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Тема: «Синтез ПД-регулятора для керування положенням стрижнів  
ядерного реактора»

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“ \_\_\_\_\_ ” \_\_\_\_\_ 2024

# QUALIFICATION PAPER

(EXPLANATORY NOTE)

FOR THE ACADEMIC DEGREE OF BACHELOR

Title: **“Synthesis of PD-controller for Position Control of Nuclear Reactor Rods”**

Submitted by: student of group CS-404 \_\_\_\_\_ Nika KRYVOSHEIA

Supervisor: PhD, Associate Professor \_\_\_\_\_ Antonina KLIPA

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# НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ

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## **ЗАВДАННЯ**

на виконання кваліфікаційної роботи

Кривошеї Ніки Ігорівни

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3. Вихідні дані роботи: математична модель, структурна схема, вимоги до запаса стійкості за фазою
4. Зміст пояснювальної записки: Теорія керування стрижнями ядерних реакторів та постановка задачі, ПД-регулятори та їх синтез різними методами, синтез ПД-регулятора для керування положенням стрижнів ядерних реакторів
5. Перелік обов'язкового ілюстративного матеріалу: Блок-схема системи керування положенням стрижнями ядерного реактора, діаграма Боде для некомпенсованої та компенсованої систем, часові характеристики.

## 6. Календарний план-графік

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2	Огляд літературних джерел	16.05.2024-20.05.2024	
3	Ознайомлення зі складовими ядерного реактора	21.05.2024-23.05.2024	
4	Дослідження ПД-регуляторів	23.05.2024-26.05.2024	
5	Синтез ПД-регулятора для контролюючих стрижнів ядерного реактора	27.05.2024-02.06.2024	
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“ \_\_\_\_\_ ” \_\_\_\_\_ 2024

**Qualification Paper Assignment for Graduate Student**

Kryvosheia Nika Ihorivna

1. The qualification paper title “Synthesis of PD-controller for Position Control of Nuclear Reactor Rods” was approved by the Rector’s order of “ 01 ” April 2024 № 511/CT.
2. The paper to be completed between: 13.05.2024 and 16.06.2024
3. Initial data for the paper: mathematical model, structural diagram, requirements for phase margin.
4. The content of the explanatory note: Theory of nuclear reactor rods control and problem statement, PID-controllers and their synthesis by different methods, Synthesis of PD-controller for position control of nuclear reactor rods.
5. The list of mandatory illustrations: Block diagram of a nuclear reactor rods position control system, Bode Diagram of an uncompensated and compensated systems, time responses.

## 6. Timetable

№	Assignment	Dates of completion	Completion mark
1	Clarification of the problem statement	13.05.2024-15.05.2024	
2	Review of literature sources	16.05.2024-20.05.2024	
3	Familiarization with the components of a nuclear reactor	21.05.2024-23.05.2024	
4	Investigation of PID-controllers	23.05.2024-26.05.2024	
5	Synthesis of a PD-controller for control rods of a nuclear reactor	27.05.2024-02.06.2024	
6	Formalization of the explanatory note	03.06.2024-09.06.2024	
7	Preparation of the report and presentation	10.06.2024-13.06.2024	

7. Assignment issue date: “13” May 2024

Qualification paper supervisor \_\_\_\_\_  
(the supervisor's signature)

Antonina KLIPA

Issued task accepted \_\_\_\_\_  
(the graduate student's signature)

Nika KRYVOSHEIA

## РЕФЕРАТ

Пояснювальна записка до дипломної роботи "Синтез ПД-регулятора для керування положенням стрижнів ядерного реактора" містить 76 сторінок, 26 ілюстрацій, 3 таблиці, 19 літературних джерел.

**Актуальність теми** полягає в тому, що функціонування ядерного реактора залежить від системи управління, тому розробка якісної системи управління відповідає за безпеку даного об'єкта.

**Об'єктом дослідження** є ядерний реактор.

**Предметом дослідження** є система управління положенням контролюючих стрижнів ядерного реактора.

**Метою роботи** є синтез ПД-регулятора для керування положенням стрижнів ядерного реактора.

**Методи дослідження** аналіз стійкості, аналіз запасів стійкості за амплітудою та фазою, аналіз перехідних характеристик, синтез регуляторів за допомогою методу частотної характеристики.

**Ключові слова:** ПД-РЕГУЛЯТОР, СИНТЕЗ, ЗАПАС СТІЙКОСТІ ЗА ФАЗОЮ, ПЕРЕХІДНІ ХАРАКТЕРИСТИКИ, ПОЛОЖЕННЯ СТРИЖНІВ, ЯДЕРНИЙ РЕАКТОР.

## ABSTRACT

Explanatory note for the qualification paper "Synthesis of PD-controller for Position Control of Nuclear Reactor Rods" includes: 76 pages, 26 figures, 3 tables, 19 references.

**The relevance of the topic** is that the functionality of a nuclear reactor depends on the control system, so the development of a high-quality control system is responsible for the safety of this facility.

**The object of the study** is nuclear reactor.

**The subject of the research** is a control system of position control of nuclear reactor control rods.

**The aim of the work** is synthesis of PD-controller for position control of nuclear reactor rods.

**Research methods** include stability analysis, analysis of stability margins by amplitude and phase, analysis of transient characteristics, synthesis of controllers using the frequency response method.

**Key words:** PD-CONTROLLER, SYNTHESIS, PHASE S MARGIN, TRANSIENT CHARACTERISTICS, ROD POSITION, NUCLEAR REACTOR.



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## INTRODUCTION

Control of nuclear reactors is one of the most important aspects of ensuring their safe and efficient operation. Nuclear reactors play a significant role in electricity generation, scientific research and other industries. Their operation is based on complex physical processes, such as uranium fission and absorption, which require careful monitoring to ensure stability and safety.

Nuclear power has a rich history that began with the discovery of nuclear fission in 1938. The first nuclear reactor was created in 1942 under the leadership of Enrico Fermi, and the commercial use of nuclear energy for electricity generation began in the 1950s. Today, nuclear reactors provide a significant portion of the world's electricity generation, offering a reliable and environmentally friendly source of energy.

The purpose of this thesis is to investigate the control theory of nuclear reactor rods and synthesize a PD-controller to control their position. In the first part of the work, the concept of a nuclear reactor, its operating principles, uranium fission and absorption processes, and the theory of reactor rod control will be considered. Particular attention will be paid to identifying the main problems that arise in the control of nuclear reactor rods.

The control of nuclear reactor rods is a critical aspect, as it affects the safety and stability of the entire reactor. The rods are used to regulate the fission reaction by controlling the number of neutrons that cause the uranium to fission. Therefore, the accuracy and reliability of the rod control system directly affect the safety of the nuclear reactor.

The second part of the paper will be devoted to the study of the structure of PID-controllers and their impact on dynamic systems. Different methods of PID-controller synthesis will be considered, in particular, methods based on the Nichols diagram. Particular attention will be paid to the synthesis of a PID-controller for controlling the position of nuclear reactor rods.

In the third part of the work, the model of nuclear reactor rods without a controller will be analyzed, a PD-controller will be synthesized using the Nichols diagram, and a model of a nuclear reactor with a PD-controller will be analyzed. Based on the analysis, a comparison of the models of nuclear reactor rods without and with a PID-controller will be made. The final section of the thesis will summarize the results of the study and provide conclusions on the effectiveness of the proposed methods for controlling nuclear reactor rods.

Thus, this thesis contains a comprehensive analysis of the theory and practice of nuclear reactor core control, with an emphasis on the use of PD-controllers to improve the efficiency and reliability of their operation.

# SECTION 1

## THEORY OF NUCLEAR REACTOR RODS CONTROL AND PROBLEM STATEMENT

### 1.1. The concept of a nuclear reactor

Nuclear power is a strategic resource in today's world, providing a reliable supply of electricity and responding to global environmental and developmental challenges. The nuclear core management system is a key element of the industry's safety and efficiency. It is responsible for regulating reactivity and ensuring stable operation of the reactor throughout its lifetime.

In addition to its role as a strategic energy resource, nuclear power stands out for its ability to meet the growing energy needs of modern society while significantly reducing greenhouse gas emissions. This is especially important in the context of global environmental challenges such as climate change, where low-carbon energy sources such as nuclear power play a vital role in mitigating the effects of fossil fuel-based energy production.

In nuclear physics and power engineering, the term "nuclear reactor" describes devices that provide controlled nuclear fission chain reactions. These reactors are used for energy production, scientific research and other technological processes related to nuclear energy.

Aerospace Control Systems Department				Explanatory Note			
Submitted	Kryvosheia N.I.			SECTION 1. THEORY OF NUCLEAR REACTOR RODS CONTROL AND PROBLEM STATEMENT		Sheet	Sheets
Supervisor	Klipa A.M.					13	76
St Inspector.	Dyvnych M.P.				404		
Head of Dep	Melnyk Yu.V.						

A specialized technical apparatus, the nuclear reactor, is engineered to initiate and manage controlled chain reactions of nuclear fission within heavy nuclei. In this process, neutrons prompt nuclear fission, resulting in the division of heavier elements into lighter ones, liberating energy and additional neutrons that can instigate further fission reactions. Modern nuclear reactors are comprised of intricate systems involving refined nuclear fuel, reaction regulation, fuel safeguarding, replacement mechanisms, and cooling systems to ensure optimal and secure operation.

Atoms serve as the fundamental constituents of matter, and their properties are contingent upon the configuration of their nuclei. These nuclei consist of positively charged protons and neutrally charged neutrons, collectively forming a nucleus encircled by electrons.

While stable atoms typically exhibit a balanced ratio of protons to neutrons, heavy elements necessitate additional neutrons to stabilize the nucleus. During the fission of heavy nuclei, the release of binding energy between nucleons occurs due to the robust nuclear forces binding them together. This liberation of energy, manifesting as heat and radiation, hinges on the principle of nuclear binding and underpins both reactor operations and nuclear detonations.

Elements with elevated atomic mass numbers exhibit diminishing stability as their mass increases. The addition of a neutron to the nucleus of an exceedingly heavy element induces further excitation of its binding energy, potentially leading to nuclear fission. Notably, this phenomenon characterizes two isotopes of uranium and two isotopes of plutonium. While only uranium-235 occurs naturally, it is found in minute quantities.

The fission of uranium-235 releases a substantial amount of binding energy, approximately 200 MeV (megaelectron volts), equivalent to around  $332 \times 10^{-12}$  J (joules). This energy can be harnessed to generate thermal energy, notably in nuclear power plants. If 1 kilogram of uranium-235 is entirely consumed within a day, it can produce thermal power reaching up to 950 MW (megawatts). Through a conventional steam cycle, this thermal energy can be converted into electrical power, catering to residential and industrial demands, totaling approximately 300 MW.

## 1.2. Principles of nuclear reactor operation

The configuration of a nuclear reactor comprises several integral components that collaborate in the electricity generation process. Beginning with the concrete chamber housing the steel vessel for nuclear reactions, we encounter the fuel rods, housing fissile materials like uranium-235 or plutonium. Interspersed among these rods are control rods, typically composed of cadmium, employed to regulate the pace of nuclear reactions.

Throughout the reaction, a substantial quantity of heat is produced, transferred to a heat exchanger via a pump. This heat exchanger, housing liquid sodium as a coolant, is linked to a steel enclosure. The resultant high-pressure steam is channeled to a turbine, where it propels the rotation of a generator under elevated pressure, facilitating electricity generation.

Upon traversing the turbine, the steam undergoes condensation back into water within a condenser. This water is then reintroduced to the heat exchanger via a pump, thereby allowing its reuse in the reaction cycle.

Thus, a typical nuclear reactor consists of the following main components, as shown in Fig. 1.2.1:

- fuel, in which fission takes place and heat is generated
- a moderator to reduce the energy of neutrons
- cooler to absorb heat
- a turbine for converting thermal energy into mechanical energy,
- generator for converting mechanical energy into electrical energy
- control rods to maintain proper neutron balance.

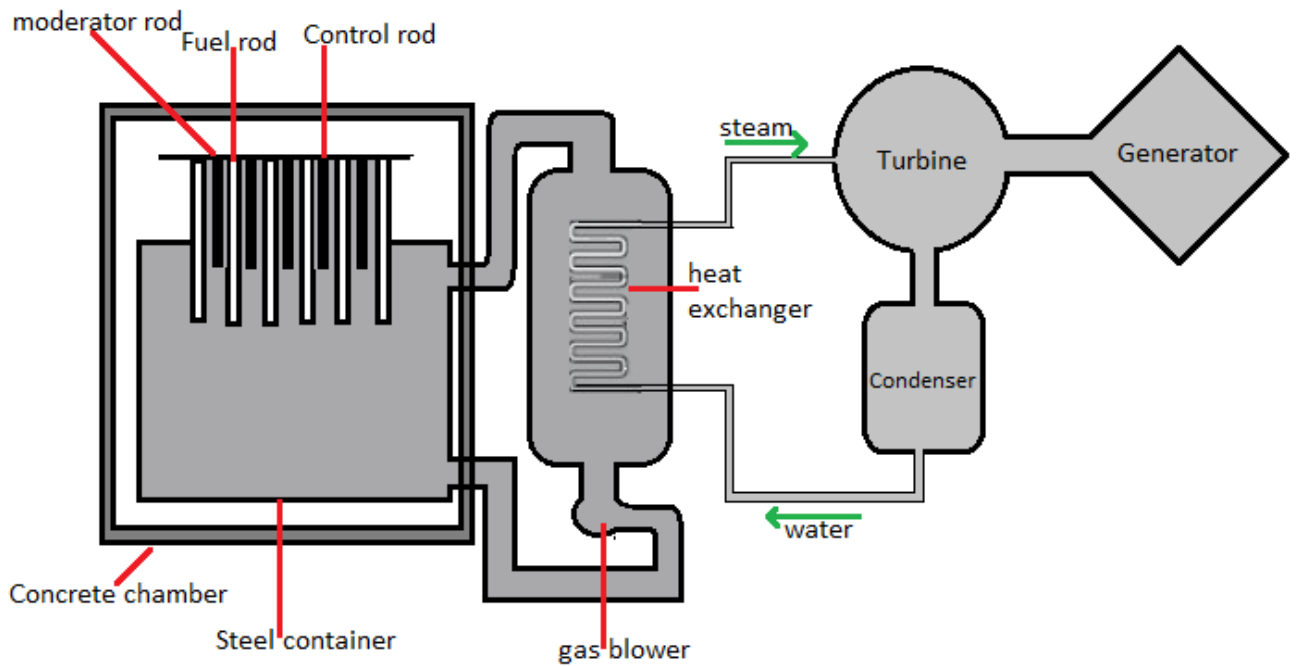


Fig. 1.2.1. Nuclear reactor construction

Let us take a closer look at each component of the nuclear reactor structure:

**The nuclear reactor.** Reactors, serving as the primary component of nuclear power plants, facilitate controlled chain reactions that emit heat. This heat is absorbed by liquid sodium, continuously circulating through a network of pipes via pumps. The key constituents of these reactors include  $U_{92}^{235}$  fuel, a moderator, control rods, and reflectors, all instrumental in facilitating this thermal generation process. The reactor core adopts a rectangular configuration, streamlining the processes of fuel replenishment and coolant circulation.

**The reflector,** in turn, reflects free neutrons, if any, from the reactor core. This prevents free neutron leakage and increases thermal efficiency. The material used for the reflector is reactor graphite, which also ensures an even distribution of thermal energy.

**Fuel.** The fuel used in the reactor consists of the isotope  $U_{92}^{235}$  and is shaped like a rod. These rods are bombarded with slow neutrons through a special neutron bombardment device.



**Moderator.** They are rod-shaped, usually made of graphite, heavy water, or beryllium. These devices slow down or reduce the speed of neutrons to increase the probability of fission. Neutrons with reduced speed are needed to cause fission.

**Control rods.** Within a reactor, a chain reaction is initiated and maintained at a steady pace during normal operation. However, in emergency scenarios, the reactor necessitates shutdown. Control rods play a crucial role in regulating the chain reaction by absorbing free neutrons. Typically crafted from cadmium or boron alloyed with steel or aluminum, these rods encompass the fuel. Additionally, safety rods (not depicted) are employed in case of unforeseen circumstances such as earthquakes, excessive power generation, control system malfunctions, or other hazardous occurrences. These safety rods are inserted either manually or automatically. While the safety rods remain within the reactor core, the chain reaction halts. Upon complete removal, the reaction recommences, initiating power generation anew.

More information about the control rods will be discussed in the next sections.

**Heat exchanger.** The reactor core produces heat energy, which is absorbed by a coolant, typically sodium metal. This sodium, now heated, circulates continuously through a heat exchanger. Within the heat exchanger, the hot metal transfers its heat to water, causing it to convert into steam under high pressure. This steam is then directed to a steam turbine via a valve for subsequent utilization.

Following heat transfer in the heat exchanger, the coolant transforms back into a colder state. After undergoing filtration to eliminate any impurities, if present, it is pumped back into the reactor for further circulation.

**Cooler.** The coolant serves to convey the heat generated within the reactor to the heat exchanger for subsequent utilization in electricity production. In instances where water serves as the coolant, it absorbs heat within the reactor and transitions into steam, directly employed in the turbine. The coolant circulates through and around the reactor core. Common coolants encompass gases (such as carbon dioxide, air, hydrogen,

helium), water, heavy water, liquid metals (like sodium, potassium sodium), and certain organic liquids.

**Shielding.** Nuclear fission or chain reactions produce radiation that can be harmful to living things. Therefore, proper shielding of the reactor is extremely important and a protective material such as concrete is used for this purpose.

**Turbine.** A steam turbine starts to rotate when it is exposed to steam. It converts this energy into mechanical energy, which is then transferred to an alternator. The generator, in turn, converts mechanical energy into electricity. Once the useful work is done, the steam is sent to a capacitor.

Heating is performed by removing steam from the turbine (turbine blowdown).

**Condenser.** The condenser separates the steam by condensing it with water taken from the water treatment chamber and the cooling tower. The condensed steam is then heated in a feedwater heater.

**Feed water heater.** The feedwater heater is used to increase the thermal efficiency of the plant. Part of the steam from the steam turbine is used to heat the water (turbine blowdown), and the resulting heated water is fed to the heat exchanger by a pump. A water treatment chamber is used to remove any impurities from the water before it is fed to the heat exchanger.

**Reflector.** The thermal shield envelops the reactor core entirely, aiding in redirecting escaping neutrons back into the core. This serves to safeguard the nuclear fuel, as the neutrons returning at reduced velocities are instrumental in sustaining the chain reaction. Nonetheless, it is imperative to ensure efficient cooling of the reflector, given that it heats up due to neutron interactions with material atoms.

**Reactor vessel.** The reactor core consists of various components, including a reflector and a shield, contained within a sturdy, thick-walled enclosure. This vessel features openings for the inlet and outlet coolant flows and is engineered to withstand elevated pressures. Positioned in the upper section of the vessel are apertures for the

insertion of control rods. Typically, the reactor core, comprising fuel and moderators, is situated in the lower portion of the vessel.

These components form an integrated system that ensures the continuous and stable operation of a nuclear reactor used to generate energy [2].

In a typical nuclear reactor setup, components are typically organized in a two-dimensional grid, where the interplay of neutrons with fuel rods, moderators, and control rods guarantees stable system operation. This arrangement optimizes neutron utilization and facilitates efficient control over reaction processes.

Neutrons initially produced in the fuel rods traverse a nearby moderator before entering subsequent fuel elements. The positioning of these elements is dictated by moderator characteristics and the desired degree of neutron energy reduction. Compact fuel rod dimensions aid in efficient heat dissipation from the system, aided by the surrounding coolant.

Frequently, the coolant doubles as a moderator, further diminishing neutron energy. Control rods, interspersed among the fuel elements, regulate neutron flux and capture surplus neutrons. Although some neutrons may escape the system, many are absorbed by reactor materials or fuel without initiating fission. Control rods are strategically configured and positioned to absorb only the requisite neutron quantity to sustain fission. The specific matrix configuration and fuel rod spacing vary depending on the properties of the reactor's fuel and moderator [3].

### **Types of rods in nuclear reactor.**

All nuclear reactors require individual components for control. Although control mechanisms can include adjusting the parameters of the cooling system or changing the level of the absorber in the coolant or moderator, the predominant approach is to use absorber assemblies such as control rods or blades. Typically, a reactor is equipped with three types of rods for distinct purposes:

- (1) safety rods for initiating and halting reactor operations
- (2) regulating rods for controlling the reactor's power output

(3) regulating rods for compensating for reactivity changes as fuel undergoes depletion from fission and neutron capture.

Safety rods play a vital role in reactor shutdowns, whether for routine maintenance or emergencies. They contain ample absorber material to halt chain reactions under any circumstances. Before fuel loading, these rods are retracted but remain on standby in case of loading errors. Once fuel is loaded, the rods are inserted and can be withdrawn during reactor operation. Fail-safe mechanisms ensure their deployment, even in the event of mechanical failures, often relying on gravity or automatic features like fuses.

In contrast, regulating rods mildly affect reactivity to prevent significant increases if accidentally fully withdrawn. They permit delayed neutrons to continue controlling power rates, ensuring safety even during operational errors.

Shim rods adjust for reactivity changes due to burnup, gradually adapting over extended periods compared to the rapid actions of safety and regulation rods. They perform optimally when movement constraints are established, often coordinating with regulating rods.

Combined rods, featuring slow withdrawal rates and rapid insertion, efficiently balance burnup effects and facilitate emergency shutdowns. Reactor operators can initiate a "scram" by fully inserting control rods for a swift shutdown to subcritical states, ensuring safety in critical situations [4].

### **1.3. Fission and absorption of uranium**

In nuclear reactors, it's feasible to induce fission in certain heavy elements, such as uranium-235, by introducing neutrons to their nuclei. This triggers the formation of lighter, more stable elements, often accompanied by the release of two to three additional neutrons. These neutrons, in turn, can instigate further fission reactions, thus initiating a chain reaction. This process can be managed, as in the controlled environment of a nuclear reactor, or uncontrolled, as in the catastrophic scenario of an atomic bomb.

Certain materials, like water, beryllium, and carbon, are adept at moderating neutron velocity without substantial energy absorption, rendering them valuable in nuclear reactors. For instance, water and graphite are recognized for their effectiveness in decelerating neutrons within these reactors.

In nuclear reactors, the combined probability of neutron absorption, encompassing both fission and capture events, for U-235 and U-238 nuclei is illustrated in Fig.1.3.1. Notably, in the resonance region where neutron and nucleus frequencies align, prominent peaks with exceptionally high capture probabilities are evident for U-238. Hence, it's advisable to steer clear of this region during the neutron deceleration process, ensuring it occurs safely away from the fuel.

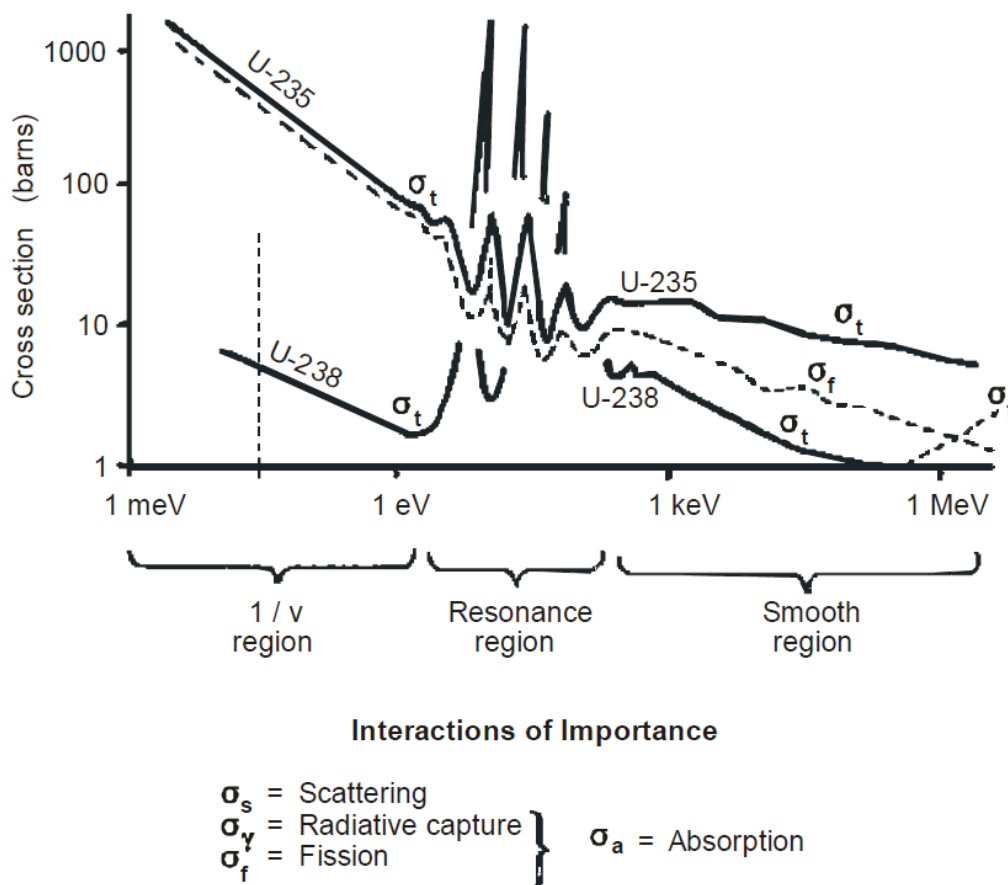


Fig. 1.3.1. Fission and absorption characteristics of uranium

In steady-state, on average, only one neutron produced by a fission has to cause fission in one subsequent nucleus to maintain the chain reaction. During fission, the released energy is predominantly converted into the kinetic energy of the fission products, and excess neutrons are removed at a high rate. High-energy fast neutrons easily penetrate fuel and other reactor materials, and their absorption probabilities vary with their energy. Adjusting the neutron energy allows you to control the fission process by slowing down the neutrons, which contributes to the efficient use of fuel [5].

#### 1.4. Theory of nuclear reactor rods control

Control rods (Fig. 1.4.1) are cylindrical tubes, usually made of neutron-absorbing materials such as boron carbide or metal alloys. They are designed to keep fission reactions within safe limits. The dimensions of control rods are usually the same as those of nuclear fuel rods. These rods in nuclear reactors are important for controlling reactor power and ensuring reactor safety.

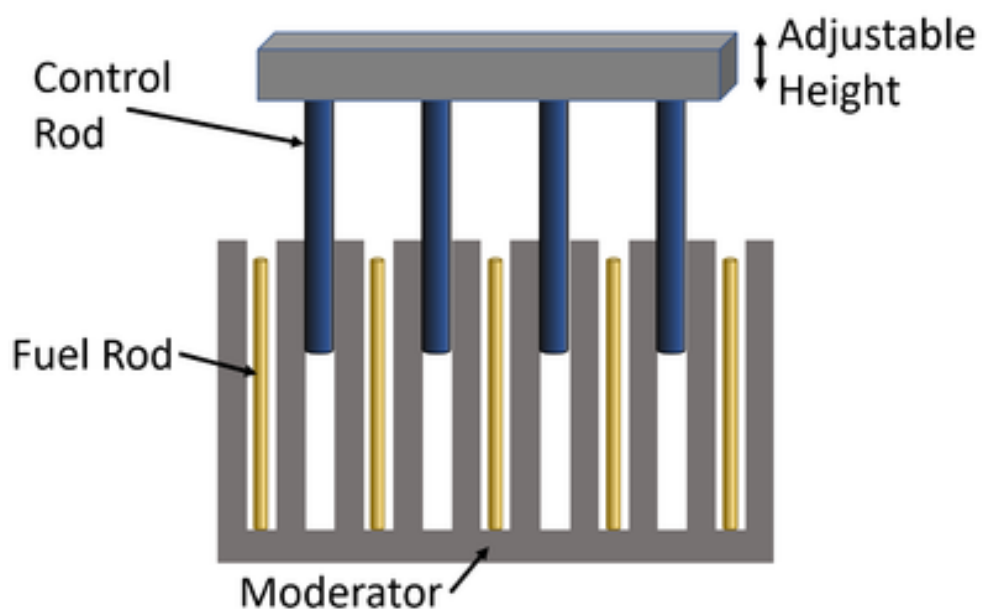


Fig. 1.4.1. Control rods

Control rods influence the number of neutrons that cause nuclear fission and thus control the reactor power. Their size and location are selected to effectively keep the reaction at the desired level. For example, greater contact of the rods with the core leads to greater neutron absorption and, consequently, a decrease in reactor power.

In other words, control rods are rods, plates, or tubes used in nuclear reactors to control the rate of fission of nuclear material such as uranium and plutonium. Several of these control rods, about the size of fuel rods, are evenly spaced and connected at one end by a metal bracket known as a spider. Typical reactors may contain about 50 such clusters with 20 individual control rods in each cluster. They are typically made of chemical elements that can absorb many neutrons without causing nuclear fission, such as boron, silver, indium, and cadmium. Since these materials have different absorption properties for neutrons of different energies, the composition of the control rods must be tuned to the specific neutron spectrum of the reactor they are to control.

The efficacy of a control rod in absorbing neutrons to regulate the fission chain reaction necessitates the selection of a material possessing high neutron absorption capacity. The neutron absorption cross section, denoted in units of barns (equivalent to  $10^{-28}$  square meters), serves as a measure of a material's neutron absorption capability. Generally, control rods are fabricated from cadmium, hafnium, or enriched boron.

In addition to material choice, mechanical characteristics and cost considerations are pivotal in control rod design. While boron-10 ranks among the most effective neutron absorbers, its brittleness renders it unsuitable for control rod fabrication. Furthermore, natural boron must undergo enrichment to attain reasonable absorption levels, a process that incurs significant expenses [6].

Reactors that use different technologies, such as light water reactors (BWR, PWR) and heavy water reactors (HWR), operate on different types of neutrons. For example, breeder reactors operate on "fast" neutrons.

The reactor is controlled by rods that define the core of the fuel column in a multi-unit reactor. Under the control rods are fuel element columns containing nuclear

fuel and waste. The control rod provides an interface for users to monitor the status of the fuel column, displaying the total heating and the relative ratio of fuel to waste. This rod also includes a "Control Rod" radiation rod that can be extended from or retracted into the fuel dispenser.

Depending on the position of the control rod in the fuel column, the level of radioactivity generated by the fuel column can vary. This affects the production of heat and electricity, as well as fuel consumption. Fully extending the control rod effectively shuts down one fuel column.

Some PWRs use special control rods that allow the core to effectively maintain a low power level. Fig. 1.4.2 shows the assembly of PWR control rods [7].

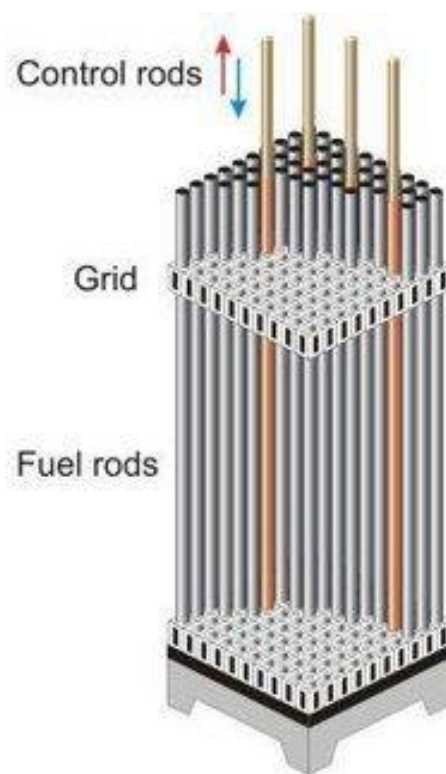


Fig. 1.4.2. Pressurized water reactor control rod assembly, above fuel element

If necessary, for example in an emergency shutdown, the control rods can quickly and completely enter the reactor, stopping chain reactions. This is an important safety mechanism that ensures that the reactor remains under control and does not become dangerously critical [8].



## 1.5. Principles of operation of control rods

The control system monitors reactor parameters such as neutron flux, power level and temperature and adjusts the position of the control rods accordingly to maintain safe and stable operation.

Control rods are an important technology for maintaining the desired state of the fission reaction in a nuclear reactor. They provide real-time control of the fission process, which is crucial both for maintaining the activity of the fission chain reaction and for preventing its uncontrolled acceleration.

The nuclear fission chain reaction serves as the foundational mechanism through which nuclear reactors generate usable energy. While uranium-235 is the predominant material undergoing fission in this chain reaction, the underlying process is broadly applicable. During fission, a U-235 atom, upon collision with an incident neutron, undergoes division into two smaller atoms (krypton K-92 and barium B-141), while simultaneously emitting an average of 2.5 new neutrons. These newly generated neutrons can in turn trigger additional fission events within other U-235 atoms, perpetuating a chain reaction that yields significant energy with each fission event [9].

Although the fission of uranium-235 typically releases an average of 2.5 neutrons, only one neutron is necessary to sustain the nuclear chain reaction at a constant rate. Control rods absorb these surplus neutrons and can be manipulated to regulate the reactor's power output. When the standard number of rods is inserted, maintaining their position at the criticality level, the power output remains unchanged. Inserting additional rods diminishes the number of neutrons, thereby reducing power output and placing the reactor below criticality. Conversely, slightly withdrawing the rods when fission surpasses criticality leads to an increase in power output. This dynamic is illustrated in Fig. 1.5.1.

Thus, a key factor in maintaining a fission chain reaction is the number of neutrons that propagate to the next generation of fissions. However, not all neutrons produced by a fission event cause the next fission event (e.g., some may simply escape

the reactor or be absorbed by non-fissionable isotopes), so it is necessary to carefully design each reactor parameter to ensure that at least one neutron from each fission event is capable of causing the next fission event. One of these adjustable parameters is the control rods.

Therefore, the most important number for nuclear power reactors is 1, as any deviation from this multiplication factor, denoted as  $k$ , signifies a reactor that is either highly inefficient or exceedingly perilous. Maintaining precisely  $k = 1$  proves challenging, as achieving this precise equilibrium is influenced by a multitude of factors. Some of these factors are inherent to the fissile fuel or reactor materials, such as the number of neutrons generated during fission or the degree of neutron absorption by fuel cladding or moderators. Nevertheless, even if the reactor is initially designed with an optimal balance, the breeding ratio inevitably fluctuates over time, given that many fission byproducts act as neutron absorbers (referred to as "poisons") and, upon accumulation, diminish the overall neutron population.

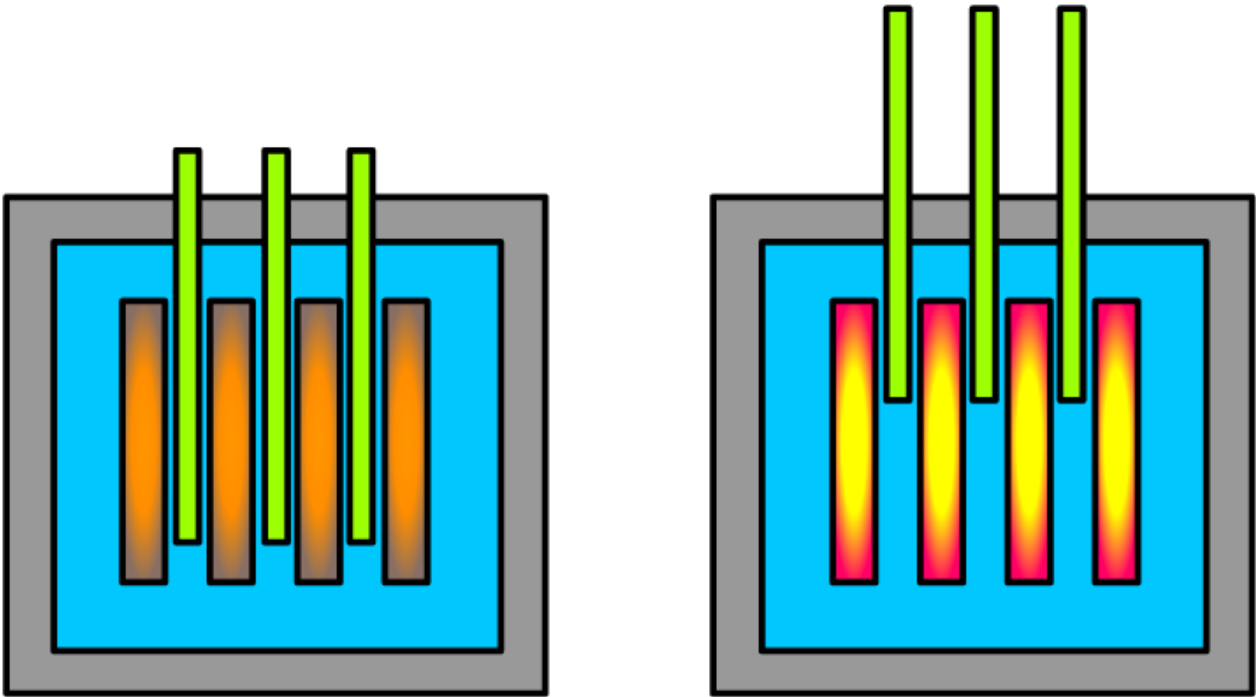


Fig. 1.5.1. A schematic illustrating the variation in reactor power output based on the insertion depth of control rods, depicted in green. On the left side, the control rods are inserted deeper than usual, leading to a decrease in reactor power output.

Conversely, on the right side, the control rods are inserted less deeply than usual, resulting in an increase in reactor power output

Thus, control rods are being used as an effective method to combat these time-dependent changes in reactors. Control rods are essentially a high-performance, neutron-absorbing mechanical structure that can be actively introduced or removed from the reactor core during the fission process. By controlling the portion of the rod that interacts with the fission reaction, the breeding ratio can be fine-tuned to maintain reactor criticality (see Fig. 1.5.1). In addition, control rods can be used to intentionally introduce rapid changes in the state of the reactor (i.e., switching the reactor on and off), especially as an emergency shutdown by fully inserting the rods.

The state of the fission chain reaction can be briefly described by the effective multiplication factor  $k$ , which indicates the change in the total number of fission events during successive generations of the chain reaction. It is defined as follows [10-11]:

$$k = \frac{\text{total number of fission events in a given generation}}{\text{total number of fission events in the previous generation}}$$

A reactor that is in a stable state (i.e., each individual fission event causes exactly one subsequent fission event) has  $k = 1$ , and such a reactor is considered critical. If  $k < 1$ , the reactor is subcritical and the chain reaction cannot be maintained. If  $k > 1$ , the reactor is supercritical and the reaction will grow exponentially [12].

## **1.6. Problem statement**

Nuclear reactors play an important role in electricity generation, which requires reliable control systems to ensure their safe and efficient operation. One of the most important components of the reactor control system is the nuclear control rods, which are designed to control reactor reactivity and power, and to shut down the reactor in an emergency. However, the effective control of these rods is challenging due to the specific characteristics of nuclear fuel and strict safety requirements for nuclear installations.

One of the main challenges in the management of nuclear control rods is the need to maintain precise control over reactor reactivity. The control rods must be able to quickly adjust the neutron flux to achieve and maintain the desired power level, while ensuring stable operation of the reactor. Achieving such precision is further complicated by the inherent characteristics of nuclear fuel, which require careful monitoring and control strategies to prevent criticality accidents or uncontrollable power excesses.

In addition, control rods must demonstrate resistance to external influences such as radiation exposure, temperature fluctuations, and electromagnetic interference. These factors can affect the performance and reliability of rod systems, requiring appropriate engineering solutions to mitigate their impact and maintain operational integrity.

Another important aspect is the limited ability to continuously replace nuclear fuel during reactor operation. Unlike conventional fuel sources such as coal or oil, nuclear fuel cannot be replenished on the fly, requiring sufficient fuel stocks to be kept in the reactor. This limitation imposes certain challenges on the design and operation of control rods, as they must effectively manage the available fuel to maintain continuous power generation without compromising safety.

Moreover, the control system should include redundant safety measures to prevent criticality accidents and mitigate the effects of unforeseen events. This includes the implementation of fault tolerance mechanisms, rapid shutdown procedures and emergency cooling systems to ensure reactor safety under all operating conditions.

Thus, the effective management of nuclear control rods in reactors requires solving multifaceted problems related to precise control of reactivity, resilience to external factors, fuel management strategies, and reliable safety measures. The development of advanced control systems capable of addressing these complex challenges while maintaining operational efficiency and safety remains one of the most important areas of nuclear reactor engineering.

## **1.7. Conclusion**

As energy technology advances, nuclear power is becoming an increasingly important component of the modern world, offering a wide range of benefits. The most important of these are low greenhouse gas emissions, which help reduce environmental pollution. In addition, nuclear power plants provide stable and reliable energy production, contributing to economic development and energy security.

Nuclear power plays a significant role in preserving the environment and combating climate change, as its use helps to reduce carbon emissions. Generating electricity without emitting harmful gases allows countries to reduce their dependence on coal and other energy sources that require excavation. Unlike fossil fuel power plants, nuclear reactors do not emit carbon dioxide (CO<sub>2</sub>) and other harmful substances into the air. Nuclear power is an important source of low-emission electricity, accounting for about 10% of global electricity production [13].

In addition, nuclear power plants provide stability in electricity production and can operate continuously, ensuring constant energy demand. Their ability to generate significant amounts of electricity makes them an important component of national energy systems, contributing to economic growth and industrial development.

Uranium, as the main fuel for nuclear reactors, is a fairly abundant element in the Earth's crust, which ensures stability in the supply of fuel for nuclear power plants.

The importance of nuclear energy in stimulating technological progress and innovation cannot be underestimated. Research in this area contributes to the emergence of new technologies and opens up new opportunities in various industries.

Thus, nuclear energy plays a key role in the modern world, offering solutions to the energy, environmental and economic challenges facing the world today. Precise control of a nuclear reactor is important for generators in the power system.

## SECTION 2

### PID-CONTROLLERS AND THEIR SYNTHESIS BY DIFFERENT METHODS

#### 2.1. PID-controller structure and its influence on a dynamic system

The control of dynamic systems is critical to ensure stability, accuracy, and efficiency in various fields of engineering and automation. Controllers play a key role in these processes by enabling systems to adapt to changing input conditions and achieve their goals.

Regulators help maintain setpoints for parameters or processes, which ensures the stability and reliability of the system under different operating conditions. For example, in temperature or pressure control systems, regulators help maintain optimal values of these parameters despite changing environmental conditions.

In addition, controllers optimize system performance by reducing energy and material consumption to achieve the desired results. This is especially important in production processes and automated systems, where effective control can have a significant impact on product quality and resource savings.

The use of controllers also increases the accuracy and resilience of the system to random influences or disturbances, resulting in more predictable and reliable operation. This is crucial in high-precision and critical applications such as aviation equipment or medical devices.

Aerospace Control Systems Department				Explanatory Note			
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Since the 1940s, the PID controller has been a cornerstone of feedback control, making significant contributions to process control. Today, these controllers dominate the landscape, constituting over 95% of control loops. Within this realm, PI controllers hold sway, showcasing both their efficiency and widespread adoption.

After the introduction of digital implementations, the structure of control systems was slightly changed, but this did not affect the basic principles of analysis and design of PID-controllers. The proportional-integral-differential controller (PID-controller) is a feedback method that consists of three components:

1. Proportional regulator (P): provides control proportional to the current error, which allows you to quickly respond to changes.
2. Integral controller (I): eliminates constant errors by accumulating the error over time.
3. Differential controller (D): takes into account the rate of change of the error, which helps to reduce overshoot and increase system stability [14].

There are many methods for designing PID-controllers for continuous systems that differ in control quality. The main characteristics of the closed-loop step response are:

1. Rise time: the period during which the system reaches a certain level from the initial value.
2. Overshoot: the maximum excess of the set point after the initial response.
3. Steady-state time: the time for the system to stabilize within a certain range around the set point.
4. Steady-state error: the difference between the set point and the final value of the system in the steady state.

These characteristics are key indicators of control quality and are used to compare different methods of PID-controller design, which allows you to choose the most appropriate method for a particular system [15].

## 2.2. Types of PID-controllers

There are the following types of controllers, and we will also consider each of them in the following subsections:

Controllers differ in the principle of interaction between the input signal (error) and the output signal (control signal). The most common are proportional-integral-differential (PID) controllers. These controllers can convert an error into a control signal in one of three ways: proportionally (P), integrally (I), or differentially (D). In addition, PID-controllers can use combinations of these control methods, such as proportional-integral (PI) and proportional-differential (PD) controllers. Each of them will be considered in the following subsections.

### 2.2.1. P-controller

The proportional controller (P-controller) is the simplest form of feedback control. It is widely used in various industrial and automated systems due to its simplicity and reliability. The main task of the P-controller is to reduce the error between the desired value (setpoint) and the actual output value of the system.

#### The principle of operation

The P-controller (Fig.2.21) operates on the basis of a proportional relationship between the error (the difference between the setpoint and the actual value) and the control signal. The formula for a proportional controller is as follows:

$$C_{out} = K_p e(t) + b,$$

where  $C_{out}$  is the output signal of the controller,

$K_p$  is the proportional gain,

$e(t)$  is the error at time  $t$ ,



$b$  is bias.

In this formula, the controller offset and gain are constants that are specific to each controller. The bias ( $b$ ) is the output signal of the controller when the error is zero. The gain ( $K_p$ ) determines how much the output signal of the controller changes when the input signal changes. In P-controllers, where signals are usually transmitted electronically, the gain relates a change in output voltage to a change in input voltage. These voltage changes have a direct effect on the properties being changed (e.g., temperature, pressure, level, etc.). Thus, the gain actually links the change in input and output properties.

If the output signal changes more than the input signal,  $K_p$  will be greater than 1. If the change in the input signal is greater than the change in the output signal,  $K_p$  will be less than 1. Ideally, if  $K_p$  is equal to infinity, the error can be reduced to zero. However, such a high gain increases the instability of the system, since zero error would mean that the measured signal is exactly equal to the setpoint. In control, absolute equality is usually not achieved; instead, some variation of the error within certain limits is allowed. Therefore, there are limits on the size of  $K_p$ , which are determined by the characteristics of the system.

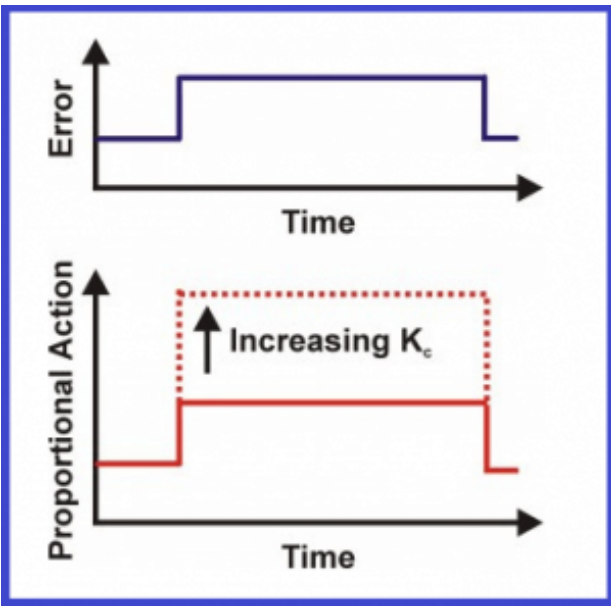


Fig. 2.2.1. Proportional Plot  $K_p$

## **Features.**

1. Easy to set up: The P-Controller is easy to set up with only one parameter proportional gain  $K_p$ . This makes it a popular choice for many applications.
2. Fast response: The P-controller provides a fast initial response to disturbances and changes in the input signal.
3. Stability: Due to its simplicity, the P-regulator provides stable control in most cases. However, if the control is excessively increased  $K_p$ , the system may become unstable.
4. Resistance to constant error: One of the disadvantages of the P-controller is that it cannot completely eliminate the steady-state error. Even with a large value of  $K_p$ , there will always be a small permanent error.

## **Application.**

P-controllers are best suited for first-order systems with a single energy storage device, such as liquid level or temperature controllers. They are also often used in systems with high dynamics, where a quick response to changes is required.

## **Disadvantages.**

**Constant error:** P-controllers cannot eliminate constant error, which may not be acceptable for some applications.

**Sensitivity to noise:** A high  $K_p$ , value can increase the sensitivity to noise, leading to oscillations and instability.

**Latency issues:** The presence of lag or dead time in the system can cause oscillations if  $K_p$ , is too high.

The P-controller is an effective and easy-to-configure tool for controlling feedback systems. It provides a quick response to changes, but has limitations in the form of constant error and sensitivity to noise. The use of a P-controller is justified in

systems where a small constant error is acceptable and high response dynamics is required.

### **2.2.2. I-controller**

An integral controller (I-controller) is an important component of a proportional-integral-differential (PID) controller. Its main task is to eliminate the system's steady-state error that occurs when using only a proportional controller.

#### **The principle of operation.**

An integral controller (Fig. 2.2.2) works by calculating the accumulated system error over a certain period of time. The formula for an integral controller is as follows:

$$C_{out} = K_i \int_0^t e(\tau) d\tau,$$

where  $C_{out}$  is the output signal of the controller;

$K_i$  is the integral gain;

$e(\tau)$  is the error at time  $\tau$ .

The integral term in a PID controller is proportional to both the error magnitude and the duration of its existence. The integral term accelerates the process of movement to a given value and eliminates the residual steady-state error that occurs when using only a proportional controller.

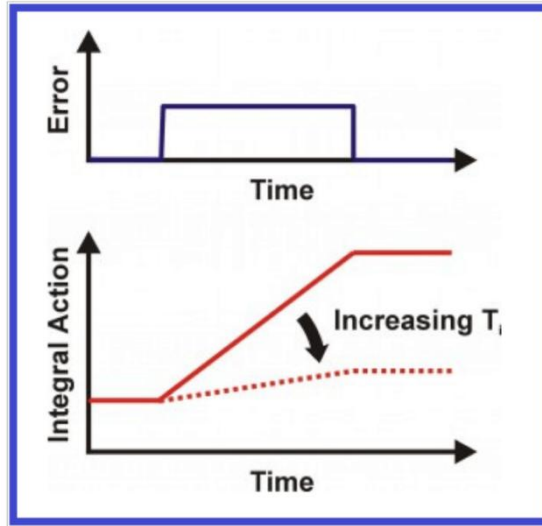


Fig. 2.2.2. Integral Plot  $K_i$

### Features.

1. Elimination of the steady-state error: The main advantage of the integral controller is its ability to eliminate the steady-state error. This is achieved by accumulating the error over time and using it to adjust the output signal.

2. Slow response: Since an integral controller takes into account errors from the past, its response to changes is slower compared to a proportional controller. This may cause the setpoint to be exceeded.

3. Dependence on the gain: Reducing the integral gain  $K_i$  increases the speed of the controller response. However, excessive reduction of  $K_i$  may reduce the ability of the controller to eliminate the steady-state error.

### Application.

Integral controllers are used mainly to reduce the steady-state error of a system. They integrate the error over time until it becomes zero. This means that even a small error value can cause a significant integral response. After the error reaches zero, the controller output signal is held at the last value to maintain a zero steady-state error. If the error is negative, the integral response will decrease.

### Impact on system stability.

An integral controller can cause system instability if the accumulated error becomes too large. This phenomenon is called "integrator hoarding" and can increase the time it takes for the controller to make the necessary changes.

The integral controller is a key element in PID-controllers that can eliminate the system's steady-state error and improve its accuracy. It works by integrating the error over time and using the accumulated error to adjust the output signal. The main advantage of the integral controller is its ability to eliminate the steady-state error, while the disadvantage is the possibility of instability due to integrator accumulation. To achieve optimal performance, the integral controller is often used in combination with other types of controllers, such as proportional or proportional-differential.

### **2.2.3. D-controller**

Derivative control (differential control) is an important component of PID-controllers. This type of control predicts process conditions by analyzing the change in error and works to minimize its changes, keeping the system in a stable state.

#### **Operation principle.**

The D-controller (Fig. 2.2.3) calculates the rate of change of the error over time and multiplies this change by the differential gain  $K_d$ . The formula for the differential term is as follows:

$$C_{out} = K_d \frac{d}{de} e(t),$$

where  $C_{out}$  is the output signal of the differential controller;

$K_d$  is the differential gain;

$\frac{d}{de} e(t)$  is the time derivative of the error.

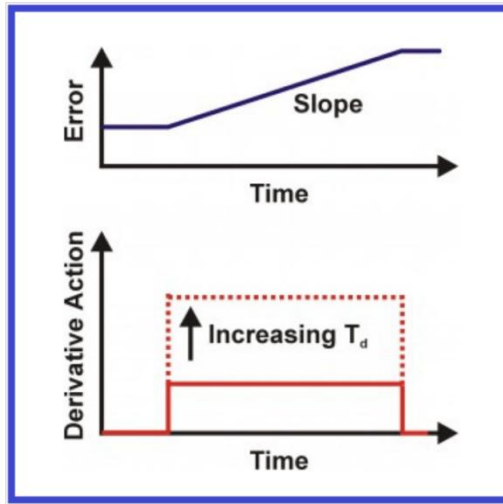


Fig. 2.2.3. Derivative Plot  $K_d$

### **Properties and applications.**

A differential controller sees how fast a process variable changes per unit of time and generates an output signal in proportion to the rate of change. This type of controller is used when a process variable starts to change at a high rate. In such cases, the D-controller moves the final actuator (such as valves or motors) in a direction that counteracts the rapid change in the process variable. It is important to note that the D-controller cannot be used alone for control, it must be combined with other types of controllers, such as P or PI.

### **Advantages.**

The main advantage of a differential controller is that it improves the transient response of the system. It predicts the behavior of the system, which helps to improve the settling time and stability of the system. This is achieved because the differential term responds quickly to changes in the error, giving the system a kind of "push" to reach the setpoint faster.

### **Limitations.**

The ideal differential controller is very sensitive and can vary widely even with small noise, so in practical implementations of PID-controllers, low-pass filtering is usually added to limit high-frequency gain and noise. It is estimated that the differential term is used in only about 20% of the implemented controllers due to its variable impact on system stability in real-world conditions. A PID-controller without a differential term is called a PI-controller.

### **Application.**

Differential control helps to reduce overshoot in the system and provide more stable operation in the presence of large external disturbances. This makes it useful for processes where a quick response to changes is important.

### **2.2.4. Proportional-integral controller (PI-controller)**

A PI-controller is a combination of proportional and integral control, in which there is no derivative (D) component, which is part of the PID system.

#### **The principle of operation.**

The control signal  $u(t)$  generated at the output of the PI-controller consists of proportional and integral components:

$$u(t) = K_p e(t) + K_i \int e(t) dt,$$

where  $K_p$  is the proportional gain;

$K_i$  is the integral gain;

$e(t)$  is the error.

The transfer function of the PI-controller is as follows:

$$W_{PI}(s) = K_p + \frac{K_i}{s}.$$

## **Properties and applications.**

A PI-controller is a form of feedback that provides faster response times than purely integral (I) control by adding a proportional component. PI control helps to prevent system oscillations and bring it back to a set point. Although the response time of a PI-controller is faster than that of an integral controller, it is still up to 50% slower than that of a purely proportional controller. Therefore, to increase the response time, the PI-controller is sometimes combined with derivative (D) control.

PI control correlates the controller output with the error and the integral of this error. Mathematically, it looks like this:

$$u(t) = K_c \left( e(t) + \frac{1}{T_i} \int e(t) dt \right) + C,$$

where  $u(t)$  is the output of the controller;

$K_c$  is the controller gain;

$T_i$  is the integral time;

$e(t)$  is the error;

$C$  is the initial value of the controller.

Integral time  $T_i$  defines the time required for the integral part of the controller to match the control provided by the proportional part of the controller.

## **Advantages and disadvantages.**

The main advantage of the PI-controller is the elimination of static error, which ensures that the system follows the set point more accurately. However, the presence of a pole at the origin can reduce the stability of the system. It is important to properly adjust the  $K_p$  and  $K_i$  coefficients to achieve the optimal balance between response speed and system stability.



The PI-controller works like a simplified PID -controller without a derivative component. It can also be seen as a combination of proportional and integral controllers. Proportional control is activated only when the system deviates from the set point, while integral control works continuously.

#### Setting up the PI-controller

The following general approach is used to configure the PI-controller:

1. Set the integral coefficient  $K_i$  to zero and the proportional coefficient  $K_p$  to one.
2. Gradually increase the proportional coefficient  $K_p$ , observing the transient response of the system, until the optimal balance between the rise time and the overshoot is achieved.
3. Gradually increase the integral coefficient  $K_i$  to achieve the nominal steady-state output value. It is important to start with a very small value because this coefficient can make the system unstable.

Thus, the PI-controller is an important part of many control systems due to its ability to effectively eliminate static error and ensure stable system operation.

### **2.2.5. Proportional-differential controller (PD-controller)**

#### **The principle of operation.**

The PD-controller is a combination of proportional and differential control, in which there is no integral component (I) of the PID system. The PD-controller combines feedback with the prediction of future process behavior. The output of the PD-controller is a linear combination of the current error and its derivative:

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt},$$

where  $K_p$  is the proportional gain;

$K_d$  is the differential gain;

$e(t)$  is the error.

The transfer function of the PD-controller is as follows:

$$W_{PD}(s) = K_p + K_d s,$$

where  $K_p$  is the gain of the proportional part;

$K_d$  is the gain of the differential part.

### **Properties and applications.**

PD-controller is used to improve system stability, dampen oscillations and predict process errors. PD control correlates the controller output with the error and its derivative.

Mathematically, it looks like this:

$$u(t) = K_c \left( e(t) + T_d \frac{de(t)}{dt} \right) + C,$$

where  $u(t)$  is the output of the controller;

$K_c$  is the proportional gain;

$T_d$  is the differential gain time;

$e(t)$  is the error;

$C$  is the initial value of the controller.

The equation shows that the PD-controller works as a simplified PID-controller without an integral component. The PD-controller can also be considered as a combination of proportional and differential controllers. Differential control predicts the error, contributing to the stability of the closed system.

The PD-controller is often used in systems where it is necessary to respond quickly to changes in error, but it is not necessary to minimize the static error. For example, in batch pH control loops where the steady-state error is not critical.

### **Setting up the PD-controller.**

The following approach is used to adjust the PD-controller:

1. Set the differential coefficient  $K_d$  to zero and the proportional coefficient  $K_p$  to one.
2. Gradually increase the proportional coefficient  $K_p$ , observing the transient response of the system, until the optimal balance between the response speed and overshoot is achieved.
3. Gradually increase the differential coefficient  $K_d$  to improve damping and reduce oscillations.

When tuning a PD-controller, it should be borne in mind that no single controller is able to meet all the requirements of the system. To achieve the required transient characteristics and reduce overshoot, a combination of several controllers is usually used. Proportional control is added to accelerate the transient, while differential control is used to reduce overshoot and oscillations.

Particular attention should be paid to the quality of the signals, as the presence of noise can lead to their amplification through differential control, which can create problems with system stability. Therefore, it is important to ensure that the signals are free of noise before they are processed by the PD-controller.

Thus, the PD-controller is an important part of many control systems due to its ability to effectively improve system stability and reduce oscillations.

### **2.2.6. PID-controller**

A proportional-integral-differential controller (PID-controller) is a universal means of automatic control that combines three types of control: proportional (P),

integral (I), and differential (D). This allows the PID-controller to provide a fast system response, minimize the steady-state error, and anticipate changes in the system.

### **The principle of operation.**

The PID-controller generates the control action  $u(t)$ , which is the sum of three components: proportional, integral, and differential:

$$u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{de(t)}{dt},$$

where  $K_p$  is the proportional gain;

$K_i$  is the integral gain;

$K_d$  is the differential gain;

$e(t)$  is the error, defined as the difference between the set value and the actual value.

The schematic of the PID-controller operation is shown in Fig. 2.2.4. Here,  $\tau(t)$  is the setpoint or desired value, while  $y(t)$  is the value of the variable known as the process variable (PV). This process is calculated as  $PV = y(t)$ , and the error is computed as  $e(t) = \tau(t) - y(t)$ . The controller performs corrective actions based on the rules of proportionality, integration, and differentiation, adjusting the control variable  $u(t)$  [15].

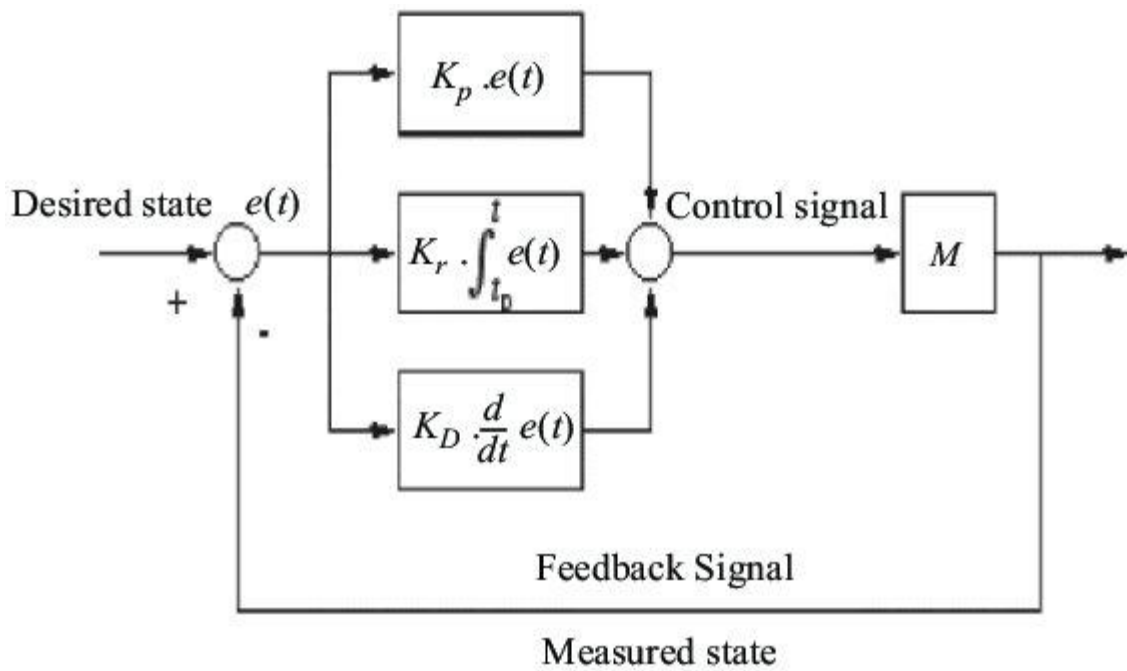


Fig. 2.2.4. A block diagram of PID control system having feedback closed loop

**Transfer function of the PID-controller.**

The ideal transfer function of a PID-controller is as follows:

$$W_{PID}(s) = K_p + \frac{K_i}{s} + K_d s.$$

However, since perfect differentiation of the error signal is impossible due to the presence of noise in real systems, the transfer function of a real PID-controller is modified by introducing an additional pole to the differential component:

$$W_{PID}(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{T_d s + 1},$$

where  $T_d$  is a time constant characterizing the inertial properties of a real differentiating link.

**Setting up the PID-controller**

To achieve the desired output of the PID-controller, it is necessary to properly adjust its parameters  $K_p$ ,  $K_i$  and  $K_d$ . The tuning process consists in determining the optimal values of the coefficients to ensure the appropriate response of the system.

### **Advantages and limitations of the PID-controller**

The PID-controller is widely used because of its ability to combine the advantages of all three types of control:

- Proportional control (P) provides a fast response to changes in the system error.
- Integral control (I) minimizes the steady-state error.
- Differential control (D) involves changes in the system, which contributes to stability.

However, the use of a differential component can lead to an increase in noise, which negatively affects the stability of the system. Therefore, the PID-controller is not recommended for use in systems with high noise levels.

### **Application of the PID-controller**

PID-controllers are widely used in industry. They are used in more than 90% of industrial controllers due to their versatility and efficiency. PID-controllers are capable of providing the required accuracy and stability, which makes them indispensable in complex automatic control systems.

A PID-controller is a powerful tool for automatic control that combines proportional, integral, and differential control. Proper tuning of the PID-controller allows you to achieve the desired system characteristics, providing fast response, minimal steady-state error and stability.

## **2.7. Tuning of PID-controller coefficients by the step response method**

The Ziegler-Nichols step response tuning method is a classical approach used to set the parameters of PID-controllers. Developed by John Ziegler and Nathaniel Nichols

in 1942, this method provides guidelines for determining the proportional gain ( $K_p$ ) integral time ( $T_i$ ), and derivative time ( $T_d$ ) based on the transient response of a process. It is particularly useful for systems where the dynamics are not well understood, providing a systematic starting point for controller tuning.

The Ziegler-Nichols step response method involves analyzing the step response of an open-loop system. The process is characterized by two key parameters derived from the step response curve: the delay time ( $L$ ) and the time constant ( $T$ ). The parameters are derived from the process's response to a unit step input, as illustrated in Fig. 2.7.1.

The Ziegler-Nichols step response method involves the following steps:

1. **Generate a Step Response:** Experimentally or through simulation, obtain the plant's response to a unit step input.
2. **Identify the S-shaped Curve:** If the plant's response is S-shaped, proceed with the method. If not, this method is not applicable.
3. **Determine Key Parameters:** Draw a tangent at the inflection point of the S-shaped curve. Identify two key parameters:
  - **Delay Time ( $L$ ):** Intersection of the tangent with the time axis.
  - **Time Constant ( $T$ ):** Intersection of the tangent with the line representing the final value of the step response.

Using these parameters, the plant's transfer function can be approximated as:

$$G(s) = \frac{K}{1 + Ts} e^{-Ls}.$$

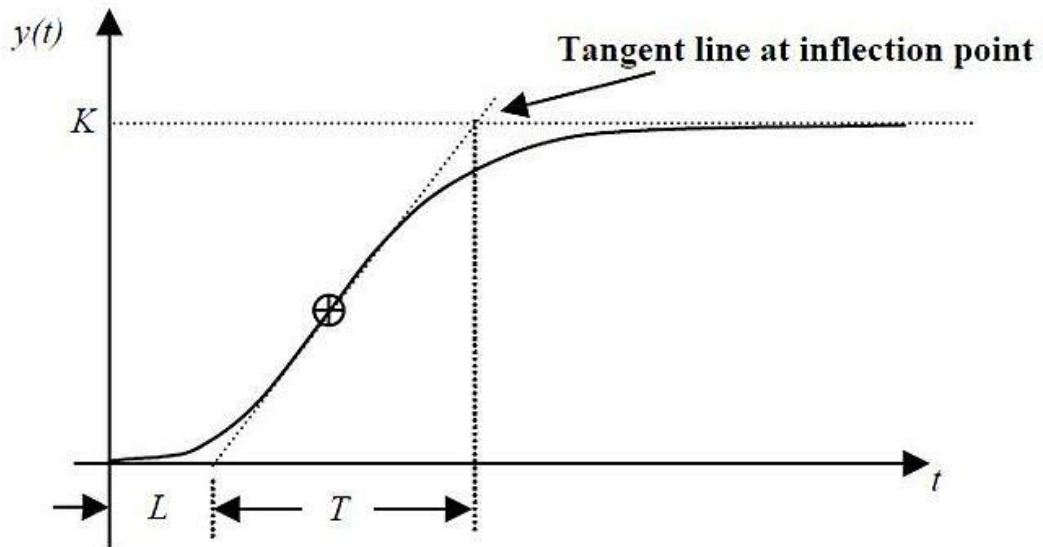


Fig. 2.7.1. Open loop system step response

### Tuning Rules

Based on the identified parameters  $L$  and  $T$ , the Ziegler-Nichols method suggests setting the PID-controller parameters according to specific formulas. The objective is to achieve a 25% maximum overshoot in the step response. The recommended settings are shown in the table below, PID parameters can be given directly as a function of  $a = KL/T$  and  $L$ , as shown in Table 2.7.1:

Table 2.7.1

Ziegler-Nichols Tuning Formula for Step Response Method

Controller	$K_p$	$T_i$	$T_d$
P	$1/a$	-	-
PI	$0.9/a$	$3L$	-
PID	$1.2/a$	$2L$	$L/2$

### Application and Advantages

The Ziegler-Nichols step response method is particularly advantageous for its simplicity and ease of application. It does not require detailed knowledge of the process



dynamics, making it accessible for a wide range of industrial applications. The method provides a good initial estimate for the PID parameters, which can then be fine-tuned for optimal performance. Provides a reasonable starting point for PID tuning with minimal process knowledge required.

But it has disadvantage Can lead to high gains, causing potential instability and excessive oscillations and best suited for processes with small dead times. Also, may result in low damping and reduced robustness against process dynamics changes. And does not define specific control objectives, potentially leading to suboptimal performance.

### **Practical Considerations**

- **Stability and Performance:** The Ziegler-Nichols method typically results in a PID-controller with about 25% overshoot in the system response. This can be acceptable for many applications, but further tuning may be necessary to meet specific performance criteria.
- **Process Suitability:** This method is best suited for processes that exhibit a clear S-shaped step response. It may not be as effective for processes with higher-order dynamics or significant nonlinearity.
- **Tuning Adjustments:** Once the initial parameters are set using the Ziegler-Nichols method, fine-tuning can be performed by adjusting  $K_p$ ,  $T_i$ , and  $T_d$  to achieve the desired transient response and steady-state performance.

In conclusion, the Ziegler-Nichols step response tuning method is a useful tool for setting PID-controller parameters, especially for processes with well-defined step responses. However, its limitations, such as high proportional gains and potential oscillatory behavior, should be considered when applying it to ensure optimal performance and stability.

## 2.8. Tuning of PID-controller coefficients by the frequency response method

The Ziegler-Nichols frequency response method is specifically designed for closed-loop PID-controllers. Its primary goal is to push the controller to the brink of instability to determine the process characteristics accurately.

This method is particularly effective when the process's dead time is relatively short compared to its time constant. However, small discrepancies between the estimated and actual process characteristics, such as gain or process delay, can result in an excessively oscillatory or unstable control loop.

In the Ziegler-Nichols frequency response method, the proportional gain ( $K_p$ ) is increased from zero until a critical value ( $K_{cr}$ ) is reached, where the output starts to exhibit sustained oscillations. To achieve this, first set the integral time ( $T_i$ ) to infinity and the derivative time ( $T_d$ ) to zero. The critical gain ( $K_{cr}$ ) and the corresponding oscillation period ( $P_{cr}$ ) can be determined using the Routh Criterion method. The PID parameters are then set using the Ziegler-Nichols formula as shown in Table 2.8.1.

Table 2.8.1

Ziegler-Nichols tuning formula for frequency method

Controller	$K_p$	$T_i$	$T_d$
P	$1/a$	-	-
PI	$0.45K_{cr}$	$P_{cr}/1.2$	-
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

In this method, the goal is to experimentally determine the point of marginal stability by gradually increasing the proportional gain until the process reaches marginal stability. At this point, the gain is called the ultimate gain ( $K_{cr}$ ), and the period of oscillation is known as the ultimate period ( $P_{cr}$ ). Refer to Fig. 2.8.1. These two

parameters define a point on the Nyquist plot. Based on this point, the controller settings can be calculated.

A significant drawback of this method is that it drives the system towards instability, which can be risky in practical applications. Additionally, the method does not provide a complete picture of the Nyquist plot, potentially resulting in a stable but highly oscillatory closed-loop system.

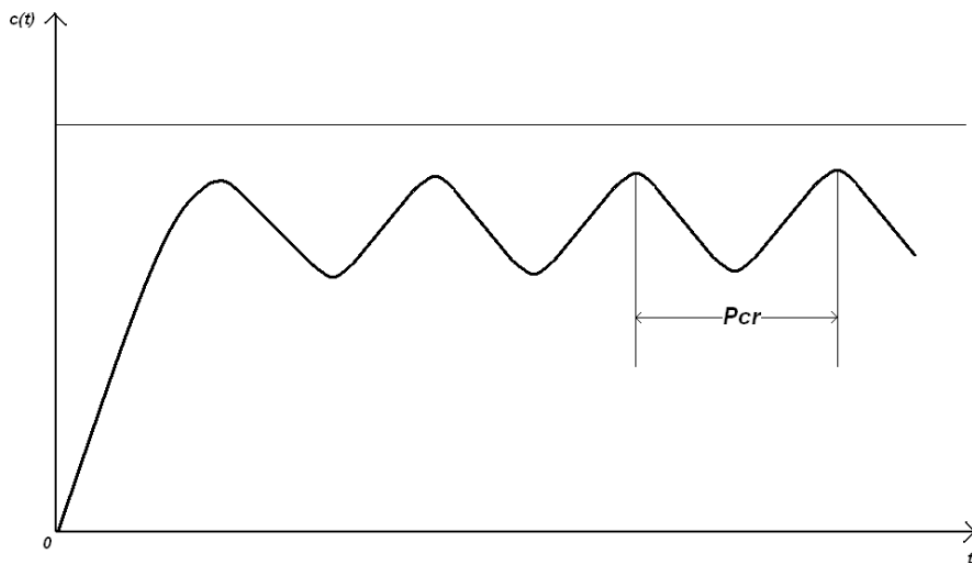


Fig. 2.8.1. The oscillation period  $P_{cr}$

In summary, the Ziegler-Nichols frequency response tuning method involves:

1. Setting  $T_i$  to infinity and  $T_d$  to zero.
2. Gradually increasing  $K_p$  until sustained oscillations occur.
3. Determining the critical gain ( $K_{cr}$ ) and the oscillation period ( $P_{cr}$ ).
4. Using these values to set the PID parameters according to the Ziegler-Nichols tuning formula.

This method provides a practical approach to tuning PID-controllers, especially when precise process dynamics are unknown. However, it requires careful handling to avoid potential instability and excessive oscillations.

## 2.8. Conclusion

In the chapter on PID-controllers, we have considered the structure of a PID-controller and its impact on a dynamic system. We also studied the types of PID-controllers and made a brief summary of the characteristics of each type of controller:

**P-controller:** Depends on the proportional coefficient, responds quickly to changes, but can leave a constant deviation.

**I-controller:** Eliminates the constant deviation by integrating the error over time, but can result in a slow response.

**D-controller:** Responds to the rate of change of the error, improving stability, but is sensitive to noise.

**PI-controller:** Combines the fast response of the P-regulator with the ability of the I-controller to eliminate constant deviation.

**PD-controller:** Combines the fast response of the P-controller with the ability of the D-controller to improve stability.

**PID-controller:** Combines all three components to provide a balance between response speed, stability, and accuracy.

Tuning the coefficients of a PID-controller is an important step in its application, and tuning methods using step and frequency response analysis make it possible to control the system accurately and efficiently, taking into account its dynamic characteristics.

PID gains can be determined using the Ziegler-Nichols tuning rules. The step response method analyzes the open-loop response by examining specific parameters  $a$  and  $\tau$ , as illustrated in Fig. 2.8.1(a). The frequency response method evaluates the process dynamics by identifying the point where the Nyquist plot of the process transfer function first intersects the negative real axis and the corresponding frequency  $\omega_c$  at which this occurs, as shown in Fig. 2.8.1(b).

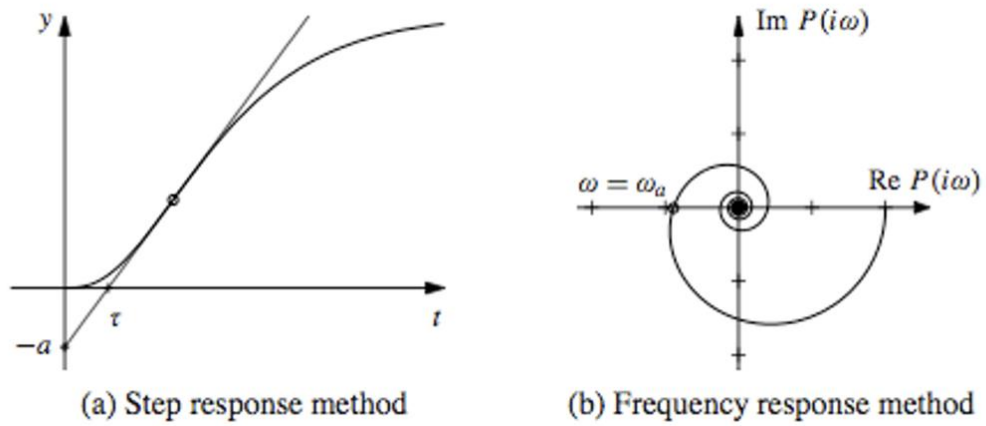


Fig. 2.8.1. Ziegler-Nichols tuning rules

This chapter has allowed us to gain in-depth knowledge of PID-controllers and their application in the control of various dynamic systems, which is an important step in the study of automatic control.

### SECTION 3

## SYNTHESIS OF PD-CONTROLLER FOR POSITION CONTROL OF NUCLEAR REACTOR RODS

The main purpose of using controllers is to reduce static error. In addition, by selecting the controller parameters, you can improve the transient characteristics: settling time and overshoot.

It should be noted that controllers that use the proportional-differential control law are important in engineering applications because they have many advantages. Their main idea is that in these controllers, their time derivative is used in the control algorithm as one that prevents the behavior of a linear system in the future, which follows from the very meaning of the term derivative. The differential control component changes the dynamics of the automatic control system, which improves it, but at the same time leaves the system statics unchanged. The effectiveness of the influence of the differential part of the PD-controller depends on the characteristics of the object, the unchanging part of the system, and on the proportionality coefficient of the derivative in the control law.

However, the PD-controller also has a disadvantage in that it is impossible to perform an indefinite differentiation operation in practice. But in today's world, with the development of technology, the inaccuracy can be as small as you like, but it also requires paying attention to the fact that unreasonably high accuracy of the differentiation implementation can lead to complication and lengthening of the system. In other words, to correctly determine the numerical value of the coefficients, the regulator must be precise, but also accurate. In this research, we consider the synthesis of a PD-controller to improve the efficiency and stability of automatic control [16].

Aerospace Control Systems Department				Explanatory Note			
Submitted	Kryvosheia N.I.			SECTION 3. SYNTHESIS OF PD- CONTROLLER FOR POSITION CONTROL OF NUCLEAR REACTOR RODS		Sheet	Sheets
Supervisor	Klipa A.M.					53	76
St Inspector.	Dyvnych M.P.				404		
Head of Dep	Melnyk Yu.V.						

### 3.1. Analysis of the nuclear reactor rods model without a controller

It is necessary to synthesize a PD-controller that can be physically implemented and that would provide the required stability margin in phase  $P_m = 55^\circ$  at a frequency of  $\omega = 3$  rad/sec for the nuclear reactor rod position control system (Fig. 3.1.1) [17] with the following transfer functions:

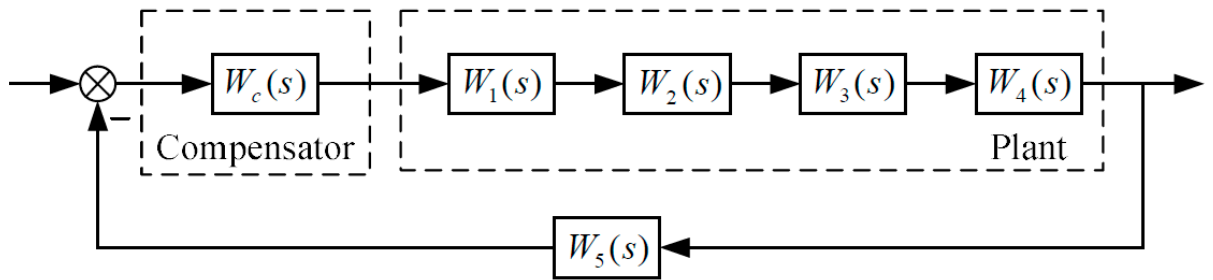


Figure 3.1.1. Block diagram of a nuclear reactor rod position control system

$$W_1(s) = 1; W_2(s) = \frac{1}{s(s+5)}; W_3(s) = 0.5;$$

$$W_4(s) = 58.2; W_5(s) = \frac{1}{0.2s+1}; W_c(s) = 1.$$

To analyze an unregulated system, we must first find the transfer function of a closed-loop system without a regulator.

$$W_{plant}(s) = W_1(s) \cdot W_2(s) \cdot W_3(s) \cdot W_4(s) = 1 \cdot \frac{1}{s(s+5)} \cdot 0.5 \cdot 58.2 = \frac{29.1}{s^2+5s};$$

$$W_{OL}(s) = W_{plant}(s) \cdot W_5(s) = \frac{29.1}{s^2+5s} \cdot \frac{1}{0.2s+1} = \frac{29.1}{0.2s^3+2s^2+5s}; \#(3.1.1)$$

$$W_{CL}(s) = \frac{W_{plant}(s)}{1+W_{OL}(s)} = \frac{\frac{29.1}{s^2+5s}}{1+\frac{29.1}{0.2s^3+2s^2+5s}} = \frac{5.82s+29.1}{0.2s^3+2s^2+5s+29.1}.$$

To determine the stability of the system, we identify the poles and zeros, if the system is stable, we can find the stability margins using the transfer function of the open-loop system.

The system is stable if all the poles of the system are negative and are on the left-half plane (LHP). If the poles of the system are positive, the system is not stable, or if the poles are on the imaginary axis, it is a marginally stable system. The zeros of the system indicate the quality of the system, i.e. whether the system will have overshoot.

The poles of the system are found by finding the roots of the denominator of the transfer function, which is a characteristic equation.

The characteristic equation of the system:  $0.2s^3 + 2s^2 + 5s + 29.1 = 0 \rightarrow$

$$p_1 = -9.0170 + 0.0000i;$$

$$p_2 = -0.4915 + 3.9868i;$$

$$p_3 = -0.4915 - 3.9868i.$$

The zeros of the system are determined by finding the roots of the numerator of the transfer function.

$$5.82s + 29.1 = 0 \rightarrow$$

$$z_1 = -5.$$

As we can see, at all the poles of the system, the real part is negative, i.e. the system is stable, and the poles also have an imaginary part, i.e. there are oscillations in the system. The zero of the system is negative, i.e. the system will have overshoot. Let us show the location of the poles and zero on the complex plane for continuous systems (Fig. 3.1.2).



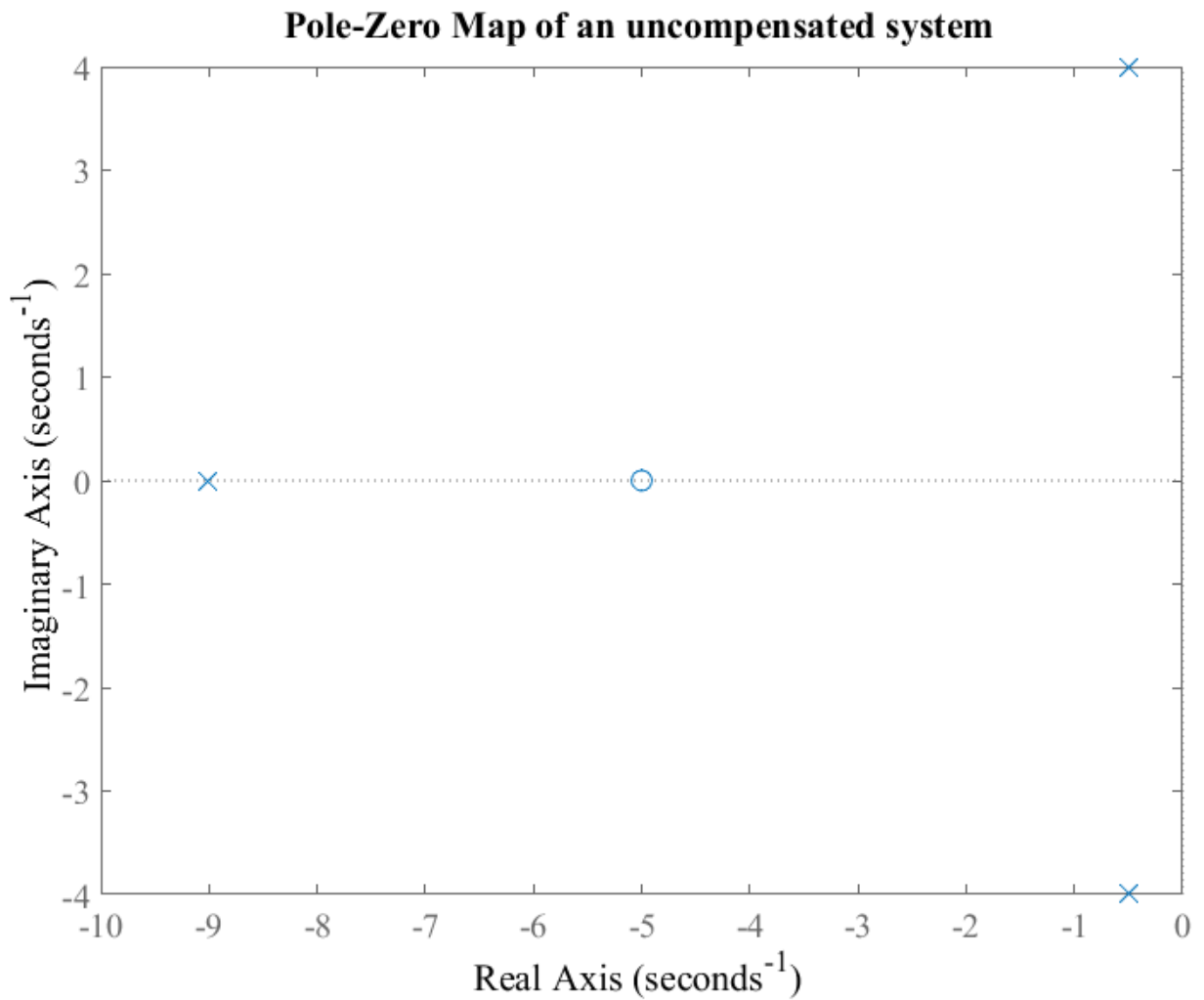


Fig. 3.1.2. Positioning poles and zeros (uncompensated system) on the complex plane

Let us plot the logarithmic frequency response (Fig. 3.1.3) of the open-loop system to make sure that the system is stable. If the open-loop system is stable, then the closed-loop system will be stable if the absolute value of the phase ordinate of the frequency response at the crossover frequency is less than 180 degrees.

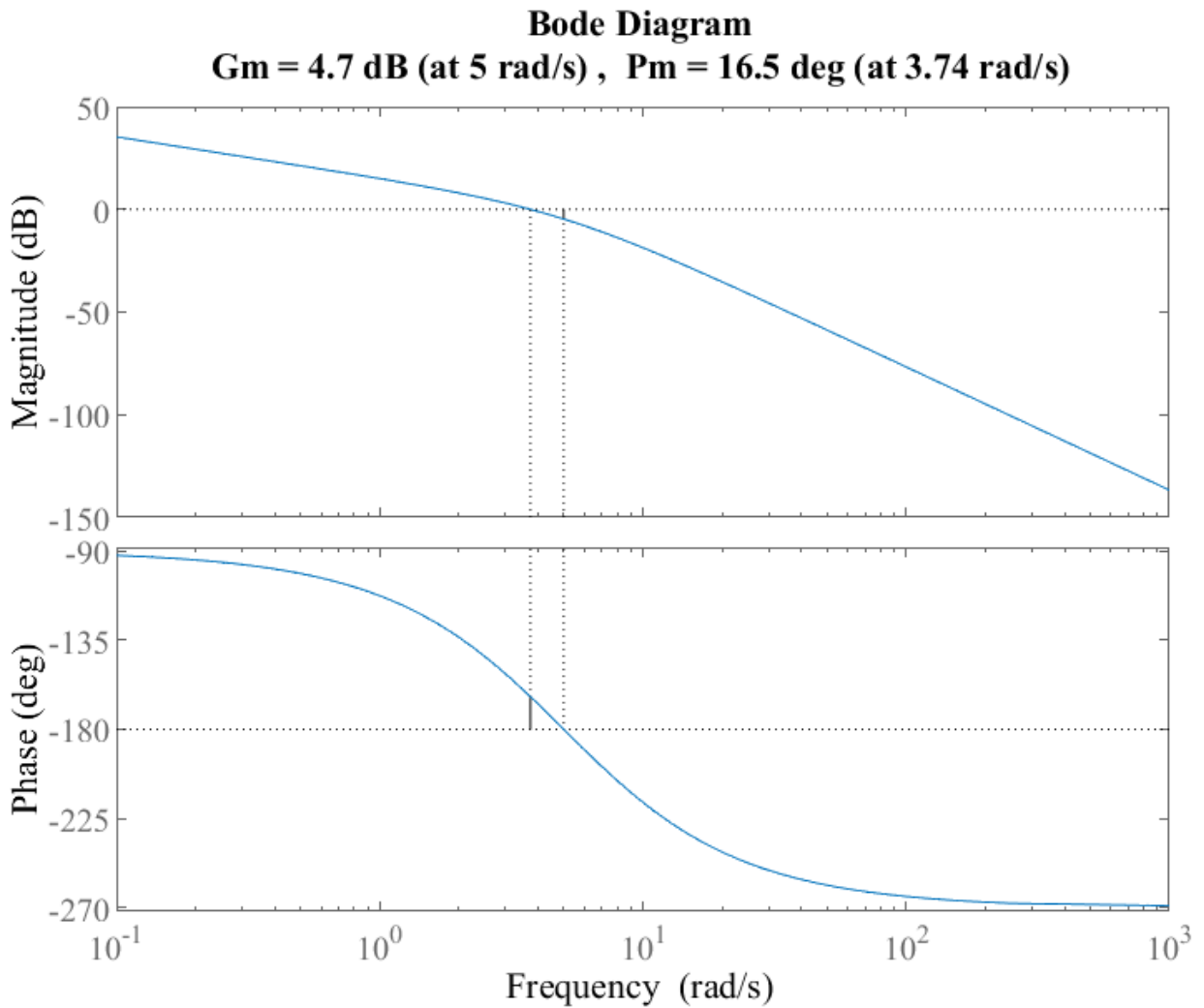


Fig. 3.1.3. Bode Diagram of an uncompensated system

From the Bode plot, we can see that the system is stable and has stability margins, but there is no phase margin at the desired frequency required for the synthesis of the PD-controller.

Let us plot the time responses (Fig. 3.1.4), i.e. step and impulse responses for the system to evaluate its quality parameters.

The step response shows how the system reacts to a change in the output value over time when a single step action is applied to the input. This response allows you to evaluate the system's settling time (the time it takes for the output to stabilize after the input displacement), transient oscillations, response speed, and other dynamic characteristics.

Impulse response shows how the system reacts to a change in the output value over time when a single impulse is applied to the input, i.e., an instantaneous impulse input (delta function), which is a short and high-amplitude function that lasts for a very short time. This characteristic allows us to analyze the system's response to instantaneous changes in the input signal, the stability of the system, and its filtering properties.

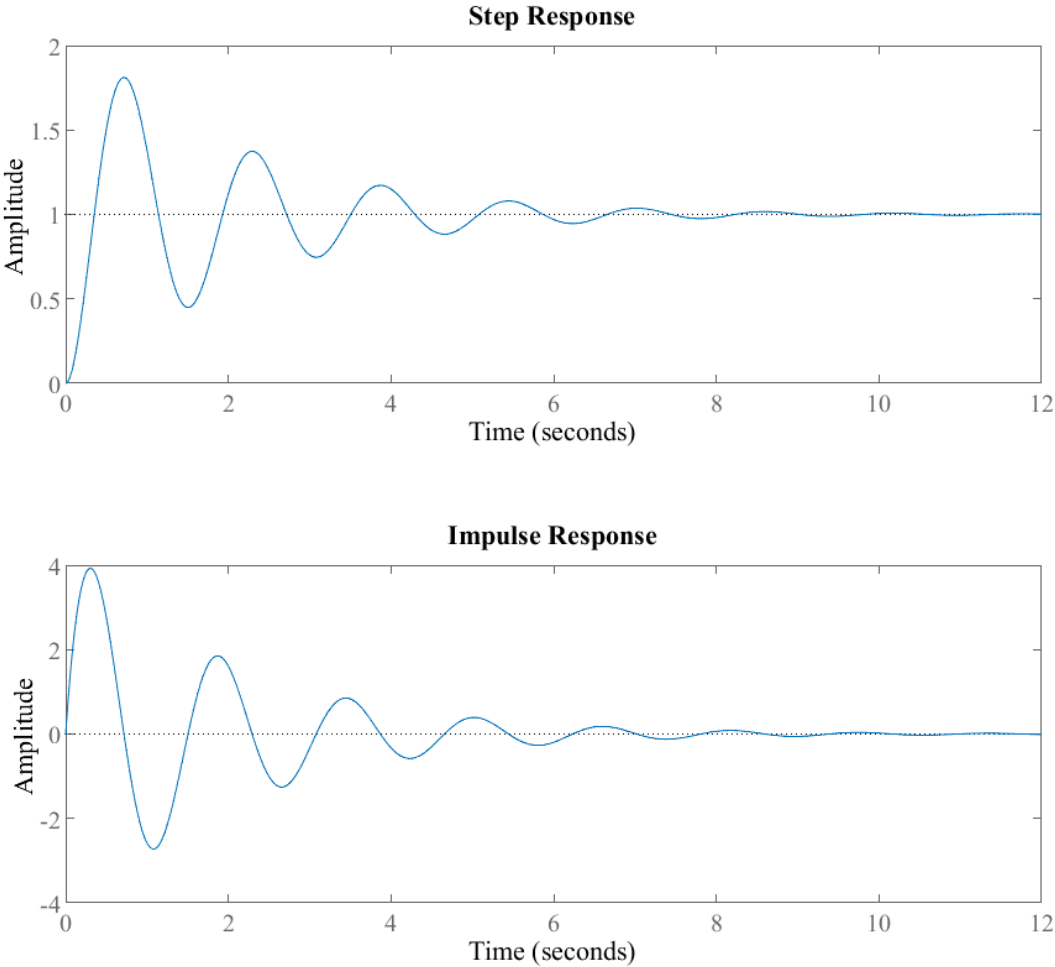


Fig. 3.1.4. Time responses of the uncompensated system

From these responses, we can see that the system has some oscillations, but has not a very long period of time to establish the system.

For a more detailed analysis of the transient responses, consider the power function (Fig. 3.1.5).

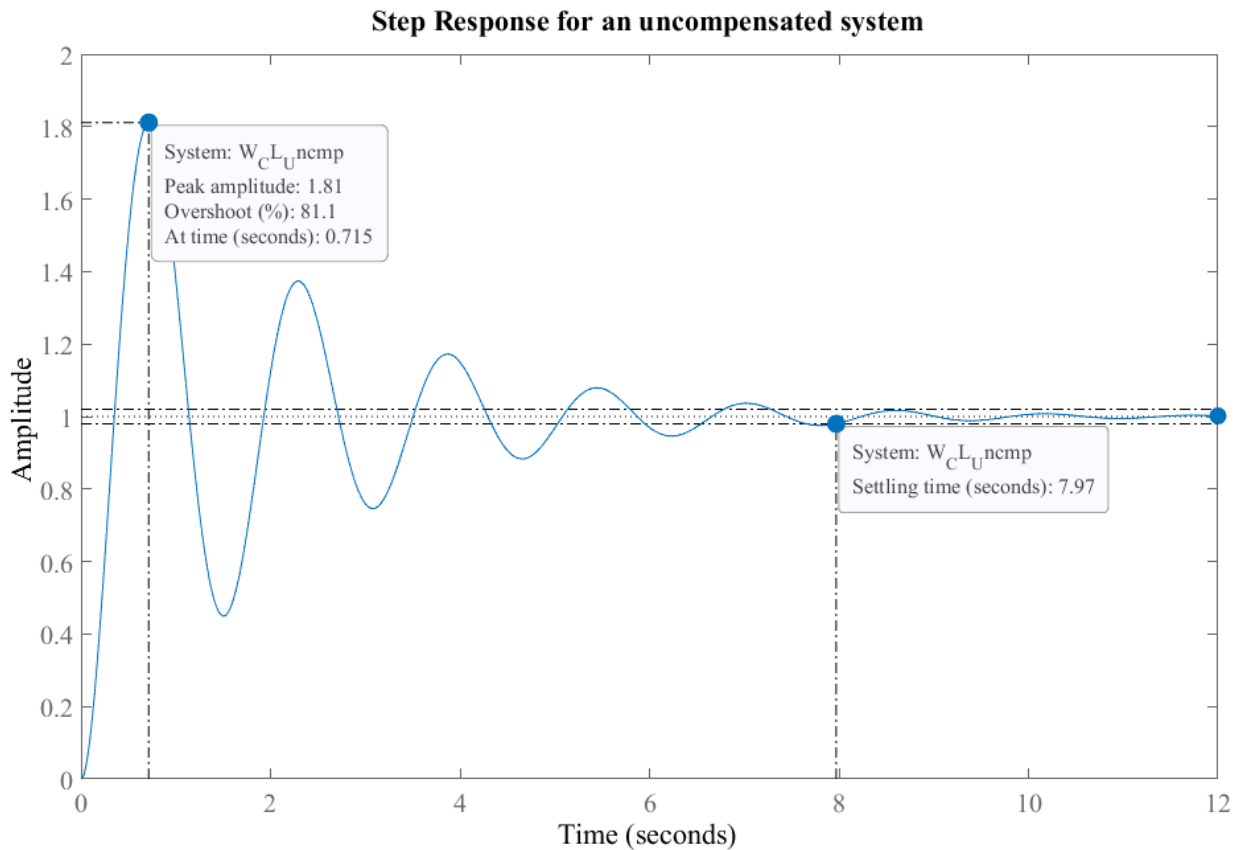


Fig. 3.1.5. Detailed step response of the uncompensated system

The step response shows the following parameters:

- Rise Time: 0.2365 sec indicates that the system reacts quickly enough to changes in the input signal and reaches a stable state.
- Settling Time: 7.9704 sec indicates that the system takes a considerable amount of time to stabilize after the input signal changes. This may indicate some instability or slowness of control.
- Overshoot: 81.1281 % indicates that the output signal exceeds its final value by a significant amount before stabilizing. Such a large overshoot may indicate instability or control deficiencies.
- Peak Time: 0.7150 sec indicates that the maximum value of the output signal is reached fairly quickly after the input signal changes.

- Steady state value: 1 indicates that the system reaches a certain steady state value of the output signal after being exposed to a step input signal.

Based on these parameters, it can be concluded that the system has a relatively fast response to changes in the input signal, but may have problems with stability and control of output signal exceedances.

Another characteristic of the system is the  $H_2$ -norm performance index.  $H_2$ -norm is a measure of the system's sensitivity to external influences such as noise and interference. It is usually measured as the square root of the integral of the squared amplitudes of the system's transfer function over all frequencies. This norm is used to determine the effectiveness of the system under variable conditions and allows you to assess how well the system maintains signal stability and quality in the presence of various sources of influence.

In our case, the  $H_2$ -norm is 3.3187, which means that your system has an average sensitivity to noise and interference. The lower the  $H_2$ -norm value, the better the system maintains signal performance.

The results of the analysis of the uncompensated system were presented at the XXIII International Scientific and Practical Conference of Higher Education Applicants and Young Scientists "POLIT. Challenges of science today" [18].

### **3.2. Synthesis of PD-controller for position control of nuclear reactor rods**

The transfer function of an ideal PD-controller is [17]:

$$W_c(s) = K_p + K_d s, \#(3.3.1)$$

where  $K_p$  is the gain of the proportional part;

$K_d$  is the gain of the differential part.

When using the ideal controller (3.1), an unlimited increase in the differential

gain leads to infinite gain at high frequencies. Therefore, in order to limit the gain at high frequencies, an additional pole is included into the differential component of the PD-controller. In this case, pole is introduced into the differential component of the PD-controller [17]. In this case, the transfer function of the real PD controller can be written as:

$$G_c(s) = K_p + \frac{K_d s}{\tau_0 s + 1}, \#(3.3.2)$$

where  $\tau_0$  is a very small value ( $\tau_0 \ll 1$ ).

In our case  $\tau_0 = 0.0001(\text{sec})$ , therefore, in the synthesis procedure, it is necessary to determine two parameters, such as  $K_p$  and  $K_d$ .

The transfer function of an open-loop uncompensated (without a controller) system is as follows:

$$W_{OL\_un}(s) = \frac{29.1}{0.2s^3 + 2s^2 + 5s}.$$

Using the transfer function (3.1) of an open-loop uncompensated (without a controller), we can start synthesizing the controller. To do this, we need to make some calculations [17].

1) Calculate the phase shift of the controller at the frequency  $\omega$ :

$$\theta = \text{arg}W_c(j\omega) = -180^\circ + P_m - \text{arg}W_p(j\omega)W_f(j\omega),$$

$$\theta = \text{arg}W_c(j\omega) = -180^\circ + 55^\circ + 151.9275^\circ = \frac{-26.92^\circ}{57.296} = 0.47 \text{ (rad)}.$$

2) Determine the gain for the proportional part of the controller:

$$K_p = \frac{\cos\theta}{|W_p(j\omega)W_f(j\omega)|} = \frac{\cos(0.47)}{1.4265} = 0.625.$$

3) Determine the gain for the differential part of the controller:

$$K_d = \frac{\sin\theta}{\omega|W_p(j\omega)W_f(j\omega)|} = \frac{\sin(0.47)}{3 \cdot 1.4265} = 0.1058.$$

After that, we can write down the transfer function of the PD-controller (2) for a given system:

$$G_c(s) = K_p + \frac{K_d s}{\tau_0 s + 1} = 0.625 + \frac{0.1058s}{0.0001s + 1} = \frac{0.1058s + 0.625}{0.0001s + 1}.$$

### 3.3. Analysis of the nuclear reactor model with PD-controller

After synthesis of the PD-controller, it must be implemented in the system to check the success of the synthesis and to achieve the required stability margin at the desired frequency.

To do this, you need to calculate the transfer function of the system with the controller.

$$W_{FFComp} = G_c \cdot W_{plant} = \frac{0.1058s + 0.625}{0.0001s + 1} \cdot \frac{29.1}{s^2 + 5s};$$

$$W_{FFComp} = \frac{3.081s + 18.19}{0.0001s^3 + s^2 + 5s};$$

$$W_{CLComp} = \frac{W_{FFComp}}{1 + W_{FFComp} \cdot W_5};$$

$$W_{CLComp} = \frac{0.6162s^2 + 6.719s + 18.19}{2e - 05s^4 + 0.2002s^3 + 2s^2 + 4.591s + 17.45}.$$

Let us analyze the system for stability using the same algorithm that was used to analyze the uncompensated system. Let us start by defining the poles and zeros.

The characteristic equation of the system:

$$0.2e - 0.5s^4 + 0.2002s^3 + 2s^2 + 8.081s + 18.19 = 0 \rightarrow$$

$$p_1 = 1.0e + 04 \cdot -1.0000 + 0.0000i;$$

$$p_2 = 1.0e + 04 \cdot -0.0006 + 0.0000i;$$

$$p_3 = 1.0e + 04 \cdot -0.0002 + 0.0003i;$$

$$p_4 = 1.0e + 04 \cdot -0.0002 - 0.0003i.$$

Zeros of the system:

$$0.6162 s^2 + 6.719 s + 18.19 = 0 \rightarrow$$

$$z_1 = -5.9030.$$

$$z_2 = -5.0000.$$

In a compensated system, as we can see, all the poles of the system have a negative real part, i.e. the system is stable, and the poles also have an imaginary part, i.e. there are oscillations in the system.

The zeros of the system are located on the negative real plane of the complex plane, i.e. they may indicate the presence of certain features in the frequency response of the system. Usually, a zero in this region can lead to the amplification of certain frequencies in the system output signal, the system will have an overshoot.

The location of the poles and zero on the complex plane for continuous systems is shown in Fig. 3.3.1.



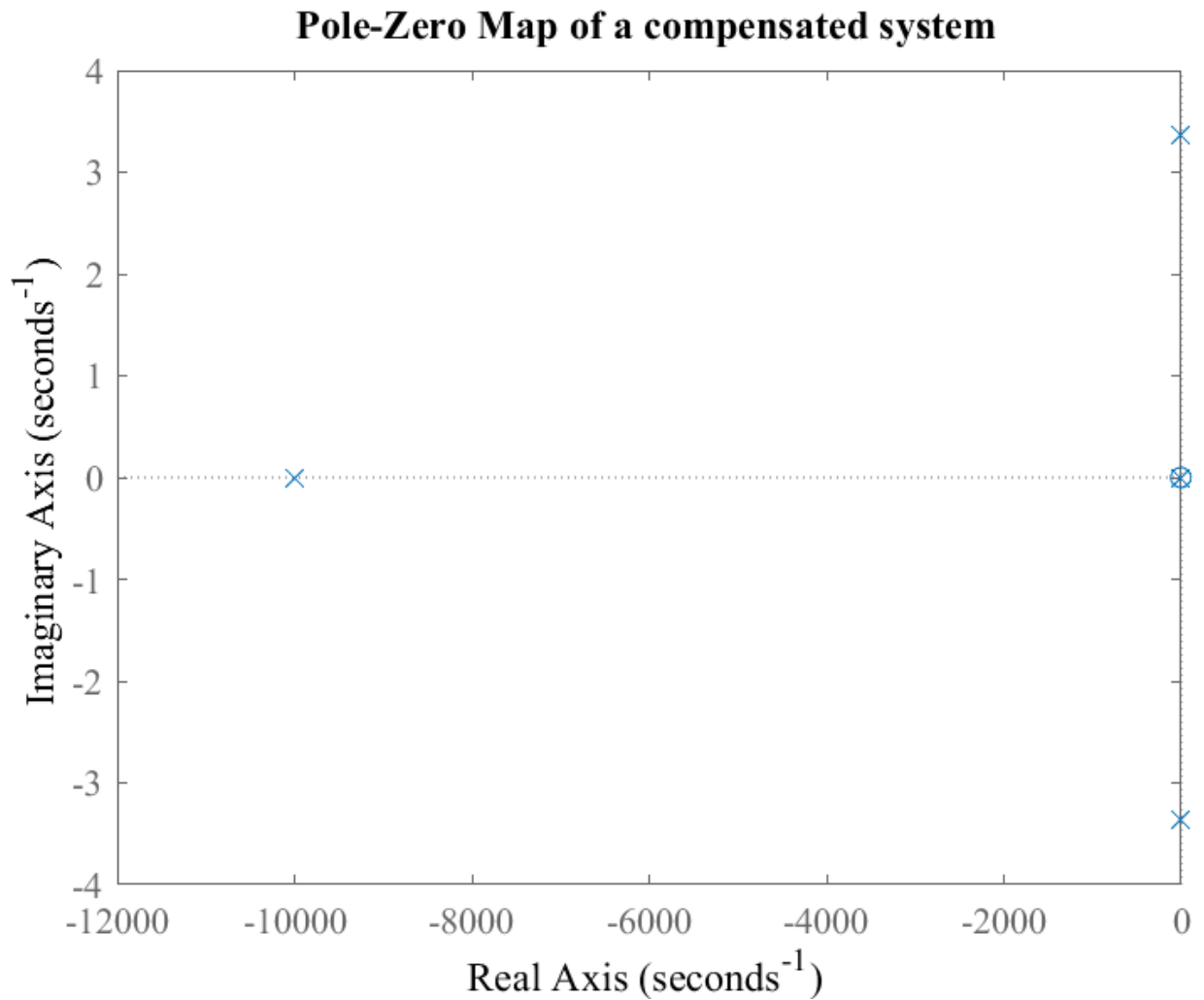


Fig. 3.3.1. Positioning poles and zeros (compensated system) on the complex plane

Now we can plot the logarithmic-frequency response (Fig. 3.3.2) to check the effect of the PD-controller on the system, i.e. whether the system has reached the desired reserve phase at the desired frequency.

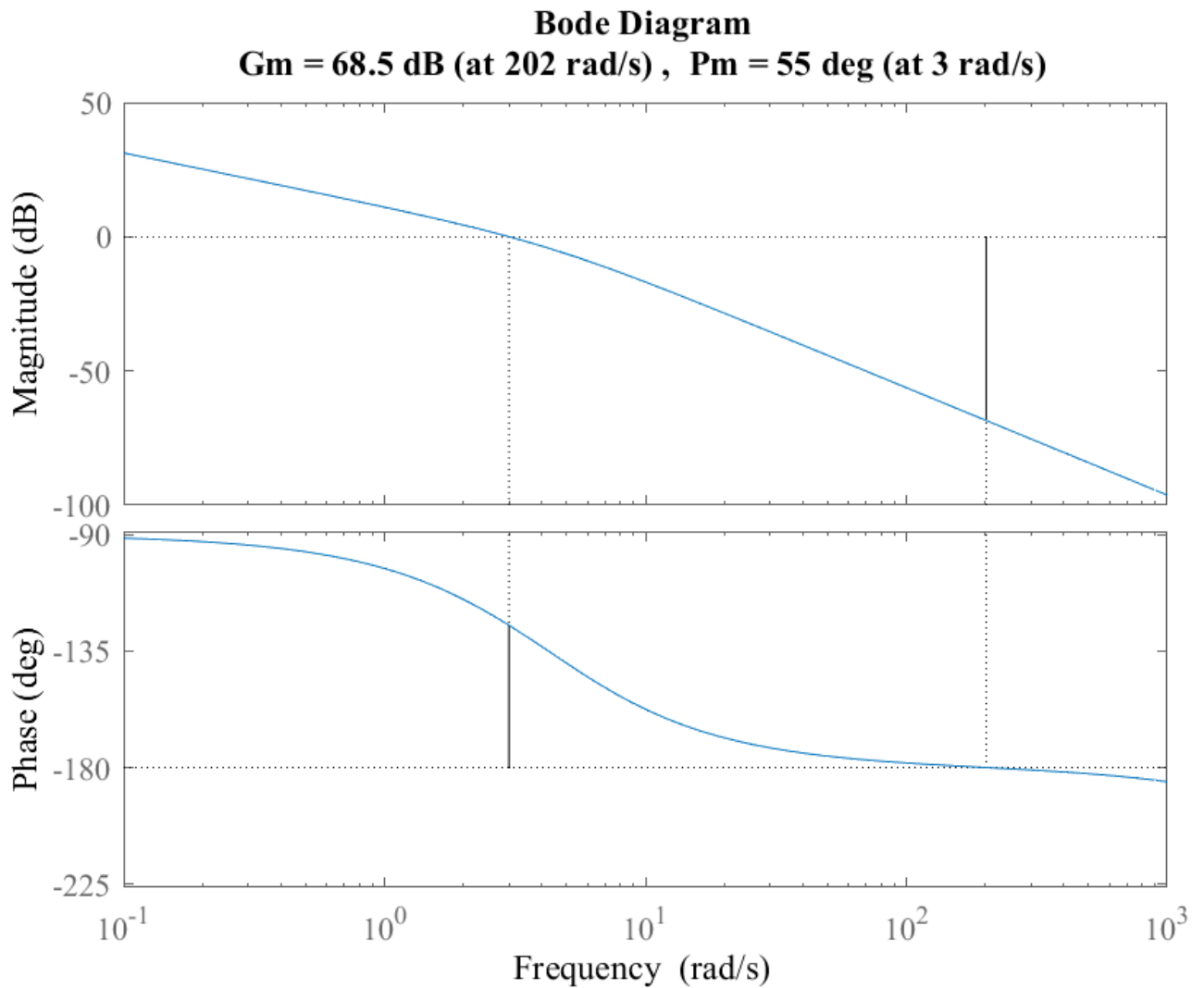


Fig. 3.3.2. Bode Diagram of uncompensated system

The bode plot shows that the system has reached the required phase stability margin, the synthesis of the PD-controller was successful.

Now we can look at the transient response (Fig. 3.3.3) of the system in more detail to understand what parameters the system now has after connecting the PD-controller.

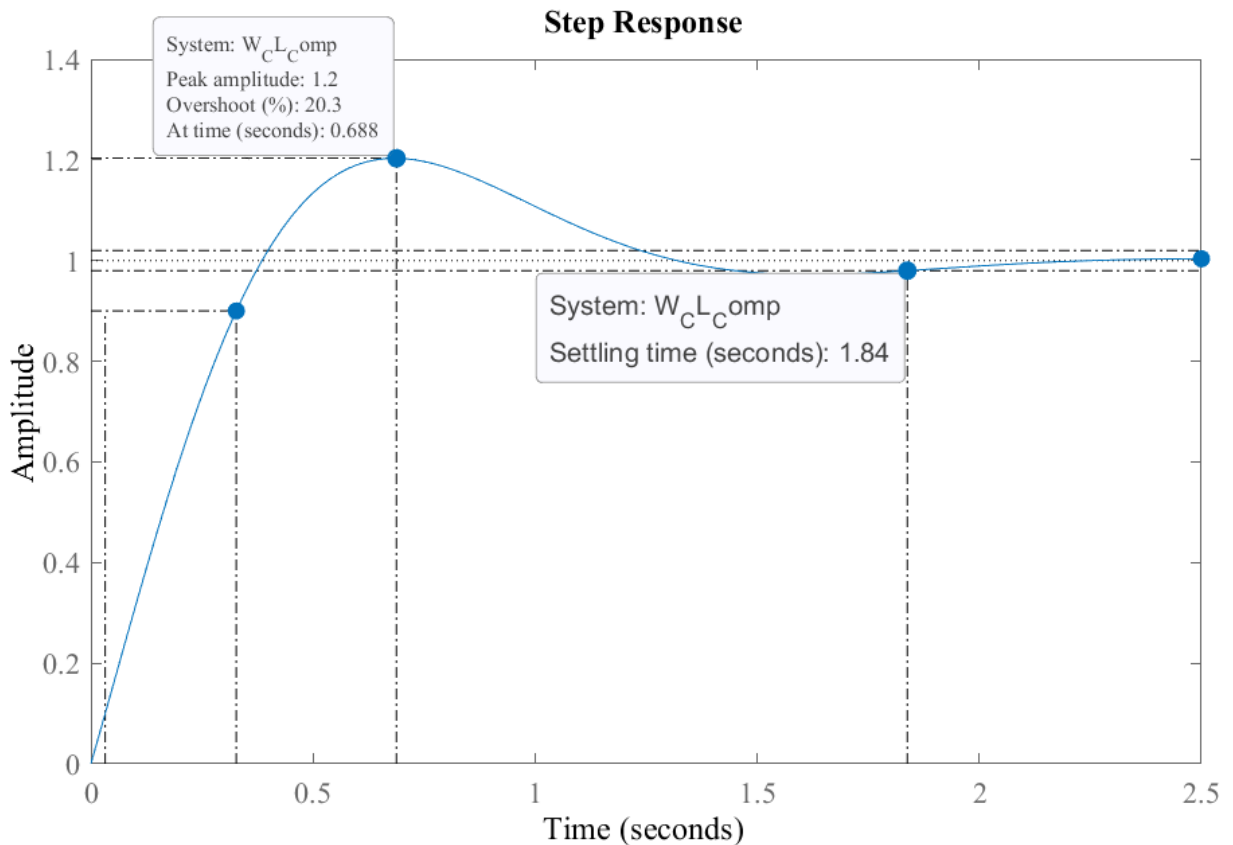


Fig. 3.3.4. Detailed step response of the compensated system

From the step response, we can determine the following parameters:

- Rise Time: 0.2952 sec means that the system reacts quickly enough to a change in the input signal and reaches a stable state.
- Settling Time: 1.84 sec is a moderate duration for the system to stabilize after an input signal change.
- Overshoot: 20.3287% indicates that the output signal temporarily exceeded the target value by this percentage before stabilizing.
- Peak Time: 0.688 sec indicates the moment when the output signal reaches the maximum value after the input signal starts to act.
- Steady state value: 1 indicates that the system reaches a certain steady state value of the output signal after the influence of a step input signal.

Additionally, let us check the H2-norm to see how the controller affects the system. The H2-norm became 1.7241, which means that there was a decrease in the H2-norm after the PD-controller was connected.

### 3.4. Comparison of the nuclear reactor rods model without and with PD-controller

After successful synthesis of the controller, the uncompensated and compensated systems can be compared to draw conclusions about the controller, its impact on the system, and what qualities of the system it has improved.

By comparing the zeros and poles of the closed-loop uncompensated and compensated systems (Table 3.4.1), the following conclusions can be drawn:

- Change of poles: When the system is compensated, the poles are located closer to the origin (negative values), which may indicate improved system stability and less sensitivity to external factors.
- Change of zeros: There is an additional zero (-5.9030), which is located closer to the origin, which also contributes to the improvement of the system's stability.

Table 3.4.1

Comparison of pole-zero characteristics

Uncompensated system		Compensated system	
$p_1 = -9.0170 + 0.0000i$	$z_1 = -5$	$p_1 = 1.0e + 04 \cdot -1.0000 + 0.0000i$	$z_1 = -5.9030$
$p_2 = -0.4915 + 3.9868i$		$p_2 = 1.0e + 04 \cdot -0.0006 + 0.0000i$	
$p_3 = -0.4915 - 3.9868i$		$p_3 = 1.0e + 04 \cdot -0.0002 + 0.0003i$	$z_2 = -5.0000$
		$p_4 = 1.0e + 04 \cdot -0.0002 - 0.0003i$	

Bode plots with gain and phase margins for the system without and with the controller are shown in Fig. 3.9. These characteristics confirm that with the synthesized PD-controller, the nuclear reactor rod position control system has the required phase stability margin, which is  $55^\circ$  at a given frequency. In addition, based on the Bode plots (Fig. 3.4.1), it can be concluded that both systems are stable.

Comparing the margins of the uncompensated and compensated systems, we can draw the following conclusions:

**Gain Margin:** The compensated system has a significantly higher gain margin of 4.7 dB (at 5 rad/s), which means it is more resistant to changes in gain than the uncompensated system (68.5 dB (at 202 rad/s).

**Phase Margin:** The compensated system has a significantly greater phase margin of  $55^\circ$  (at 3 rad/s), which means greater immunity to phase changes compared to the uncompensated system of  $16.5^\circ$  (at 3.74 rad/s).

Consequently, the compensated system with PD controller has significantly better stability characteristics (larger gain and phase margin), which indicates its better control ability and less susceptibility to unstable modes compared to the uncompensated system.

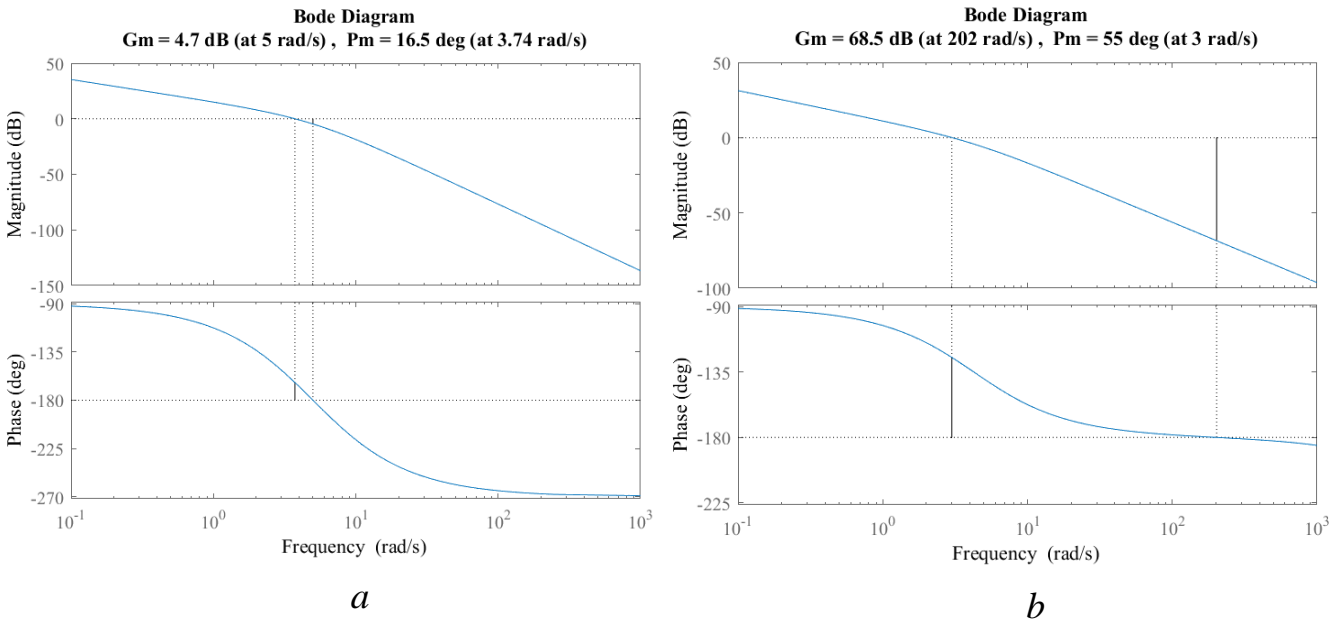


Fig. 3.4.1. Bode plots:

(a) for the uncompensated system; (b) for the compensated system

The result of the system control can be seen in the step response. The transient responses of the uncompensated and compensated closed-loop systems are shown in Fig. 3.4.2.

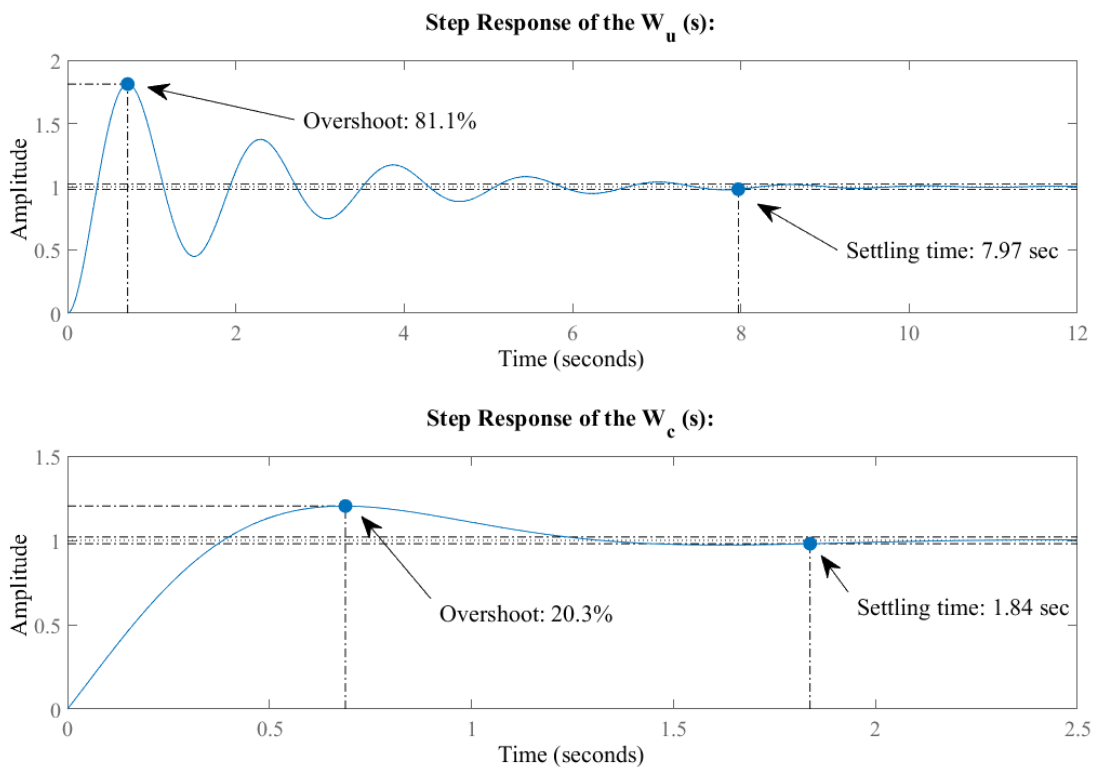


Fig. 3.4.2. Step responses of uncompensated  $W_u(s)$  and compensated  $W_c(s)$  systems

Taking into account the results obtained for the uncompensated system and the system with the PD-controller, the following conclusions can be drawn: the use of the PD-controller significantly reduces the overregulation rate, reducing it from 81.1% to 20.3% and reduced the control time from 7.97 seconds to 1.84 seconds. This demonstrates the ability of the controller to effectively compensate for emissions from the initial state of the system and improve the accuracy of response to external changes, and it also indicates a faster and more efficient adaptation of the system to new conditions, which is an important factor in many technical applications.

The results of the synthesis of PD-controller and analysis of the compensated system were presented at the XXI International Scientific and Practical Conference of Young Scientists and Students "Modern problems of scientific energy supply" [19].

### **3.5. Conclusion**

The analysis results show important advantages of using a PD-controller in a nuclear reactor position control system. Its implementation allowed to significantly improve the quality of control, reduce the level of overregulation and reduce the time of system stabilization after changes in the input signal. This demonstrates the controller's ability to effectively compensate for the impact of external factors and improve the accuracy of the system's response to changes in operating conditions. The stability margins of the system with the PD-controller significantly exceed those without compensation, which indicates greater stability and reliability of the system under various conditions.

In addition, the analysis of Bode diagrams and other characteristics showed that the system with the PD-controller has the appropriate gain and phase margins necessary for stable operation in the specified modes. These characteristics confirm the success of the controller synthesis and its compliance with the requirements of stability and reliability in control systems, in particular, in such complex technical environments as nuclear reactor control.

Thus, the use of a proportional-differential controller can effectively improve system performance, providing a more accurate and rapid response to changes, making it a promising option for implementation in automated control systems.

## CONCLUSION

In this diploma work, a comprehensive study of the theoretical and practical aspects of nuclear reactor core control using a PD-controller was conducted. First of all, the concept of a nuclear reactor and the principles of its operation, including the processes of uranium fission and absorption, were considered. Nuclear power is an important component of the modern energy system, as it provides stable and reliable electricity generation with low greenhouse gas emissions, which helps to reduce environmental pollution and combat climate change.

The first part of the paper analyzes in detail the theory of nuclear reactor core management, which is critical to ensuring the stability and safety of nuclear facilities. The main problems and challenges associated with core control were identified, and the need for accurate and reliable control to maintain stable reactor operation was emphasized.

The second part of the thesis was devoted to the study of PID-controllers, their structure and impact on dynamic systems. Different types of PID-controllers, their characteristics and impact on the system were considered. Particular attention was paid to methods of tuning the coefficients of the PID-controller, such as the Ziegler-Nichols methods, which allow for accurate and efficient control of the system, taking into account its dynamic characteristics. Based on these methods, a PID-controller was synthesized to control the position of nuclear reactor rods.

The third part of the work was devoted to the analysis of the model of the nuclear reactor core control system without a controller and with a PD-controller. After successful synthesis of the controller, the uncontrolled and controlled systems were compared. It was found that the system with the PD-controller has significantly better stability characteristics, reduced overshoot, and reduced installation time. The controlled system demonstrates increased resistance to external influences and a more accurate response to changes in input signals. The analysis of the results showed that the use of the



PD-controller significantly improves the quality of control of nuclear reactor rods, providing a more accurate and rapid response to changes.

Thus, the results of the thesis show that the introduction of a PD-controller into the nuclear reactor core control system is an effective solution to improve its performance. This makes it a promising option for implementation in automated control systems, especially in such complex technical environments as nuclear reactor control. The use of a PD-controller makes it possible to ensure more stable and reliable operation of a nuclear reactor, reducing risks and increasing its efficiency. This, in turn, contributes to energy security and sustainable development of the energy sector.

## REFERENCES

1. Hamilton L. J. Nuclear reactors. Design and construction. Nuclear engineering. Nuclear fission. – 1941.
2. Nuclear Reactor Construction and Working [Electronic resource] // Deepak Kumar Yadav. – 2021. – Access mode: <https://www.deepakkumaryadav.in/2021/05/Nuclear%20Reactor%20Construction%20and%20Working.html>
3. Garland W. J. The Essential CANDU, A Textbook on the CANDU Nuclear Power Plant Technology.
4. Nuclear reactor – Fuel types [Electronic resource] // Britannica. – Access mode: <https://www.britannica.com/technology/nuclear-reactor/Fuel-types#ref155172>
5. Chaplin R. Introduction to Nuclear Reactors / Dr. Robin Chaplin.
6. Control rod [Electronic resource] // Energy Education. – Access mode: [https://energyeducation.ca/encyclopedia/Control\\_rod](https://energyeducation.ca/encyclopedia/Control_rod)
7. Abiodun O. Control Rods Drop Failure On Reactors Stability And Safety / O. Abiodun. – 2014. – T. 27. – C. 58-73.
8. Control rods in a nuclear reactor [Electronic resource] // Rincon Educativo. – Access mode: <https://rinconeducativo.org/en/recursos-educativos/control-rods-in-a-nuclear-reactor/>
9. Elanchezhian C., Saravanakumar L., Ramnath B. V. Power Plant Engineering / C. Elanchezhian, L. Saravanakumar, B. Vijaya Ramnath. – I.K. International Publishing House, 2007.
10. Bodansky D. Nuclear Energy: Principles, Practices, and Prospects / D. Bodansky. – Springer, 2004.
11. Lamarsh J. Introduction to Nuclear Engineering / J. Lamarsh. – Addison-Wesley, 1983.

- 12.Grayson B. Fission Energy: Power from the Atom [Electronic resource] // Stanford University. – Access mode: <http://large.stanford.edu/courses/2011/ph241/grayson1/>
- 13.Nuclear Power [Electronic resource] // International Energy Agency (IEA). – Access mode: <https://www.iea.org/energy-system/electricity/nuclear-power>
- 14.Paulusová J., Dubravská M. APPLICATION OF DESIGN OF PID CONTROLLER FOR CONTINUOUS SYSTEMS / J. Paulusová, M. Dubravská. – 2012.
- 15.Introduction to PID Controllers - Theory, Tuning and Application to Frontier Areas [Electronic resource]. – 2012. – doi:10.5772/2422.
- 16.Golovinsky B.L. Theory of automatic control / B.L. Golovinsky, Y.V. Shurub, V.P. Lysenko. – K.: VC NUBiP of Ukraine, 2012.
- 17.Dorf R.C., Bishop R.H. Modern Control Systems / R.C. Dorf, R.H. Bishop. – Pearson Education, Inc., Pearson Prentice Hall, 2008. – 1046 p.
- 18.N. Kryvosheia. ANALYSIS OF UNCOMPENSATED DYNAMIC SYSTEM / N. Kryvosheia, A.Кліпа // ПОЛІТ. СУЧАСНІ ПРОБЛЕМИ НАУКИ. АЕРОНАВІГАЦІЯ. ЕЛЕКТРОНІКА. ТЕЛЕКОМУНІКАЦІЇ: XXIII міжнар. наук.-практична конф. здобувачів вищої освіти і молодих учених, 4-7 квіт. 2023 р.: тези доп. – К., 2023. – с. 145-146. – (<https://sites.google.com/npp.nau.edu.ua/faet/%D0%BD%D0%B0%D1%83%D0%BA%D0%B0/conferences/Polit>)
- 19.Kryvosheia N. Synthesis of PD-controller for Position Control of Nuclear Reactor Rods / N. Kryvosheia, A. Кліпа // Сучасні проблеми наукового забезпечення енергетики. У 2-х т. : Матеріали XXI Міжнар. наук.-практ. конф. молод. вчених і студ., м. Київ, 23–26 квіт. 2024 р. – Т. 2. – с. 258-260. - (<https://iate.kpi.ua/ua/144-konferencii-2024-roku-konferencii>)



```

%% 1 part
Pm = 55; tau = 0.0001; om = 3;
W1 = tf([1],[0 1]);
W2 = tf([1],[1 5 0]);
W3 = tf([0.5],[0 1]);
W4 = tf([58.2],[0 1]);
W12 = series(W1, W2);
W34 = series(W3, W4);
Wc = tf([1],[0 1]);
W5 = tf([1],[0.2 1]);

W_plant = series(W12,W34)
W_FF_Uncomp = series(Wc,W_plant) % FF of uncompensated
sys
W_OL_Uncomp = series(W_FF_Uncomp,W5) % Open-loop (OL)
of uncompensated sys
W_CL_Uncomp = feedback(W_FF_Uncomp,W5) % FB of
uncompensated sys

[p,z] = pzmap(W_CL_Uncomp)
figure(1), pzmap(W_CL_Uncomp)
figure(2), margin(W_OL_Uncomp)

figure(3), subplot(2,2,1), step(W_CL_Uncomp)
subplot(2,2,2), impulse(W_CL_Uncomp)

%% PD
[mag, phase] = bode(W_OL_Uncomp, om)
theta = -pi + Pm/57.296 - phase/57.296 %phase shift at
a om freq;
Kp=cos(theta)/mag %gain for the proportional part
Kd=sin(theta)/(om*mag) %gain for the differential part

% Creation of transfer function of real PD-controller
Gc=tf([Kp*tau+Kd Kp],[tau 1])
figure(4), bode(Gc)

%% Compensated system
%Transfer functions
W_FF_Comp = series(Gc,W_plant) % FF of compensated sys

```

```

W_OL_Comp = series(W_FF_Comp, W5) % FB of compensated
sys
W_CL_Comp = feedback(W_FF_Comp, W5) % CL of compensated
sys

[p2, z2] = pzmap(W_CL_Comp)
figure(5), pzmap(W_CL_Comp)
figure(6), margin(W_OL_Comp)

figure(7), subplot(2,1,1), step(W_CL_Comp)
subplot(2,1,2), impulse(W_CL_Comp)

%% Analysis of uncompensated and compensated systems
% Poles and Zeros location
[p1, z1] = pzmap(W_CL_Uncmp)
[p2, z2] = pzmap(W_CL_Comp)

% Step responses
figure(4),
subplot(2,1,1), step(W_CL_Uncmp)
subplot(2,1,2), step(W_CL_Comp)

% Main performance estimates
s1 = stepinfo(W_CL_Uncmp)
s2 = stepinfo(W_CL_Comp)

```