ON GENERALIZATION OF NAGUMO-BREZIS THEOREM

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The Nagumo-Brezis theorem gives the criterion of verifying the invariance of a set with respect to the flow generated by a vector field. For the category of MC^k-Fréchet manifolds, the existence and uniqueness of the integral curve of an MC^k-vector field was proved in [1]. Moreover, the existence of the MC^k-flow generated by an MC^k-vector field was proved in [2]. We extend the Nagumo and Brezis theorem to the category of MC^k-Fréchet manifolds. We give a criterion for a closed subset of an MC^k-Fréchet nuclear manifold to be invariant under the flow defined by an MC^k-vector field on these manifolds. Then, we will apply this result to locate critical points of real-valued mappings. Let M be a nuclear MC^k-Fréchet manifold, and D the derivative of functions.

Theorem 1. [3] *Let* $\mathsf{X} : M \to TM$ *be an* MC^1*-vector field and let* $A \subset M$ *be closed. Then,* A *is flow-invariant with respect to* X *if and only if for each* $x \in M$ *there is a chart* $(x \in U, \phi)$ *such that* $\lim_{s\to 0} t^{-1}\rho\big(\phi(x) + s\,\mathrm{D}\,\phi(x)\mathsf{X}(x), \phi(U \cap A)\big) = 0$.

Theorem 2. [3] *Let* $\varphi : M \to \mathbb{R}$ *be an* MC^1*-mapping,* $A \subset M$ *a closed subset, and* $\varphi\mid_A$ *bounded from below. Let* $\mathsf{X} : M \to TM$ *be an* MC^1*-vector field such that for each* $x \in M$ *there is a chart* $(x \in U, \phi)$ *such that* $\lim_{s\to 0} t^{-1}\varrho\big(\phi(x) + s\,\mathrm{D}\,\phi(x)\mathsf{X}(x), \phi(U \cap A)\big) = 0$*. Suppose that* $c = \inf_A \varphi(a)$ *and we have the following conditions: if* $(x_n) \subset M$ *is a sequence such that* $\varphi(x_n) \to c$ *and* $\mathrm{D}\,\varphi(x_n)(\mathsf{X}(x_n)) \to 0$*, then* (x_n) *has a convergent subsequence. Also, There is* ϵ_0 *such that for* $x \in \varphi_{c+\epsilon_0}$ $(\varphi_c = \{x \in M : \varphi(x) \leqslant c\})$*,* $\mathbf{F}(x,t)$ *(the flow generated by* X*) is defined, and* $\varphi(\mathbf{F}(x,t))$ *is non-increasing for* $t \in [0,1]$*. Then,* c *is a critical value of* φ*.*

References

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