Ministry of Science and Education of Ukraine
National Technical University "Kharkov Polytechnic Institute"
(Kharkov, UKRAINE)

National Committee of Ukraine on Theoretical and Applied Mechanics

S. Timoshenko Institute of Mechanics NAS of Ukraine (Kiev, UKRAINE)

McGill University (Montreal, CANADA)

Aberdeen University (Aberdeen, UK)

Glasgow University (Glasgow, UK)

The Third International Conference

"Nonlinear Dynamics – 2010"

Dedicated to the 125th Anniversary of the National Technical University "Kharkov Polytechnic Institute"

Proceedings

September, 21-24, 2010

THE BIFURCATION SET FOR A TWO-AXES VEHICLE MODEL WITH THE NON-LINEAR DEPENDENCE OF SLIPPING FORCES

Volodymyr G. Verbitskii[1],
Donetsk Acadamy of Automobile Transport
Donetsk, Ukraine.

Valeriy G. Khrebet ,
Highway Engineering Institute of Donetsk
National Technical University
Gorlovka, Ukraine.

Nataliya O. Velmagina,
Donetsk Acadamy of Automobile Transport
Donetsk, Ukraine.

ABSTRACT

The approach to constructing the bifurcation set of steady states for a two-axes vehicle model considering non-linear nonmonotone dependences of slipping forces is presented. Phase portraits illustrating cases of stability loss are given.

1. THEORETICAL GROUNDING.

The mathematical model of a vehicle can be presented as the dynamic system

$$\dot{x} = f\left(x, \theta, v\right). \tag{1}$$

Its steady states result from the solution of non-linear equations

$$f_i(x, \theta, v) = 0, \quad x \in R^n, \quad (i = 1, ..., n). \tag{2}$$

The system has two control parameters: longitudinal motion velocity v and the turning angle θ of front steering wheels.

In papers [1, 2] the evolution of steady states resulted from the variations of control parameters is analyzed.

Bifurcation values (v^*, θ^*) correspond to multiple solutions x^* of the system (2).

Jacobean system is altered to zero at all points of the critical set x^*:

$$J = \left\| \partial f_i / \partial x_j \right\|_{x^*} = 0, \quad x^* \in M_{kp}.$$

The system (2) with the above-mentioned equation gives rise to the critical set on the basis of the steady states manifold.

At critical points of the set the steady stationary state is eliminated (these points correspond to either fold – two-fold system solutions (2), or cusp – three-fold system solutions (2)).

Any qualitative variations of stationary states for the system of control parameters result from the birth (elimination) of two singularities.

Therefore the determination of stability boundaries considering the control parameters is of interest.

Stability boundaries can be defined by constructing the bifurcation set which divides the parameters domain into a number of domains with different stationary states, determining the zones of stability or instability.

[1] Corresponding author. Email oxsi@bigmir.net

However, the procedure of constructing the bifurcation set in the case of dependences of sideways slipping forces with evident maximum demands further development. Such necessity is explained by the existence of several branches of the bifurcation set.

2. FORMULATION OF THE PROBLEM. THE CONSTRUCTION OF THE BIFURCATION SET FOR A TWO-AXES VEHICLE MODEL.

Let's us analyze the system consisting of the body with a rear wheel axis firmly fixed and the front wheel module, its turning about the body is rigidly fixed (set by θ).

The system is subjected to the sideways reaction of the support plane – elastic wheels can move at some angle (slipping angle) to the surface of the wheel symmetry (due to elastic deformation at the point of the contact). Such situation gives rise to transverse forces resulted from the supporting plane thus interfering with sideways slipping of the wheel (slipping forces).

Let m – mass of the vehicle; J – central inertia moment of the system about the vertical axis; a, b – distance between the centre of mass of the vehicle to the middle part of the front and rear wheel axes correspondently.

Equations of the plane-parallel motion for the velocipede two-axes scheme vehicle (vertical longitudinal plane across the middle of wheel axes is the plane of the symmetry) with the constant longitudinal constituent of the mass centre velocity are

$$\begin{cases} m(\dot{u}+\omega v) = Y_1\cos\theta + Y_2; \\ J\dot{\omega} = aY_1\cos\theta - Y_2 b; \\ \delta_1 = \theta - arctg\dfrac{u+a\omega}{v},\ \delta_2 = arctg\dfrac{-u+b\omega}{v}. \end{cases} \tag{3}$$

where u – transverse constituent of the vehicle's mass centre velocity; ω – angle velocity of the vehicle about the vertical axis; δ_1, δ_2 – slipping angles for front and rear axes correspondently; Y_1 , Y_2 – sideways slipping forces as functions of slipping angles for front and rear axes correspondently.

Slipping forces are defined empirically and can be represented through different analytical dependences:

$$\begin{aligned} &-\frac{v}{g}\omega + \frac{\cos(\theta)\overline{Y_1}(\delta_1)b}{l} + \frac{\overline{Y_2}(\delta_2)a}{l} = 0, \\ &\cos(\theta)\overline{Y_1}(\delta_1) - \overline{Y_2}(\delta_2) = 0. \end{aligned} \tag{4}$$

In this case the determination of steady motion states (singularities) has the form

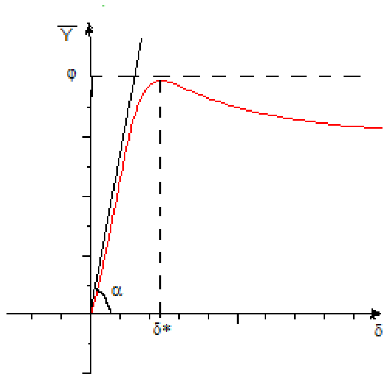
Where $\overline{Y_i}(\delta_i) = Y_i(\delta_i)/N_i$ – dimensionless sideways reactions of the support plane on the axis (N_i – vertical load on the axis).

In our paper we deal with dependences of the type

$$Y_i = \frac{\gamma_i\delta_i}{\sqrt{1+\dfrac{\left(|\delta_i|-\beta_i\right)^2}{(\beta_i)^2}}}, \tag{5}$$

which consider the nonmonotonicity of slipping forces (unlike monotone dependences at considerable slipping angle the function has descending sections).

Parameters γ_i and β_i are due to keeping geometrical characteristics of the monotone dependences, $Y_i = \dfrac{q_i\delta_i}{\sqrt{1+\dfrac{(q_i\delta_i)^2}{(\varphi_i)^2}}}$, enabling the constancy of the critical velocity for rectilinear motion, coordination of maximum values of dimensionless slipping forces (Fig.1):



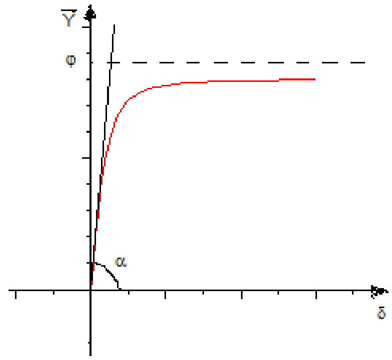


Fig.1. Non-monotone and monotone dependences of slipping angles

Let's analyze the influence of the new "geometry" of slipping forces dependences on the bifurcation set.

Earlier (for monotone dependences) different types of bifurcation sets were obtained within "geometric" approach.

The original system determining the steady states has the form

$$\bar{Y}(\delta_2-\delta_1)=\frac{v^2}{gl}(\theta+\delta_2-\delta_1), \tag{6}$$

where the left part of the equation is a non-linear function and named "stationary curve", the right part of the equation presents a straight line ("moving line").

The intersection points of the "stationary curve" and "moving line" correspond to stationary states of the system (2).

Parameters v and θ being constantly changed, the equation (6) sets the reflection of the plane with v and θ to balanced surface.

The bifurcation set (critical set) corresponds to v, θ for which "the moving straight line" contacts with "the stationary curve".

Points of the inflection of the original curve $\bar{Y}=\bar{Y}(\delta_2-\delta_1)$ correspond to the points of the bifurcation set cusp.

The triple solution for the balanced plane will correspond to the cusp, double solution – fold.

In the case of the monotone dependences of slipping forces from slipping angles of the saturation curve, the "stationary curve" can have three points of inflection, the bifurcation set – three cuspidal points.

The symmetric "cusp" corresponds to the three-fold steady state at $v = v_{кр}^{+}$ and $\theta = 0$ (the stability loss for rectilinear motion), where $V_{кр}=\sqrt{\dfrac{glq_1q_2}{q_1-q_2}}$, $q_i=\dfrac{k_i}{N_i}$ – stationary dimensionless slipping coefficients [4].

In the case of descending original dependences $\bar{Y}_i(\delta_i)$ additional points of inflection of the "moving curve" $\bar{Y}=\bar{Y}(\delta_2-\delta_1)$ come into being, resulting in the complication of the bifurcation set.

Let's analyze the method of constructing the bifurcation set for definite numeric values of γ, β:

$$Y_1=\frac{\delta\cdot 3.300062959\cdot\sqrt{2}}{\sqrt{1+\dfrac{(|\delta|-0.12)^2}{0.12^2}}}, \qquad Y_2=\frac{\delta\cdot 2.526513230\cdot\sqrt{2}}{\sqrt{1+\dfrac{(|\delta|-0.15)^2}{0.15^2}}}, \qquad \delta\in[-1;1].$$

The dependence $\bar{Y}=\bar{Y}(\delta_2-\delta_1)$ is determined by $\bar{Y}_1(\delta_1)=\bar{Y}(\delta_2)=\bar{Y}$.

Critical values of v,θ correspond to (7)

$$\frac{v^2}{gl} = \frac{dY}{d(\delta_2 - \delta_1)};$$
$$\frac{Y}{\theta + \delta_2 - \delta_1} = \frac{dY}{d(\delta_2 - \delta_1)}, \tag{7}$$

Then

$$\theta = Y \cdot Y' - (\delta_2 - \delta_1). \tag{8}$$

Therefore the system (7) gives rise to the bifurcation set having the parametric form

$$\theta = \theta(\delta_2 - \delta_1), \quad v = v(\delta_2 - \delta_1). \tag{9}$$

Sometimes Y as a parameter is more preferable to $(\delta_2 - \delta_1)$. The original dependences are $Y_1 = f_1(\delta_1)$, $Y_2 = f_2(\delta_2)$. Solving them for δ_i, we find $\delta_1 = F_1(Y_1)$, $\delta_2 = F_2(Y_2)$. Therefore $G(Y) = F_2(Y_2) - F_1(Y_1)$. In this case the final version (6) of the equation is

$$\frac{gl}{v^2} \cdot Y - \theta = G(Y). \tag{10}$$

Following the contact (Fig. 3, b) of the "stationary curve" and "moving straight line"

$$\frac{gl}{v^2} = \frac{dG}{d(Y)};$$
$$\frac{\theta + G(Y)}{Y} = \frac{dG}{d(Y)}, \tag{11}$$

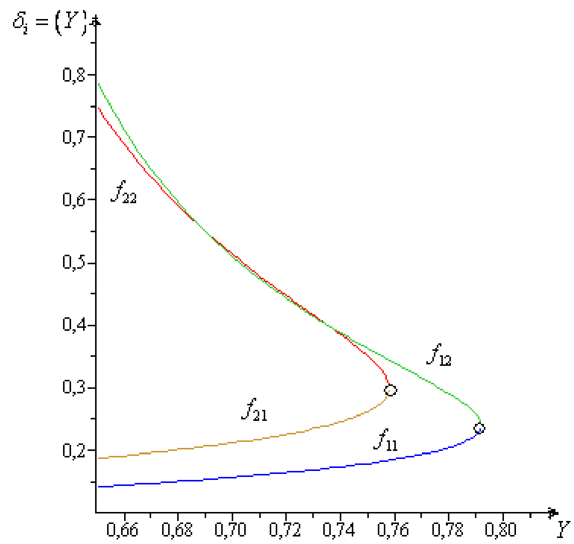
we obtain parametric equations of the bifurcation set in the form $\theta = \theta(Y)$, $v = v(Y)$

$$\theta = Y \cdot G'(Y) - G(Y);$$
$$v = \sqrt{\frac{gl}{G'(Y)}}. \tag{12}$$

Let's analyze the procedure of forming function $G(Y) = \delta_2 - \delta_1$ in the case of non-monotone dependences $Y_i(\delta_i)$.

For numerical values of β, γ we define functions $F_i(Y)$, admitting the correlation (5) for δ_i, we have two single-valued branches, connected at the points of turning (Fig. 2):

$$f_{11} = \frac{0.12\left(-|Y| + \sqrt{0.6272879344 - Y^2}\right)Y}{-Y^2 + 0.3136439672}, \quad f_{21} = \frac{0.15\left(-|Y| + \sqrt{0.5744942192 - Y^2}\right)Y}{-Y^2 + 0.2872471096},$$
$$f_{12} = \frac{0.12\left(Y + \sqrt{0.6272879344 - Y^2}\right)Y}{Y^2 - 0.3136439672}, \quad f_{22} = \frac{0.15\left(Y + \sqrt{0.5744942192 - Y^2}\right)Y}{Y^2 - 0.2872471096}.$$



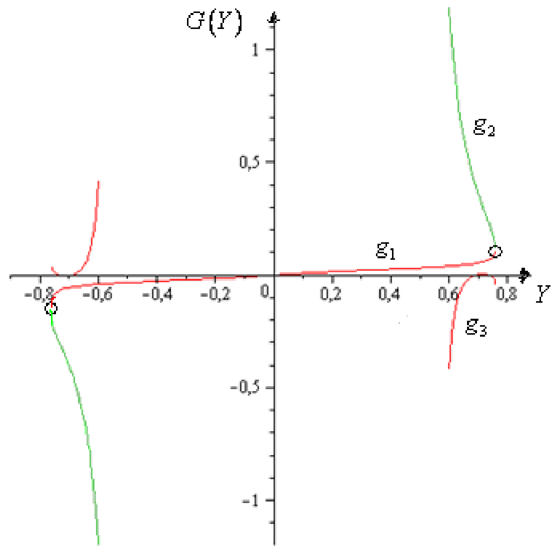


Fig. 2. Dependences of sideways slipping forces at γ_1=3.30006295, β_1=0.12 and γ_2=2.52651323, β_2=0.15.

Fig. 3 Chart of stationary curve for selected γ, β.

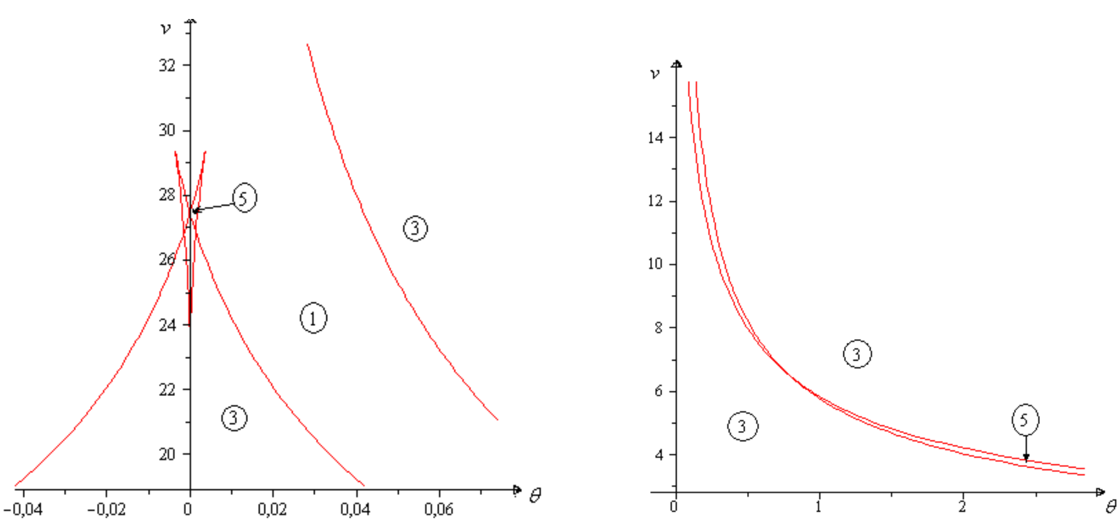
Therefore, function $G(Y) = \delta_2 - \delta_1$ is determined as the difference of corresponding single-valued branches as f_{ij}, and has three branches of single-valuedness $\{g_1, g_2, g_3\}$; g_1 and g_2 are connected at the point of turning, thus forming the "main" branch (Fig.3).

The section of the main branch up to the point of turning comes from $G(Y) = g_1 = f_{21} - f_{11}$, the second part of this branch has the form $G(Y) = g_2 = f_{22} - f_{11}$.

The additional branch of the "moving curve" is due to the descending sections of slipping forces dependences $G(Y) = g_3 = f_{22} - f_{12}$.

Every section of the function $G(Y)$ in accordance with (12) has a dual curve, presenting the part of the bifurcation set (Fig.4).

The bifurcation set divides the plane of control parameters θ, ν into domains with different number of stationary states. It is also possible to determine the number of steady and unsteady states for each domain. The critical set of parameters being intersected, the number of stationary states is changed into two states. The number of stationary states in different domains with the control parameters plane is illustrated in Fig.4.



a) b)

Fig. 4. The bifurcation set (non-linear dependence from a slipping angle) without "heel" moment: a) general set view, b) fragments of the set.

CONCLUSION

Geometrical method of determining stationary states [4] added by the algorithm of constructing the bifurcation set with Poincare's index enables us to accomplish the preliminary analysis of the quantity of stationary states and determine the stability boundaries for the plane of control parameters in the case of the non-monotone dependences of slipping forces.

Descending sections of slipping forces lead to additional branches of the bifurcation set resulted in qualitative changes of the phase portrait and in certain cases causing new dynamic effects (because of changes within the attraction domain structure for the stable motion states).

REFERENCES

[1] Verbitskii V. G. Bifurcation sets and catastrophes in manifold of the steady states of pneumowheel vehicles. Pricl. Mech. Vol. 31, №3, Kiev, pp. 89-95, 1995.

[2] Arnold V. I., Catastrophe Theory. Nauka, Moscow, 1990.

[3] Holodniok M., Klic A., Kubicek M. and Marek M., Methods of Analysing Non-linear Dynamic Systems. Mir, Moscow, 1991.

[4] Pevsner J.M. Theory of stability of automobile. Mashisdat, Moscow, 1947.