MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL AVIATION UNIVERSITY

HIGHER MATHEMATICS
PROBABILITY THEORY
RANDOM EVENTS

Method Guide to self-study
for students of majors:
192 «Building and civil engineering», 272 «Aviation Transport»,
101 «Ecology».

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Методичні рекомендації до самостійної роботи студентів містять завдання з дисципліни «Вища математика. Теорія ймовірностей. Випадкові події»,: рекомендації до самостійної роботи студентів, а також список рекомендованої літератури.

Для студентів напряму підготовки 192 «Будівництво та цивільна інженерія», 272 «Авіаційний транспорт», 101 «Екологія».

Н 65

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Method guide contains tasks for students’ self-study on the academic subject «Higher Mathematics. Probability theory. Random events» and recommended literature.

For students of majors: 192 «Building and civil engineering», 272 «Aviation transport», 101 «Ecology».

CONTENT

INTRODUCTION

Probability theory is the section of higher mathematics dealing with mathematical models when familiar regularities and strict determinism that we are accustomed to in classic mathematics are broken. Such models are used in scientific investigations, practical situations and usual life. Main problem of the probability theory is to evaluate how frequent a given random event will occur under some determinant conditions.

Probability theory emerged in the XVI century as endeavour to create the theory of game of chance with a dice, cards and roulette. Famous mathematicians Cardano, Huygens, Pascal, Fermat, Bernoulli and others were the first who laid a foundation of the probability theory. The further investigations (XVII–XX c.) were carried out by such well-known scientists as Laplace, Poisson, Gauss, Chebyshev, Markov, Kolmogorov and others.

The aim of teaching the discipline is to master the basic concepts, methods and theorems, necessary for modelling and solving applied economics and engineering problems.

The tasks of studying the educational discipline are development of students' logical and algorithmic thinking; mastering research and solving methods of mathematical problems; mastering basic skills of mathematical research applications. As a result of studying the discipline students should know basic definitions, theorems, rules and their practical application; modern methods of higher mathematics.

They will be able to use the methods of higher mathematics to study the general and special subjects and mathematical techniques for solving practical problems.

The lecturer can adjust the amount and content of tasks the student has to complete on his own while studying the relevant material.

The material of each topic corresponds to the work program of the discipline "Higher Mathematics", in particular module IV "Probability theory". Each section contains the basic methodological recommendations and tasks for independent execution, the solution of which will contribute to better understanding, assimilation and application of the basic theoretical positions.

Section 1. Statistical definition of the probability. Classical probability

Outline

1. The basic concepts of probability theory.
2. Statistical definition of the probability
3. Classical probability

Literature: [3]; [5]; [6].

Methodological recommendations

As a result of studying the materials of this section a student is **to know**: the basic concepts of probability theory, definitions of statistical and classical probability**; be able**: to calculate the probability of random events, to apply these concepts in future specialty

The basic concepts of probability theory are the following:

1. Trial (experiment, observation, test) – is invariable complex of some determinate conditions which are realized to obtain (observe or expect) some result. We denote a trail as $S, S_1, S_2,\dots$.

Note. We usually consider that the trial can be repeated any number of times without changing its conditions.

2. Event – is any possible result of the trial.

3. Random event – is a result of the trial which may occur or not.

We denote events by capital letters: $A, B, C, A_1, A_2, \dots, A_n$ and so on.

For example:

S_1 – {observation of a weather}; A – {rainy weather}; B – {fine weather};

S_2 – {test in mathematics}; C – {receiving (earning) a good mark}.

We shall consider two limit cases of random events:

certain event – is an event which occurs at every realization of the trial;

impossible event – is an event which never occurs at every realization of the trial.

For example: in trial S={throwing two dies};

A = {the sum of the numbers on both sides is equal to 1} is an impossible event;

B = {appearance of the same number on both sides} is a random event;

C = {sum of the numbers on both sides is less than 13} is an event.

Statistical definition of the probability

In practice to evaluate how frequently some random event occurs we determine the number of trials have done to obtain the necessary event. For example, we say that one from 5 days in May on average is rainy. We use also such expedient to express our confidence in some random event and say like that «I bet twenty to one that he will come».

In general, let trial S realized N times and random event A have occurred M times ($M \le N$) in total. Number M is dependent on the number of trials and is called the frequency of appearance of event A in N trials. We use also more stable characteristic the ratio M/N which is called the relative frequency of the event and denoted as $W_N(A) = \frac{M}{N}$. The relative frequency $W_N(A)$ is considered to be a measure of confidence in appearance of event A and is also called statistical (experimental) probability. Taking into account the inequality $M \le N$ we can see that $0 \le W_N(A) \le 1$.

It is found that the more is the number N of trials the more reliable evaluation we obtain. So scientist Buffon (XVIII c.) ran the experiment on tossing a coin 4040 times and obtained 2048 heads. The relative frequency of the head became $W = 0.507$. Another mathematician Pearson (XIX – XX c.) repeated this experiment 12000 times and obtained W=0.501 and enlarged the number of the trails up to 24000 times and obtained 12012 heads, that is, $W = 0.5005$. It is reasonable to suppose that W approaches 0.5 as N becomes infinitely large.

Using the relative frequency we introduce the following cardinal notion, abstract concept of probability.

Definition. If trial S is repeated many-many times without changing the experimental conditions and there exists a limit of relative frequency as $n \to \infty$, this limit is called the probability of event A in the trial S and denoted:

$$P(A) = \lim_{n \to \infty} W(A)$$

Note. This definition is called statistical definition of the probability. It supposes that the limit of the relative frequency exists, if it is not so than such event is beyond our scope.

From properties of limits of the inequality $0 \le W_n(A) \le 1$ it follows that the value of probability is not negative and never exceeds one. It follows also that zero probability does not mean that the event is impossible and analogously the unit probability does not indicate that the event is certain. Though the probability of an impossible event is zero and probability of a certain event is equal to one.

By using statistical probability (relative frequency) we can calculate the probability of events only approximately on the base of experiment repeated many times. But many experiments such as testing one person with the same questions, destruction of a certain type of equipment cannot be repeated under the same conditions and some experiments are very expansive.

Classical probability

We consider another idea giving us the way to calculate the probability exactly in many cases without doing experiment. It is so-called classical approach. We have to introduce some auxiliary notations.

Definition. Events A and B are called *exclusive* (disjoint, incompatible) in trial S, if they cannot occur in the same realization of the trial.

Events A_1, A_2, …, A_n are called *mutually exclusive* in trial S if any two of them are exclusive.

Examples: 1) In the trial S={tossing a coin} the events A={to obtain the heads} and B={to obtain the tails} are exclusive.

2) In the trial S={rolling a dice} the events A_1={to obtain one}, A_2={to obtain two}, …, A_6 ={to obtain 6} are mutually exclusive.

Definition. Events A_1, A_2,…,A_n are called *exhaustive events* in trail S if in every realization of the trial S at least one of them undoubtedly occurs.

This group of the events contains all possible results of the trial S.

Definition. If any of events A_1, A_2,…,A_n has no preference to occur in the trial S then these events are called *equally possible* or *equally likely*.

Example. In trial S={drawing one card from a pack of 36 cards} the following events A={ace of clubs}, B={king of hearts}, C={knave of spades}, D={ten of diamonds} are mutually exclusive, equally possible but are not exhaustive.

Definition. If we compose a set of events $\{\omega_1, \omega_2, \dots, \omega_n\}$ which are mutually exclusive, exhaustive, and equally possible in trial S then we say that the trial is represented by the *classical model* with *elementary events* $\omega_k \left(k=1, 2, \ldots, n\right)$. We denote a classical model as $\Omega_S = \{\omega_1, \omega_2, \ldots, \omega_n\}$.

Notes. There exist trials which do not admit of composing any classical model but it is possible to construct classical models for many trials. For example, in trial *S*={drawing one card from a pack of 36 cards} the following sets of events will be classical models of the same trial *S*:

1) $\Omega_1 = \{\omega_1, \omega_2\}$ where ω_1={draw any card of red suit}, ω_2={draw any card of black suit};

2) $\Omega_2 = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ where ω_1={draw any card of spade}, ω_2={draw any card of clubs}; ω_3={draw any card of diamond}, ω_4={draw any card of hearts}.

3) $\Omega_3 = \{\omega_1, \omega_2, \ldots, \omega_8, \omega_9\}$ where ω_1={draw the six of any suit}, ω_2={draw the seven of any suit }, …, ω_8={draw the king of any suit}, ω_9={draw the ace of any suit}.

Definition. Let $\Omega_S = \{\omega_1, \omega_2, \ldots, \omega_n\}$ be a classical model of trial *S* and *A* be random event such that appearance of elementary event ω_k implies obligatory the event *A* then ω_k is said to be a *favourable* event with respect to event *A*.

Example. The trial *S*={ There are 5 balls marked 1, 2, 3, 4, 5 in a box. The balls number 1, 3, 5 are black and the others are white. One ball is drawn}. The event *A*={draw the black ball}, the event *B*={draw the white ball}. Then $\Omega_3 = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ where ω_1={draw number 1}, ω_2={draw number 2},…, ω_5={draw number 5} is a classical model and ω_1, ω_3, ω_5 are favourable to *A*, and ω_2 , ω_4 are favourable to *B*.

Definition 7. If we have managed to build a classical model for trial *S* with *n* elementary events and *m* of them are favourable to *A*, then the probability of event *A* is equal to the following ratio:

$$P(A) = m/n. \tag{1.1}$$

This is so called classic definition of the probability.

Example. In the previous example there are 5 elementary events in the classical model and 3 of them are favourable to the event *A* and 2 are favourable to the event *B*. Therefore, *P*(*A*)=3/5 and *P*(*B*)=2/5. According to the statistical definition of the probability it means that in 50 repeating trials we get a red ball approximately 30 times and 20 times a black one.

Self-test assignment

1.1. Three dies are thrown one time each. What is the probability that on three dies equal numbers will appear or all three numbers will be different?

1.2. Haw many 3-figure numbers can be composed from figures 1, 2, 3, 4, 5, 6 when all figures in a composed one are different?

1.3. A set of integers is given $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. From this set four figures are randomly taken one by one and then put in a raw. What is the probability to get years 1945 or 1985?

1.4. There are the following letters on 9 cards: *T, E, L, E, P, H, O, N, E*. these cards are shuffled, then they are randomly put in a line. What is the probability to get the word *TELEPHONE*?

1.5. The coin is tossed two times. What is the probability that the heads will be the first to appear or it will occur after the second tossing?

1.6. A coin and a die are thrown up once each. What is the probability that 5 will appear on the side of the die and heads will fall out on the coin or an even number will appear on the side of the die and heads will fall out on the coin?

1.7. There are 33 letters of the Ukrainian alphabet written on several cards. A random card is taken out and the letter on it is copied and the card is returned to its place. 7 letters are written. What is the probability that words «Україна» or «Держава» will be written?

1.8. There is a set of 6 samples with numbers from 1 to 6. All samples are taken one by one at random. Find the probability that the numbers of samples will be located in the increasing order.

1.9. How many 2-digit numbers can we form from figures 1, 2, 3, 4, 5, 6?

1.10. Haw many 3-digit numbers can be composed from figures 1, 2, 3, 4, 5, 6 when all the figures in the composed ones are different?

1.11. How many 5-digit numbers can be formed from figures: a) 1, 2, 3, 4, 5; b) 0, 1, 2, 3, 4? The figures are different in each number.

1.12. A coin was tossed up 3 times. Describe the events: event A – the heads were observed at least 2 times; event B – in all 3 times the tails were observed.

Section 2. Basic combinatorial problems

Outline.

1. The main principle of combinatorics.
2. Combinations, arrangements, permutations**.**

Literature: [1]; [3]; [5]; [6]; [8].

Methodological recommendations

As a result of studying the materials of this section a student **is to know:** the main principle of combinatorics; formulas of combinations, arrangements, permutations**; be able:** to calculate the probability of random events; to apply these concepts in future specialty.

In many cases it is unnecessary to list the set of elementary events of the classical model in a given trial. It is sufficient by using the rules of combinatorics to calculate the total number n of these events and number m of the events favourable to a given event A. Recall some of them.

The main principle of combinatorics (Rule of Product)**.** If we have two sets $A=\{a_1, a_2, ..., a_m\}$ and $B=\{b_1, b_2, ..., b_n\}$ then the set of all possible ordered pairs (a_i, b_j), $i=1,2,...,m,\quad j=1,2,...,n,$ is called Cartesian product of the two sets A and B and denoted as $A\times B$. By an ordered pairs of objects we mean that (AB) and (BA) are different pairs.

Let us calculate the number of ordered pairs from two sets: {a, b, c} and {a, b, c, d}. We take the first object from one set and the second object from the other set:

aa ab ac ad
ba bb bc bd
ca cb cc cd.

The number N of the different pairs of elements is equal to the product of corresponding numbers of elements of the sets A and B:

$$N\left(A\times B\right)=N\left(A\right)\cdot N\left(B\right)=mn.$$

Analogues rule is true for more than two sets.

Example. Let there be two sets of figures A={1, 2, 3, 4, 5} and B={0, 1, 2, 3, 4, 5, 6}. We compose a two-figured number when the first figure is randomly taken from A and the second from B. Calculate the probability that the number will be even.

Solution. Compose the classical model including all possible ordered pairs. The number n of the pairs is the following product:

$$n = N(A) \cdot N(B) = 5 \cdot 7 = 35\,.$$

Among them the number of the favourable pairs will be the product $N(A)$=5·4 (the number of even figures in B). Thus

$$m = 5 \cdot 4 = 20\,.$$

And the probability will be

$$P = \frac{m}{n} = \frac{20}{35} = \frac{4}{7}\,.$$

Example. (Numerical lock). A lock can be opened by choosing only a certain combination of one figure from each of four disks. On each disk there are 10 figures. What is the probability of event A={the lock will be opened at the first attempt}?

Solution. By Product Rule number of possible events: $n = n_1 \cdot n_2 \cdot n_3 \cdot n_4$. Only one of them is favourable event to A. Then:

$$P(A) = \frac{1}{10^4} = 10^{-4}\,.$$

***Combinations*.** The number of different combinations of n elements taken k at once can be calculated by the following formula:

$$C_n^k = \frac{n!}{k!(n-k)!} = \frac{n(n-1)(n-2)...(n-k+1)}{1 \cdot 2 \cdot 3 \cdot ... \cdot k}\,. \qquad (1.2)$$

Note. If we are interested in combinations we do not take into account the order of the objects. Two combinations are considered to be different if they differ by their elements only.

Example. Let the trial S={take three balls at once from the five balls marked as 1, 2, 3, 4, 5}. Find the probability of the event A={sum of the numbers on three drawn balls is equal to 8}.

Solution. Compose mental classical model including all possible combinations of 5 different digits taken 3 at once. The number of such combinations will be

$$n = C_5^3 = \frac{5!}{(5-3)2!} = \frac{4 \cdot 5}{2} = 10\,.$$

Among them only 2 are favourable to event *A*. They are {1+3+4} and {1+2+5}. So *m*=2. Therefore, the probability of event *A* will be

$$P(A) = 2/10 = 0.2\,.$$

Arrangements. The number of different arrangements of *n* elements taken *k* at once can be calculated by the following formula:

$$A_n^k = \frac{n!}{(n-k)!} = n(n-1)(n-2)...(n-k+1) \qquad (1.3)$$

Note. Two arrangements are considered to be different if they differ by the elements or by their order in the sequence.

Example. There are 5 cards with letters A, B, C, D, E and we draw three of them one by one. What is the probability that the drawn cards compose the word «ACE»?

Solution. The order of drawing cards is important. The number of such arrangements will be

$$n = A_5^3 = \frac{5!}{(5-3)!} = 3\cdot 4\cdot 5 = 60\,.$$

Only one among them gives the word «ACE». So *m*=1 and the probability is

$$P(A) = 1/60.$$

***Permutations*.** The special case of arrangement of *n* elements taken *n* at once is called permutation. The number of different permutations can be calculated by replacing *k* by *n* in formula (1.2). We get the following formula:

$$P_n = n! = 1\cdot 2\cdot 3\cdot ...\cdot n\,, \quad (0! = 1)\,.$$

Note. Two permutations of the same elements are different if they differ by the order of the elements.

Example. Let all cards in the previous example be drawn one by one. What is the probability that they will be drawn in the alphabet order?

Solution. $n = P_5 = 5! = 120$; $m = 1$. Then $P(A) = 1/120$.

Let's consider also some problems on combinations with two or more elements.

Example. Let there be a set of figures {1, 2, 3, …, 8, 9} and we take randomly two figures from the set. What is the probability that the sum of two drawn figures will be even?

Solution. Here the order of taken figures is immaterial and so we compose the classical model including all possible combinations of 9 elements taken 2 at once. Their number will be:

$$n = C_9^2 = \frac{9!}{2!\cdot 7!} = 36.$$

The sum of two figures is even if they are both either even or odd. The number *m* of combinations when two figures are even or odd will be:

$$m = m_1 + m_2;$$

$$m_1 = C_4^2 = \frac{4!}{2!\cdot 2!} = 6; \; m_2 = C_5^2 = \frac{5!}{2!\cdot 3!} = 10.$$

Thus the sought for probability will be

$$P = \frac{m}{n} = \frac{6+10}{36} = \frac{4}{9}.$$

Answer: 4/9.

The Rule of Product is true not only for ordered pairs, triples but for multiples as well.

Example. Let there be a trial $S =$ {from a box containing 5 white and 4 black balls; 4 balls are drawn randomly}. What is the probability that the event *A*={to take 3 white and 1 black balls} will occur?

Solution. $n = C_9^4 = \dfrac{9!}{4!\cdot 5!}$;

$$m = m_1 \cdot m_2 = C_5^3 C_4^1 = \frac{5!}{3!\cdot 2!}\cdot\frac{4!}{3!\cdot 1!}.$$

$$P(A) = \frac{m}{n} = \frac{5!\cdot 4!}{3!\cdot 2!\cdot 3!\cdot 1!}\cdot\frac{4!\cdot 5!}{9!} = \frac{20}{63} = 0.32.$$

Answer: 0.32.

Self-test assignment

2.1. A box contains 10 red, 8 green and 12 blue balls, which are identical by touch. Two balls are randomly taken out. What is the probability that they will be green, if we already know that they are not blue?

2.2. There are 20 balls of the same size in the urn. 8 of them are red, 7 – blue and 5 – green. 3 balls are randomly taken out of the urn. What is the probability that all of them will be of different colors or all of them will be red?

2.3. Among 10 electric motors of the same kind that came to the storehouse 7 are standard, and the others – have defects. Three electric motors are taken randomly. What is the probability that all of them will be ready for operation or all of them will not be reliable?

2.4. There are 12 pencils of the same size in the pencil-box. 8 pencils are red, the other 4 are blue. We take 3 random pencils. What is the probability that they'll be of the same color?

2.5. Among 12 items 4 are defective. 2 random items are taken from 12. What is the probability that among them at least one will be standard?

2.6. Among 10 water pumps 3 have some defects. 4 pumps were randomly selected. Calculate the probability of the random events: A_1 – 4 pumps will be without defects; A_2 – 3 pumps will be without defects and 1 will have a defect; A_3 – 2 pumps will be without defects and 2 with defects; A_4 – 1 pump will be without defects and the rest with defects.

2.7. There are 13 identical parts in the box, 8 of which are standard and the rest are defective. 4 random parts are taken out of the box. What is the probability that 4 standard parts were taken out or 2 defective and 2 standard ones?

2.8. There are 20 students in a group. Among them there are 10 boys and 10 girls. From the list 5 random students are taken. What is the probability that all of them will be boys or all of them will be girls?

2.9. There is a set of 15 items: 7 items of the 1st grade, 6 items of the 2nd grade and 2 items of the 3rd grade. 5 random items have been chosen. Find the probability that among them there will be no items of the 3rd grade.

2.10. One day 10 friends came to a restaurant. The owner offered them to come to him every day and each time to take seats at the same table in different order; after all combinations are exhausted, he will give the friends free meals. When will this day come?

2.11. In how many ways can 15 participants of the competition be awarded with gold, silver and bronze medals?

2.12**.** The car number consists of 2 letters and 5 figures. Find the quantity of all possible car-marks, if 32 letters of the Ukrainian alphabet are used.

2.13. In a bank's branch office there are 15 workers, 3 of them have no adequate qualification. How many lists of workers can be formed: a) of 8 workers; b) of 6 qualified workers; c) of 9 workers and two of them without adequate qualification?

2.14. There are 30 students in a group. How many teams of two students on duty can we form, if: a) one of them should be the head; b) they are both equal?

2.15. An army detachment consists of 3 officers, 6 sergeants and 60 soldiers. How many detachments is it possible to choose consisting of 1 officer, 2 sergeants and 20 soldiers?

2.16**.** Morse code consists of 2 symbols (a dot and a dash). How many letters can be represented, when every letter contains not more than 5 symbols?

2.17. Suppose a club has 16 members; how many different four-member committees can be formed?

2.18. A boy can choose 7 stamps of a set of different 11 stamps. How many different selections can he make?

2.19. A group consists of 5 boys and 8 girls. How many 5-member teams can be made to contain: a) no girls, b) no boys, c) at least one boy, d) at least one member of each sex?

2.20. A student has forgotten the last three figures of the phone-number and he tries the figures at random. Find the probability that the student will get the right number, when a) these three figures are different; b) these three figures can be repeated.

Section 3. Geometrical probability

Outline

1. Geometrical probability
2. Paradox of zero probability

Literature: [1]; [3]; [5]; [6]; [8].

Methodological recommendations

As a result of studying the materials of this section a student is to **know:** problems on *geometrical* probability; **be able:** to calculate the probability of random events; to apply these concepts in future specialty.

Some problems on the theory of probability can be solved by geometrical methods. They are the so called problems on *geometrical* probability. Let's consider this method.

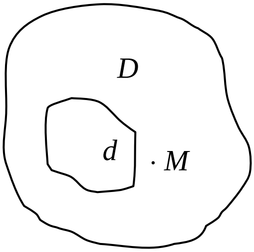

Fig. 1

Let it be region D with area S_D, including another region d with area S_d (Fig. 1). We put point M occasionally inside D. If the probability of the event $A = \{$point M is in the smaller region $d\}$ does not depend on location of d then it can be calculated by the following formula:

$$P(A) = \frac{S_d}{S_D}.$$

Example (the problem of appointment). Two people make an appointment within $12^{00} – 13^{00}$ o'clock. They agreed that the first to come would be waiting for 15 minutes and then would go away if the meeting failed.

What is the probability of the event $A = \{$the appointment will happen$\}$?

Solution. Use Cartesian coordinates *Oxy* and *x*-axis corresponding to the time of the first coming, *y*-axis – to the second coming (Fig. 2).

Then the appointment will happen if the following inequality is true:

$$|y - x| \le \frac{1}{4} \Rightarrow -0.25 \le y - x \le 0.25;$$

$$y \ge x - 0.25;$$

$$y \le x + 0.25.$$

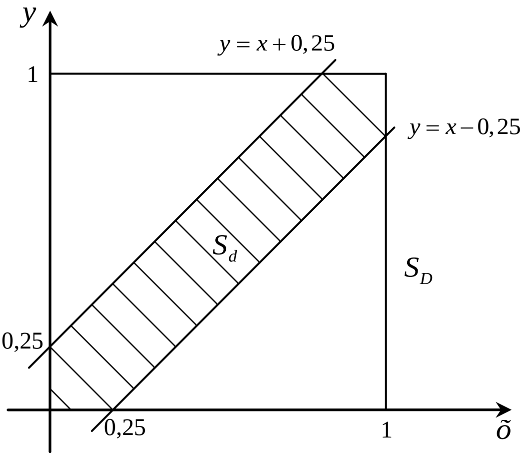

Fig. 2

Use the definition of geometrical probability. Here *D* is a square with a unit side. Then $S_D = 1$ and *d* is a dashed region (Fig. 2) with an area $S_d = 1 - (3/4)^2 = 7/16$.

Answer: $P(A) = \dfrac{7}{16} \approx \dfrac{1}{2}$.

Paradox of zero probability

If in the previous problem the time of waiting is 0. Then we have $S_d = 0$ and $P(A) = 0$. But it does not mean that the appointment does not happen.

Note. Zero probability of an event does not exclude appearance of the event.

Self-test assignment

3.1. On the segment [–1; 1] there have randomly been taken 2 real numbers. Find the probability that their sum is positive and their product is negative.

3.2. Two real numbers *p* and *q* are chosen from the interval [–1; 1]. Find the probability that the equation $x^2+px+q = 0$ has real positive roots.

3.3. Two planes are arriving at the airport at any time between 12^{00} and 12^{30}. Find the probability that the plane which arrived later will not have to wait for permission to land, if the next landing can be made no earlier than 10 minutes after the previous one.

3.4. Two real numbers p and q are chosen from the interval [–1; 1]. Find the probability that equation $x^2+px+q = 0$ has real roots of the same sign.

3.5. Two people make an appointment within $18^{00} - 19^{00}$. The first come will be waiting for 20 minutes and then will go away if the meeting fails. What is the probability that the meeting will not happen?

3.6. A dot is put inside a circle with radius R. Find probability that the point will be inside the inscribed a) square; b) right triangle. It is considered that probability of hit of the point in a part of the circle is proportional to the area of this part and does not depend on its location in relation to the circle.

3.7. A shooter fires a target 20 times and 2 misses are registered. Find the frequency of hitting the target.

3.8. Two random numbers are taken from the interval [0; 3]. Find the probability that their difference is less than 1 and their product is more than 2.

3.9. Two points: are randomly put onto the segment. The segment length is 12 sm. What is the probability that the length of each of 3 obtained segments will be not less than 3 sm?

3.10. 2 airplanes arrive to the airport zone at the random time moments between 12 *AM* and 1 *PM*. The landing of the airplane that arrives the second can take place not earlier than 10 minutes past the 1^{st} airplane landing had been completed. What is the probability that the second airplane wouldn't wait for landing?

3.11. Two real numbers are chosen at random from segment [-2; 1]. Find the probability that the your some is positive and product is negative.

3.12. Two random numbers are taken from the interval [0; 1]. Find the probability that their sum is not less than 1 and their product is not greater than 1.

Section 4. Basic operations on events. Theorems of addition and multiplication of probabilities

Outline

1. Basic operations on events.
2. Conditional probability. Dependent and independent events
3. Theorems of addition and multiplication of probabilities.

Literature: [3]; [5]; [6]; [7].

Methodological recommendations

As a result of studying the materials of this section a student **is to know:** definitions of conditional probability, dependent and independent events; basic operations on events, theorems of addition and multiplication of probabilities; **be able:** to calculate the probability of random events; to apply these concepts in future specialty.

Basic operations on events

A pictorial representation of the operation is obtained by using Venn diagrams or Eler's circles (Fig.3, Fig.4, Fig.5, Fig.6).

Definition. *The union* (logical sum) of two events A and B denoted as $A+B$ or $A\cup B$ and read A ***or*** B is the event consisting of occurrence of either A or B or both events together in one and the same trial S.

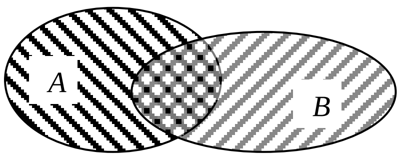

Fig.3

Example. A = {tomorrow it will be windy};

B = {tomorrow it will be snowy};

$A+B$ = {tomorrow there will be wind or snow or wind with snow}.

Definition. *The intersection* (logical product) of two events A and B, denoted as $A\cdot B$ or $A \cap B$ and read «A ***and*** B» is the event consisting in occurrence of both events A and B together in one and the same trial S.

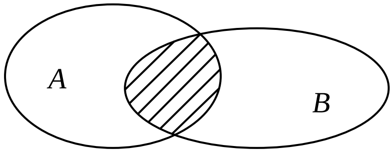

Fig. 4

Example. A = {tomorrow it will be windy};

B = {tomorrow it will be snowy};

$A \cdot B$ = {tomorrow there will be a snowstorm}.

Definition. *The difference* of two events A and B, denoted $A - B$ or $A \setminus B$ is the event consisting of occurrence event A and not occurrence B in one and the same trial S.

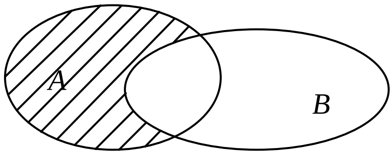

Fig. 5

Example. S = {rolling a die};

A = {even number}; B = {number is 2}; $A–B$ = {numbers 4 or 6}.

Definition. *The complement* of event A is any event except A, denoted $\overline{A}$ and read "not A". The event $\overline{A}$ is named also *opposite* to event A.

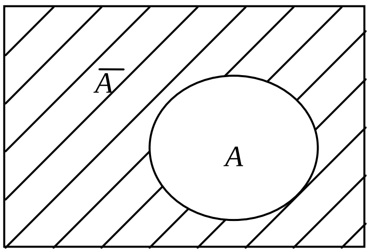

Fig.6

Example. A = { windy};

$\overline{A}$ = {any weather without wind, calm weather}.

De-Morgan's law. For opposite events the following formulas are true:

$$\overline{A+B} = \overline{A} \cdot \overline{B}, \qquad \overline{AB} = \overline{A} + \overline{B}$$

Conditional probability. Dependent and independent events

Example. Consider trial S = {drawing out two balls from the urn with 1 black and 4 white balls without replacement}; and possible events = {first ball is black}; B = {second ball is black}.

1) Let's calculate the probability of event B provided that event A has already occurred. It is obvious that the probability of B under this condition is equal to zero: $P(B/A)=0$.

2) But when event A has not occurred (the first is not black, that is white) then the probability of B under this condition is equal to $1/4$: $P(B/\overline{A})=1/4$.

3) The probability of event B without any conditions (the absolute probability is equal to 1/5: $P(B)=1/5$.

Definition. The probability of occurrence of event B in trial S provided that another event A has taken place is called conditional probability of the event B, and is denoted: $P(B/A)$, $P_A(B)$ or $P(B|A)$. This expression is read «conditional probability of B given A».

Definition. Event B is called dependent on another event A in trial S if the conditional probability of B depends on whether the event A has taken place or not. On the contrary, when $P(B/A)=P(B/\overline{A})=P(B)$ then A and B are called independent events.

Example. Let's consider the previous example when the first ball is replaced in the urn. The trial S = { there is 1 black and 4 white balls in the urn; let's draw out one ball and put it back into the urn then draw out the other ball}. A = {first ball is black}; B = { second ball is black }.

Then $P(B/A)=P(B/\overline{A})=P(B)=1/5$.

Note. If event B is dependent on A, then event A is dependent on B. We may say that A and B are mutually dependent events.

Theorems of addition and multiplication of probabilities

Let two dependent events A and B may occur in trial S and probabilities $P(A)$, $P(B)$, $P(A/B)$, $P(B/A)$ are known. Then it is possible to calculate the probability of the product of events A and B by using the following theorem.

Theorem (multiplication of probabilities). The probability of product of two events *A* and *B* equals the probability of one of them multiplied by conditional probability of the other:

$$P(A\cdot B)=P(A)\cdot P(B/A)=P(B)\cdot P(A/B).$$

Notes.

1) If there are more than two random events $A, B, C, \ldots, K, L$ then $P(A\cdot B\cdot C\ldots K\cdot L)=P(A)P(B/A)P(C/AB)\ldots P(L/ABC\ldots K)$.

2) If events *A* and *B* are independent, that is $P(A)=P_B(A)$, $P(B)=P_A(B)$, then $P(A\cdot B)=P(A)\cdot P(B)$.

3) If events *A* and *B* are exclusive then they are dependent

$$P(A/B)=0 \text{ and } P(B/A)=0.$$

Therefore $P(A\cdot B)=P(A)P(B/A)=P(B)P(A/B)=0,$

which is natural, so their occurrence in one and the same trail is impossible.

Example. Among seven wares there are three defective ones. When drawing three wares one by one without returning find the probability of event *A*={the sequence will be like the following: defective – normal – defective}.

Solution. Let us denote events: A_1 – the first taken ware is defective; A_2 – the second ware is normal; A_3 – the third one is defective. Then probability of event *A* can be calculated by the theorem of product of probabilities:

$$P(A)=P(A_1A_2A_3)=P(A_1)P(A_2/A_1)P(A_3/A_1A_2),$$

$$P(A)=\frac{3}{7}\cdot\frac{4}{6}\cdot\frac{2}{5}=\frac{4}{35}=0.114.$$

Answer: 0.114.

Theorem (addition of probabilities). The probability of a sum of two events *A* and *B* equals the sum of probabilities of these events without the probability of their product:

$$P(A+B)=P(A)+P(B)-P(A\cdot B).$$

Note. If events *A* and *B* are exclusive, then $P(A\cdot B)=0,$ that is

$$P(A+B)=P(A)+P(B).$$

Example. *S* = {drawing two balls out of an urn containing two black and three white balls};

A = {the first ball is black, the second is white};

B = {the first ball is white, the second is black};

$A+B$ = {to get two balls of different color}.

Events A and B are exclusive:

$$P(A+B)=P(A)+P(B)=\frac{2}{5}\cdot\frac{3}{4}+\frac{3}{5}\cdot\frac{2}{4}=\frac{12}{20}=\frac{3}{5}.$$

Example. S = {two archers shoot at one target};

A = {the first one hits the target}, $P(A)=0.9$;

B = {the second one hits}, $P(B)=0.8$;

$A+B$ = {the target has been destroyed}.

Events A and B are compatible but independent:

$$P(A+B)=P(A)+P(B)-P\left(AB\right)=0.9+0.8-0.9\cdot 0.8=0.98.$$

Example. S = {drawing one card out of 36 card pack};

A = {to get a king of any suit};

B = {to get any card of spades};

$A+B$ = {king or any card of spades}.

Events A and B are compatible but independent.

Calculate $P(A)=\frac{4}{36}=\frac{1}{9}$; $P(B)=\frac{9}{36}=\frac{1}{4}$.

Then $P\left(A+B\right)=P\left(A\right)+P\left(B\right)-P\left(AB\right)=\frac{1}{9}+\frac{1}{4}-\frac{1}{36}=\frac{1}{3}$.

Theorem. The probability of the sum of two opposite events A and $\overline{A}$ equals one:

Proof. Because the sum of events A and $\overline{A}$ is a certain event (U) then

$$P\left(A+\overline{A}\right)=P\left(U\right)=1.$$

Corollary. From the previous theorem it follows that

$$P\left(A+\overline{A}\right)=P\left(A\right)+P\left(\overline{A}\right)=1 \text{ and } P\left(\overline{A}\right)=1-P\left(A\right).$$

This formula is used for calculation of the probability of opposite events and in particular for the probability of occurrence at least one of independent events.

Let $A_1, A_2, \ldots, A_n$ be independent events. Then the event A={occurrence at least one of events $A_1, A_2, \ldots, A_n$ } is equivalent to their sum:

$$A=A_1, A_2, \ldots, A_n.$$

Find the opposite event by using the law of de-Morgan:

$$\overline{A}=\overline{A_1+A_2+\ldots+A_n}=\overline{A}_1\cdot\overline{A}_2\cdot\ldots\cdot\overline{A}_n.$$

Note that the opposite events $\overline{A}_1, \overline{A}_2, \ldots, \overline{A}_n$ are also independent.

Then $P(A)=1-P(\overline{A})=1-P(\overline{A}_1)\cdot P(\overline{A}_2)\cdot\ldots\cdot P(\overline{A}_n)$.

Example. Find the probability to hit a target at least one time of three shots (event A), if the probability to hit the target at the first shot (event A_1) makes 0.7; at the second one (event A_2) – 0.8, at the third one (event A_3) – 0.85.

Solution. It is true that events $A_1, A_2, \ldots, A_n$ and their opposite events are independent. Then

$$P(\overline{A})=P(\overline{A}_1)\cdot P(\overline{A}_2)\cdot P(\overline{A}_3)=(1-0.7)(1-0.8)(1-0.85)=0.009$$

From which we will find the probability of event A:

$$P(A)=1-P(\overline{A})=1-0.009=0.991.$$

Answer: 0.991.

Self-test assignment

4.1. Find the probability of reliable work of electric circuit after turning on if the probability that one element will break down after turning on is constant and equal to 0.1. The elements are connected in the circuit as follows (Fig.7):

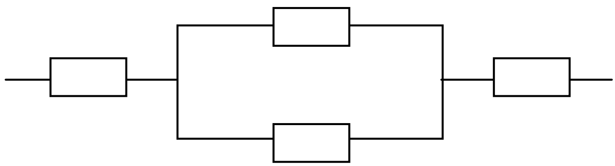

Fig.7

4.2. The probability is equal to 0.1 that a shooter will hit a target of 10 at one shot; the probability to hit a target of 9 is 0.3; the probability to hit less than 9 is 0.6. Find the probability that the shooter will hit between 9 and 10 at one shot.

4.3. Spare parts have to pass three independent technological operations. The probability to get waste products on the first operation is equal to 0.05; on the second and the third – 0.02 and 0.01 respectively.

What is the probability to get standard part after carrying out of three operations?

4.4. There are 2 autonomous burglar alarms in a car. The probability that in an attempt to steal the car the first device will be activated is 0.95; the second – 0.9. Find the probability that: only one device will be activated; at least one will be activated.

4.5. A device consists of three independent parts. When at least one is broken, the whole device is out of service. The probability of proper 24 hour operation for the first part is 0.9; for the second and third – 0.85 and 0.75 respectively. What is the reliability that the device will operate for 24 hours?

4.6. Values of the probabilities $P = 0.2$; $P(A) = 0.5$; $P(B) = 0.4$ are known. Define if events A and B are dependent. Calculate $P(A+B)$, $P(A/B)$, $P(B/A)$.

4.7. Two sets of integers are given: $\Omega_1 = \{1,2,3,4,5\}$, $\Omega_2 = \{1,2,3,4\}$. We take one random figure from each set. Describe random events: A – a sum of figures will be multiple by 2; B – a sum of numbers will be multiple by 3. Check up if random events A and B are dependent. Calculate probabilities $P(A+B)$, $P(AB)$.

4.8. The probability is 0.9 that a car engine will work after starting. Find the probabilities of the following events: 1) the engine will work after the second starting; 2) the engine will not work after more than two startings.

4.9. Each of three shooters fires a target once. The probability to hit the target is equal to 0.5 for the first shooter, it is equal to 0.7 for the second and 0.8 – for the third shooter. Find the probability that one or two shots hit the target.

4.10. A piece of equipment consists of two devices. Intactness of each device is a necessary condition for work of the whole unit. Reliability of the first device is equal to 0.9; and of the second – 0.8. During testing, it has been broken. Find the probability that both devices have been broken, if it is definitely known, that the first device has been broken?

4.11. Find the probability of reliable work of electric circuit after turning on, if the probability that one element will break after turning on is constant and equals 0.2. The elements are connected in the circuit as follows (Fig.8):

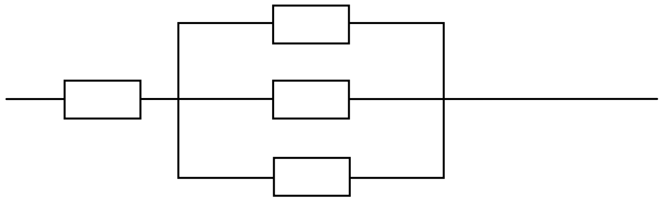

Fig.8

4.12. Find the probability of reliable work of electric circuit after turning on network, if the probability that one element will break after turning on is constant and is equal to 0.2. The elements are connected by the circuit in the following way (Fig.9):

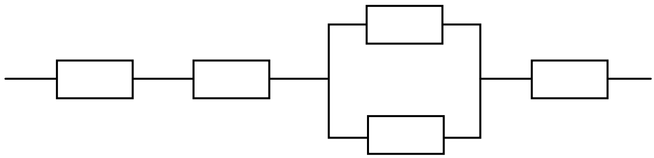

Fig. 9

4.13. There are two sets of integers

$\Omega_1 = \{1,2,3,4,5,6,7,8\}$, $\Omega_2 = \{1,2,3,4,5,6\}$.

One number is taken from each. Find out if random events A and B are dependant, when: A is the product of taken numbers which is multiple of 3, B is the product of taken numbers which is multiple of 7. Calculate the probabilities $P(A+B)$, $P(AB)$.

4.14. The probability to hit the target at every shot $p = 0.9$. Calculate the probability that out of 5 shots there will be three successive misses.

4.15. The probability to hit a target at one shot is a constant value that is equal to 0.9. How many shots must be fired for the probability to hit the target at least once be equal to 0.9999?

4.16. There are made 3 rocket shots fired at a military airplane. The probability to hit the plane from the first shot is equal to 0.95; from second and third – 0.85 and 0.7 respectively. If the rocket hits the plane it will be destroyed. What is the probability that the plane will be destroyed?

4.17. Three parts were made by three machines: one by each. The probability that the part made by the first machine is defective is 0.05; by the second machine – 0.07; by the third machine – 0.1. Find the probability that among made parts: a) only one is defective; b) at least one is defective.

4.18. An airport makes 3 flights to city A during 24 hours. The probability of delay of the first flight according to meteorological conditions is equal to 0.05; of the second – 0.1 and of the third – 0.15. Find the probability that: a) only 2 flights will be delayed; b) all flights will be made on time.

4.19. There are 10 white, 15 black, 20 blue and 25 red cards of identical size in an urn. One random card is taken. Find the probability that the card will be: a) white, black, blue, red; b) white or black, blue or red; c) white, or black, or blue.

4.20. In an urn there are 30 identical marbles which are numbered from 1 to 30. One random marble is taken out of the urn. What is the probability that the number of the marble will be multiple of 3 or 5?

4.21. A worker operates three machines which work independently one of the other. The probability that within one hour the 1-st machine doesn't need attention of the worker equals 0.9; the 2-nd – 0.8; the 3-rd – 0.85. What is the probability that within one hour a) three machines will work properly; b) three machines will need attention of the worker; c) at least one machine will need attention of the worker.

4.22. A student is looking up some formula in 3 reference books. The probability that this formula is in the 1-st, 2-nd and 3-rd books equals 0.6; 0.7; 0.8 respectively. What is the probability that this formula is: a) in one book only; b) only in two books; c) in all three books?

4.23. Let *A, B, C* – be some random events. Write down the expressions for the events which are:

a) event *A* will be the only to happen;
b) events *A* and B will happen but event *C* will not happen;
c) at least one event will happen;
d) no event will happen;

e) not more than two events will happen.

4.24. An urn contains 6 white and 5 black balls. 3 random balls are taken out. Find the probability that only white balls are taken, if they are taken out: a) without putting them back; b) with putting them back.

4.25. A set of balls consists of 9 black and 3 white ones. The set was occasionally divided into three equal parts. Find the probability that one white ball will be in each part.

4.26. A die is thrown up. Event A – the number of the points on the side is even, event B – the number is less than three. Describe the operations with the events A and B, $A \cdot B$, $A+B$.

4.27. A gardener has planted 10 apple-tree transplants during autumn. Each of the transplants may put root with equal probability. What is the probability that 6 or 2 out of 10 transplants will not root next spring?

4.28. In a group 70% of the students speak English, 40% – speak French and 25% – speak English and French. Find the probability that the random student from this group: a) does not speak English but speaks French; b) speaks English but does not speak French; c) speaks neither English no French; d) speaks English or French; e) speaks English and does not speak French or does not speak English and speaks French.

4.29. Suppose an urn contains seven black balls and five white balls. We draw two balls from the urn without replacement. Assuming that each ball in the urn is equally likely to be drawn out, what is the probability that was drawn out balls are black?

4.30. The probability of hitting the target at one firing from two cannons is equal to 0.38. Find probability of hitting the target by the first cannon, if it is known that for the second cannon this probability equals 0.8.

Section 5. Total probability and Bayes' formula

Outline

1. Formula of total probability.
2. Bayes' formula.

Literature: [1],[3]; [5]; [6]; [7]; [8].

Methodological recommendations

As a result of studying the materials of this section a student **is to know:** concepts of the probabilities of hypotheses and conditional probabilities, formulas of total probability and Bayes' formula**; be able:** to calculate the probability of random events; to apply these concepts in future specialty.

Let event A take place together with one of events H_1, H_2, …, H_k, which are exhaustive and exclusive. Events H_k (k = 1, 2 …, n) are named *hypotheses*. If the probabilities of hypotheses and conditional probabilities of event A given each hypothesis are known, the probability of event A in trial S (so called *total probability*) is calculated by the formula:

$$P(A) = P(H_1)P(A/H_1) + P(H_2)P(A/H_2) + \ldots + P(H_n)P(A/H_n)$$

If it is known, that event A has happened with probability $P(A)$, then the conditional probability of each hypothesis $H_к$ given A can be calculated by the following Bayes' formula:

$$P(H_k / A) = \frac{P(H_k)P(A/H_k)}{P(A)} \quad (k = 1, 2 ..., n).$$

Example. The probability of DC team to lose the match against UM team in rainy weather makes 0.5; and if there is no rain – 0.6. The probability of rain on the day of the match makes 0.2. a) Find the probability of not to be defeated by DC team. b) A DC team won the match. What is the probability that the match was took place in rainy weather?

Solution. a) Let event A – be avoidance of defeat by DC team. We will suggest two hypotheses:

H_1 – during the match there will be rainy weather; H_2 – during the match there will be no rain.

According to the problem corresponding probabilities are: $P(H_1) = 0.2$; $P(H_2) = 0.8$; $P(A/H_1) = 0.5$; $P(A/H_2) = 0.4$. After the formula of total probability we will find the probability for DC team to win:

$$P(A) = P(H_1)P(A/H_1) + P(H_2)P(A/H_2),$$

$$P(A) = 0.2 \cdot 0.5 + 0.8 \cdot 0.4 = 0.42 .$$

b) By Bayes' formula we find the probability that there was rain during the match in which DC team won the match:

$$P(H_1 / A) = \frac{P(H_1)P(A/H_1)}{P(A)} = \frac{0.2 \cdot 0.5}{0.42} = 0.238 \cdot$$

Answer: a) 0.42. b) 0.238.

Self-test assignment

5.1. The ratio of lorries driving through a petrol station to cars is 5:9. The probability that a lorry will drive to petrol station is equal to 0.25; that a car – 0.157. The vehicle has driven to the petrol station. What is the probability that it is a lorry?

5.2. A farm has prepared wheat seeds for sowing, among them there are 85% of the first grade, 10% of the second and 5 % of the third grade. The probability, that from one seed of the I-st grade will grow the ear of wheat with not less than 30 seeds, is equal to 0.65; for the II-nd and III-rd grades – 0.15 and 0.1 respectively. What is the probability that a randomly taken out ear of new crop will contain not less than 30 seeds?

5.3. Spare parts made by three workshops are sent to assemble radio receivers. 45% of all parts are made by the 1st factory, 45% – by the 2nd and 30% – by the 3rd. We know that the amount of non-standard parts produced by the 1st factory is 0.3%, by the second – 0.5%, by the third – 0.9%, respectively. One part of those received for assembling was found non-standard. What is the probability that the part was made by the 2nd factory?

5.4. Three workers produce the same spare parts. Their productivity is characterized as 10:8:12. The probability for each worker to make a defective part is equal to 0.02; 0.07 and 0.01, respectively. What is the probability that random part will be standard?

5.5. During a day the first worker produced 80 items, the second – 60 items. Defective items made by the first worker on average make 5% , by the second – 8%. Produced items are put into one container. A random item from the container appeared defective. What is the probability, that it was produced by the first worker?

5.6. Watches are manufactured by 3 factories. The first factory manufactures 45% of all watchers for sale, the second – 35%, and the third – 20%. The first factory produces 80% of the I-st grade production, the 2-nd – 90% and the 3-rd – 75%. A sold watch is of the I-st grade.

Find the probability that this watch has been manufactured by the second factory?

5.7. In a specialized hospital 55% of patients on average come with type *A* disease, 30% – with type *B*; 15% of patients have type *C* disease. The probability of complete recovery from the *A*-type disease is equal to 0.8; from diseases of *B*-type and *C*-type this probability is equal to 0.9 and 0.7, respectively. A patient who was at the hospital recovered. What is the probability that he underwent treatment for the *A*-type disease?

5.8. The storehouse has received identical items 40% of which had been produced by factory № 1, 35% – by factory № 2 and 25% – by factory № 3. 2%, 5% and 9% of defective items were accounted to the factories, respectively. What is the probability that a random item will be valid?

5.9. Spare parts of the same type produced by 3 factories are supplied to a warehouse for repairing aircraft. The first factory supplies 45% of all parts, the second – 30% and the third – 25%. The probability of manufacturing high quality parts for each factory equals 0.9; 0.95; 0.85, respectively. a) Determine the average percentage of defective parts that are stored in the warehouse. b) A random part is found defective. Find the probability that it is made by the first factory.

5.10. According to statistical data the probability of being caught in flight by the storm in some region at high altitudes is equal to 0.4; at average altitudes – 0.6; at low altitudes – 0.8. In this region 10 % of flights are carried out at high altitude, 30 % – at average and 60 % – at low altitude. a) Find the probability that the plane, which carries out flight in this region, will not be caught by a storm. б) The plane has not been caught by a storm. At what most probable altitude did it fly?

5.11. There are 5 rifles and 3 of them have optical sight. The probability that a shooter hits a target by the rifle with optical sight is equal to 0.95 and without optical sight – 0.75. Find the probability that the target will be hit when the shooter will make one shoot by an occasionally taken rifle.

5.12. There are 12 items from factory №1, 20 items from factory №2 and 18 items from factory №3. The probability that the item made at factory №1 is of good quality is equal to 0.9; for the items from factories

№2 and №3 these probabilities are equal to 0.6 and 0.9, respectively. Find the probability that occasionally taken out item is of good quality.

5.13. 5% of all men and 0.25% of women are colorblind. An occasionally chosen person is colorblind. What is the probability that a) this person is a man; b) this person is a woman? It is supposed that the quantity of men and women is equal.

5.14**.** The workshop services planes during 3 shifts. The I-st shift service 40% of all planes, the II-nd – 10% and the III-rd – 50% . The rate of not qualitative servicing is equal to: 2% for the I-st shift; 1% for the II-nd shift; 4% for the III-rd shift. 1) What is the probability that random plane will be properly serviced. 2) A randomly chosen plane has been properly serviced. What is the probability that the plane was serviced by the III-rd shift?

5**.**15. During transportation of a box with 21 standard and 10 non-standard spare parts, one of them was lost, it's unknown which. After that one of the spare parts randomly taken out of the box turned out standard. Find the probability that the lost spare part was: a) standard; b) non-standard.

5.16. In an urn which contains two balls a white ball has been put in, then from the urn one ball was randomly taken out. Find the probability that the taken ball is white, if all possible initial ball sets are equally probable.

5.17. There are 11 spare parts of the same type in a box; 7 of them are standard and the others are defective. From the box three ones were randomly taken out and were not returned. What is the probability to take randomly a standard part after that?

5.18. There are three urns. In the first one there are 6 white and 4 black balls, in the second – 8 white and 2 black ones and in the third – 1 white and 1 black balls. Somebody has at random taken 3 balls from the first urn and 2 balls from the second one and put them into the third urn. What is the probability to take from the third urn after that one white ball?

5.19. Two automatic devices produce identical components, which go into one container. Productivity of the first automatic device is greater than that of the second one. The first automatic device makes 60% of components of good quality and the second – 84%. The component taken at random from the container was of a good quality. Find the probability that this component was made by the first automatic device.

5.20. There are 3 sets of components with 20 ones in each set. The quantity of standard components in the first, second and third sets are equal to 20, 15, 10, respectively. From the set, which was chosen at random, one component was taken out and it was standard. This component was returned into the set and then another one was taken out at random, which was also standard. Find the probability that the last component was taken from the third set.

5.21. Consider two different departments of the bank. There are 32 employees in the first department and 40 employees in the second. It is known that 75% of employees of the first department and 50% of employees of the second department are women. Two improve work efficiency in the bank people management department moved two persons from the second department to the first and then fired one employee from the first department. Assuming that the probability to be moved and the probability to be fired are equal for each employee, compute:

a) the probability out of that to moved employees there is only one women;

b) the probability that the fired from the first department employee was the one that was moved from the second department, if we know that people management department fired a man.

5.22. The battery of three cannons shoot a target and two cannons hit it. Find the probability that the first cannon hit the target, if probabilities of hitting by the first, second or third cannons are equal to 0.4; 0.3; 0.5, respectively.

Section 6. Bernoulli's formula

Outline

1. The *scheme of successive independent tests or Bernoulli scheme*.
2. The most probable number of successes.

Literature: [3]; [5]; [7]; [8].

Methodological recommendations

As a result of studying the materials of this section a student **is to know:** the *scheme of successive independent tests, formulas of* the most probable number of successes; **be able:** to calculate the probability of random events; to apply this concepts in future specialty.

Suppose trial *S* consists of the repeating *n* times of one and the same dealing, and event *A* can appear in each repeating with the same probability *p*, which does not depend on the other results (we will name the event *A* as *success*). For example, the trial *S* is tossing a coin three times (n=3) and appearance of the heads (event *A*) we call as success. Such series of *n* repeating is named as the *scheme of successive independent tests or Bernoulli scheme*. The event *A* in *n* repeating the dealing can appear *к* times in general, *к* = 0, 1, 2,…, *n*. Denote the probability of appearance of event *A k* times in *n* independent repeating $P_n(k)$. Then this probability can be calculated by Bernoulli formula:

$$P_n(k) = C_n^k p^k q^{n-k},$$

when ($q = 1 - p$), $C_n^k = \dfrac{n!}{k!(n-k)!}$.

Example. Three shooters fire at the target. The probability to hit the target for each shooter is equal to 0.8. Find the probabilities for the following events: 1) nobody hits the target; 2) only one shooter hits the target; 3) two shooters hit the target; 4) all shooters hit the target.

Solution. By Bernoulli's formula we obtain:

$P_3(0) = C_3^0 \cdot 0.8^0 \cdot 0.2^3 = 0.008$; $P_3(1) = C_3^1 \cdot 0.8 \cdot 0.2^2 = 0.096$;

$P_3(2) = C_3^2 \cdot 0.8^2 \cdot 0.2 = 0.384$; $P_3(3) = C_3^3 \cdot 0.8^3 \cdot 0.2^0 = 0.512$.

It is important that $P_3(0) + P_3(1) + P_3(2) + P_3(3) = 1$, it means all considered hits are exhaustive events.

Note. The most probable number of successes $k_{\max}$ can be calculated from the condition: $np - q < k_{\max} < np + p$; or by the following expression $k_{\max} = [np + p]$, it means $k_{\max}$ is the greatest integer number less or equal to $np + p$.

For example, if a shooter fired at a target five times and the probability to hit the target *p*= 0.8 at every shot then the most probable number of hits $k_{\max} = [5 \cdot 0.8 + 0.8] = [4.8] = 4.$ The corresponding probability of 4 hits at 5 shots by Bernoulli's formula will be $P_5(4) = 0.41.$

If in this problem *p*= 0.5 then $np + p = 5 \cdot 05 + 0.5 = 3$ (the integer) and $np - q = 2.$ Thus $k_{\max} = 2$ or $k_{\max} = 3$ and $P_5(2) = P_5(3) = 0.31.$

Self-test assignment

6.1. What event is the more probable: to win an equal opponent in three games of four or five of eight (drawn game is excluded)?

6.2. The probability is equal to 0.1 that one sign will be wrong in the message. What is the probability that the message of 10 signs: 1) will not have a mistake; 2) will have 3 mistakes?

6.3. A worker provides service of five automatic machines. The probability is equal to 0.7 that one machine will on average require attention of the worker during his shift. What is the probability that at least one machine requires attention of the worker? Find the most probable number m_0 of machines which require attention of the worker throughout the shift.

6.4. Under certain technological conditions 75% of manufactured carriages are of top quality. Find the probability that among five carriages chosen for a test there will be: 1) only three ones of the top quality; 2) not less than four ones of the top quality.

6.5. A device consists of *n* irrespectively working mechanisms. The reliability that each mechanism will work for time *t* is constant and equals 0.9. Calculate number *n* if the most probable number of reliable mechanisms is 20.

6.6. A coin was tossed five times. Find the probability that the "heads" will fall out: 1) one time; 2) five times.

6.7. The probability that the basketball player will hit the basket with one throw equals 0.8. What is the probability that after five throws the basketball player will hit the basket: 1) two times; 2) not less than two times?

6.8. The wholesale store services 6 shops. Every shop can order servicing the next day with the probability 0.8 irrespective of other shops. Calculate the probability that orders will be made by: 1) four shops; 2) not less than four shops.

6. 9. The probability to win with one lottery ticket equals 1/7. Find the probability that among five acquired tickets there are: 1) 2 losing tickets; 2) 2 winning tickets. Find the most probable number of winning tickets.

6.10. The probability that random part is non-standard is equal to 0.1. Find the probability that among 5 random parts the number of non-standard ones will be: 1) not more than two; 2) not less than two. Find the most probable number of non-standard parts among 5 taken.

6.11. 5% of items produced by a factory turned out defective. Find the probability that among 5 random items there are: 1) no defective items; 2) two defective items.

6.12. A factory produces TV-sets 85% of which are of top quality. From a batch of TV-sets 7 random ones were taken. What is the probability that among taken TV-sets there will be of top quality: 1) 4; 2) not less than 3?

6.13. A group of items contains 30% of non-standard ones. Find the probability that among 5 random items there will be: a) only one non-standard item; b) at least one non-standard item.

6.14. A worker serves 3 automatic machines. The probability that a defective part will be produced by the first machine is equal to 0.02; by the second – 0.05 and by the third – 0.1. The productivity of the first machine is 3 times as high as that of the second, and the productivity of the third is 2 times lower than that of the second. All parts are in the same container. a) Find the probability that random part will be defective. b) A random part is found defective. Which machine was it most probably made by?

6.15. There are 20 white and 5 black balls in an urn. Occasionally 6 balls were taken one by one and then put back into the urn. What is the probability that among taken balls there will be: a) only 4 white balls; b) not fewer than four white ones?

6.16. How many times is it necessary to throw up a die so that the most probable number of points 2 will be equal to 32?

6.17. There have been sown 28 barley seeds with the same probability to come out for each seed. Find this probability, if the most probable numbers of positive results are 17 and 18.

6.18. A coin is tossed up five times. Find the probability that the heads will come out: a) less than two times; b) not less than two times.

6.19. There are five children in a family. Find the probability that among these children there are: a) two boys; b) not more than two boys; c) more than two boys; d) not fewer than two and not more than three boys. The probability that a boy was born is equal to 0.51.

6.20. The department of technical control checks up a set of 10 spare parts. The probability that a part is standard is equal to 0.75. Find the most probable number of parts which will be found standard.

6.21. How many independent tests should be carried out to get most probable number of appearance of the event 25, when the probability of appearance of this event in each test is equal to 0.4?

6.22. What is the probability of appearance of the event in each of 39 independent tests, if most probable number of appearance of this event in these tests equals 25?

6.23. A battery fired 6 shots at an object. The probability to hit it by one shot is equal to 0.3. Find: a) the most probable number of hits; b) the probability of this number; c) the probability that the object will be destroyed by at least two hits.

6.24. There is a set of wares of two grades in a store, thus the wares of the second grade are 1.5 times more than those of the first grade. Find the probability that among three wares taken at random at least one will be of the first grade.

6.25. Find the probability that event A will appear not less than three times in four independent tests, if the probability of appearance of this event in one independent test is equal to 0.4.

Section 7. Local and integral De Moivre-Laplace Formulas

Outline

1. Local De Moivre-Laplace formula.
2. Integral De Moivre-Laplace formula.

Literature: [1, c. 51–69]; [3]; [5]; [6]; [7]; [8].

Methodological recommendations

As a result of studying the materials of this section a student **is to know:** Local and integral De Moivre-Laplace formulas, **properties of** Gauss' function and Laplace' function; **be able:** to calculate the probability of random events; to apply these concepts in future specialty, to apply Appendixes 1and 2.

Local De Moivre-Laplace formula

When variable n is large ($np > 5$ and $nq > 5$) the calculation of probability $P_n(k)$ by Bernoulli formula becomes not convenient and this probability can be found approximately by asymptotical formula (the greater n the higher accuracy). Such formula was developed by two famous mathematicians De Moivre Abraham (England) and Laplace Pierre Simon (France) and is named after them. It is a local De Moivre-Laplace formula:

$$P_n(k) \approx \frac{1}{\sqrt{npq}}\varphi(x) \quad \text{, where } x = \frac{k-np}{\sqrt{npq}} \text{ and}$$

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

The function $\varphi(x)$ (called Gauss's function) is evenly and quickly decreases, so $\varphi(4)$=0.0001. The values of the function $\varphi(x)$ can be found from the table (see Appendix 1).

Note. Stirling's formula: $n\,! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ may be used.

Example. Find the probability that when tossing a coin hundred times 50 heads will appear.

Solution. First of all we find the argument $x = \frac{50 - 100\cdot 0.5}{\sqrt{100\cdot 0.5\cdot 0.5}} = \frac{0}{5} = 0$.

Then Gauss function can be calculated or taken from the table:

$\varphi(0)=\frac{1}{\sqrt{2\pi}}\approx 0,4$. According to local De Moivre-Laplace formula we obtain: $P_{100}(50)=\frac{1}{5}0.4=0.08$.

Note that this probability slightly changes with changing k:

K	50	49 51	48 52	47 53	46 54	45 55	44 56	…	40 60
$P_{100}(k)$	8%	8%	7.4%	6.1%	6%	5%	4%		1%

Integral De Moivre-Laplace formula

To calculate the probability of appearance of event A from k_1 to k_2 times when repeating the dealing n times ($np>5$ and $nq>5$) in Bernoulli scheme we use the following approximate formula (integral De Moivre-Laplace formula):

$$P_n(k_1\le k\le k_2)=\hat{O}(x_2)-\hat{O}(x_1),$$

where $x_{1,2}=\frac{k_{1,2}-np}{\sqrt{npq}}$, $\hat{O}(x)=\frac{1}{\sqrt{2\pi}}\int_0^x e^{-\frac{t^2}{2}}dt$.

Note. The function $\hat{O}(x)$ is called Laplace' function and is odd $\hat{O}(-x)=-\hat{O}(x)$. Furthermore, $\hat{O}(0)=0$; $\hat{O}(-x)=-\hat{O}(x)$; $\hat{O}(\infty)=\frac{1}{2}$ and for $x>5$ $\hat{O}(x)$ undertakes ≈ 0.5. The values of the function can be found from the table (see Appendix 2).

Example. Find the probability that in a sequence of 100 tossings of a coin the heads will appear from 40 to 60 times.

Solution. According to the condition of the problem: $n=100$, $k_1=40$, $k_2=60$, $np=50$, $p=\frac{1}{2}$, $q=\frac{1}{2}$, $\sqrt{npq}=5$. Then we find the arguments $x_1=\frac{40-50}{5}=-2$ and $x_2=\frac{60-50}{5}=2$ $x_1=\frac{60-50}{5}=2$. From the appendix we take: $\hat{O}(2)=0.477$. Since this function is odd

$\hat{O}(-2) = -0.477$. By integral De Moivre-Laplace formula we obtain: $P_{100}(40 \div 60) = 0.477 - (-0.477) = 0.954$.

Self-test assignment

7.1. Among stamped bolts 10% are on average defective. 900 bolts were randomly chosen. What is the probability that the number of standard ones will be not more than 850?

7.2. The probability is on average equal to 0.99 that turnstile-automat in metro will work properly after the magnetic card is passed over. What is the probability that while passing 400 passengers, who use cards, the automat will work properly from 360 to 380 times?

7.3. The probability is 0.05 that according to technical control a produced automatic machine-tool needs additional regulation. How many machine-tools must be tested so that the most probable number of the machine-tools which do not need any regulation may equal 80?

7.4. First-grade items of goods produced by a factory constitute 75%. What is the probability that among 400 items not more than 280 will be of the first-grade?

7.5. The probability of appearance of some random event in every independent experiment is constant and equals 0.9. How many experiments are needed when the most probable number of appearance of this event is equal to 36?

7.6. What is the value of the probability that a produced part will be standard if after testing 300 parts, the most probable number of standard ones equals 200?

7.7. The probability that a student will pass an exam in mathematics equals 0.67. Find the probability that out of 100 students of the same year of study an exam will be passed: 1) by not less than 60 students; 2) by the number from 50 to 80 students.

7.8. The probability that a customer visiting a shoe shop will buy something is equal to 0.2. What is the probability that out of 200 customers 30 will buy something?

7.9. The probability, that each passenger who wants to book a ticket to *N* airport equals 0.1. Find the probability that tickets to *N* airport out of 100 passengers will be bought by: a) less than 15 people; b) from 5 to 12 people c) more than 20 people.

7.10. An airline provides 400 flights during a month. The probability of full commercial loading of each flight is equal to 0.8. Find the probabilities that during the month with the full loading will be made: a) not less than 300 flights; b) more than half of flights.

7.11. According to the statistical data 1% of passengers cancel their flight. Find the probability that out of 300 passengers having tickets will cancel their flight: a) not more than 5 passengers; b) not less than 3 passengers.

7.12. A technical control department usually approves 90% of factory production. How many items is it necessary to produce to have not less than 200 items approved?

7.13. According to statistical data 5% of flights which are carried out by airline are delayed for technical reasons. Find the probability that out of 400 schedule flights there will be delayed for technical reasons: a) not more than 3% of flights; б) not less than 10% of flights.

7.14. The probability that a student is present at a lecture equals 0.8. Find the probability that out of 100 students at the lecture will be present: a) 75; b) not less than 75 and not more than 90; c) not less than 75; d) not more than 74 students.

7.15. The department of technical control is testing 900 wares. The probability that one ware is standard is equal to 0.9. With the probability 0.9544 find the limits of the interval which contains the number of standard wares among the tested ones.

7.16. In some region the number of diabetics is 0.2% of all inhabitants. 4000 people were examined at random. What is the probability that among the examined people the number of diabetics will be: a) 4 people; b) from 3 to 6 people; c) not more than 4 people?

7.17. The probability of appearance of an event in each of 2100 independent trials is equal to 0.7. Find the probability that the event will appear: a) not less than 1470 and not more than 1500 times; b) 1478 times; c) not more than 1469 times.

7.18. The probability of appearance of an event in each independent test equals 0.8. How many tests are needed to expect that the event will appear not less than 75 times with probability 0.9?

Section 8. Poisson formula

Outline

1. Poisson formula.
2. Elementary flow of events.
3. Probability of the deviation.

Literature: [1]; [3]; [5]; [6]; [7]; [8].

Methodological recommendations

As a result of studying the materials of this section a student **is to know:** Poisson formula, elementary flow of events, probability of the deviation **be able:** to calculate the probability of random events; to apply these concepts in future specialty

Poisson formula

We consider an event to be rare if the probability of its appearance in each repeating of the dealing is very low $\left(p \le 0.01\right)$. But when repeating many times $\left(n \ge 100\right)$ in accordance with Bernoulli scheme the probability $P_n\left(k\right)$ may become considerable. Such probability is calculated approximately by the following Poisson's formula (if product $np \le 20$):

$$P_n\left(k\right) \approx \frac{(np)^k}{k!} e^{-np}.$$

Note. When the event A is a failure of an equipment then for k=0 we have so called reliability of the equipment $P_n\left(0\right) = e^{-np}$.

Example. There are 1500 pages in a dictionary. The probability of a printing error on one page is 0.001. To find the probability that in a dictionary: a) there will be only three errors; b) there will be none; c) there will be at least one error.

Solution. a) Here the probability of event in one experiment $p = 0.001 < 0.01$ and $pn = 1500 \cdot 0.001 = 1.5 < 20$. Then by the Poisson formula, we find

$$P_{1500}(3) = \frac{1{,}5^3}{3!} e^{-1{,}5} \approx 0.125.$$

b) The probability that there will be no errors in the dictionary, that is $k = 0$, is found after the same formula

$$P_{1500}(0) = \frac{1{,}5^0}{0!} e^{-1{,}5} = \frac{1}{1 \cdot \sqrt{e^3}} \approx \frac{1}{4.48} \approx 0.223.$$

c) Event A – there will be at least one error in the dictionary, is opposite to the event – there are no errors in the dictionary.

Therefore $P(A) = 1 - P_{1500}(0) = 1 - 0.223 = 0.777$.

Answer: a) 0.125; b) 0.223; c) 0.777.

Elementary flow of events

Let event A repeat and appears at random moments $\tau_1, \tau_2, \ldots, \tau_n$. Denote the mean number of random events appearing during time T as λ. For example, appearance of radioactive pulses recorded by Geiger-counter or calls at a telephone-exchange.

Definition. A sequence of events which appear at a random moment is called the flow of events.

Definition. The mean number λ of the events per time unit is called intensity of the flow.

Let the event flow be under the following conditions:

1. Uniformity. Probability of the appearance of event A during the interval Δt does not depend on location of interval Δt but depends only on Δt length and intensity λ.

For example, probabilities of the appearance of event A within the intervals of the time (1; 7) and (10; 16) with $\Delta t = 6$ are the same.

2. Ordinariness. The appearance of more than one event during any infinitesimal interval of time is impossible. It means that the coincidence of some events is impossible.

3. Independency. The appearance of the event during interval Δt is independent of the number of appearing events before this interval.

Definition. The event flow under conditions of uniformity, ordinariness and independency is called elementary flow of events or Poisson flow.

Let k – be the number of events occurred during time T in elementary flow of events. Than the probability of such occurrence can be found by Poisson formula which is a mathematical model of the elementary flow of events:

$$P_T(k) = \frac{(\lambda T)^k}{k!} e^{-\lambda T} .$$

Example. The mean number of calls at a telephone-exchange during 1 minute is equal to 3. Find the probability that during 5 minutes: a) 2 calls; b) less than 2 calls; c) not less than 2 calls; d) 15 calls will appear.

Solution: $\lambda = 3,\ T = 5,\ k = 2,\ \lambda T = 15$.

a) $P_5(2) = \dfrac{15^2 e^{-15}}{2!} = 0.00003$;

b) $P_5(k<2) = P_5(0) + P_5(1) = e^{-15} + 15e^{-15} = 0.000045$

(almost impossible event);

$P_5(k<2) = 1 - P_5(k<2) = 1 - 0.000049 = 0.999995$

(almost certain event).

c) $P_5(15) = \dfrac{15^{15} e^{-15}}{15!}$.

Using Stirling's formula $15! \approx \sqrt{2\pi \cdot 15} \cdot 15^{15} \cdot e^{-15}$ we obtain

$$P_5(15) \approx \frac{15^{15} e^{-15}}{\sqrt{2\pi \cdot 15} \cdot 15^{15} \cdot e^{-15}} = \frac{1}{\sqrt{30\pi}} \approx 0.1 .$$

Probability of the deviation

Let probability p of event A be given in each repeating trial. Let's make n repetitions and event A will appear k times, $0 \le k \le n$. Suppose that event A appears k times $(0 \le k \le n)$. Take into account that relative frequency $\dfrac{k}{n} \xrightarrow[n\to\infty]{} p$ and $0 \le \dfrac{k}{n} \le 1$.

Find the probability that relative frequency with n repetitions differs from the given probability by less than ε. That is, we have to find the probability of the inequality:

$$\left|\frac{\kappa}{n}-p\right|<\varepsilon \Rightarrow -\varepsilon<\frac{\kappa}{n}-p<\varepsilon \Rightarrow -\varepsilon n<\kappa-np<\varepsilon n \Rightarrow \underbrace{np-\varepsilon n}_{\kappa_1}<\kappa<\underbrace{np+\varepsilon n}_{\kappa_2}.$$

We use integral De Moivre-Laplace formula:

$P(k_1 \le k < k_2) = \hat{O}(x_2) - \hat{O}(x_1)$ where

$$x_1 = \frac{k_1 - np}{\sqrt{npq}} = \frac{np - n\varepsilon - np}{\sqrt{npq}} = -\frac{n\varepsilon}{\sqrt{npq}} = -\sqrt{\frac{n}{pq}}\varepsilon;$$

$$x_2 = \frac{k_2 - np}{\sqrt{npq}} = \frac{np + n\varepsilon - np}{\sqrt{npq}} = \sqrt{\frac{n}{pq}}\varepsilon;$$

$$P\left(\left|\frac{k}{n}-p\right|<\varepsilon\right) = 2\hat{O}\left(\sqrt{\frac{n}{pq}}\varepsilon\right).$$

This formula also enables us to estimate how many trials it is necessary to provide with approximately known probability.

Example. The spare part is not standard with *p=0.1*. How many spare parts is it necessary to check so that the relative frequency of defective ones differs with respect to *p=0.1* not greater than 0.03 with probability 0.9544.

Solution: $p = 0.1;\ q = 0.9;\ \varepsilon = 0.03;$

$$P\left(\left|\frac{m}{n}-p\right|\le\varepsilon\right) = 0.9544;\quad P\left(\left|\frac{m}{n}-p\right|\le\varepsilon\right) = 2\hat{O}\left(\varepsilon\sqrt{\frac{n}{pq}}\right);$$

$$0.9544 = 2\hat{O}\left(\frac{0.03\sqrt{n}}{\sqrt{0.1\cdot 0.09}}\right);\quad \hat{O}\left(0.1\sqrt{n}\right) = 0.4772.$$

From the table $\hat{O}\left(2\right) = 0.4772;$ that is:

$$2 = 0.1\sqrt{n} \Rightarrow \sqrt{n} = 20 \Rightarrow n = 400.$$

It means, if we take 400 spare parts, then from $28 = (0.1 - 0.03)\cdot 400$ to $52 = (0.1 - 0.03)\cdot 400$ defective ones will be found among them. We can assert this result with probability 0.9544.

Self-test assignment.

8.1. The probability of surviving a bacterium after radioactive irradiating is on average equal to 0.004. Find the probability that after radioactive irradiating 500 bacteria at least 5 of them will survive.

8.2. To estimate the probability, that absolute value of difference between a number of thread breaks and the average number of breaks on 1000 spindles will be less than 10, if it is known that on average 7% of threads are broken.

8.3. Edition of a book is 50000 copies. The probability that there is a defect of stitching a book is 0.0001. Find the probability that the edition contains 5 off-grade stitched copies.

8.4. The probability that any computer will be chosen for testing equals 0.02. Find the probability that from 4000 computers in the warehouse there have been tested from 100 up to 200 ones.

8.5. The probability that a passenger will miss the train is equal to 0.007. What is the probability that out of 20000 passengers from 1000 to 4000 people will miss the train?

8.6. An instrument has six identical safety devices. The probability that the device will break down after 1000 working hours is constant value and equals 0.4. Find the probability that after 1000 working hours will break down: 1) 3 devices; 2) not more than 3 devices.

8.7. Among identical details parts there are 90% of standard ones and 10% of non-standard ones. 400 random parts are taken. Find the needed quantity of standard parts m_i for the probability of inequality $\left(m_i < m < 40\right)$ to be equal to 0.99?

8.8. Testing of valves showed that 90% of them pass through guaranteed service life. How many valves should be tested if the probability is 0.999 that deviation of relative frequency of finding valves working properly within guaranteed service life from probability of this event taken by absolute value does not exceed $\varepsilon = 0.001$?

8.9. The probability of appearing random event A in each of 900 independent trials is constant and equals 0.8. What is the value of number $\varepsilon > 0$ if the probability of inequality $\left|W\left(A\right) - 0{,}8\right| < \varepsilon$ is not less than 0.999?

8.10. The probability that a device will fail during its reliability testing is equal to 0.1. How many devices must be checked if we know that

$P\left(\left|W(A)-p\right|<\varepsilon\right)=0.99$; where $\varepsilon=0.001$, and the relative frequency of devices failure is $W(A)$.

8.11. A die is thrown up 10000 times. Calculate the probability that the frequency of appearing number 6 differs in absolute value from the probability of this event by not more than 0.01.

8.12. A radio set consists of 1000 elements. The probability of breaking each element during a day is equal to 0.002 and does not depend on other elements. Find the probability of breaking during the day: a) only 2 elements; b) not less than 2 elements.

8.13. A die was rolled 1000 times. With probability 0.95 find the limits between which the number of four points will come out.

8.14. A shop got 1000 bottles of mineral water. The probability that during transportation one bottle will be broken is equal to 0.003. Find the probability that the shop got broken bottles: a) only two; b) less than two.

8.15. In a lot of food jars each one can become deformed with the probability 0.003. Calculate the probability that among 1000 jars will become deformed: 1) more than two jars; 2) at least one jar.

8.16. A coin is tossed up 20 times. What is the probability that the heads will come out 7 or 17 times?

8.17. A device contains plenty of independently working elements with the same (very low) probability of breakage. Find the average number of broken elements during time T when it is known with probability 0.98 that during this time at least one element will be broken.

8.18. During rush-hours one passenger passes in 1 second on average through the underground station. The flow of passengers is considered as the simplest one. What is the probability that for 5 seconds through the station will pass: a) 4 passengers; b) from one to five passengers?

8.19. During 1 minute 2 calls are on average received at an automatic telephone exchange (ATX). Find the probability that during 4 minutes will be received: a) three calls; b) less than three calls; c) not less than three calls.

8.20. During 6 hours 13000 impulses entered a receiver. What is the probability that during 5 seconds it will receive: a) 4 impulses; b) not more than 4 impulses?

8.21. Rezerford observed radioactive matter on average radiate 3.87 alpha-particles in the interval of 7.5 seconds. Find the probability that per 1 sec this matter will radiates at least one alpha-particle.

8.22. Probability of hitting the target at every shot is equal to 0.001. Find the probability of hitting the target by two bullets and more, if the number of shots is equal to 5000.

8.23. The mean value of morbific microbes density in one cubic meter of air is equal to 100. There is a sample of 2 cubic decimetre of air for a test. Find the probability that at least one microbe will be found in the sample.

8.24. In the table of random numbers the figures are grouped in twos. Find approximate value of the probability that couple 09 will appear not less than 2 times among 100 couples of the figures.

8.25. During type-setting a book, any letter can be composed wrong with probability 0.0001. After type-setting, a proof-reader finds every mistake with probability 0.9. After the proof-reader the text is read by the author, who finds the rest of mistakes with probability 0.5. Find the probability that not more than 5 mistakes will remain in the book of 100000 printing signs.

8.26. How many times is it necessary to throw up a die so that with the probability $p = 0.997$ the inequality $\left|\frac{m}{n} - \frac{1}{6}\right| \leq$ 0.001 will take place?

8.27. The probability of appearance of an event equals 0.8 in each of 625 independent tests. Find the probability that relative frequency of appearance of the event will differ from its probability in absolute value by not more than 0.04.

8.27. The probability of appearance of an event in each of 900 independent trials equals 0.5. Find the positive number ε that absolute value of deviation of relative frequency of the event from its probability 0.5 does not exceed this number ε with probability 0.7698.

APPENDIX 1

Values of function $\varphi(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

x	0	1	2	3	4	5	6	7	8	9
0.0	0.3989	3989	3989	3988	3986	3984	3982	3980	3977	3973
0.1	3970	3965	3961	3956	3951	3945	3939	3932	3925	3918
0.2	3910	3902	3894	3885	3876	3868	3857	3847	3836	3825
0.3	3814	3802	3790	3778	3765	3752	3739	3726	3712	3697
0.4	3683	3668	3652	3637	3621	3605	3589	3572	3555	3538
0.5	3521	3503	3485	3467	3448	3429	3410	3391	3372	3352
0.6	3332	3312	3292	3271	3251	3230	3209	3187	3166	3144
0.7	3123	3101	3079	3056	3034	3011	2989	2966	2943	2920
0.8	2897	2874	2850	2827	2803	2780	2756	2732	2709	2685
0.9	2661	2637	2613	2589	2565	2541	2516	2492	2468	2444
1.0	0.2420	2396	2371	2347	2323	2299	2275	2251	2227	2203
1.1	2179	2155	2131	2107	2083	2059	2036	2012	1989	1965
1.2	1942	1919	1895	1872	1849	1826	1804	1781	1758	1736
1.3	1714	1691	1669	1647	1626	1604	1582	1561	1539	1518
1.4	1497	1476	1456	1435	1415	1394	1374	1354	1334	1315
1.5	1295	1276	1257	1238	1219	1200	1182	1163	1145	1127
1.6	1109	1092	1074	1057	1044	1023	1006	0989	0973	0957
1.7	0940	0925	0909	0893	0878	0863	0848	0833	0818	0804

1.8	0790	0775	0761	0748	0734	0721	0707	0694	0681	0669
1.9	0656	0644	0632	0620	0608	0596	0584	0573	0562	0551
2.0	0.0540	0529	0519	0508	0498	0488	0478	0468	0459	0449
2.1	0440	0431	0422	0413	0404	0396	0387	0379	0371	0363
2.2	0355	0347	0339	0332	0325	0317	0310	0303	0297	0290
2.3	0283	0277	0270	0264	0258	0252	0246	0241	0235	0229
2.4	0224	0219	0213	0208	0203	0198	0194	0189	0184	0180
2.5	0175	0171	0167	0163	0158	0154	0151	0147	0143	0149
2.6	0136	0132	0129	0126	0122	0119	0116	0113	0110	0107
2.7	0104	0101	0099	0096	0093	0091	0088	0086	0084	0081
2.8	0079	0077	0075	0073	0071	0069	0067	0065	0063	0061
2.9	0060	0058	0056	0055	0053	0051	0050	0048	0047	0046
3.0	0.0044	0043	0042	0040	0039	0038	0037	0036	0035	0034
3.1	0033	0032	0031	0030	0029	0028	0027	0026	0025	0025
3.2	0024	0023	0022	0022	0021	0020	0020	0019	0018	0018
3.3	0017	0017	0016	0016	0015	0015	0014	0014	0013	0013
3.4	0012	0012	0012	0011	0011	0010	0010	0010	0009	0009
3.5	0009	0008	0008	0008	0008	0007	0007	0007	0007	0006
3.6	0006	0006	0006	0005	0005	0005	0005	0005	0005	0004
3.7	0004	0004	0004	0004	0004	0004	0003	0003	0003	0003
3.8	0003	0003	0003	0003	0003	0002	0002	0002	0002	0002
3.9	0002	0002	0002	0002	0002	0002	0002	0002	0001	0001

APPENDIX 2

Values of function $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2} dt$

X	0	1	2	3	4	5	6	7	8	9
0.0	0.00000	00399	00798	01197	01595	01994	02392	02790	03188	03586
0.1	03983	04380	04776	05172	05567	05962	06356	06749	07142	07535
0.2	07926	08318	08706	09095	09483	09871	10257	10642	11026	11409
0.3	11791	12172	12552	12930	13307	13683	14058	14431	14803	15173
0.4	15542	15910	16276	16640	17003	17364	17724	18082	18439	18793
0.5	19146	19497	19847	20194	20540	20884	21226	21566	21904	22240
0.6	225075	22907	23237	23565	23891	24215	24537	24857	25175	25490
0.7	25804	26115	26424	26730	27035	27337	27637	27935	28230	28524
0.8	28814	29103	29389	29673	29955	30234	30511	30785	31057	31327
0.9	31594	31859	32121	32381	32639	32894	33147	33398	33646	33891
1.0	0.34134	34375	34614	34850	35083	35314	35543	35769	35993	36214
1.1	36433	36650	36864	37076	37286	37493	37698	37900	38100	38298
1.2	38493	38686	38877	39065	39251	39435	39617	39796	39973	40147
1.3	40320	40490	40658	40824	40988	41149	41309	41466	41621	41774
1.4	41924	42073	42220	42364	42507	42647	42786	42922	43056	43189
1.5	43319	43448	43574	43699	43822	43943	44062	44179	44295	44408
1.6	44520	44630	44738	44845	44950	45053	45154	45254	45352	45449
1.7	45543	45637	45728	45818	45907	45994	46080	46164	46246	46327
1.8	46407	46485	46562	46638	46712	46784	46856	46926	46995	47062
1.9	47128	47193	47257	47320	47381	47441	47500	47558	47615	47670

2.0	0.47725	47778	47831	47822	47932	47982	48030	48077	48124	48169
2.1	48214	48257	48300	48341	48382	48422	48461	48500	48537	48574
2.2	48610	48645	48679	48713	48745	48778	48809	48840	48870	48899
2.3	48928	48956	48983	49010	49036	49061	49086	49111	49134	49158
2.4	49180	49202	49224	49245	49266	49286	49305	49324	94343	49361
2.5	49379	49396	49413	49430	49446	49461	49477	49492	49506	49520
2.6	49534	49547	49560	49573	49585	49598	49609	49621	49632	49643
2.7	49653	49664	49674	49683	49693	49702	49711	49720	49728	49736
2.8	49744	49752	49760	49767	49774	49781	49788	49795	49801	49807
2.9	49813	49819	49825	49831	49836	49841	49846	49851	49856	49861
3.0	0.49865	3.4	0.49966		3.8	0.49993		5.0	0.4999997	
3.1	0.49903	3.5	0.49977		3.9	0.49995				
3.2	0.49931	3.6	0.49984		4.0	0.499968				
3.3	0.49952	3.7	0.49989		4.5	0.499997				

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