

A.V. Goncharenko, Dr  
(National Aviation University, Ukraine)

## **Applicability of the multi-optional uncertainty conditional optimality doctrine to the neuron firing model**

*It is made an attempt to discover an explainable plausible reason for a neuron model activation function, or a squashing function, of a sigmoid type function like logistic function, substantiation in terms of the multi-optional conditional optimality doctrine for the special hybrid-optimal effectiveness functions uncertainty.*

**Introduction.** The most up-to-date research in the area of neural networks deals with the extremely complex processes occurring in the networks neuron connections [1]. The neural network structures of the modeled systems and processes can be of different nature. It is always important to apply a suitable approach for such modeling. In the sphere of the aircraft airworthiness support, a significant influence of the aeronautical engineering maintenance technologies uncertainty is a crucial thing.

**State of the problem.** The uncertainty measure in the given consideration is the entropy of the special hybrid-optimal effectiveness functions. Such kind of entropy originates from the Jaynes' principle [2-4], being adapted to the subjective entropy maximum principle [5], with the implementation possibilities to the applicable fields of aviation industry as that follows the readings of the references [6-34].

The mentioned research scientific gap is the neural networking model that takes into account the significant role of the uncertainty when considering the neuron activation function.

**Purpose of the paper.** The presented paper is aimed at the sigmoid neuron activation functions modeling on the basis of the developed doctrine about the multi-optional functions entropy conditional optimality.

**Problem setting.** The problem statement for the current state would be as to find a value extremized with the known view expression used as a neuron model activation function. Consider sigmoid function, for instance, logistic function [1].

It is generally accepted that activation functions or squashing functions have the view of a logistic function [1, p. 47, (1.12)]:

$$\phi(v) = \frac{1}{1 + \exp(-av)}, \quad (1)$$

where  $v$  is induced local field or activation potential of the neuron,  $a$  is slope parameter of the sigmoid function.

**Hybrid Multi-Optional Functions Optimization Doctrine. Methods I.** In order to reveal the optimality of equation (1) [1], it is applied the prototype model of subjective analysis [5], being preceded with the Jaynes' principle [2-4]:

$$\Phi_\pi = \alpha H_\pi + \beta \varepsilon + \gamma N, \quad (2)$$

where  $\Phi_\pi$  is objective functional;  $H_\pi$  is subjective entropy;  $\varepsilon = \varepsilon(\pi, U, \dots)$  is the function of subjective effectiveness depending upon preferences  $\pi$ , utilities functions

$U$ , etc.;  $N$  is normalizing condition;  $\alpha, \beta, \gamma$  are structural parameters (Lagrange multipliers, weight coefficients or endogenous parameters of psych).

On conditions of the objective functional (2) extremum existence:

$$\frac{\partial \Phi_\pi}{\partial \pi_i} = 0, \quad (3)$$

it yields the so called functions of the individual subjective preferences [5]:

$$\pi_i = \frac{\exp(\beta U_i)}{\sum_{j=1}^N \exp(\beta U_j)}. \quad (4)$$

**Methods II.** Now, the evolution of the proposed at this paper approach from the subjective analysis (2-4) [5] to the hybrid multi-optimal functions optimization doctrine implies the use of the hybrid multi-optimal functions, as an objectively existing characteristic of a phenomena, instead of the subjectively preferred by a human functions, since no one chooses the objectively existential reality [11-34].

**Neuron Model Sigmoid Activation Function. Methods III.** Accordingly to the introduced hybrid multi-optimal functions entropy conditional optimization doctrine, the objective functional is being constructed in the following way, [11-21]:

$$\Phi_h = - \sum_{i=1}^n h_i \ln h_i + \beta \sum_{i=1}^n h_i v_i + \gamma \left( \sum_{i=1}^n h_i - 1 \right), \quad (5)$$

where  $h_i$  is the hybrid multi-optimal function (objective fundamental value of the process) deemed to be relevant to the induced local field or activation potential  $v_i$ .

The necessary conditions of functional (5) extremum existence, absolutely like (3) for (2) yield

$$h_i = \frac{\exp(\beta v_i)}{\sum_{j=1}^n \exp(\beta v_j)}. \quad (6)$$

For any two activation potentials [1, p. 43, (1.3)]  $v_1$  and  $v_2$ , at  $n=2$

$$h_1 = \frac{\exp(\beta v_1)}{\exp(\beta v_1) + \exp(\beta v_2)}, \quad h_2 = \frac{\exp(\beta v_2)}{\exp(\beta v_1) + \exp(\beta v_2)}. \quad (7)$$

If each of the induced local fields  $v_1, v_2, \dots, v_i$  is compared with the threshold activation potential  $v_0$ ,

$$\Phi_{h_{(0,i)}} = - \left( h_{0/i} \ln h_{0/i} + h_{i/0} \ln h_{i/0} \right) + \beta \left( h_{0/i} v_0 + h_{i/0} v_i \right) + \gamma \left( h_{0/i} + h_{i/0} - 1 \right). \quad (8)$$

$$h_{0/i} = \frac{1}{1 + \exp[\beta(v_i - v_0)]}. \quad (9)$$

Comparing equations (9) and (1) one can notice that

$$\beta = -a, \quad v = v_i - v_0. \quad (10)$$

The hybrid-optimal functions entropy

$$H_h = -\sum_{i=1}^n h_i \ln h_i, \quad (11)$$

serves as a measure of uncertainty of the hybrid-optimal functions  $h_i$ . Unfortunately, such measure of uncertainty as expression (11) does not show the direction of the uncertainty and its relative value.

**Methods IV.** In order to bypass such a difficulty it is proposed to apply the hybrid combined relative pseudo-entropy function developed in reference [22]:

$$\bar{H}_{\max} = \frac{H_{\max} - H_h}{H_{\max}} \cdot \frac{\Delta h}{|\Delta h|}. \quad (12)$$

Here in expression (12)  $H_{\max}$  is the maximal possible entropy (uncertainty) of the hybrid-optimal functions  $h_i$ ,  $H_h$  is the factual entropy (11),

$$\Delta h = \sum_{j=1}^M h_j^+ - \sum_{k=1}^L h_k^-, \quad (13)$$

where  $h_j^+$  and  $h_k^-$  are positive and negative properties hybrid-optimal functions respectively,  $M$  and  $L$  are numbers of the positive and negative properties options:

$$M + L = n. \quad (14)$$

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