APPLICATIONS OF THE DEFINITE INTEGRAL TO ECONOMICS

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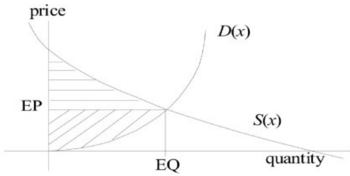
The price of an item determines the supply and the demand for the item. As price increases demand for the item usually falls. Conversely, as the price increases, the quantity producers are willing to supply will increase. Let $P_c = D(x)$ be the price at which consumers are willing to pay if there is a quantity of x items produced. Let $P_x = S(x)$ be the price at which producers are willing to supply items with a production of x items. D(x) is a decreasing function and S(x) is an increasing function. Consider the graph below.



The quantity where the demand and supply functions meet is called the equilibrium quantity (EQ), and the corresponding price is called the equilibrium price (EP). The actual point of intersection is called the equilibrium point. We assume that the market establishes itself in such a manner that the equilibrium point is achieved. There are consumers willing to pay more than the equilibrium price for the product. When the product sells at the equilibrium price, these consumers realize a savings. The difference between what these consumers would be willing to pay and what they have to pay is called the consumers' surplus. Geometrically, this represents the area shaded with horizontal lines above. This area is calculated with an integral as Consumers' surplus is equal to

$$\int_0^{EQ}[D(x) - EP]dx.$$

Similarly, we can talk about the producers' concerns. At an established equilibrium price, there are producers, who would be willing to supply the item at a lower price. The difference between what the producers earn and what they would have been willing to accept is called the producers' surplus. Geometrically, this represents the area shaded with slanted lines in the previous graph. This area is calculated with an integral as Producers' surplus is equal to

$$\int_0^{EQ}[EP - S(x)]dx.$$

Consider the following calculations. Suppose the supply curve is given by

$$D(x) = 2 - \frac{x}{2} \text{ and let } S(x) = \frac{x^2}{4}.$$

Setting the two functions equal to find the equilibrium points gives us

$$2 - \frac{x}{2} = \frac{x^2}{4} \Rightarrow x^2 + 2x - 8 = 0 \Rightarrow (x + 4)(x - 2) = 0.$$

We select the positive solution $x = 2$.

Then the consumers' surplus is equal to

$$\int_0^2 \left(\left(2-\frac{x}{2}\right)-1\right)dx = \int_0^2 \left(1-\frac{x}{2}\right)dx = \left(x-\frac{x^2}{4}\right)|_0^2 = 1-0=1.$$

The producers' surplus is calculated as

$$\int_0^2 \left(1-\frac{x^2}{4}\right)dx = \left(x-\frac{x^3}{12}\right)|_0^2 = 2-\frac{8}{12}=\frac{4}{3}.$$

A set of payments made at regular time intervals is called an annuity. The time during which payments are made is called the term of the annuity. Assume the payments are all equal in size. Assume that we can earn continuous interest on these payments. If *P* is the payment, is the continuous interest rate, *t* is the term in years, and m is the number of payments per year, then the future value of the annuity, *A* is derived an integral on

Future Value is equal to $A=\frac{mP}{r}(e^{rt}-1)$.

In a similar manner, we can derive the equation for the present value of the annuity as

Present Value is equal to $PV=\frac{mP}{r}(1-e^{-rt})$.

Determine the present and future values of an annuity of $300 every month for 20 years if the continuous interest rate is 6%. Using the above equations gives us a present value of

$$PV=\frac{12*300}{0.06}(1-e^{-0.06*20})\approx 41928.35\ .$$

Using the formula to determine the future value of the payments will lead us to

$$A=\frac{12*300}{0.06}(e^{0.06*20}-1)\approx 139207.02.$$

The future value is much larger than the present value. This is because of the 20 years worth of interest it will have accrued.

A Lorentz curve can be used to describe what portion of a population has what portion of that population's wealth. In particular, if $f(x)$ is a Lorentz function, then we define $f(x)$ as the proportion of total income earned by the poorest *x*% of the population. In these terms, $f(0.6)=0.48$ means the poorest 60% of the population earns only 48% of the total income. If $f(x)=x$, we would see that the poorest *n*% of the population would have *n*% of the wealth. If we measure the deviation of the Lorentz curve $f(x)$ from the pure socialist model $y=x$, we can see how far from this socialist state the economy will lie. This measurement of the Lorentz curve $f=x$ is called the Gini index and is defined as

Gini index is equal to $2\int_0^1[x-f(x)]dx$.

Suppose the Lorentz curve for a society is given by $f(x)=\frac{14}{15}x^2+\frac{1}{15}x$. A quick calculation reveals $f(0.5)\approx 0.27$. Thus, the poorest half of the population controls approximately 27% of the society's wealth. To compute the Gini index, we calculate the following integral.

Gini index is equal to $2\int_0^1\left[x-(\frac{14}{15}x^2+\frac{1}{15}x)\right]dx = 2(\frac{7}{15}x^2-\frac{14}{45}^3)|_0^1\approx 0.31$.

The Gini index of 0.31 can be construed to represent a 31% deviation from a uniform distribution of a society's wealth.

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