MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL AVIATION UNIVERSITY

Faculty of Transport, Management and Logistics

Higher Mathematics Department

METHODICAL GUIDANCE TO THE STUDENTS' SELF-STUDY

on

«Higher Mathematics»

Field of study: 27 «Transport»
Speciality: 275 «Air Transport Technologies»
Specialization: 275.04 «Air Transport Technologies»

Educational Professional programs: «Air Transportation Management»
«Multimodal Transport and Logistics»
«Onboard Support of Air Passenger Transportation»

Developed by: Assosiate Professor I. Klyus

Methodical guidance to the students' self-study

was considered and approved
by the meeting of the Higher Mathematics Department,
Minutes № ___ of _____2021

Head of Department_____I. Lastivka

METHODICAL SUPPORT OF ARRANGEMENT OF STUDENTS' SELF-STUDY

1. Higher mathematics. Part 1: Manual/ Denisiuk V.P., Grishina L.I., Karupu O. W., Oleshko T.A., Pakhnenko V.V., Repeta V.K. – Kyiv: NAU, 2006.

2. Higher mathematics. Part 2: Manual/ Denisiuk V.P., Demydko V.G., Repeta V.K. – Kyiv: NAU, 2009.

3. Higher mathematics. Part 3: Manual/ Denisiuk V.P., Grishina L.I., Karupu O. W., Oleshko T.A., Pakhnenko V.V., Repeta V.K. – Kyiv: NAU, 2006.

4. Mathematical analysis: Manual / V. P. Denisiuk, V. G. Demydko., O. V. Karupu, T. A. Oleshko, V. V. Pakhnenko, V. K. Repeta. – Kyiv: NAU, 2013. – 396 p. 5.Higher mathematics. Linear algebra. Algebra of vectors. Elements of analytic geometry: Method Guide / compilers: A.O.Antonova, I. S. Klyus, I. O. Lastivka, V. I. Trofymenko. – К. : NAU, 2018. – 60 p

6. Higher mathematics. Introduction to mathematical analysis: Method Guide / compilers: I. S. Klyus, I. O. Lastivka. – К. : NAU, 2019. – 48 p.

7. Higher mathematics. Differential calculus of one variable: Method Guide / compilers: I. S. Klyus, I. O. Lastivka. – К. : NAU, 2021. – 48 p.

8. Higher mathematics. |Integral calculus: Method Guide / compilers: I. S. Klyus, I. O. Lastivka. – К. : NAU, 2021. – 72 p.

Module №1 „Elements of Linear Algebra,Vector Algebra and Analytical Geometry."

Topic 1. 1. Elements of Linear and Vector Algebra

1. Concepts, definitions, formulations:

1. Determinants of the 2nd, the 3rd and the *n– th* orders.
2. Matrices. Linear operations with matrices. Multiplication of matrices.
3. Inverse matrix
4. Definite, indefinite, consistent, inconsistent SLAE.
5. Matrix form of SLAE.
6. Gauss' method of SLAE solution.
7. Kronecker-Capelli theorem usage in SLAE investigation.
8. Geometrical vector. Vector addition and subtraction operations, multiplication by scalar.
9. Linear dependence and independence of vectors.
10. Cartesian coordinate system (CCS).
11. Dot product of two vectors.
12. Cross product of two vectors.

13. Triple product

2. Proofs and conclusions

1. Properties of determinants (2nd and 3rd orders).
2. Matrix addition and multiplication properties.
3. Existence of an inverse matrix.
4. Inverse matrix method of SLAE solution.
5. Cramer's Theorem.
6. Kronecker-Capelli Theorem.
7. Projection of vector on axis.
8. Representation of a vector in terms of base vectors.
9. Properties of a dot product; calculation by coordinates.
10. Properties of a cross product; calculation by coordinates.
11. Properties of a triple product; calculation by coordinates

3. Assignments

1. Calculate the determinants of order 2, 3 and *n*, to be able to lay out a determinant by the elements of any row or column, to reduce determinant to the triangle form.
2, Find the matrix sum, difference and product.
3. Find the matrix rank.
4. Find an inverse matrix.
5. Solve the square systems by Cramer's method, through inverse matrix.
6. Solve the square systems by Cramer's method, through inverse matrix.
7. Solve the arbitrary SLAE by Gauss' method.
8. Analyse SLAE on the consistence (compatibility) according to Kronecker-Capelli Theorem.
9. Analyse SLAE on the consistence (compatibility) according to Kronecker-Capelli Theorem.
10. Find the eigenvalues and eigenvectors of matrix.
11. Find the vector coordinates, it's length, unit vector. Find the angle between vectors.
12. Find the vector sum, difference, dot and cross products.
13. Calculate the area of the triangle, volume of pyramid.
14. Be able to represent the vector in terms of base vectors.
15. Be able to use the condition of two vectors perpendicularity

Topic 1. 2. Analytical Geometry

1. Concepts, definitions, formulations:

1. Different equations of a straight line (typical problems of finding equations of a straight line).

2. A plane. Different equations of a plane (typical problems on finding of equations of a plane).

2. Proofs and conclusions

1. Different forms of the equation of a straight line on a plane (general, symmetric, parametric, passing through two points, in slope — intercept form, in «segments», normal).
2. Mutual location of two straight lines. An angle between two straight lines. Conditions of parallelism and perpendicularity.
3. Distance from a point to a straight line.
4. Equation of a plane passing through a point perpendicularly to a given vector.
5. Equation of a plane passing through three given points.
6. Symmetric equations of a straight line in space.

3. Assignments

1. To work out the equation of a straight line passing through two points, through one point in the given direction.
2. To work out equations of a plane passing through a point perpendicularly to a vector, through three points.
3. To find angles between straight lines and planes.
4. To find an intersection point of a straight line and a plane.

Module №2 „Introduction to mathematical analysis. Differential calculus of the function of one variable"

Topic 2.1. Introduction to Mathematical Analysis

1. Sets. Classification of numerical sets. Operations on sets. The modules of a real number.
2. A sequence.
3. A function. Classification of functions. The elementary functions. An inverse function. A composite function.
4. The Limit of a numerical sequence. The Limit of a function. Infinitesimals.
5. Continuity. Continuity of a function at a point and on an interval. Properties of continuous functions. Points of discontinuity and its classification.

2. Proofs and conclusions

1. Theorems about limits.
2. The first and the second honorable limits.
3. Theorems about equivalent infinitesimals.

3. Assignments

1. Evaluate the limits.
2. Evaluate the limits using the equivalent infinitesimals.
3. Investigate functions for continuity.

Topic 2. 2. Differential Calculus of the Function of One Variable

1. Concepts, definitions, formulations:

1. Definition of a derivative. Geometrical and physical interpretation.
2. A table of derivatives. Rules of differentiation.
3. A connection between continuity and differentiability.
4. A differential. Geometrical interpretation of a differential.
5. The usage of the differentials.
6. Evaluation of the first and higher order derivatives.
7. Leibniz's formula.
8. Lagrange's formula.
9. L'Hospital's rule for expansion of indeterminate forms $\left[\frac{0}{0}\right]$ or $\left[\frac{\infty}{\infty}\right]$.
10. Taylor's formula.
11. Maclaurin's formula.
12. Investigation for function increase and decrease on the given interval.
13. Investigation of a function for extremum.
14. Minimum and maximum values on the interval.
15. Concavity intervals. Inflection points.
16. Asymptotes.
17. Plan of graph construction.

2. Proofs and conclusions

1. The derivatives of elementary functions.
2. The first order and higher order derivatives of the parametric functions.
3. Theorem about continuity of differentiable functions.
4. Geometrical interpretation of the first order differential.
5. Equation of a tangent line and a normal to the curve.
6. Lagrange's and Fermat's theorems.
7. L'Hospital's rule for expanding of indeterminate form $\left[\frac{0}{0}\right]$.
8. The necessary monotony conditions.
9. The necessary and sufficient extremum conditions.
10. Curve asymptotes seeking rule.

3. Assignments

1. Find the derivatives of functions.
2. Find the derivatives of composite functions, implicit functions and parametric functions.
3. Find the differentials of functions.
4. Find the derivatives and the differentials of higher order.
5. Solve tasks for geometrical and physical interpretation of a derivative.
6. Investigate elementary functions.
7. Sketch the graphs of elementary functions.

8. Find different limits with the help of L'Hospital's rule.
9. Find intervals of function increase and decrease, local extremum.
10. Find concavity intervals.
11. Find graph asymptotes.
12. Construct the graph.

Module №3 "Differential calculus of the function of several variables. Integral calculus"

Topic 3.1. Differential Calculus of the Functions of Several Variables

1. Concepts, definitions, formulations:

1. Functions of several variables.
2. Limit, and continuity of the functions of several variables.
3. Partial derivatives.
4. Differential.
5. Relative extrema.
6. Tangent plane and normal to a surface.
7. Gradient.
8. Extrema on a polygon.

.2. Proofs and conclusions

1. Functions of several variables. Domain of a function of several variables.
2. Properties of continuous of the functions of several variables.
3. Formulas for calculation of partial derivatives.
4. Differential. Properties and calculation.
5. Relative extrema. Necessary and sufficient conditions.
6. Tangent plane and normal to a surface.
7. Gradient. Properties and calculation.
8. E extrema on a polygon.

3. Assignments

1. Finding domain of a function of several variables.
2. Finding the first and higher order partial derivatives and the differentials.
3. Finding partial derivatives of the composite functions.
4. Implicit partial differentiation.
5. Finding equations of tangent plane and normal to a surface.
6. Finding gradient.
7. Finding relative extrema and extrema on a polygon.

Topic 3. 2. Integral Calculus of the Function of One Variable

1. Concepts, definitions, formulations:

1. Antiderivative. Indefinite integrals. Table of integrals. Evaluating techniques.
2. Polynomial functions. Rational functions.

3. Integrating of rational functions by partial fractions.
4. Integrals involving powers of trigonometric functions.
5. Integrating of irrational functions.
6. Definite integrals. Newton-Leibniz fundamental theorem.
7. Properties of definite integrals. Evaluating techniques.
8. Application of the definite integrals

2. Proofs and conclusions

1. Concepts of antiderivative and the indefinite integral. The table of the integrals.
2. The substitution technique.
3. Integration by parts.
4. Integrating of partial fractions. Integrating of rational functions.
5. Integrals involving powers of trigonometric functions.
6. Integrating of irrational functions.
7. Definite integrals. Newton-Leibniz fundamental theorem.
8. Properties of definite integrals.
9. Improper integrals. Convergence and evaluating.
10. Application of the definite integrals in geometry and mechanics.

3. Assignments

1. Find indefinite integrals applying table of integrals.
2. Find indefinite integrals applying substitution technique.
3. Find indefinite integrals applying integration by parts.
4. Find integrals of rational functions by partial fractions.
5. Find integrals involving powers of trigonometric functions.
6. Find integrals of irrational functions.
7. Find definite integrals applying Newton-Leibniz formula..
8. Find definite integrals applying evaluating techniques.
9. Investigate improper integrals for convergence. Find improper integrals.
10. Apply definite integrals for solving geometric and mechanical problems.

Module №4 "Improper and double integrals. Differential equations. Some numerical methods"

Topic 4.1. Improper and double integrals

1. Concepts, definitions, formulations:

1. Improper integrals. Convergence of improper integrals.
2. Double integral, properties, geometric content, calculation.
3. Conditions of the existence and properties
4. Change of variable in double integral. Transition to polar coordinates.

Proofs and conclusions

1. Improper integrals. Convergence and evaluating
2. The rule of reducing a double integral to a repeated one.
3. Application of double integral.
4. The rule of reducing a double integral to a repeated one.

Assignments

1. Investigate improper integrals for convergence. Find improper integrals.
2. Reduce double integrals to the definite integrals and calculate them.

Topic 4.2. Differential Equations

1. Concepts, definitions, formulations:

1. Differential equations of the first order. General definitions. Integral curve. Cauchy problem.
2. Differential equations of the first order: separable equation, homogeneous differential equation, linear differential equations of the first order, Bernoulli equation, exact differential equations.
3. Differential equations of higher order. Basic concepts and definitions.
4. Differential equations which allow reduction of order.
5. Linear differential equations of order *n*.
6. Linear homogeneous differential equations with constant coefficients.
7. Linear non-homogeneous equations. Method of undetermined coefficients.
8. Statement of the problem of numerical integration.
9. Formulas of rectangles.
10. The formula of trapezoids.
11. Simpson's formula (parabola).
12. Classification of numerical methods for solving the Cauchy problem.
13. Modifications of the Euler method.
14. Modified Euler method.
15. Improved Euler-Cauchy method followed by iterative processing.
16. Runge – Kutta method

2. Proofs and conclusions

1. Differential equations of the first order. General and particular solutions of differential equation. Cauchy problem.
2. Separable equation.
3. Homogeneous differential equation.
4. Linear differential equations of the first order.
5. Bernoulli equation.
6. Exact differential equations.

7. Differential equations of higher order.
8. Linear differential equations of order *n*. Fundamental system of solutions. Structure of the general solution of the homogeneous linear differential equation of order *n*.
9. Method of variation of constants.
10. Linear homogeneous differential equations with constant coefficients. The structure of the general solution of a linear homogeneous equations.
11. Linear non-homogeneous equations. The structure of the general solution of a linear non-homogeneous equations.
12. Method of undetermined coefficients.
13. Derive the formulas of rectangles for the approximate calculation of definite integrals.
14. Derive the formula of trapezoids for the approximate calculation of definite integrals.
15. Derive the formula of parabolas for the approximate calculation of definite integrals.
16. What methods are single-point?
17. What methods are multipoint?
18. What methods are methods of forecasting and correction?
19. Feature of the mathematical model of Euler's method.
20. Geometric interpretation of Euler's method.
21. Algorithm of Euler's method.

3. Assignments

1. Finding general solutions of the differential equations of the first order: separable equation, homogeneous differential equation, linear differential equations of the first order, Bernoulli equation, exact differential equations.
2. Finding the particular solution of the differential equation through the given point. Linear differential equations with constant coefficients.
3. Finding the most general solution of the simultaneous equations.
4. Some applications of differential equations.

Literature

1.Denisiuk V. P. Higher mathematics. Part 2: Manual / V. P. Denisiuk, V. G. Demydko, V. K. Repeta. — K. : NAU, 2009. — 248 p.

2.Ластівка І.О., Коновалюк В.С., Ковтонюк І.Ю., Паламарчук Ю.А., Петрусенко В.П., Чуб Л.О. Вища математика. Модуль 3. Невизначений та визначений інтеграли: Навч. посібник– К.:Книжкове вид-во НАУ, 2007. – 208 с.

Guidelines

1. Elaboration of lecture material
2. Preparation for practical classes
3. Doing homework for practical classes

4. Completion of individual homework
5. Elaboration of literature.