NATIONAL AVIATIONAL UNIVERSITY

Faculty of Transport, Management and Logistics

Higher Mathematics Department

Methodical guidance to the students' self-study on "Higher Mathematics"

Field of study:	15 «Automation and Instrumantation»
Specialty:	151 «Automation and Computer Integrated Technologies»
Educational Professional Programs:	«Computer-aided Control Systems and Automatics» «Computer-Integrated Technological Processes and Production» «Information Technologies and Aviation Computer Systems Engineering»

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Methodical guidance to the students' self-study was considered and approved by the meeting of Higher Mathematics Department

Minutes № ____ of «___»_____20__

Head of the Department________ I. Lastivka

METHODICAL SUPORT OF ARRANGEMENT OF STUDENTS' SELF-STUDY

1. Higher mathematics. Part 1: Manual/ Denisiuk V.P., Grishina L.I., Karupu O. W., Oleshko T.A., Pakhnenko V.V., Repeta V.K. – Kyiv: NAU, 2006.
2. Higher mathematics. Part 2: Manual/ Denisiuk V.P., Demydko V.G., Repeta V.K. – Kyiv: NAU, 2009.
3. Higher mathematics. Part 3: Manual/ Denisiuk V.P., Grishina L.I., Karupu O. W., Oleshko T.A., Pakhnenko V.V., Repeta V.K. – Kyiv: NAU, 2006.
4. Mathematical analysis: Manual / V. P. Denisiuk, V. G. Demydko., O. V. Karupu, T. A. Oleshko, V. V. Pakhnenko, V. K. Repeta. – Kyiv: NAU, 2013. – 396 p.
5. Higher mathematics. Part 4:

Module №1 "Elements of Linear and Vector Algebra and Analytical Geometry."

Topic 1. 1. Elements of Linear and Vector Algebra

1. Concepts, definitions, formulations:

1. Determinants of the 2nd, the 3rd and the *n– th* orders.
2. Matrices. Linear operations with matrices. Multiplication of matrices.
3. Inverse matrix
4. Definite, indefinite, consistent, inconsistent SLAE.
5. Matrix form of SLAE.
6. Gauss' method of SLAE solution.
7. Kronecker-Capelli theorem usage in SLAE investigation.
8. Geometrical vector. Vector addition and subtraction operations, multiplication by scalar.
9. Linear dependence and independence of vectors.
10. Cartesian coordinate system (CCS).
11. Dot product of two vectors.
12. Cross product of two vectors.
13. Triple product

2. Proofs and conclusions

1. Properties of determinants (2nd and 3rd orders).
2. Matrix addition and multiplication properties.
3. Existence of an inverse matrix.
4. Inverse matrix method of SLAE solution.
5. Cramer's Theorem.
6. Kronecker-Capelli Theorem.
7. Projection of vector on axis.

8. Representation of a vector in terms of base vectors.
9. Properties of a dot product; calculation by coordinates.
10. Properties of a cross product; calculation by coordinates.
11. Properties of a triple product; calculation by coordinates

3. Assignments

1. Calculate the determinants of order 2, 3 and *n*, to be able to lay out a determinant by the elements of any row or column, to reduce determinant to the triangle form.
2, Find the matrix sum, difference and product.
3. Find the matrix rank.
4. Find an inverse matrix.
5. Solve the square systems by Cramer’s method, through inverse matrix.
6. Solve the square systems by Cramer’s method, through inverse matrix.
7. Solve the arbitrary SLAE by Gauss’ method.
8. Analyse SLAE on the consistence (compatibility) according to Kronecker-Capelli Theorem.
9. Analyse SLAE on the consistence (compatibility) according to Kronecker-Capelli Theorem.
10. Find the eigenvalues and eigenvectors of matrix.
11. Find the vector coordinates, it’s length, unit vector. Find the angle between vectors.
12. Find the vector sum, difference, dot and cross products.
13. Calculate the area of the triangle, volume of pyramid.
14. Be able to represent the vector in terms of base vectors.
15. Be able to use the condition of two vectors perpendicularity

Topic 1. 2. Analytical Geometry

1. Concepts, definitions, formulations:

1. Different equations of a straight line (typical problems of finding equations of a straight line).
2. Curves of the second order: a circle, an ellipse, a hyperbola, a parabola (their standard equations).*
3. A plane. Different equations of a plane (typical problems on finding of equations of a plane).
4. Cylindrical, conic surfaces.
5. Surfaces of revolution.
6. Method of sections.

2. Proofs and conclusions

1. Different forms of the equation of a straight line on a plane (general, symmetric, parametric, passing through two points, in slope — intercept form, in «segments», normal).
2. Mutual location of two straight lines. An angle between two straight lines. Conditions of parallelism and perpendicularity.
3. Distance from a point to a straight line.

4. Equation of a plane passing through a point perpendicularly to a given vector.
5. Equation of a plane passing through three given points.
6. Equation of a circle, an ellipse, a parabola.
7. Symmetric equations of a straight line in space.

3. Assignments

1. To work out the equation of a straight line passing through two points, through one point in the given direction.
2. To work out equations of a plane passing through a point perpendicularly to a vector, through three points.
3. To find angles between straight lines and planes.
4. To find an intersection point of a straight line and a plane.
5. To reduce equations of the second order to the standard form and to sketch their graphs.

Module №2 " Introduction to Mathematical Analysis. Differential Calculus of the Functions of One and Several Variable "

Topic 2. 1. Introduction to Mathematical Analysis.

1. Concepts, definitions, formulations:

1.Sets. Classification of numerical sets. Operations on sets. The modules of a real number.
2. A sequence.
3. A function. Classification of functions. The elementary functions. An inverse function. A composite function.
4. The Limit of a numerical sequence. The Limit of a function. Infinitesimals.
5. Continuity. Continuity of a function at a point and on an interval. Properties of continuous functions. Points of discontinuity and its classification.

2. Proofs and conclusions

1. Complex number.
2. Module and argument of a complex number
3. Theorems about limits.
4. The first and the second honorable limits.
5. Theorems about equivalent infinitesimals.

3. Assignments

1. Operate with complex numbers
2. Evaluate the limits.
3. Evaluate the limits using the equivalent infinitesimals.
4. Investigate functions for continuity.

Topic 2.2 Differential Calculus of the Function of One Variable

1. Concepts, definitions, formulations:

1.Definition of a derivative. Geometrical and physical interpretation.
2. A table of derivatives. Rules of differentiation.
3. A connection between continuity and differentiability.
4. A differential. Geometrical interpretation of a differential.
5. The usage of the differentials.
6. Evaluation of the first and higher order derivatives.
7. Leibniz's formula.
8. Lagrange's formula.
9. L'Hospital's rule for expansion of indeterminate forms $\left[\frac{0}{0}\right]$ or $\left[\frac{\infty}{\infty}\right]$.
10. Taylor's formula.
11. Maclaurin's formula.
12. Investigation for function increase and decrease on the given interval.
13. Investigation of a function for extremum.
14. Minimum and maximum values on the interval.
15.Concavity intervals. Inflection points.
16. Asymptotes.
17. Plan of graph construction.

2. Proofs and conclusions

1. The derivatives of elementary functions.
2. The first order and higher order derivatives of the parametric functions.
3. Theorem about continuity of differentiable functions.
4. Geometrical interpretation of the first order differential.
5. Equation of a tangent line and a normal to the curve.
6. Lagrange's and Fermat's theorems.
7. L'Hospital's rule for expanding of indeterminate form $\left[\frac{0}{0}\right]$.
8. The necessary monotony conditions.
9. The necessary and sufficient extremum conditions.
10.Curve asymptotes seeking rule.

3. Assignments

1. Find the derivatives of functions.
2. Find the derivatives of composite functions, implicit functions and parametric functions.
3. Find the differentials of functions.
4. Find the derivatives and the differentials of higher order.
5. Solve tasks for geometrical and physical interpretation of a derivative.
6. Investigate elementary functions.
7. Sketch the graphs of elementary functions.
8. Find different limits with the help of L'Hospital's rule.
9. Find intervals of function increase and decrease, local extremum.
10. Find concavity intervals.

11. Find graph asymptotes.
12. Construct the graph.

Topic 2. 3. Functions of several Variables

1. Concepts, definitions, formulations:

1. Functions of several variables.
2. Limit, and continuity of the functions of several variables.
3. Partial derivatives.
4. Differential.
5. Relative extrema.
6. Tangent plane and normal to a surface.
7. Gradient.
8. Extrema on a polygon.

.**2. Proofs and conclusions**

1. Functions of several variables. Domain of a function of several variables.
2. Properties of continuous of the functions of several variables.
3. Formulas for calculation of partial derivatives.
4. Differential. Properties and calculation.
5. Relative extrema. Necessary and sufficient conditions.
6. Tangent plane and normal to a surface.
7. Gradient. Properties and calculation.
8. E extrema on a polygon.

3. Assignments

1. Finding domain of a function of several variables.
2. Finding the first and higher order partial derivatives and the differentials.
3. Finding partial derivatives of the composite functions.
4. Implicit partial differentiation.
5. Finding equations of tangent plane and normal to a surface.
6. Finding gradient.
7. Finding relative extrema and extrema on a polygon.

Module №3 „ Integral Calculus of Functions of One Variable. ”

Topic 3. 1. Complex numbers.

1. Concepts, definitions, formulations:

1. Complex numbers.
2. Forms of the complex numbers.
3. Operations with them

2. Proofs and conclusions

1. Complex number.
2. Module and argument of a complex number

3. Assignments

1. Operate with complex numbers

Topic 3. 2. Integral Calculus of the Function of One Variable

1. Concepts, definitions, formulations:

1. Antiderivative. Indefinite integrals. Table of integrals. Evaluating techniques.
2. Polynomial functions. Rational functions.
3. Integrating of rational functions by partial fractions.
4. Integrals involving powers of trigonometric functions.
5. Integrating of irrational functions.
6. Definite integrals. Newton-Leibniz fundamental theorem.
7. Properties of definite integrals. Evaluating techniques.
8. Improper integrals. Convergence of improper integrals.
9. Application of the definite integrals

2. Proofs and conclusions

1. Concepts of antiderivative and the indefinite integral. The table of the integrals.
2. The substitution technique.
3. Integration by parts.
4. Integrating of partial fractions. Integrating of rational functions.
5. Integrals involving powers of trigonometric functions.
6. Integrating of irrational functions.
7. Definite integrals. Newton-Leibniz fundamental theorem.
8. Properties of definite integrals.
9. Improper integrals. Convergence and evaluating.
10. Application of the definite integrals in geometry and mechanics.

3. Assignments

1. Find indefinite integrals applying table of integrals.
2. Find indefinite integrals applying substitution technique.
3. Find indefinite integrals applying integration by parts.
4. Find integrals of rational functions by partial fractions.
5. Find integrals involving powers of trigonometric functions.
6. Find integrals of irrational functions.
7. Find definite integrals applying Newton-Leibniz formula..
8. Find definite integrals applying evaluating techniques.
9. Investigate improper integrals for convergence. Find improper integrals.

10. Apply definite integrals for solving geometric and mechanical problems.

Module №4 „ Differential Equations . Series."

Topic 4.1. Differential Equations

1. **Concepts, definitions, formulations:**

1. Differential equations of the first order. General definitions. Integral curve. Cauchy problem.
2. Differential equations of the first order: separable equation, homogeneous differential equation, linear differential equations of the first order, Bernoulli equation, exact differential equations.
3. Differential equations of higher order. Basic concepts and definitions.
4. Differential equations which allow reduction of order.
5. Linear differential equations of order *n*.
6. Linear homogeneous differential equations with constant coefficients.
7. Linear non-homogeneous equations. Method of undetermined coefficients.
8. Systems of the differential equations. Normal system of differential equations.
The method of elimination and integration combinations of solutions of systems of differential equations in normal form.
9. System of differential equations with constant coefficients.

2. Proofs and conclusions

1. Differential equations of the first order. General and particular solutions of differential equation. Cauchy problem.
2. Separable equation.
3. Homogeneous differential equation.
4. Linear differential equations of the first order.
5. Bernoulli equation.
6. Exact differential equations.
7. Differential equations of higher order.
8. Linear differential equations of order *n*. Fundamental system of solutions. Structure of the general solution of the homogeneous linear differential equation of order *n*.
9. Method of variation of constants.
10. Linear homogeneous differential equations with constant coefficients. The structure of the general solution of a linear homogeneous equations.
11. Linear non-homogeneous equations. The structure of the general solution of a linear non-homogeneous equations.
12. Method of undetermined coefficients.
13. Systems of the differential equations. Normal system of differential equations.
The method of elimination and integration combinations of solutions of systems of differential equations in normal form.

3. Assignments

1. Finding general solutions of the differential equations of the first order: separable equation, homogeneous differential equation, linear differential equations of the first order, Bernoulli equation, exact differential equations.
2. Finding the particular solution of the differential equation through the given point. Linear differential equations with constant coefficients.
3. Finding the most general solution of the simultaneous equations.
4. Some applications of differential equations.

Topic 4. 2. Series

1. Concepts, definitions, formulations:

1. Number series. Principal concepts and definitions.
2. The necessary condition for convergence.
3, Tests for convergence of positive terms series (comparison test, D'Alembert's test, Cauchy's test).
4. Alternating series. Leibniz test.
5. Absolute and conditional convergence.
6. Functional series. General definitions.
7. Uniform convergence. Weierstrass' test. Properties of uniformly convergent series. 8.
9. Power series. Abel's theorem. Interval and radius of convergence of a power series.
10. Taylor's and Maclaurin's series.
11. Expansion of a function in a series.
12. Applications of the series.
13. Trigonometric Fourier series. Fourier coefficients. Dirichlet's theorem.
14. Fourier series for 2π- and 2l-periodic functions.
15. Fourier series for odd and even functions.
16. Fourier series for functions defined on a segment [0; *l*] or on arbitrary segment [a; b]. Complex form of Fourier series.
17. Fourier integral. Fourier integral for odd and even functions.
18. Complex form of Fourier integral. Fourier transformation.

2. Proofs and conclusions

1. Properties of number series.
2. The necessary condition for convergence.
3. Tests for convergence of positive terms series (comparison test, D'Alembert's test, Cauchy's test).
4. Alternating series. Leibniz test.
5. Absolute and conditional convergence
6. Domain of convergence for functional series.
7. Weierstrass' test for uniform convergence convergence.
8. Power series. Abel's theorem. Interval and radius of convergence of a power series.
9. Formulas of expansion of a function into Taylor's and Maclaurin's series.

10. Expansion of a function in a series.
11. Trigonometric Fourier series. Fourier coefficients. Dirichlet's theorem.
12. Formulas of expansion of a function into Fourier series for 2π-periodic functions.
13. Formulas of expansion of a function into Fourier series for 2l-periodic functions.
14. Formulas of expansion of a function into Fourier series for 2π- and 2l-periodic functions.
15. Formulas of expansion into Fourier series for odd and even functions.
16. Formulas of expansion into Fourier series for functions defined on a segment [0; *l*] or on arbitrary segment [a; b].
17. Formulas of expansion of a function into complex Fourier series.
18. Formulas of expansion of a function into Fourier integral.
19. Formulas of direct and inverse Fourier transformation.

3. Assignments

1. Investigate number series applying sufficient condition for divergence and necessary condition for convergence.
2. Investigate positive terms series for convergence applying comparison test, D'Alembert's test, Cauchy's test.
3. Investigate alternating series for convergence applying Leibniz's test.
4. Investigate number series for absolute and conditional convergence.
5. Investigate functional series for uniform convergence applying Weierstrass' test.
6. Finding interval and radius of convergence of a power series. Taylor's and Maclaurin's series.
7. Finding expansion of a function into Taylor's and Maclaurin's series.
Applications of the series.
8. Finding expansion of a function into Fourier series.
9. Finding expansion of a function into Fourier integral.

Module №5 "Multiple, Curvilinear and Surface Integrals. Elements of field"

Topic 5. 1. Multiple Integrals

1. Concepts, definitions, formulations:

1. Double integral, properties, geometric content, calculation.
2. Conditions of the existence and properties
3. Change of variable in double integral. Transition to polar coordinates.
4. Triple integral, properties, geometric content, calculation.
5. Conditions of the existence and properties. Cylindrical and Spherical coordinates.

Proofs and conclusions

1. The rule of reducing a double integral to a repeated one.
2. Application of double integral.
3. The rule of reducing a double integral to a repeated one.
4. Application of triple integrals.

Assignments

1. Reduce double integrals to the definite integrals and calculate them.
2. Reduce triple integrals to the definite integrals and calculate them

Topic 5. 2. Curvilinear and Surface Integrals. Elements of the field theory.

1. Concepts, definitions, formulations:

1. Linear integrals of the first and second type. Definition, properties, calculation.
2. Green's formula.
3. Conditions for a line integral being path-independent.
4. Integration of total differentials
5. Calculation of the force field using a curvilinear integral of the second kind.
6. Surface integrals of the first and second type. Definitions, properties, reduction to a double integral.
7. Ostrogradsky's formula — Gauss.
8. Stokes formula.
9. Scalar and vector fields; their description using scalar and vector functions.
10. Examples of physical scalar and vector fields. Geometric characteristics of the field.
11. Scalar field level lines and surfaces.
12. Vector lines vector field.
13. Differential characteristics of the field.
14. Direction Derivative of the scalar field.
15. Gradient.
16. Divergence, rotor vector field.
17. Classification of vector fields.
18. Hamilton operator.
19. Integral characteristics of the field.
20. Flow of a vector field through a surface.
21. Circulation vector field.
22. Ostrogradsky-Gauss formula; its vector record; physical meaning.
23. Stokes formula; its vector record; physical meaning.
24. Potential field potential; its finding by means of a curvilinear integral.

2.Proofs and conclusions

1. The rule of reducing of a curvilinear integral of the first kind to an ordinary definite integral.
2. The rule of reducing of a curvilinear integral of the second kind to an ordinary definite integral.

3. The rule of reducing a surface integral to a double.
4. Green's formula.
5. Necessary and sufficient conditions for the vanishing of a curved Integral over a closed loop.
6. Ostrogradsky-Gauss formula.
7.The formula for direction derivative; its expression through a gradient.

3. Assignments
1. Reduce curvilinear and surface integrals to definite integrals and calculate them.
2. When calculating multiple integrals, use polar, cylindrical and spherical coordinates.
3. Find the derivative in the direction, gradient
4. Find the divergence, rotor.
5. Determine the type of vector field.
6. Finding flow, work, circulation, potential.

Module №6 "The theory of the function of a complex variable. Operational calculus"

Topic 6. 1. The theory of the function of a complex variable

1. Concepts, definitions, formulations:
1.Function of a complex variable, boundary and continuity;
2.Fundamental elementary functions and their properties;
3.Derivative function of a complex variable, Cauchy-Riemann conditions;
4.Analytical, harmonic function;
5.Integral of the function of a complex variable;
6. Cauchy's integral theorem and Cauchy's formula;
7.Taylor and Laurent series;
8. Isolated singular points;
9.Residues;

Proofs and conclusions
1. Allocate real and imaginary parts of function;
2. Differentiate and integrate the function;
3.Restore the analytical function in its real or imaginary parts.
4.Find isolated points and classify them;
5. Find residues of functions;

Assignments
1. Apply the Cauchy formula to calculate integrals in a closed loop;
2. Decompose functions into a series of Laurens;
3. Calculate the integrals using the residues.

Topic 6.2. Operational calculus.

1. Concepts, definitions, formulations:

1. Definition of the original, image, Laplace transform;
2. Basic elementary functions;
3. Theorems of linearity, similarity, displacement, delay;
4. Theorems on differentiation and integration of the original and the image;
5. Image convolution function.

2.Proofs and conclusions

1.Find images of originals;
2.Find original images.

3.Assignments

1.Apply the operating method to solve differential equations and systems of differential equations.

Module №7 «Fundamentals of Theory of Probability and Mathematical Statistics»

Topic 7. 1. Fundamentals of Theory of Probability

1. Concepts, definitions, formulations:

1.Basic formulas of combinatorics;
2.Basic concepts of probability theory and methods for calculation the probabilities of random events;
3.Main characteristics of random variable;
4.Laws of probability distribution of discrete random variables;
5.Laws of probability distribution of continuous random variables
6. Main characteristics of the system of two random variables

Proofs and conclusions

1. Calculate the probabilities of random events;
2. Find the numerical characteristics of discrete and continuous random variables;
3.Make the laws of distribution of two-dimensional random variable;

Assignments

1. Apply the methods of probability theory to calculate probabilities of random events;
2. Calculate the numerical characteristics of discrete random variables;
3. Calculate the numerical characteristics of continuous random variables.

Topic 7. 2. Fundamentals of Mathematical Statistics

1. Concepts, definitions, formulations:

1.Populations and samples.

2. Variational series.
3. Polygon and histogram, empirical distribution function, sample characteristics.
4. Statistical testing: general concepts.
5. Statistical estimation of population's parameters.
6. Interval statistical estimation of numerical characteristics and parameters of distribution of general population.
8.Normal population distribution: confidence intervals for estimation of expectation and standard deviation.
9. Statistical hypotheses.

Proofs and conclusions

2.Accuracy and reliability of statistical estimation.
3.Statistical criterion.
4. Parametric and nonparametric statistical hypotheses.

Assignments

1. Construct Variational series;
2.Construct Polygon and histogram, empirical distribution function;
3.Calculate Sample characteristics.
4. Perform statistical analysis of the sample.
5.Construct the critical area.
6. Apply General algorithm for testing statistical hypotheses.