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Mathematical Writing Style

The basic problem in writing mathematics is the same as in writing biology, writing a novel, or writing directions for assembling a harpsichord: the problem is to communicate an idea. To do so, and to do it clearly, you must have something to say, and you must have someone to say it to, you must organize what you want to say, and you must arrange it in the order you want it said in. You must write it, rewrite it, and re-rewrite it several times, and you must be willing to think hard about and work hard on mechanical details such as diction, notation, and punctuation.

All mathematicians, even very young students very near the beginning of their mathematical learning, know that mathematics has a language of its own (in fact it is one), and an author must have thorough mastery of the grammar and vocabulary of that language as well as of the vernacular. There is no Berlitz course for the language of mathematics; apparently the only way to learn it is to live with it for years. What follows is not, it cannot be, a mathematical analogue of Fowler, Roger, and Webster, but it may perhaps serve to indicate a dozen or two of the thousands of items that those analogues would contain.

These hints are presented as a source of ideas on mathematical style.

- Two basic rules are:

1. Have mercy on the reader.
2. Have mercy on the editor/publisher.

We will illustrate these as we move along.

- General Flow of the Paper.

- **Definition**: *All* basic definitions should be given if they are not a standard part of the literature. It is perhaps best to err on the side of making life easier on the reader by including a bit too much as opposed to too little (Rule 1).

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- As a very general rule, the definitions should go *before* the results that they are used in (Rule 1).
- The "quantifiers" should always be clear (Rule 1). Some examples to avoid:

* "We have f(x) = g(x) (x ∈ X)." What does the parenthesis mean? That f(x) = g(x) for all x ∈ X, or, for some x ∈ X?

* The word "constant" is terribly ambiguous. It is important to make explicit *exactly* which variables the constant depends on.

- **Theorem/Proposition/Lemma/Corollary**: Give clear and unambiguous statements of results. These are what other people are reading your paper for; so you should ensure that these, at least, can be understood by the reader (Rule 1).

***** The statement of the Theorem/Proposition/Lemma/Corollary should *not* include comments (except for an occasional brief remark in parenthesis) or examples.

- If you use or quote an important result of another person, you should give a reference. You should avoid giving the impression that such a result is obvious, a generally accepted fact, due to you, and so on.

* A reference to a book should always give the page!

* Try to avoid using "by the proof of" when the proof is in the paper and the statements can be rewritten to be *directly* quoted.

* A "well-known" result that is *not* in the literature should be proved if needed (Rule 1).

- **Proof**: A proof should give enough information to make the theorem believable *and* leave the reader with the confidence (as well as the ability) to fill in details should it be necessary (Rule 1).
- Whatever format or style you choose to adopt, especially if it deviates from the publisher's style, make sure that it is consistent. This is mostly a difficulty with books (Rule 2).

* If one proof ends with a "QED," then they all should, etc.

* If you leave a blank page at the end of one chapter so the next one can start on an odd-numbered page, then make sure you always do.

- **References**: The references should have a consistent (and preferably accepted) style for the entries (Rule 2).
- **T_EX:** In general, advanced $T_EX$ing should be left to the experts; i.e., as a typesetter or page designer the author should tread lightly. Remember, the more one messes with the T_EX-file, the less portable the manuscript will be. Your article may *not* be accepted at the first place you send it, make sure you can easily resubmit it elsewhere (Rule 2). Moreover, playing with the T_EX increases the likelihood that the final output will look different on different systems. (Rule 0: have mercy on the author!)

- Writing a paper or book entails making choices of what material is important and what can be skipped. It is impossible to cover all possible results and so the material needs to be covered in a well thought out manner. A paper or book should *not* be considered an opportunity for showing off (Rule 1).

- Other comments:

- One should, of course, observe the usual conventions in terms of spelling, punctuation, and the other basic elements of style. Use complete sentences, with subject, *verb*, and complement (Rule 1).

* Words like "then", "and", or "or" should not be replaced by a comma. It is bad to write "If $x = 2$, $y = 3$, $z = 4$" meaning "If $x = 2$ and $y = 3$, then z is equal to 4" (or "If $x = 2$ and $y = 3$, then $z = 4$").

- Use the present – not the past – form.

* As an example of bad writing, we have: "We have proved that *f(x)* was equal to *g(x)*…". This is corrected to: "We have proved that *f(x)* is equal to *g(x)*…".

- Straightforward computations (such as the product of matrices) may be left to the reader. **However:** those which are necessary for the main results should be given in complete detail (so that the reader has the option of checking them or not; Rule 1).

- Do not simply state "*X* is isomorphic to *Y*" unless it is completely obvious. Rather, it will be much easier on the reader if you state "the function $f: X \to Y$ is an isomorphism" where *f* is *explicitly* given (Rule 1).

- One should avoid giving the reader the impression that the subject matter can be mastered only with great pain. In fact, this is an *ideal* way to lose readers (or audiences!).

- One should avoid using abbreviations like "w.r.t." (with respect to), "iff" (if and only if), and "w.l.o.g." (without loss of generality). They simply do not look very nice (and "iff" is offensive! – Rules 1 and 2).

- You should *not* begin a sentence with a math symbol. This can confuse the printer as well as the reader (Rules 1 and 2).

* As an example of such bad writing, we have: "… we want to prove the continuity of $f(x) = 2\cos^2 x \cdot \sin x$, cos x being continuous…". This is corrected to: "…$f(x) = 2\cos^2 x \cdot \sin x$. Since cos x is continuous…".

- If your paper raises a natural question, and you don't know the answer, by all means *say so*! This may turn out to be more interesting than the theorems that you prove.

* Conversely, refrain from making "conjectures" too hastily. Use instead the words "question" or "problem." Remember that a good "question" should be answerable by "yes" or "no." To ask "under what conditions does A hold" is not a question worth printing.

- It is often helpful to begin a new section of the paper with a summary of the general setting.

- After the paper is finished, it should be reread (and, perhaps, rewritten) from the reader's point of view (Rule 1).

- A good way to begin is to use a standard classic of mathematical exposition (e.g., Bourbaki-Algebra, works by Serre, Atiyah or Milnor) as a basic model.

When you've written everything you can think of, take a day or two to read over the manuscript quickly and to test it for the obvious major points that would first strike a stranger's eye. Is the mathematics good, is the exposition interesting, is the language clear, and is the format pleasant and easy to read? Then proofread and check the computations; that's an obvious piece of advice, and no one needs to be told how to do it. "Ripening" is easy to explain but not always easy to do: it means to put the manuscript out of sight and try to forget it for a few months. When you have done all that, and then re-read the whole work from a rested point of view, you have done all you can. Don't wait and hope for one more result, and don't keep on polishing. Even if you do get that result or do remove that sharp corner, you'll only discover another mirage just ahead.

To sum it all up: begin at the beginning, go on till you come to the end, and then, with no further ado, stop.

Literature

1. Goss D. (1996), Basic structures of function field arithmetic, Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)], **35**, Berlin, New York. **2**. Higham N. J. Handbook of Writing for the Mathematical Sciences, SIAM, 1993. **3**. Halmos P. How to write mathematics, Enseign. Math. **16** (1970), pp. 123-152. **4**. Zucker S. Variation of a mixed Hodge structure II, Invent. Math. **80** (1985), p. 545.