МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ
Факультет аеронавігації, електроніки та телекомунікацій
Кафедра аерокосмічних систем управління

ДОПУСТИТИ ДО ЗАХИСТУ
Завідувач кафедри АКСУ
___________Юрій МЕЛЬНИК
“_____” ____________ 2024 р.

КВАЛІФІКАЦІЙНА РОБОТА

(ПОЯСНЮВАЛЬНА ЗАПИСКА)
ВИПУСКНИКА ОСВІТНЬОГО СТУПЕНЯ
«БАКАЛАВР»

Тема: «Дискретний комплементарний фільтр для визначення орієнтації»

Виконавець: студент групи СУ-404Ба_____________ Микита ГРАНОВСЬКИЙ
Керівник: _______________________________________Лев РИЖКОВ

Нормоконтролер:_________________________________Микола ДИВНИЧ

Київ 2024

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL AVIATION UNIVERSITY

Faculty of Air Navigation, Electronics and Telecommunications

Aerospace Control Systems Department

APPROVED FOR DEFENCE

Head of the ACS Department

____________ Yurii MELNYK

“_____” ____________ 2024 y.

QUALIFICATION WORK

(EXPLANATORY NOTE)

FOR THE ACADEMIC DEGREE OF BACHELOR

Title: «Discrete complementary filter for determining orientation»

Submitted by: student of group CS-404: Mykyta HRANOVSKYI.

Supervisor: Lev RYZHKOV

Standards inspector: Mykola DYVNYCH

Kyiv 2024

НАЦІОНАЛЬНИЙ АВІАЦІЙНИЙ УНІВЕРСИТЕТ

Факультет аеронавігації, електроніки та телекомунікацій

Кафедра аерокосмічних систем управління

Спеціальність: 151 Автоматизація та комп’ютерно-інтегровані технології

ЗАТВЕРДЖУЮ

Завідувач кафедри АКСУ

________ Юрій МЕЛЬНИК

“ ____ ” __________ 2024 р.

ЗАВДАННЯ

на виконання кваліфікаційної роботи

Грановського Микити Євгеновича

1. **Тема кваліфікаційної роботи:**«Дискретний комплементарний фільтр для визначення орієнтації» затверджена наказом ректора від «13» квітня 2024 р. № 507/ст.
2. **Термін виконання роботи:** з 10.03.24 по 10.06.24.
3. **Вихідні дані роботи:** 3.1 Склад комплементарного фільтра – датчик кутової швидкості, акселерометр 3.2. Похибки визначення кутів – не більша $0{,}5^0$
4. **Зміст пояснювальної записки:**

Розділ 1. Основи визначення орієнтації;Розділ 2. Використання комплементарного фільтра в авіації;Розділ 3. Дискретний комплементарний фільтр.

5. **Перелік обов’язкового ілюстративного матеріалу:** Графіки результатів моделювання та розрахунків. Матеріали презентації в PowerPoint.

6. **Календарний план-графік**

№ пор.	Завдання	Термін виконання	Відмітка про виконання
1.	Отримання завдання	10.05.2024 – 11.05.2024	
2.	Формування мети та основних завдань дослідження	12.05.2024 – 13.05.2024	
3.	Аналіз існуючих методів визначення орієнтації	14.05.2024 – 22.05.2024	
4.	Розробка та моделювання комплементарного фільтра	23.05-2024 – 26.05.2024	
5	Дослідження впливу параметрів фільтра та компенсація похибок	27.05.2024-06.06.2024	
6	Підготовка презентації та роздаткового матеріалу	07.06.2024-11.06.2024	

7. **Дата видачі завдання:** “10” травня 2024 р.

Керівник кваліфікаційної роботи ________________ Лев РИЖКОВ
(підпис керівника)

Завдання прийняв до виконання ________________ Микита ГРАНОВСЬКИЙ
(підпис випускника)

NATIONAL AVIATION UNIVERSITY

Faculty of Air Navigation, Electronics and Telecommunications

Aerospace Control Systems Department

Specialty: 151 Automation and Computer-integrated Technologies

APPROVED BY

Head of the ACS Department

______ Yurii MELNYK

"____" __________2024

Qualification Paper Assignment for Graduate Student

Hranovskyi Mykyta Evgenovich

1. **The qualification paper title** «Discrete complementary filter for determining orientation» was approved by the Rector's order of “ 13 ” April 2024 № 507/ст.
2. **The paper to be completed between: 10.05.24 and 10.06.24**
3. **Initial data for the paper:** 3.1 Composition of the complementary filter - angular velocity sensor, accelerometer 3.2. Angle determination errors are no more than 0.50
4. **The content of the explanatory note:**

Chapter 1. Basics of determining orientation; Chapter 2. The use of a complementary filter in aviation; Chapter 3. Discrete complementary filter.

5. **The list of mandatory illustrations:** Graphs of modeling results and calculations. Presentation materials in PowerPoint.

6. Timetable:

№	Assignment	Dates of completion	Completion mark
1.	Receiving a task	10.05.2024 – 11.05.2024	
2.	Formation of the purpose and main objectives of the study	12.05.2024 – 13.05.2024	
3.	Analysis of existing orientation determination methods	14.05.2024 – 22.05.2024	
4.	Development and modeling of a complementary filter	23.05-2024 – 26.05.2024	
5	Study of the effect of filter parameters and error compensation	27.05.2024-06.06.2024	
6	Preparing a presentation and handouts	07.06.2024-11.06.2024	

7. Assignment issue date: “10” May 2024 y.

Qualification paper supervisor ______________________ Lev RYZHKOV
(signature)

Issued task accepted ______________________ Mykyta HRANOVSKYI
(signature)

РЕФЕРАТ

Текстова частина роботи: 42 с., 24 рис., 19 літературних джерела.

Об'єкт дослідження – дискретний комплементарний фільтр для визначення орієнтації рухомого об'єкта.

Предмет дослідження – системи визначення орієнтації рухомих об'єктів з використанням дискретних комплементарних фільтрів.

Мета роботи – дослідження дискретного комплементарного фільтра для визначення орієнтації рухомого об'єкта, а також оцінка його ефективності у практичному застосуванні в авіації.

Методи дослідження – теоретичний аналіз і огляд літератури, математичне моделювання та числове моделювання у середовищі Matlab-Simulink, експериментальні дослідження на основі реальних вимірювань, аналіз отриманих результатів і їх інтерпретація.

У роботі досліджено дискретний комплементарний фільтр для визначення орієнтації рухомого об'єкта, проведено моделювання у Matlab-Simulink, експериментальні дослідження на реальних даних та оцінено можливість його застосування у авіаційних системах навігації.

ABSTRACT

Text part of the work: 42 p., 24 fig., 19 literature sources.

Object of research – discrete complementary filter for determining the orientation of a moving object.

Subject of research – systems for determining the orientation of moving objects using discrete complementary filters.

Purpose of the work – research of a discrete complementary filter for determining the orientation of a moving object, as well as evaluating its effectiveness in practical application in aviation..

Research methods – theoretical analysis and review of the literature, mathematical modeling and numerical modeling in the Matlab-Simulink environment, experimental studies based on real measurements, analysis of the obtained results and their interpretation.

In the work, a discrete complementary filter for determining the orientation of a moving object was investigated, simulations were carried out in Matlab-Simulink, experimental studies were conducted on real data, and the possibility of its application in aviation navigation systems was evaluated.

Content

Introduction

Actuality of theme

In the modern conditions of the development of aviation and cosmonautics, determining the orientation of a moving object is one of the key tasks. Flight safety, navigation accuracy, and the efficiency of managing moving objects depend on the accuracy and reliability of the orientation determination system. Modern measurement systems such as gyroscopes and accelerometers provide a high level of accuracy, but they have their limitations and can be subject to various types of noise and errors.

Complementary filters, in particular discrete complementary filters, are effective tools for combining data from different sensors to improve the accuracy of orientation determination. The use of such filters makes it possible to compensate for the shortcomings of individual measuring devices and obtain more accurate results. This is especially important in conditions of dynamic changes, when the object is in motion, and its orientation can change quickly and unpredictably.

The use of discrete complementary filters in aviation makes it possible to increase the accuracy of navigation and control systems of aircraft. This has a direct impact on flight safety, reducing maintenance and operation costs, as well as on the development of new technologies in the field of avionics.

Thus, the topic of the thesis "Discrete complementary filter for orientation determination" is relevant and timely. It is aimed at solving important practical problems in the field of aviation technologies and has significant potential for further research and improvement of data processing methods from various measuring devices.

The purpose and tasks of the work

The aim of the thesis is the development and research of a discrete complementary filter for determining the orientation of a moving object, as well as an assessment of its effectiveness in practical application in aviation.

To achieve the goal, the following tasks must be solved:

1. Overview of existing methods of orientation determination and their comparative analysis.
2. Development of a mathematical model of a discrete complementary filter.
3. Modeling the operation of the filter in the Matlab-Simulink environment.
4. Study of the effect of filter parameters on its accuracy.
5. Assessment of the possibility of using the developed filter in navigation and control systems of aircraft.

Research methods

The following research methods are used in the thesis:

- Theoretical analysis and literature review on the research topic.
- Mathematical modeling and numerical modeling in the Matlab-Simulink environment.
- Analysis of the obtained results and their interpretation.

Structure of work

The thesis consists of three main sections. The first chapter covers the basics of orientation determination, including the principles of operation of gyroscopes and accelerometers, a comparison of Kalman and complementary filters, and the basics of discrete transformation.

The second chapter deals with the use of the complementary filter in aviation. This section examines the following issues in detail:

- The influence of the frequency of oscillating units on the operation of the complementary filter.
- Modeling of the complementary filter in the Matlab-Simulink environment.
- Effect of acceleration on the accuracy of the complementary filter.
- Compensation of offset of sensor zeros.

- Effect of gain coefficients on filter accuracy.

The third section deals with the discrete complementary filter, including the following aspects:

- Synthesis of the system for determining the orientation of a moving body.
- Selection of filter parameters to reduce the influence of linear accelerations.
- Implementation of discrete-type transmission functions.
- Synthesis of a discrete complementary filter.

Chapter 1: Basics of orientation determination

1.1 Gyroscopes and accelerometers

Determining the orientation of a moving object is a key aspect in many industries, including aviation, robotics, marine navigation, and other areas where accuracy and reliability are critical. Gyroscopes and accelerometers are the main sensors used for this purpose. In this subsection, we will consider the principles of their operation and their role in orientation determination systems.

1.1.1 Principle of operation of gyroscopes

Gyroscopes are devices used to measure the angular velocity of an object. They are based on the principle of conservation of angular momentum. There are several types of gyroscopes, including mechanical, optical, and MEMS gyroscopes.

- Mechanical Gyroscopes: This is the oldest type of gyroscope, which uses a rotating disk that maintains its orientation due to the high speed of rotation. The errors of such gyroscopes are usually caused by mechanical wear and friction.
- Optical gyroscopes: Use interferometry to measure angular velocity. The most famous types are fiber optic and laser gyroscopes. They provide high accuracy and have no moving parts, which reduces their wear.
- MEMS gyroscopes: Use microelectromechanical systems (MEMS) to measure angular velocity. They are compact, lightweight, and cost-effective, but can have larger errors compared to optical gyroscopes.

Gyroscopes are used to measure angular velocities in three axes: rotation around the X-axis, the Y-axis, and the Z-axis. The output from the gyroscope is integrated to obtain the angular position of the object. However, integration of angular velocity can lead to accumulated errors due to sensor drift, so gyroscopes are usually used in conjunction with other sensors to improve accuracy.

1.1.2 Principle of operation of accelerometers

Accelerometers measure the linear acceleration of an object. They are based on the principle of measuring the force acting on a mass that is fixed by a spring. There are different types of accelerometers, including mechanical, piezoelectric, and MEMS accelerometers.

- Mechanical accelerometers: Use a pendulum mechanism to measure acceleration. They are simple in design, but can be quite bulky.
- Piezoelectric accelerometers: Use piezoelectric materials that generate an electrical signal in response to mechanical deformation caused by acceleration. They are accurate and resistant to high temperatures, but cannot measure constant accelerations.
- MEMS accelerometers: Use microelectromechanical systems to measure acceleration. They are compact, light and cost-effective, but can have larger errors compared to piezoelectric accelerometers.

Accelerometers measure acceleration in three axes: X, Y, and Z. They are used to determine the inclination of an object by measuring the components of gravitational acceleration. However, accelerometers are sensitive to dynamic accelerations such as acceleration and deceleration, which can lead to errors in orientation determination.

1.1.3 Use of gyroscopes and accelerometers in orientation determination systems

Gyroscopes and accelerometers are often used together to improve orientation accuracy. Gyroscopes provide high accuracy in the short-term measurement of angular velocities, but their data can be prone to drift in the long term. Accelerometers, on the other hand, provide stable long-term measurements of an object's tilt, but can be sensitive to dynamic accelerations.

The combination of data from gyroscopes and accelerometers allows you to compensate for the shortcomings of each of the sensors and get more accurate results. This approach is the basis for complementary filters, which will be discussed in detail in the following sections.

Understanding the principles of operation of gyroscopes and accelerometers, as well as their characteristics and limitations, is essential to the development of effective orientation detection systems. In the following subsections, we will look at how these sensors are used in complementary filters and how they can be combined to achieve high accuracy and reliability in various applications.

1.2 Kalman filter and complementary filter

In systems for determining the orientation of a moving object, various filtering methods are used to process data from gyroscopes and accelerometers. One of the most popular methods is the Kalman filter and the complementary filter. In this subsection, we will consider the basics of each of them and conduct a comparative analysis.

1.2.1 Basics of the Kalman filter

The Kalman filter is an optimal recursive filter used to estimate the state of a system based on incomplete or noisy measurements. It was developed by Rudolf Kalman in 1960 and has become the basis for many applications in various fields, including aviation, navigation, financial analysis and others.

The basic idea of the Kalman filter is that it uses a model of the system state and measurements to obtain an estimate of the current state of the system with a minimum root mean square error. The Kalman filter consists of two main steps: prediction and update.

- Prediction stage: In this stage, the filter uses the system dynamics model to predict the next state of the system based on the previous state and control influences.

$$\hat{x}_{k|k-1} = F\hat{x}_{k-1|k-1} + Bu_k$$
$$P_{k|k-1} = FP_{k-1|k-1}F^T + Q$$

where $\hat{x}_{k|k-1}$ — predicted state, F — transition function matrix, B — management matrix, u_k — control vector, $P_{k|k-1}$ — predicted error covariance, Q — process noise covariance.

- Update phase: In this phase, the filter uses new measurements to update the estimate of the system state and reduce uncertainty.

$$K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T + R)^{-1}$$
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - H\hat{x}_{k|k-1})$$
$$P_{k|k} = (I - K_kH)P_{k|k-1}$$

where K_k — the Kalman coefficient, H — matrix of measurements, z_k — vector of measurements, R — covariance of measurement noise, I — unit matrix.

The Kalman filter is a powerful data processing tool, but its implementation can be challenging due to the need to accurately model system dynamics and noise.

1.2.2 Basics of the complementary filter

Комплементарний фільтр є більш простим у реалізації, ніж Калманівський фільтр, і часто використовується для обробки даних з гіроскопів та акселерометрів. Основна ідея комплементарного фільтра полягає в тому, що він комбінує дані з різних сенсорів, використовуючи їх комплементарні властивості.

Комплементарний фільтр припускає, що похибки гіроскопа є низькочастотними, а похибки акселерометра — високочастотними. Тому фільтр використовує низькочастотну складову сигналу з акселерометра та високочастотну складову сигналу з гіроскопа для отримання точної оцінки орієнтації.

The basic equations of the complementary filter for estimating the tilt angle θ look like this:

$$\theta_{filt} = (1-\alpha)(\theta_{filt} + \omega_{gyro}\Delta t) + \alpha\theta_{acc}$$

where θ_{filt} — estimated angle of inclination, ω_{gyro} — angular velocity from the gyroscope, Δt — time interval, θ_{acc} — tilt angle from the accelerometer, α — filter coefficient, which defines the frequency properties of the filter.

Parameter α defines the weight given to each sensor. Low values give more weight to the gyro data, while high values α give more weight to the data from the accelerometer. Value α is usually chosen experimentally to achieve optimal filter performance.

1.2.3 Comparison of Kalman and complementary filters

- Difficulty of implementation: Kalman filter is more difficult to implement because it requires accurate modeling of system dynamics and noise. The complementary filter is simpler and does not require complex calculations.

- Accuracy: The Kalman filter provides an optimal estimate of the system state with a minimum root mean square error. The complementary filter can be less accurate, but provides sufficient accuracy for many practical applications.
- Flexibility: The Kalman filter can be adapted for different types of systems and measurements, while the complementary filter is usually used to process data from specific sensors (gyroscopes and accelerometers).
- Computational resources: The complementary filter requires less computational resources, making it more attractive for resource-constrained embedded systems.

Both Kalman and complementary filters have their advantages and disadvantages. The choice of filter depends on specific requirements and application conditions. In many cases, the complementary filter is quite effective and simple to implement, which makes it a popular choice for systems for determining the orientation of moving objects. In the following sections, we will consider in more detail the use of the complementary filter, its modifications and applications in aviation systems.

1.3 Discrete transformation

Discrete conversion is a key element in digital signal processing, particularly in the implementation of discrete complementary filters. In this subsection, we will review the basic concepts and principles of the discrete transform that are necessary to understand and implement the discrete complementary filter.

1.3.1 Basic concepts of discrete transformation

Discrete transformation involves the transformation of continuous signals into discrete signals consisting of a set of values taken at specific moments in time. The basic concepts of discrete transformation include:

- Sampling: This is the process of converting a continuous signal into a discrete one by sampling the signal values at fixed moments of time. The sampling rate determines how often the signal values are taken.

$$x[n] = x(nT_s)$$

where x[n] — discrete signal, x(t) — continuous signal, T_s — discretization period, n — discrete time index.

- Quantization: This is the process of converting the continuous values of a discrete signal into a finite set of values. Quantization results in errors called quantum errors.
- Reconstruction: This is the process of converting a discrete signal back into a continuous one. Perfect reconstruction is possible only if the sampling rate satisfies the Nyquist-Shannon theorem.

1.3.2 Laplace transform and Z-transform

Various mathematical tools are used to analyze and process discrete signals, including the Laplace transform and the Z-transform.

- Laplace transform: Used to analyze continuous systems. The Laplace transform transforms differential equations into algebraic ones, which greatly simplifies their solution.

$$X(s) = \int_0^\infty x(t)e^{-st}dt$$

where X(s) — function image x(t), s — complex variable.

- Z-transformation: Used to analyze discrete systems. The Z-transformation transforms difference equations into algebraic ones.

$$X(z) = \sum_{n=0}^\infty x[n]z^{-n}$$

where X(z) — image of a discrete function x[n], z — complex variable.

The Z-transform is a discrete analogue of the Laplace transform and is used to analyze the stability and frequency characteristics of discrete systems.

1.3.3 Discrete systems and filters

Discrete signal processing systems are used to filter, analyze and process discrete signals. The main components of discrete systems are discrete filters.

- Discrete filters: Used to highlight the desired signal components or to suppress unwanted noise. There are two main types of discrete filters:

- Finite Impulse Response (FIR) Filters: These filters have an impulse response that ends after a finite number of steps. FIR filters are stable and have a linear phase characteristic.
- Infinite Impulse Response (IIR) Filters: These filters have an impulse response that does not end after a finite number of steps. IIR filters can be more efficient, but can be unstable and have a non-linear phase response.

An understanding of the principles of discrete transformation is necessary for the design and implementation of discrete complementary filters. Discretization, quantization, Z-transformation are the main components that ensure efficient processing of discrete signals. In the following sections, we will consider how these principles are applied to the synthesis of a discrete complementary filter to determine the orientation of a moving object.

Chapter 2: Use of the complementary filter in aviation

2.1 Introduction to the study of the operation of the complementary filter

The complementary filter is a simple and effective tool for combining data from different sensors, such as gyroscopes and accelerometers, to determine the orientation of a moving object. In aviation systems, orientation determination is critical to ensure flight safety, navigation accuracy, and effective aircraft control. In this section, we will consider the principles of operation of the complementary filter, its application in aviation systems, and the results of studies conducted to evaluate its effectiveness.

2.1.1 The principle of operation of the complementary filter

As mentioned in the previous section, the complementary filter combines data from gyroscopes and accelerometers to obtain an accurate estimation of orientation. Gyroscopes provide high-frequency measurements of angular velocity, which can be subject to drift over time. Accelerometers, on the other hand, provide low-frequency measurements of an object's tilt, but can be sensitive to dynamic accelerations. The complementary filter uses these complementary properties to obtain accurate orientation data.

The basic equations of the complementary filter for estimating the tilt angle θ look like this:

$$\theta_{filt} = (1-\alpha)(\theta_{filt} + \omega_{gyro}\Delta t) + \alpha\theta_{acc}$$

where:

- θ_{filt} — estimated angle of inclination,

- ω_{gyro}— angular velocity from the gyroscope,

- Δt— time interval,

- θ_{acc} — tilt angle from the accelerometer,

- α — filter coefficient, which determines the frequency properties of the filter.

Parameter α defines the weight given to each sensor. Low values α give more weight to gyroscope data, while high values give more weight to accelerometer data. Value α is usually chosen experimentally to achieve optimal filter performance.

2.1.2 Application of the complementary filter in aviation systems

In aviation systems, accurate orientation determination is critical for safe and efficient aircraft control. The complementary filter allows you to combine the high accuracy of gyroscopes in short-term measurements with the stability of accelerometers in long-term measurements. This ensures high accuracy and reliability of orientation determination even in difficult conditions.

A complementary filter is used in such systems as:

- Autopilots: To accurately determine the position and orientation of the aircraft.
- Navigation systems: To provide accurate orientation and position data during flight.
- Flight control systems: To stabilize and control the movement of the aircraft.

2.1.3 Review of studies on the operation of the complementary filter

Studies of the operation of the complementary filter include modeling its operation, evaluating the influence of various parameters on the accuracy and stability of the filter, as well as experimental studies based on real data. Below are key aspects of such studies:

- Modeling: Modeling the complementary filter in the Matlab-Simulink environment allows you to evaluate its effectiveness and optimize the filter parameters. Modeling includes the creation of a model of a moving object with gyroscopes and accelerometers, as well as the implementation of a complementary filter algorithm for sensor data processing.
- Experimental studies: Conducting experiments using real data from sensors allows you to evaluate the performance of the complementary filter in real conditions. This includes measuring the angular velocities and accelerations of a moving object, processing the data with a complementary filter, and comparing the obtained results with real orientation values.
- Effect of parameters: Investigation of the effect of filter parameters, such as the filter coefficient α, sampling frequency and characteristics of the sensors allow to optimize the operation of the filter and achieve high accuracy of orientation determination.

The complementary filter is an effective tool for determining the orientation of moving objects in aviation systems. It combines the high accuracy of gyroscopes with the stability of accelerometers, providing reliable results even in difficult conditions. Research and modeling of the operation of the complementary filter allow to optimize its parameters and increase the accuracy of determining the orientation, which is critical for the safety and efficiency of aircraft control.

2.2 Influence of the frequency of oscillating units on the operation of a complementary filter

We will accept x_0 and amplitudes 650, 16. The corresponding scheme is shown in the figure 1.1:

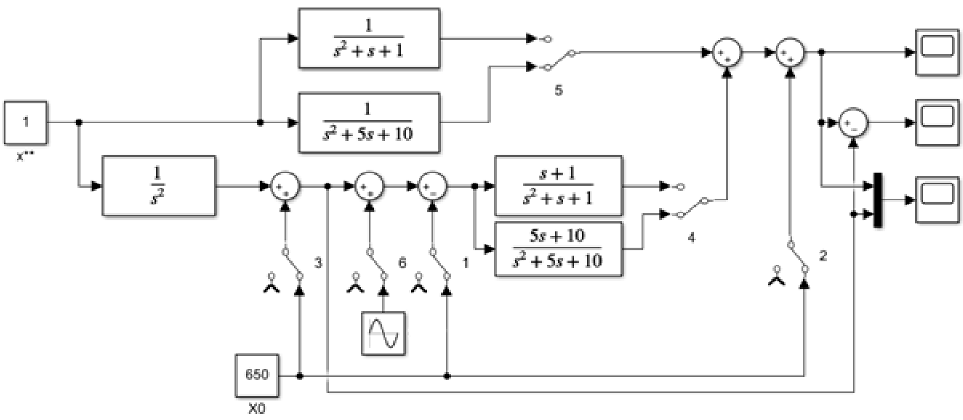


Fig. 1.1

We will assume that switched on switches 4 and 5 correspond to the functions of lower frequencies, and switched on - to higher frequencies. The following measurements were made:

- influence of the initial value x_0 (650) on the output signal without compensation of its influence and with compensation for the transfer functions given in the description;

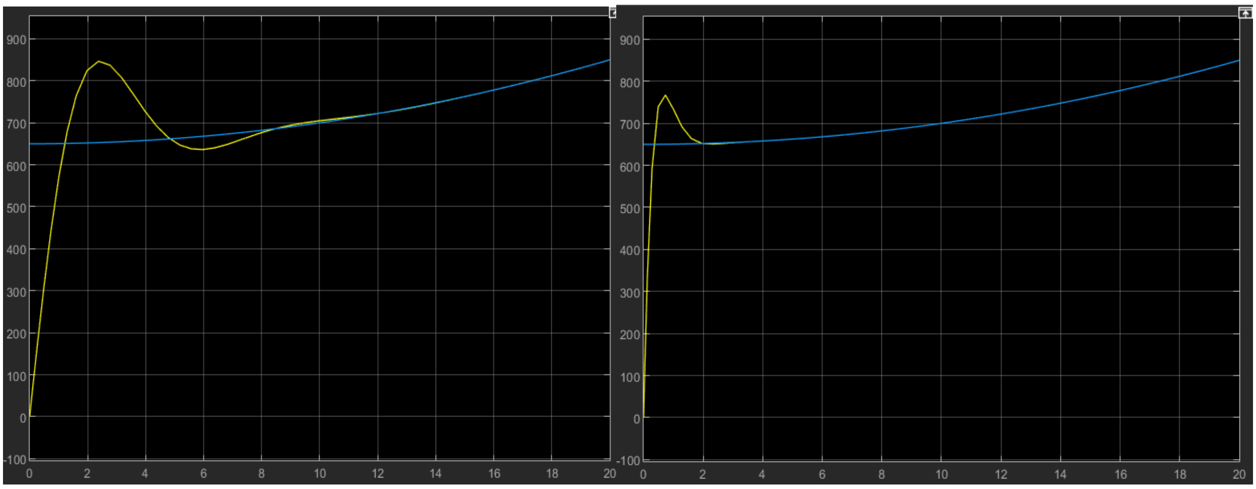


Fig.1.2, Without compensation for lower frequency and for higher frequency

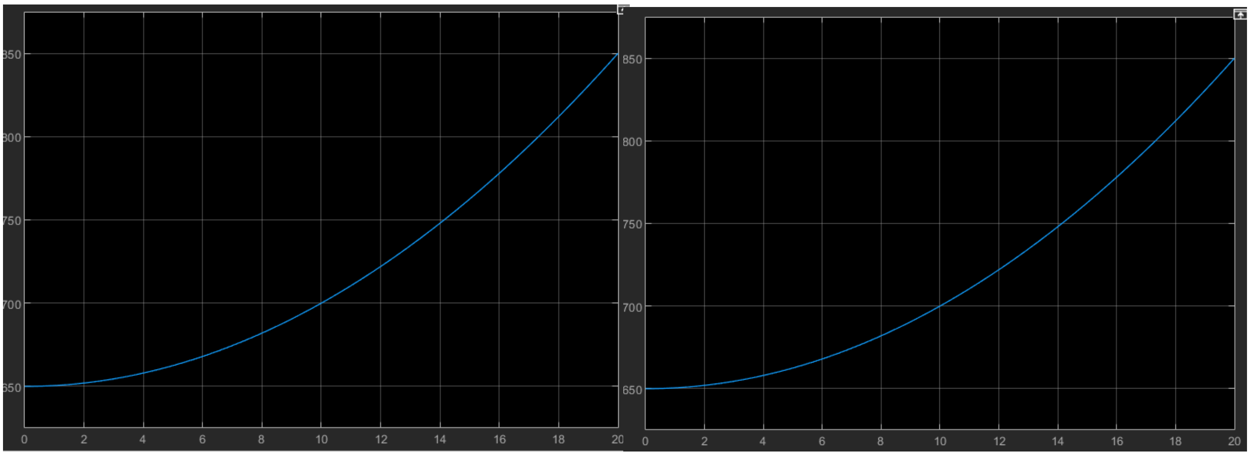
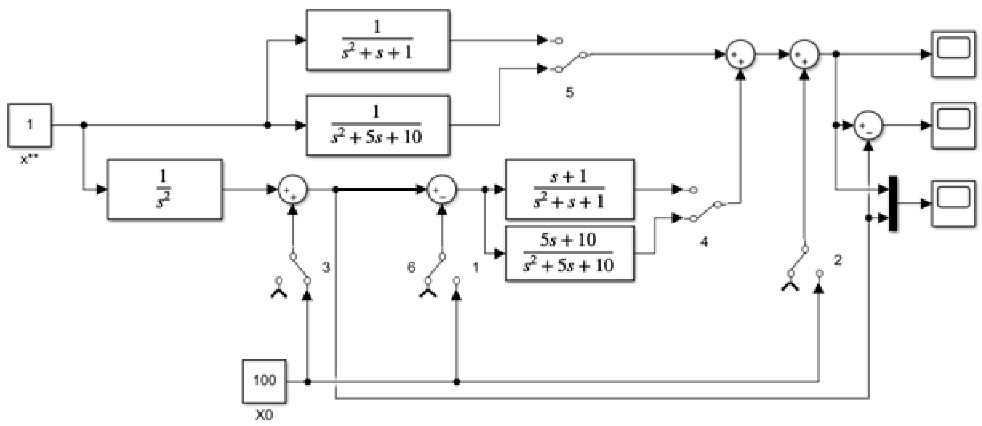


Fig. 1.3, With compensation for lower frequency and for higher frequency

We see that the adjusting links do not affect the dependence of the output parameter on the measured variable. There is a reduction in the influence of high-frequency interference components. In the transient process, there is an input signal error. If keys 1 and 2 are switched, then the blue curve coincides with the yellow one. This error can be reduced by increasing the frequency of self-oscillations of the correcting links and the damping coefficient.

2.3 Modeling of the complementary filter in the Matlab-Simulink environment

Let's consider an example: $\ddot{x} = 1м/с^2; x_0 = 100м; t = 20c$. The simulation scheme in the Matlab-Simulink environment is shown in Fig. 1.4.



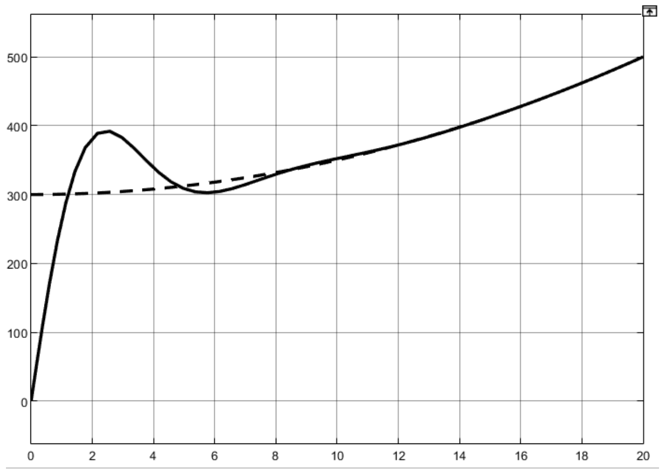
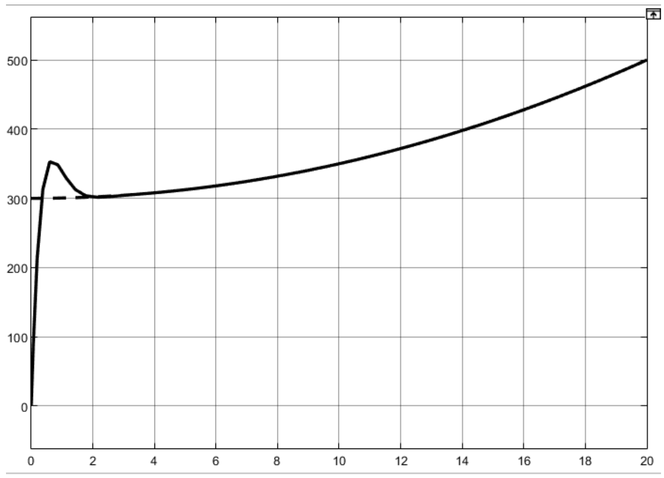


Fig.1.4, The general scheme according to which further modeling takes place. Switches 1 and 2 are responsible for compensation; switches 4 and 5 are responsible for decreasing or increasing the frequency of the function.

The dashed line is an input variable, solid – output.



Two processes were simulated with different frequencies of natural oscillations and damping coefficients. According to the simulation results, we can see that the error can be reduced by increasing the frequency of the self-oscillations of the correcting links and the damping coefficient - which is what we tried to achieve in the course of this experiment.

2.4 Effect of acceleration on the accuracy of the complementary filter

In general, accelerometers, regardless of the principle of operation, estimate the apparent acceleration - the vector difference between the true acceleration and the acceleration of free fall $f = a - g$. The direction of the imaginary acceleration does not coincide with the vertical and, as a result, errors due to accelerations appear in the estimates of roll and pitch.

The simplest method was developed for aircraft gyroverticals - to reduce or completely turn off the control angular speed in the presence of accelerations or angular speed of the course (which indicates a maneuver - a turn). It is not always possible to distinguish the true accelerations from the free-fall acceleration projections, caused, for example, by the tilt of the gyro platform, which needs to be compensated for. Therefore, the method is unreliable.

A more accurate method is based on the use of external speed measurements from the GNSS receiver. If the speed v is known, then it can be differentiated and the value of the acceleration can be obtained $\dot{v}$. Then the difference … - … will be exactly equal - … regardless of body movement. It can be used as a vertical standard.

For example, you can set the control angular velocities in the form:

$$\omega_N' = gk_g\left(\hat{f}_E - \dot{v}_E\right), \omega_E' = -gk_g\left(\hat{f}_N - \dot{v}_N\right)$$

2.5 Offset of sensor zeros

The disadvantage and feature of MEMS gyroscopes and accelerometers - sensors is the instability of zero shifts over time and at different temperatures. Calibration is not enough to compensate for them - you need to compensate for them during operation.

2.5.1 Gyroscopes

The calculated position of the connected coordinate system deviates from the true position with an angular velocity determined by two opposing factors - the zero displacement of the gyroscope and the control angular velocity: $\delta\omega - \omega'$. If the correction system managed to compensate for the zero drift of the gyroscope, then in the set mode $\delta\omega = \omega'$.

That is, in the control speed ω'зthere is information about an unknown active disturbance $\delta\omega$. Therefore, compensatory estimation can be applied - we do not know the magnitude of the disturbances directly, but we know what compensating controls are needed to compensate for them. This is the basis for estimating the zero displacement of the gyroscope.

2.5.2 Accelerometers

Estimating the displacement of the zeros of the accelerometer is much more difficult. In rectilinear motion, the effect of the zero offset of the accelerometers cannot be distinguished from the inclination of the carrier or the misalignment of the installation of the measuring unit on it. No additional error to ω' accelerometers do not add. The error appears only when turning, which allows you to separate and independently evaluate the errors of gyroscopes and accelerometers.

2.6 Filter gain coefficients

The gain of the filter β represents all the measurement errors of the zeros of the gyroscope, expressed as the value of the derived quaternion.

Sources of error: sensor noise, approximating filter, quantization errors, calibration errors, sensor installation and alignment errors, non-orthogonality of sensor axes, and frequency response.

The gain of the filter ς я represents the rate of convergence to compensate for non-zero gyroscope measurement errors and is represented as the quaternion derivative.

These errors represent the displacement of the gyroscope. Determine β and ς it is possible with the help of angular values ω_β and $\omega_{\dot{\varsigma}}$, where ω_β - average estimate of zero measurement error on each axis; $\omega_{\dot{\varsigma}}$ - estimated drift speed (distance) along each axis.

$$\beta = \left\| \frac{1}{2}\hat{q} \otimes [0 \quad \underline{\omega_b} \quad \underline{\omega_b} \quad \underline{\omega_b}] \right\| = \sqrt{\frac{3}{4}\underline{\omega_b}}$$

$$\varsigma = \sqrt{\frac{3}{4}\underline{\omega_{\dot{\varsigma}}}}$$

2.6.1 Influence of the amplification factor on the accuracy of the complementary filter

The results of the study of the influence of the coefficient β on the accuracy of the filter are shown in the form of graphs in fig. 10. Experimental data were processed limiting values β in the range from 0 to 0.5. It is worth noting that for each implementation of the filter there is an optimal value β, which is high enough to compensate for error accumulation and low enough to keep unnecessary noise out of the calculator.

2.6.2 Influence of measurement frequency on accuracy

Figure 1.5 shows the results of the study of the influence of the frequency of measurements on the accuracy of the complimentary filter. The experiment was conducted with the optimal value of the amplification factor β for higher and lower oscillation frequency.

The change in the measurement frequency was simulated by passing part of the measurements at a frequency from 1 Hz to 512 Hz. It can be seen in Fig 1.5 that the proposed filter achieves the same level of accuracy at a frequency of 50 Hz as at a frequency of 512 Hz. Both implementations of the faltar achieve a reduction of static error (in a stationary state) < 2 degrees and dynamic error (in a moving state) < 7 degrees at a measurement frequency of 10 Hz. This level of accuracy is quite acceptable.

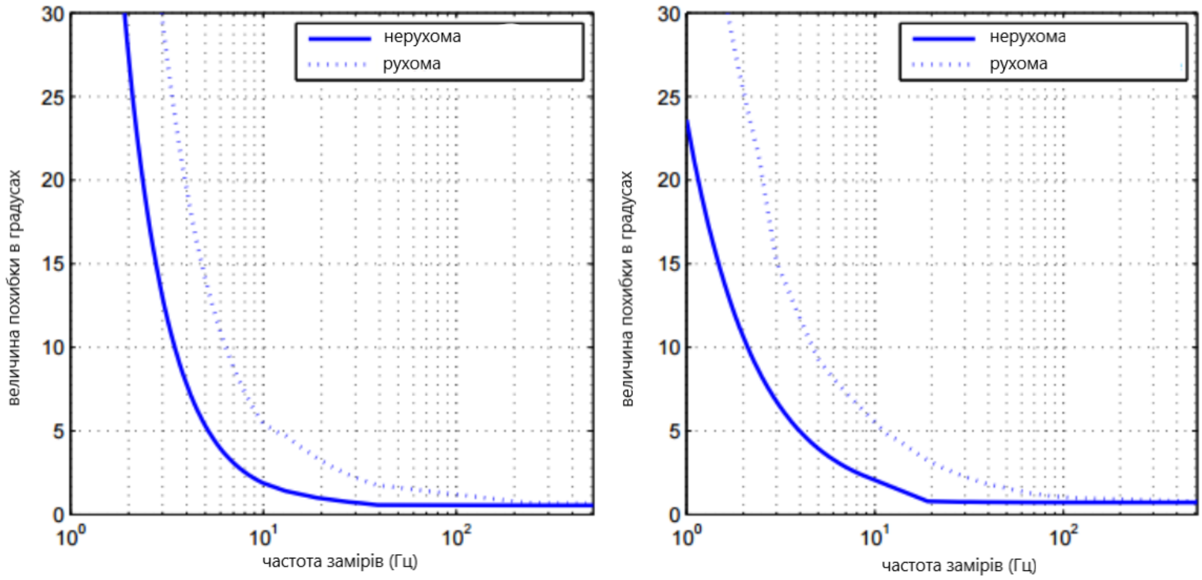


Fig. 1.5

Chapter 3: Discrete complementary filter

3.1 Synthesis of the system for determining the orientation of a moving body

The synthesis of the system for determining the orientation of a moving body is an important step in the development of a discrete complementary filter. This process includes choosing the structure of the system, modeling its operation, analyzing the effect of various parameters on accuracy and stability, and optimizing the settings to achieve high efficiency. In this subsection, we review the main stages of system synthesis, including structure selection, modeling, and analysis.

3.1.1 Selection of the structure of the system for determining the orientation of a moving body

Currently, the complementary filter scheme shown in fig. is most often used to measure the angular position of the Fig. 2.1.

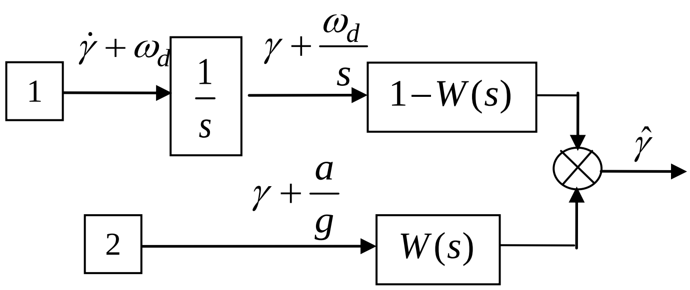


Fig. 2.1

Here 1 – gyroscope (angular velocity sensor), 2 – accelerometer. γ – rotation angle of the object; $\hat{\gamma}$ – output; ω_d – gyroscope drift angular velocity; a – acceleration of the object.

If accepted $W(s)=\dfrac{1}{Ts+1}$, we will have the following scheme (Fig. 2.2)

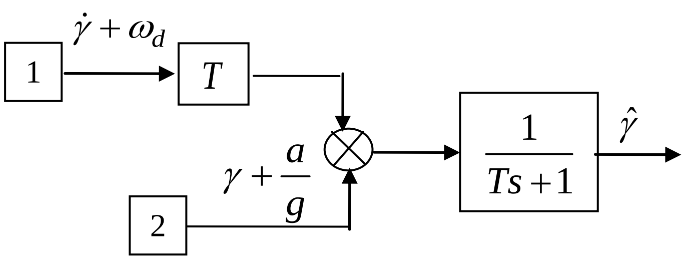


Fig. 2.2

It is significant that in this scheme there is no integration operation of the signal from the gyroscope. At the same time, due to the proper selection of the time constant, the influence of high-frequency disturbances of the accelerometer can be significantly reduced.

At the same time, in real operating conditions, the maximum error of the accelerometer (as an angle meter) is often determined by the linear acceleration of the object. This happens when accelerating and stopping the object, when turning the object.

3.1.2 Analysis of the influence of accelerations on the accuracy of the complimentary filter

We will analyze the impact of such accelerations on the accuracy of the complementary filter and consider the issue of choosing the filter structure from this point of view.

Let's write down the expression for the filter error

$$\delta = \frac{T}{Ts+1}\omega_d + \frac{1}{Ts+1}\frac{a}{g}, \qquad (3.1)$$

where ω_d – drift angular velocity; a – acceleration of the object.

Let us assume that the acceleration of the object takes place during a short period of time τ (a few seconds). To reduce its influence, the time constant of the aperiodic link should be significantly longer τ.

We will accept $\frac{a}{g} = 0,5;\ \tau = 5c;\ T = 100c;\ \omega_{\partial} = 1 \cdot 10^{-4}\, 1/c$. That is, the angle between the full acceleration and the vertical position is $26,6^0$.

That is, the angle between the full acceleration and the vertical position is

$$\delta_a \approx \frac{a}{g}\frac{\tau}{T}e^{-\frac{t}{T}}. \qquad (3.2)$$

For accepted data, we get $\delta_{am} = \frac{a}{g}\frac{\tau}{T} = 1,43^0$. The error due to the drift of the gyroscope becomes significant: the static error due to the drift is equal to $\delta_{\text{гст}} = T\omega_d = 0,57^0$. The simulation results are shown in Fig. 2.3.

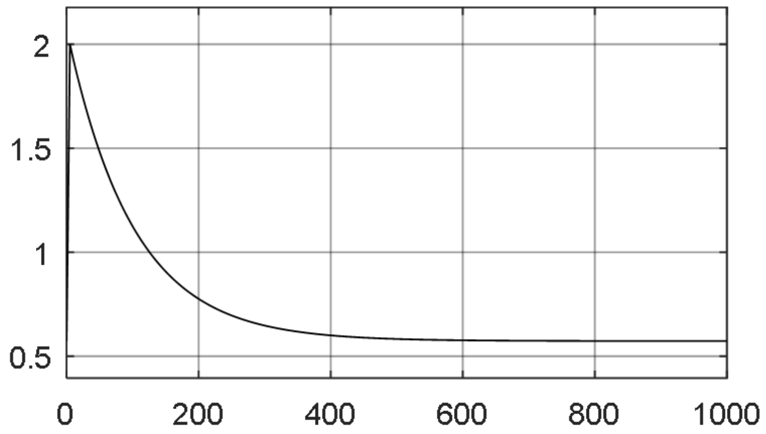


Fig. 2.3.

Thus, the use of a complementary filter based on an aperiodic link does not provide high accuracy in the presence of linear accelerations of the object.

Let's change the structure of the filter and accept $W(s) = \dfrac{r_1 s + r_2}{s^2 + r_1 s + r_2}$. We will have the following equation for the error:

$$\delta = \frac{s}{s^2 + r_1 s + r_2}\omega_d + \frac{r_1 s + r_2}{s^2 + r_1 s + r_2}\frac{a}{g}. \qquad (3.3)$$

We see that there is no static error due to the drift of the gyroscope.

3.2 Selection of filter parameters to reduce the influence of linear accelerations

Let's consider the choice of parameters from the point of view of reducing the influence of linear accelerations. For this, the period of own oscillations $\dfrac{2\pi}{\sqrt{r_2}}$ should be much more time τ. We will accept $\dfrac{2\pi}{\sqrt{r_2}} = 100\tau$, namely $r_2 = \left(\dfrac{2\pi}{100\tau}\right)^2 = 1{,}6 \cdot 10^{-4}\, 1/c^2$.

We will accept $r_1 = 1 \cdot 10^{-2}\, 1/c$.

The modeling scheme is shown in Fig 2.5. The simulation results are shown in Fig 2.4.

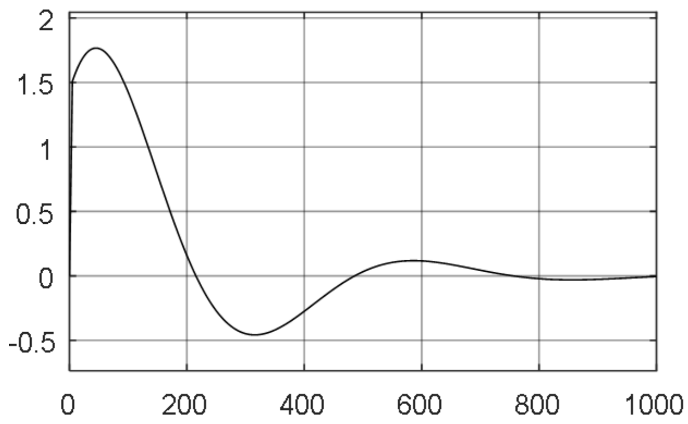


Fig. 2.4.

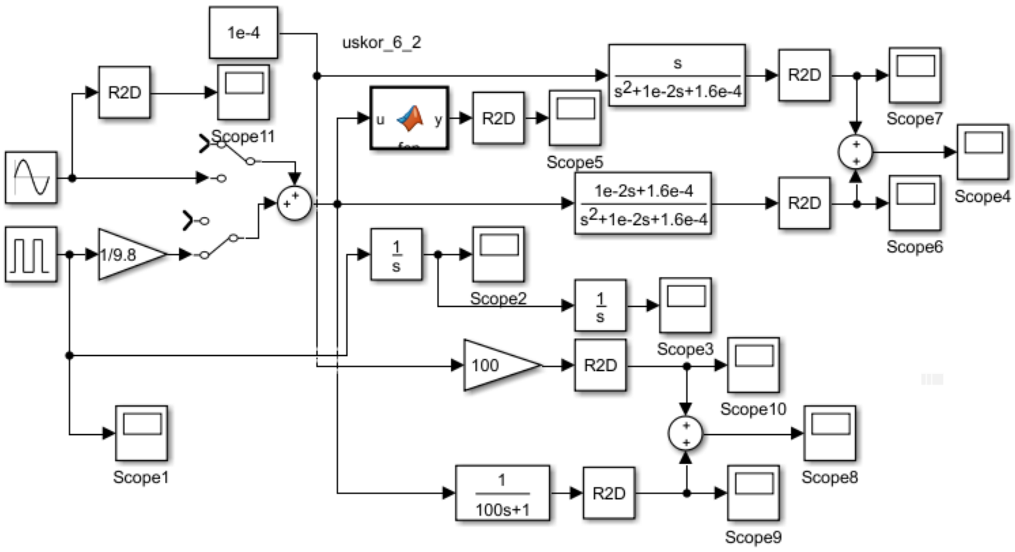


Fig. 2.5

If we consider that for small values τ can be accepted $\frac{r_2}{s^2+r_1s+r_2}\frac{a}{g} \approx \frac{r_2}{s^2+r_1s+r_2} s\frac{a\tau}{g}$, is an expression (3.3) can be written as:

$$\delta \approx \frac{s}{s^2+r_1s+r_2}\omega_d + \frac{r_2}{s^2+r_1s+r_2}\frac{a}{g} \approx \frac{r_2}{s^2+r_1s+r_2} s\frac{a\tau}{g} = \frac{s}{s^2+r_1s+r_2}\omega_{de},$$

where $\omega_{de} = \omega_d + \frac{a}{g}r_2\tau$.

That is, the presence of short-term acceleration is equivalent to an increase in gyroscope drift. The simulation results are shown in Fig. 2.6.

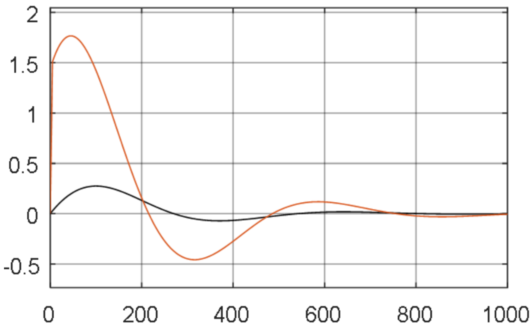


Fig. 2.6.

3.3 Implementation of discrete-type transfer functions

The implementation of discrete-type transfer functions is a key aspect of the development of a discrete complementary filter for determining the orientation of a moving body. Discrete transfer functions allow digital processing of signals received from gyroscopes and accelerometers, which ensures high accuracy and stability of the filter. In this subsection, we will consider the basic principles of discretization of transfer functions, their implementation in digital systems, and examples of use in Matlab-Simulink.

3.3.1 Link of the 1st order

We have the equation

$$y(s) = \frac{k}{Ts+1} x(s) \tag{4.1}$$

or

$$T\dot{y} + y = kx \,. \tag{4.2}$$

For a discrete system, we have the following expression for the step derivative n

$$\dot{y}(n) = \frac{y(n+1) - y(n)}{\tau}, \tag{4.3}$$

where τ – sampling time.

Let's write equation (4.2) like this

$$T \frac{y(n+1) - y(n)}{\tau} + y(n) = kx(n) \,. \tag{4.4}$$

From here we find

$$y(n+1) = k\frac{\tau}{T}x(n) + \left(1 - \frac{\tau}{T}\right)y(n)\,. \qquad (4.5)$$

or

$$y(n) = k\frac{\tau}{T}x(n-1) + \left(1 - \frac{\tau}{T}\right)y(n-1)\,. \qquad (4.6)$$

Let's enter the bias symbol one step z, namely, $y(n+1) = z\,y(n)$. Then, $y(n-1) = z^{-1}\,y(n)$, and the equation (4.6) takes shape

$$y(n) = \frac{k}{T}z^{-1}x(n) + \left(1 - \frac{\tau}{T}\right)z^{-1}y(n) \qquad (4.7)$$

or

$$y(n) = z^{-1}\left(\frac{k}{T}x(n) + \left(1 - \frac{\tau}{T}\right)y(n)\right). \qquad (4.8)$$

Equation (4.8) corresponds to the following scheme (Fig. 2.11) in the Simulink environment (parameter $\tau = 0.04$ is entered into the workspace before modeling).

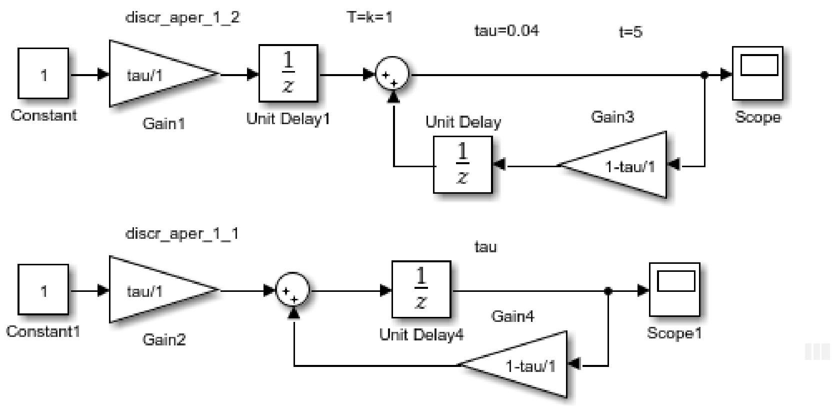


Fig. 2.11

Systems of this structure are called recursive filters.

3.3.2 Link of the 2nd order.

We have the equation

$$y(s) = \frac{k_1 s + k_2}{a_1 s^2 + a_2 s + a_3}x(s) \qquad (4.9)$$

or

$$a_1\ddot{y}+a_2\dot{y}+a_3y=k_1\dot{x}+k_2x\,. \qquad (4.10)$$

We have

$$\dot{y}(n)=\frac{z-1}{\tau}y(n);\ \ \ddot{y}(n)=\left(\frac{z-1}{\tau}\right)^2 y(n)=\frac{z^2-2z+1}{\tau^2}y(n) \quad , \qquad (4.11)$$

where z^2 – two-step bias symbol, namely $y(n+2)=z^2\,y(n)$.

Then equation (4.10) can be written as follows

$$\left(a_1\frac{z^2-2z+1}{\tau^2}+a_2\frac{z-1}{\tau}+1\right)y(n)=\left(k_1\frac{z-1}{\tau}+k_2\right)x(n) \qquad (4.12)$$

or

$$\left[Z^2+\left(-2+\frac{a_2}{a_1}\tau\right)Z+1-\frac{a_2}{a_1}\tau+\frac{\tau^2}{a_1}\right]y(n)=\left[\frac{k_1\tau}{a_1}Z+\frac{\tau}{a_1}\left(k_2\tau-k_1\right)\right]x(n)\,. \qquad (4.13)$$

Equation (4.13) will be rewritten in the form

$$y(n)=\frac{1}{a_1}z^{-1}\left\{\left[k_1\tau+\left(k_2\tau^2-k_1\tau\right)z^{-1}\right]x(n)+\left[2a_1-a_2\tau-\left(a_1-a_2\tau+a_3\tau^2\right)z^{-1}\right]y(n)\right\} (4.14)$$

Equation (4.14) corresponds to the following scheme (Fig. 2.12) in the Simulink environment

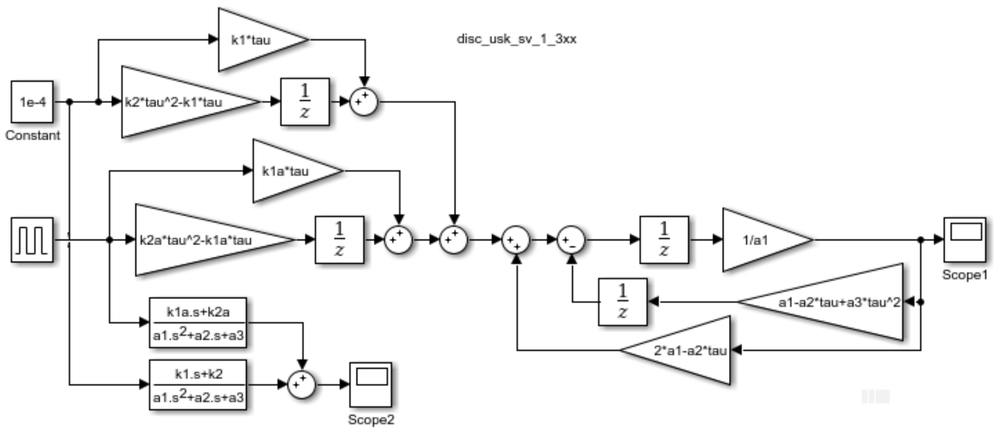


Fig. 2.12

Equation (4.14) with acceleration corresponds to the following scheme (Fig. 2.13) in Simulink

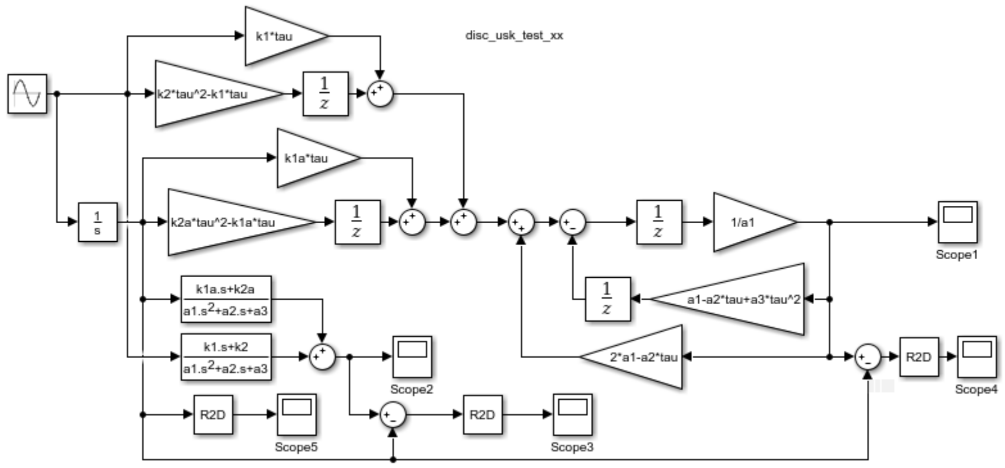


Fig. 2.13

In the analysis of the complementary filter is accepted: $a_1 = k_1 = 1;\ k_2 = 0;\ k_{1a} = a_2;\ \ k_{2a} = a_3.$

Next, we will change the notation and accept the following transfer functions:

- for the gyroscope

$$W_г(s) = \frac{s}{s^2 + b_1 s + b_2};$$

- for the accelerometer

$$W_a(s) = \frac{b_1 s + b_2}{s^2 + b_1 s + b_2}.$$

$$y(n) = z^{-1}\left\{\left(1 - z^{-1}\right)\tau G(n) + \left[b_1\tau + \left(b_2\tau^2 - b_1\tau\right)z^{-1}\right]A(n) + \left[2 - b_1\tau - \left(1 - b_1\tau + b_2\tau^2\right)z^{-1}\right]y(n)\right\}$$

$$\gamma(n) = \tau G(n-1) - \tau G(n-2) + b_1\tau A(n-1) + \left(b_2\tau^2 - b_1\tau\right)A(n-2) +$$
$$+\left(2 - b_1\tau\right)y(n-1) - \left(1 - b_1\tau + b_2\tau^2\right)y(n-2)$$

The corresponding equation corresponds to the following scheme (Fig. 2.14) in Simulink:

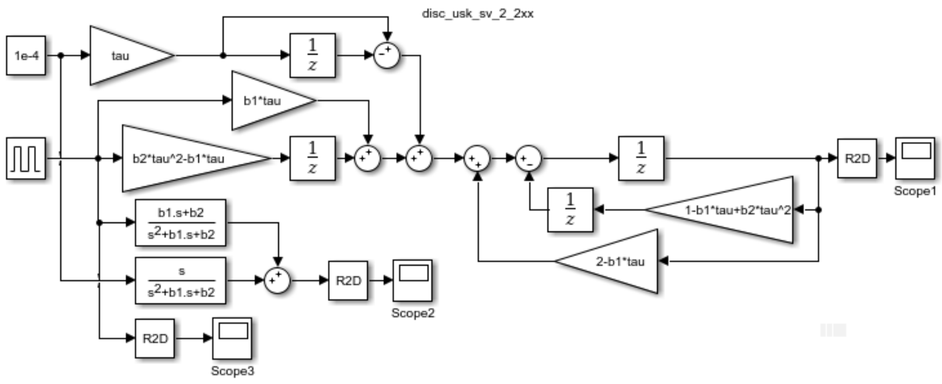


Fig. 2.14

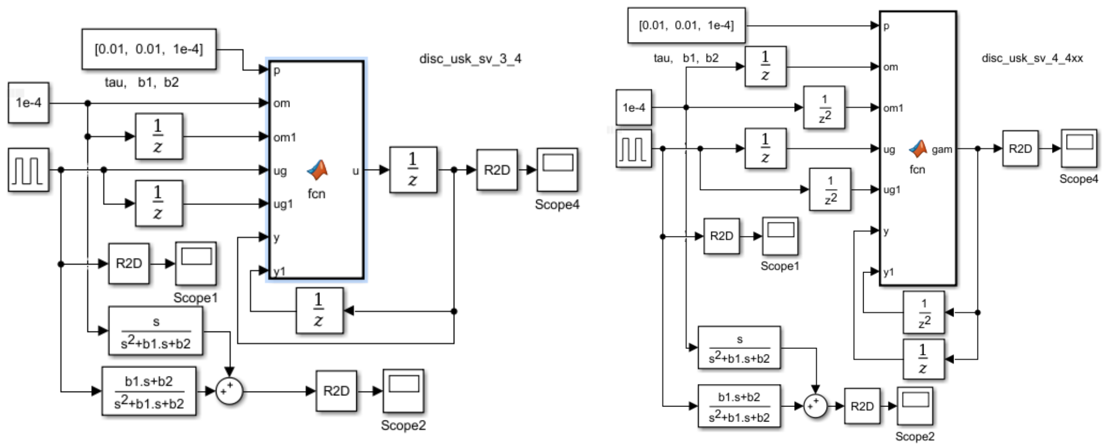


Fig. 2.14-a;b

3.4 Synthesis of a discrete complementary filter

Let us now consider a discrete complementary filter. Let's write it down

$$\hat{\gamma} = \frac{1}{Ts+1}(TG + A) \qquad (5.1)$$

or

$$T\dot{\hat{\gamma}} + \hat{\gamma} = TG + A. \qquad (5.2)$$

Equations (5.1) or (5.2) define the analog filter equations.

To switch from an analog filter to a discrete filter, write:

$$\dot{\hat{\gamma}} = \frac{\hat{\gamma}(n) - \hat{\gamma}(n-1)}{\tau},$$

де τ – sampling time.

We have a differential equation

$$T\frac{\hat{\gamma}(n)-\hat{\gamma}(n-1)}{\tau}+\hat{\gamma}(n-1)=TG(n)+A(n)\,. \qquad (5.3)$$

From here we get the equation of the discrete complementary filter

$$\hat{\gamma}(n)=\left(1-\frac{\tau}{T}\right)\hat{\gamma}(n-1)+\tau\left[G(n)+\frac{1}{T}A(n)\right]. \qquad (5.4)$$

We note that, if necessary, it is possible to synthesize a complementary filter that is astatic with respect to the drift of the gyroscope. Let's take the transfer function $W(s)$ as

$$W(s)=\frac{a_2 s+1}{a_1 s^2+a_2 s+1}\,. \qquad (5.5)$$

Then $1-W(s)=\frac{a_1 s^2}{a_1 s^2+a_2 s+1}$, and the diagram shown in Figure 2.15 will look like:

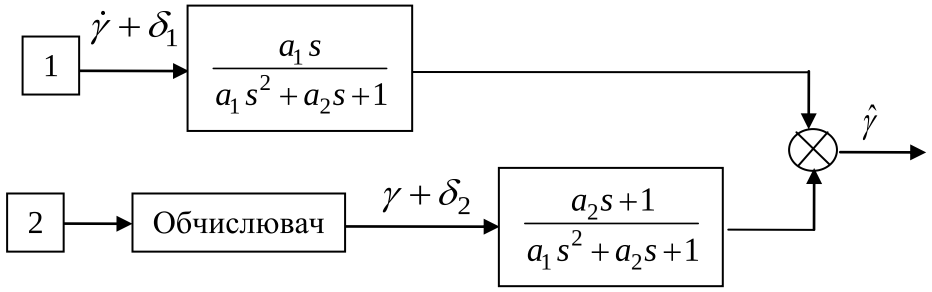


Fig. 2.15. The final scheme of the complexing filter

We have:

$$\hat{\gamma}=\gamma+\frac{a_1 s}{a_1 s^2+a_2 s+1}\delta_1+\frac{a_2 s+1}{a_1 s^2+a_2 s+1}\delta_2\,. \qquad (5.6)$$

In fig. 2.16 gives the errors of calculating the angle for $\delta_1=1\cdot 10^{-4}\,1/c$ when using a filter with a transfer function (3) from *T*=1sec (solid curve), and when using the filter with the transfer function (14) from $a_1=0{,}8 сек^2$; $a_2=0{,}1 сек$ (dashed curve).

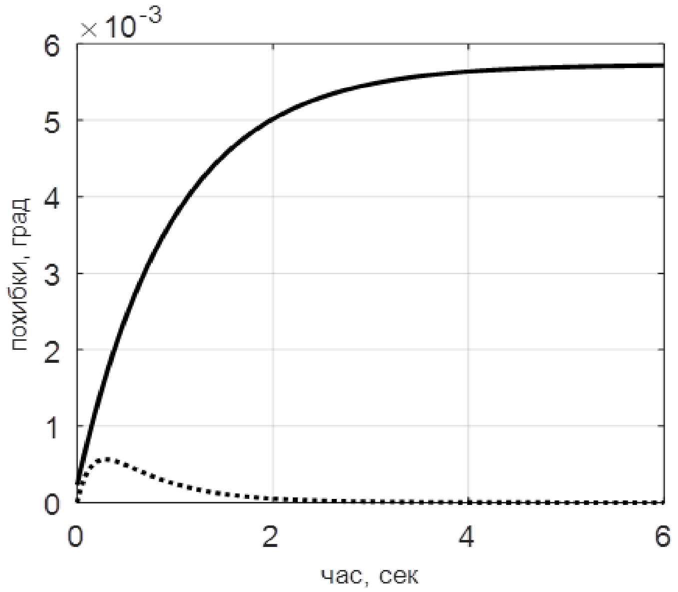


Fig. 2.16, Angle calculation errors

Conclusions

In the course of the diploma work on the topic "Discrete complementary filter for orientation determination", several important results were achieved, which have theoretical and practical significance for navigation systems and control of moving objects in aviation.

First, a detailed review of modern orientation determination methods was conducted, including the use of gyroscopes and accelerometers, as well as their combination using complementary filters. Gyroscopes have been found to provide high accuracy in short-term measurement of angular velocities, but are prone to drift over time. Accelerometers, in turn, provide stable tilt measurements, but are sensitive to dynamic accelerations. A complementary filter effectively combines these data, reducing the influence of errors from each of the sensors.

Secondly, the simulation of the operation of the complementary filter was carried out in the Matlab-Simulink environment, which allowed for a detailed analysis of the influence of various parameters on the accuracy and stability of the filtering. Simulations have shown that the correct selection of oscillator frequencies and amplification factors is critical for achieving high accuracy of orientation determination.

Thirdly, a detailed analysis of the methods of compensation for sensor zero displacement and optimization of the parameters of the complementary filter was carried out. The use of temperature compensation, regular calibration, and adaptive algorithms have been shown to significantly reduce measurement errors and increase system stability.

Thanks to the synthesis of the orientation detection system, an effective discrete complementary filter was developed, which provides high accuracy and stability even in complex dynamic conditions. This filter can be used in aircraft navigation and control systems, which has important practical significance for the aviation industry.

Based on the conducted research, it can be concluded that the discrete complementary filter is a powerful tool for determining the orientation of moving objects. Further research can be aimed at improving the methods of adaptive adjustment of filter parameters and integration of data from other sensors, such as magnetometers and GPS, to increase the accuracy and reliability of the system.

References

1. Copeland, J. "Digital Signal Processing". — M.: Mir, 2015. — 368 p.
2. Kalenov, N.E. "Modeling and optimization of control systems". — St. Petersburg: Peter, 2016. — 432 p.
3. Oreshkin, V.I. "Fundamentals of the theory of automatic control". — M.: Fizmatlit, 2014. — 512 p.
4. Simon, D. "Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches." — Wiley, 2006. — 552 p.
5. Welch, G., & Bishop, G. "An Introduction to the Kalman Filter." — UNC-Chapel Hill, 2006. — 16 p.
6. Grewal, M. S., & Andrews, A. P. "Kalman Filtering: Theory and Practice Using MATLAB." — Wiley, 2015. — 584 p.
7. Bar-Shalom, Y., Li, X. R., & Kirubarajan, T. "Estimation with Applications to Tracking and Navigation." — Wiley, 2001. — 584 p.
8. Sola, J. "Quaternion kinematics for the error-state Kalman filter". — 2017. — arXiv preprint arXiv:1711.02508.
9. Madgwick, S. O. H. "An efficient orientation filter for inertial and inertial/magnetic sensor arrays." — Report x-io and University of Bristol (UK), 2010.
10. Titterton, D.H., & Weston, J.L. "Strapdown Inertial Navigation Technology." — 2nd ed., The Institution of Engineering and Technology, 2004. — 578 p.
11. Mahony, R., Hamel, T., & Pflimlin, J.M. "Nonlinear complementary filters on the special orthogonal group". — IEEE Transactions on Automatic Control, 2008, 53(5), 1203-1218.
12. Shin, E. H. "Estimation techniques for low-cost inertial navigation." — University of Calgary, 2005. — 242 p.
13. Vagy, A., "Inertial Navigation Systems with Geodetic Applications". — 2nd ed., Springer, 2006. — 508 p.
14. Müller, P. "Fundamentals of the theory of automatic control." — St. Petersburg: BHV-Petersburg, 2017. — 432 p.

15. Farrell, J. A. "Aided Navigation: GPS with High Rate Sensors." — McGraw-Hill, 2008. — 512 p.
16. Zhao, Y., & Gao, J. "Design and Implementation of a Complementary Filter for Attitude Estimation of a Fixed-Wing UAV." — IEEE Access, 2019, 7, 33123-33131.
17. Textbook of Matlab-Simulink. — Moscow: Dialektika, 2018. — 560 p.
18. Kok, M., Hol, J. D., & Schön, T. B. "Using Inertial Sensors for Position and Orientation Estimation." — Foundations and Trends® in Signal Processing, 2017, 11(1-2), 1-153.
19. Skog, I., & Handel, P. "In-Car Positioning and Navigation Technologies." — Springer, 2015. — 144 p.