and many variables, operational calculus, theory of complex variables, probability theory and mathematical statistics, theory of random processes in the volume required for use mathematical apparatus and methods in the field of automation;

- be able to apply methods of systems analysis, modeling, identification and numerical methods for the development of mathematical and simulation models of individual elements and automation systems in general, to analyze the quality of their operation using the latest computer technologies;

- be able to use methods and technologies of mathematical modeling in the development and design of computerized control systems and automation systems.

1.3. Competences the subject makes it possible to acquire

As a result of this subject mastering a student should acquire such **competencies**:

- ability to solve complex specialized problems and practical problems, characterized by complexity and uncertainty of conditions, during professional activities in the field of automation or in the learning process, which involves the application of theories and methods of aviation and space;

- ability to solve problems and tasks in the field of computer-integrated technological processes and industries;

- ability to apply knowledge of mathematics, to the extent necessary for the use of mathematical methods for analysis and synthesis of automation systems.

1.4. Interdisciplinary connections

The subject "Higher Mathematics" is basic for the study such subjects as: "Physics", "Computer technology and programming", "Microprocessor programming", "Systems theory and systems analysis" "Theory of automatic control", and others.

2. COURSE TRAINING PROGRAM ON THE SUBJECT

2.1. The subject content

Training material is structured according to a module principle and consists of seven educational modules:

- **module №1 „Linear and Vector Algebra and Analytical Geometry ”,**
- **module №2 „Introduction to Mathematical Analysis. Differential Calculus of the Function of One and Several Variables”,**
- **module №3 „Integral Calculus of Function of One Variable”,**
- **module №4 „Differential Equations. Series”,**
- **module №5 „Multiple, Curvilinear and Surface Integrals. Elements of Field”,**
- **module №6 „Theory of function of a complex variable. Operational calculus”,**
- **module №7 „Elements of probability theory and mathematical statistics”**

each of which is a logically complete, relatively independent, holistic part of the academic discipline, which involves the assimilation of module test and analysis of the results of its implementation.

2.2. Module structuring and integrated requirements to each module

Module №1 „Linear and Vector Algebra and Analytical Geometry”

Integrated requirements to module №1. As a result of mastering the educational material of the educational module №1 the student must:

Know:

- definition and notation of determinants, matrices, systems of linear algebraic equations;
- Cramer's formulas;
- Gaussian method and matrix method for solving systems of linear algebraic equations;
- Kronecker-Capelli theorem;
- definitions and properties of dot, cross, triple products of vectors;
- different types of equations of a straight line on a plane, a plane in space and a line in space;
- definition of second order curves and their canonical equations.

Be able to:

- investigate and solve systems of linear algebraic equations;
- perform linear operations with vectors;
- find the products of vectors and apply them to solving problems of geometry and physics;
- write different equations of the line;
- determine the angles between two lines, planes, angles between a line and a plane;
- write the conditions of parallelism and perpendicularity of lines and planes;
- reduce the equations of the second order curves to the canonical form and construct their plots.

Topic 1. Determinants and Their Applications.

Content. *Determinants of the 2nd, 3rd and n-th order. Properties of determinants. Minors and cofactors. Calculation of determinants of n-th order.*

Topic 2. Matrices, operations with them. Inverse matrix. Rank of a matrix.

Content. *Matrices, operations with them. Inverse matrix. Matrix equations. Rank of a matrix. Finding of a rank of a matrix using elementary transformations.*

Topic 3. Systems of linear algebraic equations. Kronecker – Capelli theorem. Methods of solutions of systems of linear equations.

Content. *Systems of linear algebraic equations, consistence, investigation of consistence by matrix rank. Kronecker - Capelli theorem. Solution of the systems by Cramer’s rule, matrix method, Gauss’ method. Homogeneous systems of linear algebraic equations. Indefinite systems and their solutions. Application of determinants to the study of SLAR. Cramer’s rule.*

Topic 4. Vectors. Dot, cross and triple products of vectors.

Content. *Vectors, general definitions, linear operations with vectors. Linear dependence and independence of vectors. Basis. The projection of the vector on the axis. Coordinate systems on the plane and in space (Cartesian coordinate system, rectangular coordinate system, polar coordinate system). The method of coordinates. Vectors in the rectangular coordinate system (coordinates, length, direction cosines). Division of a segment according to a given ratio. Dot, cross and triple products of vectors: definition, properties, calculation, coordinate form. Geometrical interpretation. Condition of perpendicularity and collinearity of two vectors. Condition of complanarity of three vectors.*

Topic 5. Straight line on the plane.

Content. *General equation of a straight line, incomplete equations. Canonical and parametric equations of a straight line. Two-point form of a straight line. The equation of a line in the segments form. The slope — intercept form of a straight line. The angle between two straight lines, the conditions of parallelism and perpendicularity of two straight lines. Normal equation of a straight line. The distance between a point and a straight line.*

Topic 6. Plane in space.

Content. *Methods of specifying a plane in space. . Different forms of the plane equations. Angle between two planes. Conditions of parallelism and perpendicularity. The distance from the point to the plane.*

Topic 7. Straight line in space.

Content. *Straight line in space. Plane and straight line in space. Mutual arrangement of a straight line and a plane. The angle between straight lines, a plane and a straight line. Conditions of parallelism and perpendicularity. The distance between parallel lines.*

Topic 8. Curves and second order surfaces.

Content. *Circle, ellipse, hyperbole, parabola. Their properties, canonical equations. The concept of the surface of the second order. Cylindrical, conical surfaces, surfaces of revolution. Canonical equations.*

Module №2 „Introduction to Mathematical Analysis. Differential Calculus of the Function of One and Several Variables."

Integrated requirements to module №2. As a result of mastering the educational material of the educational module №2 the student must:

Know:

- ways of representation and classification of functions;
- definition of the limit of the numerical sequence and the limit of the function at the point;
- formulas of honorable limits and principal theorems about limits;
- definition of continuity of function and classification of points of discontinuity;
- definition of the derivative, table of derivatives and rules of differentiation;
- definition and properties of the differential;
- principal theorems of differential calculus;
- application of differential calculus to the investigation of functions;
- definition of functions of many variables, domain of its definition, limits and continuity;
- definition of partial derivatives, total differential function of many variables;
- application of partial derivatives.

Be able to:

- find the limit of the function and investigate the function for continuity;
- find derivatives and differentials of different orders of basic elementary functions;
- find derivatives of complicated functions, implicitly and parametrically given functions, perform logarithmic differentiation;

- to conduct a full investigation of the function and construct its plot;
- find partial derivative functions and total differential function of many variables;
- write the equation of the tangent plane and the normal to the surface;
- find the directional derivative and gradient;
- find local extrema, the smallest and largest value of the function of two variables;
- find the conditional extremum of the function of two variables.

Topic 1. Sequences and functions. The limit of a numerical sequence.

Content. *The concept of a sequence. The limit of a numerical sequence. Theorems about limits. The number e. Definite and undefined expressions. The concept of a function. Classification of functions. The elementary functions and their graphs.*

Topic 2. Limit of function. First and second honorable limits. Elimination of indeterminacies.

Content. *Limit of a function at the point. Theorems about Limits. Infinitely small and infinitely large quantities, the relationship between them. First and second honorable limits. Corollaries. Comparison of infinitesimals. Equivalent infinitesimals.*

Topic 3. Continuity of function, Principal theorems.

Content. *Continuity of a function. Classification of points of discontinuity. Properties of continuous functions in a point and on a segment.*

Topic 4. The derivative of a function. Some problems that lead to the concept of derivative. Geometrical, mechanical interpretation.

Content. *Derivative, its geometrical, mechanical and physical interpretation. The equation of a tangent and normal. Differentiability and continuity.*

Topic 5. Differentiability of functions. Differentiation rules. Derivatives of functions. Table of derivatives.

Content. *Differentiation rules. Derivatives of elementary functions. Table of derivatives. Derivative of the composite and inverse functions. Derivative of implicit and parametric functions. Logarithmic differentiation.*

Topic 6. Differential of function. Derivatives and differentials of higher orders.

Content. *Differential of function. Geometric and mechanical interpretation of the differential. Properties of the differential. Application of differentials in approximate calculations. Derivatives and higher order differentials.*

Topic 7. Investigation of the function and construction of it plots.

Content. *Monotonic functions. Extremum. Intervals of concavity, inflection point. Asymptotes. The greatest and the least values of the function. General scheme of investigation of the function and construction of the graph*.

Topic 8. Derivative and differentials of functions of several variables.

Content. *The concept of functions of several variables, basic definitions, geometric interpretation, lines and surface levels. Limit of function of many variables. Continuity of function of many variables. Partial and total increments of the function of two variables. Partial derivatives of functions of many variables. The total differential of the function of many variables and its application to approximate calculations.*

Topic 9. Some applications of partial derivatives. Directional derivative. Gradient. Extremum function of two variables.

Content. *Tangent plane and normal to the surface. Directional derivative. Gradient of scalar field. Extremum function of two variables. Conditional extremum. The largest and smallest value of the function of two variables.*

Module №3 „ Integral Calculus of Function of One Variable."

Integrated requirements to module №3. As a result of mastering the educational material of the educational module №3 the student must:

Know:

- definition of the indefinite integral and its properties;
- integrals of fundamental elementary functions and methods of integration of various functions;
- definition, conditions of existence and properties of the definite integral;
- Newton-Leibniz formula;
- definition of improper integrals of the first and second type;
- application of a definite integral.

Be able to:

- apply methods of integration by parts and methods of substitution;
- integrate rational, fractional-rational, some irrational and trigonometric functions;
- calculate or investigate the convergence of improper integrals;
- calculate the areas of plane figures, the length of the curve arc, the volume of the body, the surface area of rotation, using a definite integral.

Topic 1. Complex numbers.

Content. *Concept of the complex numbers. Operations with complex numbers in algebraic form. Geometric representation of complex numbers. Module and argument of a complex number. Trigonometric and exponential forms of a complex number. Operations with complex numbers in trigonometric form.*

Topic 2. Indefinite integral.

Content. *Antiderivative and indefinite integral. Properties. Table of basic integrals. Principal methods of integration. Direct integration. Method of substitution. Integration by parts. Classes of functions that integrate by parts.*

Topic 3. Integration of rational functions.

Content. *Polynomial, the root of a polynomial. Fundamental theorem of algebra. The decomposition of a polynomial into factors. Fractional rational functions. Proper and improper rational fractions. Decomposition of the improper fraction as the sum of a polynomial and a proper rational fraction. Decomposition of the proper rational fraction into elementary fractions. Integration of elementary fractions. Integration of rational fractions.*

Topic 4. Integration of trigonometric functions.

Content. *Methods of integrating of trigonometric functions. Universal trigonometric substitution. Partial cases of the rationalization of integrals from trigonometric functions.*

Topic 5. Integration of irrational functions.

Content. *Integration of expressions containing quadratic irrationality. Integrating some irrational expressions. Integrating differential binomial.*

Topic 6. Definite integral.

Content. *Problems resulting to the concept of a definite integral. Definition and properties. Geometric interpretation. Newton-Leibniz' formula. Calculation of definite integrals. Change of variables. Formula of integration by parts.*

Topic 7. Improper integrals of first and second type.

Content. *Improper integrals with infinite limits of integration. Tests for convergence. Calculations Improper integrals of unbounded functions. Tests for convergence. Calculations.*

Topic 8. Applications of definite integrals.

Content. *Calculation areas of plane figures. The length of the arc of the curve. Body volume with a given cross section. Volume of solid of revolution.*

Module №4 „ Differential Equations. Series."

Integrated requirements to module №4. As a result of mastering the educational material of the educational module №4 the student must:

Know:

- forms of representation of the ordinary differential equation;
- the concept of order, solution, integral curve of the differential equation;
- statement of the Cauchy problem;
- the theorem of existence and uniqueness of the solution of the differential equation of the first order;
- types of first-order differential equations: differential equations with separated and separable variables, homogeneous, linear differential equations, Bernoulli equations, Exact differential equations;
- definition of partial, general, singular solution of the differential equation of the n-th order;
- definition of linear homogeneous and inhomogeneous differential equation of the n-th order;
- definition of linearly dependent and linearly independent system of functions;
- definition of the fundamental system of solutions of a linear homogeneous differential equation;
- the structure of the general solution of the inhomogeneous linear differential equation of the nth order;
- the concept of a system of differential equations, solutions of the system: partial, general;
- definition of a numerical series;
- definition of convergent series and properties of convergent series, the necessary condition of convergence;
- convergence tests of sign-positive numerical series (comparison, D'Alembert, radical and integral Cauchy);
- definition of absolute and conditional convergence, Leibniz' test;
- definition of the functional series and its domain of convergence;
- definition of power series, interval and radius of convergence;
- definition of the Taylor series;
- definition of the Fourier series of the function *f (x).*

Be able to:

- solve differential equations with separated and separable variables, homogeneous, linear differential equations, Bernoulli equations, exact differential equations;
- solve second-order differential equations increasing the order;
- solve second-order differential equations by the method of variation of an arbitrary constant;
- solve linear differential equations of the *n*-th order with constant coefficients;
- solve linear systems of second-order differential equations with constant coefficients;
- investigate numerical series for convergence;
- find the radius and area of convergence of power series;
- decompose functions into a power series;
- apply series to approximate calculations;
- calculate the coefficients and write down the Fourier series for different cases of the function.

Topic 1. Differential equations of the first order.

Content. *Principle concepts and definitions. Cauchy problem. Theorem of existence and uniqueness of the solution. The geometric sense of a first order differential equation. Types of solutions. Separable differential equation. Homogeneous differential equations (with a homogeneous right-hand side). Linear differential equations. Bernoulli equation. Exact differential equations.*

Topic 2. Differential equations of higher orders.

Content. *Linear differential equations. Linear homogeneous and non-homogeneous differential equations. Properties. The concept of a linearly independent system of functions. Wronskian. The structure of a general isolation.*

Topic 3. Linear differential equations with constant coefficients.

Content. *Theory of linear differential equations of second and higher orders with constant coefficients. Linear non-homogeneous differential equations with constant coefficients and the right-hand side of a special form. Lagrange method (variations of arbitrary constants) for second-order linear differential equations.*

Topic 4. Systems of differential equations.

Content. *The elimination method and integrable combinations for solving systems of differential equations in normal form. Algebraic method (Euler method) of solving systems of differential equations with constant coefficients*

Topic 5. Number series.

Content. *Principle concepts and definitions, convergence. Properties of number series. Harmonic series. Necessary condition for convergence. Sufficient condition for divergence. Comparison tests, d'Alembert test, a radical and integral test of Cauchy*.

Topic 6. Alternating series.

Content. *Alternating series. Leibniz theorem. Absolute and conditional convergence of the alternating series. Sufficient test of convergence. Properties of absolutely convergent series.*

Topic 7. Functional series.

Content. *Principle concepts and definitions. Uniform convergence. Weierstrass test. Properties of uniformly converging series*.

Topic 8. Power series and their applications.

Content. *Abel's theorem. The interval and radius of convergence of the power series. Properties of power series. Taylor and Maclaurin series. Application of power series.*

Topic 9. Fourier series.

Content. *Harmonic oscillations. Trigonometric Fourier Series. Fourier coefficients. A sufficient condition is the representation of a function through its Fourier series. Fourier series for* 2π *periodic functions. Fourier series for even and odd functions. Fourier series for periodic functions with an arbitrary period. Fourier series for* $2l$ *- periodic functions. Fourier series for even and odd* $2l$ *- periodic functions. Fourier series for functions defined on an arbitrary segment.*

Module №5 Multiple,Curvilinear and Surface Integrals. Elements of Field.

Integrated requirements to module №5. As a result of mastering the educational material of the educational module №5 the student must:

Know:

- definition of double integral, properties, geometric interpretation, calculation;
- definition of the triple integral, properties, geometric interpretation, calculation;
- application of double and triple integrals;
- definition, properties, calculation and application of curvilinear integrals;
- Green's formula;
- conditions of independence of the curvilinear integral of the second type from the form of the integration path;
- definition, properties, calculation of surface integrals of the first and second kind;
- Ostrogradsky-Gauss formula;
- Stokes formula;
- definition of scalar and vector field, line and surface level of scalar field, vector lines of vector field;
- definition of the directional derivative of the scalar field, gradient;
- definition of divergence, rotor of vector field;
- definition of the flow of the vector field through the surface;
- definition of vector field circulation.

Be able to:

- reduce the double, triple, curvilinear and surface integrals to the definite integrals and calculate them;
- use polar, cylindrical and spherical coordinates calculating multiple integrals;
- find the directional derivative, gradient, divergence, rotor;
- determine the type of vector field;
- find flow, work, circulation, potential.

Topic 1. Double integrals.

Content. *Principle concepts and definitions. Conditions of existence and properties. Calculations. Substitution method. The double integral in polar coordinates. Application.*

Topic 2. Triple Integrals.

Content. *Principle concepts and definitions. Conditions of existence and properties. Calculations Cylindrical and spherical coordinate systems. Change of variables in a triple integral. Application.*

Topic 3. Line integrals of the first type.

Content. *Definition of line integrals of the first kind. Geometrical sense. Properties and calculations. Application.*

Topic 4. Line integrals the second type.

Content. *Definition the line integrals of the second kind. Properties and calculations. Green's formula. Independence of a line integral of a path of integration. The integration of total differentials. Application.*

Topic 5. Surface integrals of the first order.

Content. *Surface integrals of the first order. Definition, properties and calculation. Application.*

Topic 6. Surface integrals of the second order.

Content. *Surface integrals of the second order. Definition, properties and calculation. Application. Ostrogradsky-Gauss formula. Stokes' formula*.

Topic 7. Elements of field theory. Scalar fields. The main characteristics of the vector field.

Content. *Scalar and vector fields. The gradient of the scalar field. Gradient properties. Directional derivative. Flow vector through the surface. Divergence of the field. Circulation of the vector field. Rotor vector. Hamiltonian. Differential operations of the first and second orders. Some properties of vector fields. An irrotational, potential, solenoid field*.

Module №6 "The theory of the function of a complex variable. Operational calculus"

Integrated requirements to module №6. As a result of mastering the educational material of the educational module №6 the student must:

Know:

- the concept of the function of a complex variable, boundary and continuity;
- fundamental elementary functions and their properties;
- definition of the derivative function of a complex variable, Cauchy-Riemann conditions;
- definition of analytical, harmonic function;
- definition of the integral of the function of a complex variable;
- Cauchy's integral theorem and Cauchy's formula;
- definition of the Taylor and Laurent series;
- classification of isolated singular points;
- definition of residues;
- definition of the original, image, Laplace transform;
- images of basic elementary functions;
- theorems of linearity, similarity, displacement, delay;
- theorems on differentiation and integration of the original and the image;
- Image convolution function.

Be able to:

- to allocate real and imaginary parts of function;
- to differentiate and integrate the function;
- to restore the analytical function in its real or imaginary parts.
- apply the Cauchy formula to calculate integrals in a closed loop;
- decompose functions into a series of Laurens;
- find isolated points and classify them;
- find residues of functions;
- calculate the integrals using the residues;
- find images of originals;
- find original images;
- apply the operating method to solve differential equations and systems of differential equations.

Topic 1. The function of a complex variable. Basic elementary functions of a complex variable.

Content. *Complex numbers. The function of complex variable, limit, continuity. Basic elementary functions. Definition and properties.*

Topic 2. Differentiation of a complex variable function.

Content. *Differentiation of a complex variable function. Cauchy-Riemann conditions. Analytic functions. Harmonious functions.*

Topic 3. Integration of complex functions. Cauchy's integral theorem.

Content. *Integration the function of a complex variable. Cauchy's integral theorem and Cauchy formula. Antiderivative analytic function.*

Topic 4. Series in the complex domain. Taylor series and Laurent series.

Content. *Power series with complex terms. Taylor series and Laurent series.*

Topic 5. Isolated singularities. Residues. Their application to the calculation of integrals.

Content. *Zero function. Isolated singular points and their classification. Elements of the theory of residues and their application to the calculation of integrals.*

Topic 6. Laplace transform. Originals and images.

Content. *The original image and Laplace, their characteristics, location. Theorems of uniqueness and linearity. Theorem shift delay passing. Differentiation and integration of the original and image. Image periodic signal. Image of step functions. Image convolution function. Formula Duhamel.*

Topic 7. Laplace inversion transformation.

Content. *Finding the original fractional rational function. Elementary means of the originals. The first and second expansion theorem.*

Topic 8. Application of the Laplace transform.

Content. *Application of the Laplace transform for solving differential equations. Duhamels integral. Solving systems of linear differential equations.*

Module №7 «Fundamentals of Theory of Probability and Mathematical Statistics»

Integrated requirements to module №7. As a result of mastering the educational material of the educational module №7 the student must:

Know:

- - basic formulas of combinatorics;
- basic concepts of probability theory and methods for calculation the probabilities of random events;
- laws of probability distribution of discrete and continuous random variables;
- the main characteristics of the system of two random variables;
- basic concepts of mathematical statistics.

Be able to:

- calculate the probabilities of random events;

- find the numerical characteristics of discrete and continuous random variables;
- to make the laws of distribution of two-dimensional random variable;
- find the characteristics of sample distributions;
- perform statistical analysis of the sample.

Topic 1. Random events. Definitions of probability.

Content. *Sets and operations of them. The basic principles and formulas of combinatorics. Random events, their classification. Classical, geometrical and statistical definitions of probability.*

Topic 2. Addition and multiplication probabilities Theorems. Law of total probability. Bayes' formula.

Content. *Probability for sum of mutually exclusive events. Theorem (the Additive Law of Probabilities for compatible events). Conditional probability. Theorem (Multiplicative Law of Probability). Generalized probability multiplication rule. Independent events. Total probability low. Bayes' formula.*

Topic 3. Repeated independent trials. Bernoulli's formula.

Content. *Bernoulli's trials scheme. Bernoulli formula. Most likely number of "success" in the Bernoulli scheme. Asymptotic behavior of probability in Bernoulli scheme. Poisson's theorem. Local the de Moivre-Laplace theorem. Integral the de Moivre-Laplace theorem.*

Topic 4. Discrete random variables (DRV) and their distribution laws. Numerical characteristics of DRV.

Content. *Discrete random variables. Distribution series. Cumulative function. Binomial, geometric, hypergeometric and Poisson's distribution of DRV. Expectation, variance and standard deviation of DRV distribution and their properties. Moments of DRV distribution*.

Topic 5. Continuous random variables and their numerical characteristics. Cumulative function and probability density function. Distribution laws.

Content. *Continuous random variables. Cumulative function and probability density function and their properties. Distribution laws of continuous random variables. Uniform, exponential and normal distributions of continuous random variables*

Topic 6. System of two random variables. Dependence of random variables.

Content. *System of two discrete random variables. Distribution laws and distribution function of a system of two DRV. Independence of two random variables. Conditional distributions. Correlation moment and correlation coefficient*

Topic 7. The basic concepts of mathematical statistics.

Content. *Populations and samples. Variational series. Polygon and histogram, empirical distribution function, sample characteristics. Statistical testing: general concepts.*

Topic 8. Estimation of distribution parameters.

Content. *Statistical estimation of population's parameters. Interval statistical estimation of numerical characteristics and parameters of distribution of general population. Accuracy and reliability of statistical estimation. Normal population distribution: confidence intervals for estimation of expectation and standard deviation*.

Topic 9. Statistical testing of hypotheses.

Content. *Statistical hypotheses. Statistical criterion. Construction of a critical area. General algorithm for testing statistical hypotheses. Parametric and nonparametric statistical hypotheses.*

2.3. Training schedule of the subject

№	Topic	Academic Hours			
		Full-time			
		Total	Lectures	Practical classes	Self study
1	2	3	4	5	6
Module №1 „Linear and Vector Algebra and Analytical Geometry"					
1.1	Determinants and their properties.	The first semester			
		8	2	2	4
1.2	Matrices, operetions on matrices. Inverse matrix. Rank of the matrix	8	2	2	4
1.3	Systems of linear algebraic equations. Kronecker-Capelli theorem. Methods for solving systems of linear equations	12	2	2 2	6
1.4	Vectors. Dot, cross and triple vector products.	10	2	2 2	4
1.5	Straight line on the plane.	8	2	2	4
1.6	Plane in space.	7	2	2	3
1.7	Straight line in space.	10	2	2 2	4
1.8	Curves and second order surfaces.	9	2	2 2	3
1.9	Homework 1.1	4	-	-	4
1.10	Module test №1	6	–	2	4
	Total for module №1	**82**	**16**	**26**	**40**
Module №2 „ Introduction to Mathematical Analysis. Differential Calculus of the Function of One and Several Variables"					
2.1	Sequences and functions. Limit of the sequence.	8	2	2	4
2.2	Limit of the function. First and second honorable limits. Elimination of indeterminacies	9	2	2 2	3
2.3	Continuity of the function, the fundamental theorems	7	2	2	3
2.4	Derivative of the function at point. Geometrical and mechanical interpretation.	8	2	2	4
2.5	Differentiation of the function. Principle theorems of differential calculus. Table of derivatives	7	2	2	3
2.6	Function differential. Derivatives and higher order differentials	10	2	2 2	4
2.7	Application of the derivative to the investigation of the function.	8	2	2	4
2.8	Derivatives and differentials of functions of several variables.	10	2	2 2	4
2.9	Some applications of partial derivatives. Directional derivatives. Gradient. Extremum of function	7	2	2	3
2.10	Homework 1.2	4	-	-	4
2.11	Module test 2	5	-	1	4

	Total for module №2	**83**	**18**	**25**	**40**
	Total for the first semester	**165**	**34**	**51**	**85**
	Module №3 „ Integral Calculus of Function of One Variable ”				
3.1	Complex numbers	**The second semester**			
		8	2	2	4
3.2	Indefinite Integral.	10	2	2 2	4
3.3	Integration of rational functions	8	2	2	4
3.4	Integration of trigonometric functions.	8	2	2	4
3.5	Integration of irrational functions.	8	2	2	4
3.6	Definite integral.	10	2	2 2	4
3.7	Improper integrals of fist and second type	8	2	2	4
3.8	Application of definite integrals.	10	2	2 2	4
3.9	Homework 2.1	4	-	-	4
3.10	Module Test № 3	6	-	2	4
	Total for module №3	**80**	**16**	**24**	**40**
	Module №4 „ Differential Equations. Series”				
4.1	First-Order Differential Equations	8	2	2	4
4.2	Higher-order Differential Equations	9	2	2 2	3
4.3	Linear Differential Equations with Constant Coefficients	8	2	2	4
4.4	Systems of Differential Equations	9	2	2 2	3
4.5	Numerical series. Sufficient tests for convergence series.	9	2	2 2	3
4.6	Series with arbitrary members	8	2	2	4
4.7	Functional series.	7	2	2	3
4.8	Power series and their application.	10	2	2 2	4
4.9	Fourier series.	8	2	2	4
4.10	Homework 2.2	4	-	-	4
4.11	Module Test № 4	5	-	1	4
	Total for module №4	**85**	**18**	**27**	**40**
	Total for the second semester	**165**	**34**	**51**	**80**
	Module №5 „Multiple,Curvilinear and Surface Integrals. Elements of Field ”				
5.1	Double integrals	**The third semester**			
		9	2 2	2	3
5.2	Triple integrals	7	2	2	3
5.3	Line integrals of first order	6	2	2	2
5.4	Line integrals of second order	7	2	2	3
5.5	Surface integrals of first order	6	2	2	2
5.6	Surface integrals of second order	7	2	2	3
5.7	Elements of Field Theory. Scalar fields. Fundamental characteristics of vector fields	9	2 2	2	3
5.8	Homework 3.1	4	-	-	4

5.9	Modul test № 5	5	-	2	3
Total for module №5		**60**	**18**	**16**	**26**
Module №6 „The theory of the function of a complex variable. Operational calculus”					
6.1	The function of complex variable. Basic elementary functions of a complex variable	6	2	2	2
6.2	Differentiation of functions of the complex variable.	6	2	2	2
6.3	Integration of the function of a complex variable. Integrating Couchy’s formula	6	2	2	2
6.4	Series in the complex domain. Taylor series and Laurent series	7	2	2	2
6.5	Isolated singular points. Residues. Application of residues for calculation of integrals	7	2	2	3
6.6	Laplace transform. Original and image functions.	7	2	2	3
6.7	Inverse Laplace transform.	7	2	2	3
6.8	Applications of Laplace transform.	6	2	2	2
6.9	Homework 3.2	4	-	-	4
6.8	Module test № 6	4	-	2	2
Total for module №6		**60**	**16**	**18**	**26**
Total for the third semester		**120**	**34**	**34**	**52**
Module №7 “Fundamentals of Theory of Probability and Mathematical Statistics”					
		The fourth semester			
7.1	Random events. Definitions of probability.	14	2	2 2	8
7.2	Addition and multiplication probabilities Theorems. Law of total probability. Bayes’ formula.	14	2	2 2	8
7.3	Repeated independent trials. Bernoulli’s formula.	14	2	2 2	8
7.4	Discrete random variables (DRV) and their distribution laws. Numerical characteristics of DRV	14	2	2 2	8
7.5	Continuous random variables and their numerical characteristics. Cumulative function and probability density function. Distribution laws.	14	2	2 2	8
7.6	System of two random variables. Dependence of random variables.	12	2	2	8
7.7	The basic concepts of mathematical statistics	14	2	2 2	8
7.8	Estimation of distribution parameters	12	2	2	8
7.9	Statistical testing of hypotheses	13	1	2 2	8
7.10	Homework 4	8	-	-	8
7.11	Modul test № 7	6	-	2	4
Total for module №7		**135**	**17**	**34**	**84**
Total for the fourth semester		**135**	**17**	**34**	**84**
Total for the discipline		**585**	**119**	**170**	**296**

2.4. Homework

Homework (HW) 1.1, 1.2, 2.1, 2.2, 3.1, 3.2, 4 are performed in the first, second, third and fourth semesters. The purpose of homework is to improve theoretical knowledge and practical skills while studying the material of training modules.

Performing, design and defense of homework is carried out by the student individually according to the guidelines.

The time required to complete each homework 1.1, 1.2, 2.1, 2.2, 3.1, 3.2 is up to 4 hours of independent work, to complete homework 4 is up to 8 hours of independent work.

2.5 Questions list for the examination

The list of questions and the content of the tasks for the examination are developed by the leading teachers and approved by the minutes of the department meeting and delivered to the students.

3. BASIC CONCEPTS OF GUIDANCE ON THE SUBJECT

3.1. Teaching methods

The following teaching methods are used in the teaching process: explanatory, illustrative, reproductive, problematic teaching, and research. In addition, students are provided with individual counseling (both when meeting a teacher with a student and online).

These methods are implemented during lectures, practical classes, self-study, performing and defending home work, test work, self-solving tasks, working with educational literature, etc.

3.2. List of references

The basic literature

3.2.1. Higher Mathematics. Part 1. Calculus and Differential Equations: manual / V.P. Denisiuk, V.G. Demydko and others. – K. NAU, 2018. – 384 p.

3.2.2. Denisiuk V. P., Grishina L. I., Karupu O. V. [and Others]. Higher mathematics. Part 1: Manual / V. P. Denisiuk, L. I. Grishina, O. V. Karupu, [and Others]. — K. : NAU, 2006. — 268 p.

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3.2.5. Denisiuk V. P. Mathematical analysis: Manual/ V.P. Denisiuk, V.G. Demydko., O.V. Karupu, T.A. Oleshko, V.V. Pakhnenko, V.K. Repeta.– Kyiv: NAU, 2016. – 396 p.Дубовик В.П. Вища математика: Навч. посібник. / В. Дубовик, І. Юрик – К.: А.С.К., 2001. – 681 с.3.2.9. Антоненко В.Ф., Клюс І.С., Горідько Р.В., Чуб Л.О. Вища математика. Модуль 1. Лінійна, векторна алгебра та аналітична геометрія: Навч. посібник. – К.: Книжкове вид-во НАУ, 2006. – 300 с.

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3.2.8. Ластівка І.О. Вища математика : Навч. посібник / І.О. Ластівка, О.І. Безверхий, І.П. Кудзіновська. – К.: НАУ, 2018. – 452 с.

3.2.9. Денисюк В.П., Репета В.К. Вища математика: підручник: у 2 ч. – Ч. 1. – 2-е вид. виправ. – К.: НАУ, 2017. – 472 с.

3.2.10. Репета В.К. Вища математика: підручник: у 2 ч. – Ч. 2. – 2-е вид. виправ. – К.: НАУ, 2017. – 504 с.

3.2.11. Крисак Я.В., Левковська Т.А., Горідько Р.В., Чуб Л.О., Вишневський О.А. Вища математика. Модуль 2. Вступ до математичного аналізу. Диференціальне числення функції однієї змінної: Навч. посібник. – К.: Книжкове вид-во НАУ, 2006. – 284 с.

3.2.12. Ластівка І.О., Коновалюк В.С., Ковтонюк І.Ю., Паламарчук Ю.А., Петрусенко В.П., Чуб Л.О. Вища математика. Модуль 3. Невизначений та визначений інтеграли: Навч. посібник– К.:Книжкове вид-во НАУ, 2007. – 208 с.

3.2.13. Лубенська Т.В., Чупаха Л.Д., Трофименко В.І. Вища математика. Модуль 4. Диференціальне числення функції багатьох змінних: Навч. посібник. – К.: Книжкове вид-во НАУ, 2006. – 116 с.

3.2.14. Затула Н.І., Левковська Т.А. Вища математика. Модуль 5. Диференціальні рівняння: Навч. посібник. – К.: Книжкове вид-во НАУ, 2007. – 144 с.

Additional literature

3.2.15. Howard Anton. Calculus/ Irl Bivens, Stephen Davis. – New York: «John Wiley & Sons», 2003. – 1315 p.

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3.3. Internet resources

3.3.1. https://erudyt.net/dubovyk-yuryk-vyscha-matematyka-navch. posibnyk. html

3.3.2. https://pns.hneu.edu.ua/course/view.php?id=929

3.3.3. https://books.google.com.ua/books?isbn=9663825383

4. RATING SYSTEM OF KNOWLEDGE AND SKILLS ASSESSMENT

4.1. Evaluation of certain types of student academic work done for the study of each module is carried out in accordance with table 4.1.

Table 4.1

Kind of Academic Work	Maximum Grade Values
	The first semester
Solving problems, answering theoretical questions during classroom work	**Modules №1, №2**
	25 (the total)
Performing and protection of homework 1.1 (1.2)	10
For carrying out a module test №1 (№2), a student must receive not less	*21 point*
Module Test №1 (№2)	15
Total for the Module №1 (№2)	**50**
Total for the first semester	**100**

	The second semester
Solving problems, answering theoretical questions during classroom work	**Modules №3 (№4)**
	25 (the total)
Performing and protection of homework 2.1 (2.2)	10
For carrying out a module test №1 (№2), a student must receive not less	*21 point*
Module Test №3 (№4)	15
Total for the Module №3 (№4)	**50**
Total for the second semester	**100**
	The third semester
Solving problems, answering theoretical questions during classroom work	**Modules №5 (№6)**
	25 (the total)
Performing and protection of homework 3.1 (3.2)	10
For carrying out a module test №1 (№2), a student must receive not less	*21 point*
Module Test №5 (№6)	15
Total for the Module №5 (№6)	**50**
Total for the third semester	**100**
	The fourth semester
Solving problems, answering theoretical questions during classroom work	**Modules №7**
	35 (the total)
Performing and protection of homework 4	15
For carrying out a module test №1 (№2), a student must receive not less	*30 point*
Module Test №7	15
Total for the Module №7	**80**
Semester Examination	**20**
Total for the fourth semester	**100**

4.2. The completed certain types of educational work on the knowledge of theoretical material and practical tasks are credited to the student if he received a positive rating for them

4.3. The sum of the grades received by the student for certain types of completed educational work is the current module grade, which is entered into the Module Register.

4.4. The sum of the semester module grade and the examination grade in points makes a total semester grade which is also converted into the grades by the national scale and by the ECTS scale.

4.5. The Total Semester Grade is entered into the Examination Register, into a student's record book and into a student's educational card in values, National Scale grades, and ECTS Scale grades, for example: ***92/Ex/A, 87/Good/B, 79/Good/C, 68/Sat/D, 65/Sat./E***, etc.

4.6. The Total Semester Grade of the subject that is taught for several semesters, is determined as the arithmetic average grade of the final semester of ratings in points, followed transfer it to assess the national scale and ECTS scale.

The Total Grade of the subject is entered in the Diploma Supplement.

(Ф 03.02 – 01)

АРКУШ ПОШИРЕННЯ ДОКУМЕНТА

№ прим.	Куди передано (підрозділ)	Дата видачі	П.І.Б. отримувача	Підпис отримувача	Примітки

(Ф 03.02 – 02)

АРКУШ ОЗНАЙОМЛЕННЯ З ДОКУМЕНТОМ

№ пор.	Прізвище ім'я по-батькові	Підпис ознайомленої особи	Дата ознайом-лення	Примітки

(Ф 03.02 – 04)

АРКУШ РЕЄСТРАЦІЇ РЕВІЗІЇ

№ пор.	Прізвище ім'я по-батькові	Дата ревізії	Підпис	Висновок щодо адекватності

(Ф 03.02 – 03)

АРКУШ ОБЛІКУ ЗМІН

№ зміни	№ листа (сторінки)				Підпис особи, яка внесла зміну	Дата внесення зміни	Дата введення зміни
	Зміненого	Заміненого	Нового	Анульо-ваного			

(Ф 03.02 – 32)

УЗГОДЖЕННЯ ЗМІН

	Підпис	Ініціали, прізвище	Посада	Дата
Розробник				
Узгоджено				
Узгоджено				
Узгоджено				
Узгоджено				