MACHINE ELEMENTS

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Syllabus

- Lectures – 34 hours
- Laboratory and practical classes - 17 hours
- Self study – 93 hours
- Total – 144 hours

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Syllabus

- *Module № 1* - Transmissions of power
- *Module №2* - Machine elements for carrying and transmitting rotatory power and joints
- *Module №3* - Term project
- *Exam*

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Term project

Analysis and design of a mechanical drive

- Explanatory note
- Drawings
  - Designing of a speed reducer
  - Designing of a mechanical drive
  - Shop drawing of a gear
  - Shop drawing of a shaft

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Reference material

1. Березовский Ю.Н., Чернилевский Д.В. “Детали машин”, М.,1983
2. Berezovsky Yu., Chernilevsky D, Petrov M. “Machine Design”, 1988
3. Иванов М.Н. “Детали машин”, 1991
4. Чернавский С.А. “Курсовое проектирование деталей машин”, 1987
5. Цехнович Л.И. “Атлас конструкции редукторов”, 1990
6. Kryzhanovskyi A., Pavlov V.,Kornienko A., Bashta O., Babenko E. “Machine Elements. Term paper designing”, 2011

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Lecture 1

MACHINE ELEMENTS. MAIN DEFINITIONS

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Definition of "Machine Elements"

"Machine elements" is a subject about fundamentals of machine design.

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What is a Machine?

Machine is a technical device in which mechanical motions are performed in order to replace or facilitate human's physical and mental work.

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What is an Element?

Element is an elementary part of machine that is produced without any assembling operations.

Elements are divided into

- simple (screws, nuts, keys, ball ,etc.)

- elaborate in shape (crankshaft, gearbox, toothed wheel, etc.).

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What is a Subassembly?

Subassembly is a combination of assembly units and elements which perform the identical functions (bearings, clutches, speed reducers).

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Classification of Machine Elements

Machine elements are divided into two groups:

- *Group I* – general purpose machine elements;
- *Group II* – special purpose machine elements.

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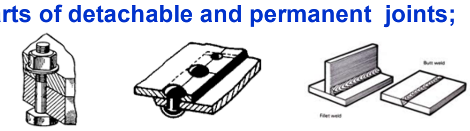
General Purpose Machine Elements

General purpose machine elements are used practically in all machines.

They include:

- Parts of detachable and permanent joints;



- Parts of friction and gear drives;

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General Purpose Machine Elements

They include:

- Shafts and axels;

- Bearings;

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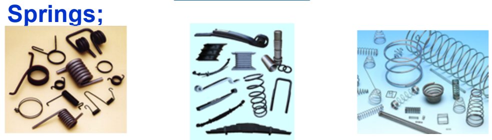
General Purpose Machine Elements

They include:

- Couplings and clutches;

- Springs;



- Machine frames.

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Special Purpose Machine Elements

Special purpose machine elements **are employed in individual types of machines.**

Examples:

- Pistons;
- Valves;
- Propellers;
- Turbines, etc.

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Structure of the course "Machine Elements"

- Part I – Transmissions of Power;
- Part II – Machine Elements for Carrying and Transmitting Rotatory Motion;
- Part III – Joints.

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Criteria of Serviceability of Machine Elements

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Definition of Serviceability

Serviceability is ability of an element or machine to perform its function according to specifications.

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Criteria of Serviceability

- Strength
- Rigidity
- Wear resistance
- Vibration resistance
- Heat resistance
- Corrosion resistance

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Criteria of Serviceability

- *Strength* is ability of an element to resist breakdown and plastic deformation under applied load.
- *Rigidity* is ability of an element to resist changing the shape under applied load.
- *Wear resistance* is ability of an element to resist superficial deterioration in friction that distorts the original geometry and surface conditions of an element.

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Criteria of Serviceability

- *Vibration resistance* is ability of an element to resist vibrations.
- *Heat resistance* is ability of an element to operate at elevated temperature during expending service life.
- *Corrosion resistance* is ability of an element to resist corrosion which can be defined as a process of destruction of metal superficial layers owing to the oxidation.

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Evaluation of strength

$$\sigma \le [\sigma], \qquad \tau \le [\tau],$$

where
σ and $[\sigma]$ are correspondingly acting and allowable normal stresses (due to tension, compression and bending);
τ and $[\tau]$ are correspondingly acting and allowable tangential stresses (due to shear and torsion).

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Strength conditions

$$\sigma = \frac{N}{A} \le [\sigma];$$

$$\tau = \frac{Q}{A} \le [\tau];$$

$$\tau = \frac{T}{W_p} \le [\tau];$$

$$\sigma = \frac{M}{W} \le [\sigma].$$

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Determination of allowable stresses

$$[\sigma] = \frac{\sigma_{lim}}{S}$$

$$[\tau] = \frac{\tau_{lim}}{S}$$

where
σ_{lim} and τ_{lim} are correspondingly limit normal and limit tangential stresses;
S is safety factor

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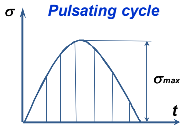
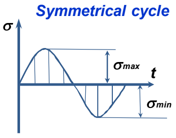
Determination of allowable stresses

Pulsating cycle



R=0, $\sigma_{lim}=\sigma_0$, $\sigma_{peak}=\sigma_{mean}=\sigma_{max}/2$

Symmetrical cycle



R=-1, $\sigma_{lim}=\sigma_{-1}$, $\sigma_{peak}=\sigma_{max}$, $\sigma_{mean}=0$

Asymmetry factor

$$R = \frac{\sigma_{min}}{\sigma_{max}}.$$

Peak stress

$$\sigma_{peak} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Mean stress

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

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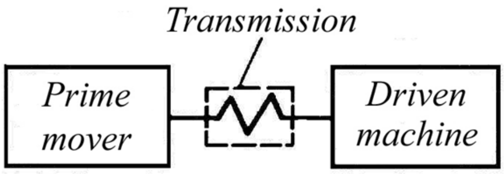
TPANSMISSION OF POWER

TRANSMISSIONS

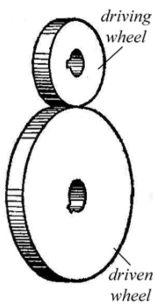


Transmission is a mechanism intended to transmit power from the prime mover (an engine or a motor) to the driven machine.

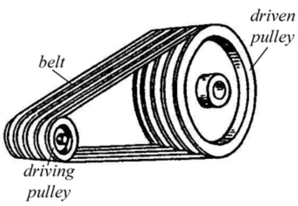
CLASSIFICATION OF TRANSMISSIONS

Transmissions by friction

Friction drive



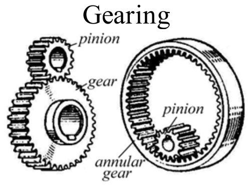
Belt drive



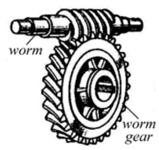
CLASSIFICATION OF TRANSMISSIONS

Transmissions by engagement

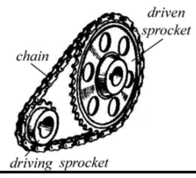
Gearing



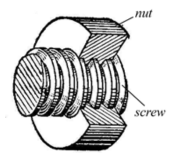
Worm gearing



Chain drive



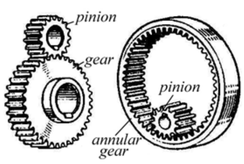
Power screw



CLASSIFICATION OF TRANSMISSIONS

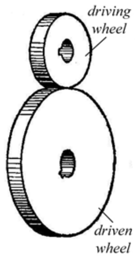
Transmissions with direct contact

Gearing



Worm gearing

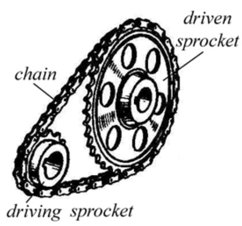
Friction drive



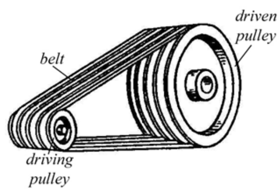
CLASSIFICATION OF TRANSMISSIONS

Transmissions by flexible connection

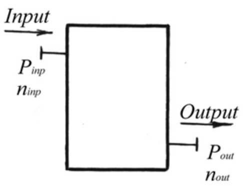
Chain drive



Belt drive



BASIC PARAMETERS OF POWER TRANSMISSIONS



P_{inp} – input power in kW;
P_{out} – output power in kW;
n_{inp} - rotational speed of the input shaft in rpm or
ω_{inp} - angular velocity of the input shaft in sec^{-1};
n_{out} - rotational speed of the output shaft in rpm or
ω_{out} - angular velocity of the output shaft in sec^{-1}.

FORCE AND KINEMATIC RELATIONS IN POWER TRANSMISSIONS

1. Efficiency

$$\eta = \frac{P_{out}}{P_{inp}} < 1;$$

$$\eta = \eta_1 \cdot \eta_2 \cdot \ldots \cdot \eta_n.$$

2. Velocity ratio

$$u = \frac{n_{inp}}{n_{out}} = \frac{\omega_{inp}}{\omega_{out}};$$

$$u = u_1 \cdot u_2 \cdot \ldots \cdot u_n.$$

3. Peripheral speed

$$V = \omega \cdot \frac{d}{2}.$$

4. Turning force

$$F_t = \frac{P}{V} = \frac{2 \cdot T}{d}.$$

5. Input and output torques

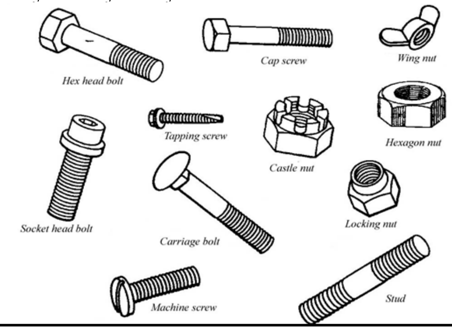
$$T_{inp} = \frac{P_{inp}}{\omega_{inp}} = F_t \cdot \frac{d}{2};$$

$$T_{out} = T_{inp} \cdot \eta \cdot u.$$

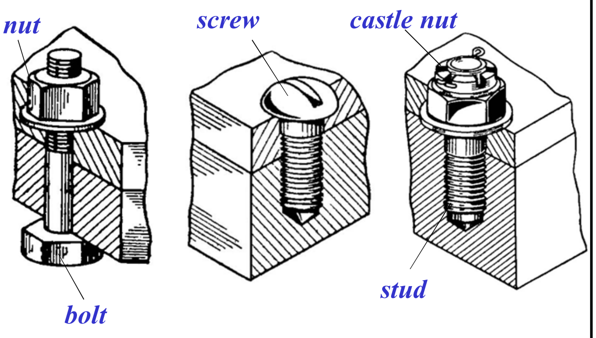
THREADED JOINTS

THREADED JOINTS

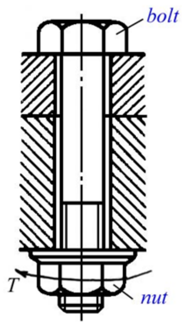
Threaded joints are joints that are made with threaded fastening elements such as bolts, screws, studs, nuts, etc.



THREADED JOINTS



BASIC FASTENING ELEMENTS OF THREADED JOINTS

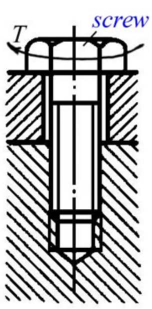


Bolt is a fastening element which is intended to be used with a *nut*.

Bolts are tightened by torque ***T*** on the nut.

Bolts are used to fasten together relatively thin parts whose material is insufficiently strong for a proper thread.

BASIC FASTENING ELEMENTS OF THREADED JOINTS

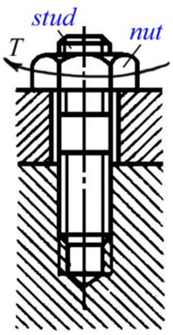


Screw is a product whose primary purpose is assembly into a *tapped hole*.

Screws are tightened by torque ***T*** on the head.

Screws are used to fasten elements of quite large thickness. But materials of these elements have to be sufficiently strong. Otherwise the internal thread in a tapped hole wears out quickly.

BASIC FASTENING ELEMENTS OF THREADED JOINTS



Stud is a threaded rod whose one end assembles into a *tapped hole* and the other end receives a *nut.*

Stud is tightened by a torque on the nut.

Studs are used when a tapped hole does not provide needed durability of the thread.

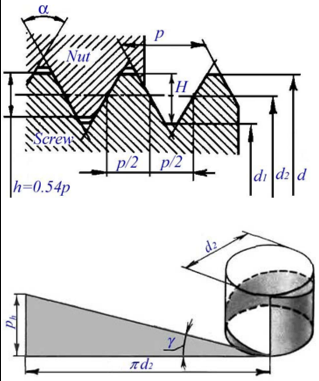
ADVANTAGES OF THREADED JOINTS

- Threaded joints offer a high load-carrying capacity and reliability;
- They can easily be assembled and disassembled;
- Worn threaded fasteners can readily be replaced;
- They are inexpensive to make;
- They use standard fasteners and permit a high degree of interchangeability.

DISADVANTAGES OF THREADED JOINTS

- Threaded joints experience considerable stress concentration in the thread which markedly reduces their strength, especially under cyclic loads.

GEOMETRY OF THE THREAD



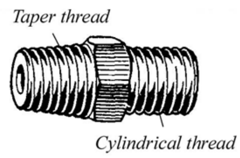
d – major diameter of the thread;
d_1 – minor diameter of the thread;
d_2 – mean diameter of the thread;
H – whole depth of the thread;
h – working depth of the profile;
p – pitch of the thread;
p_h – lead of he thread ($p_h = p \cdot n$, where n is number of starts);
α – thread angle;
γ – lead angle;

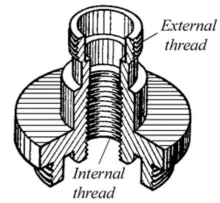
$$tg\,\gamma = \frac{p_h}{\pi \cdot d_2} = \frac{p \cdot n}{\pi \cdot d_2}.$$

CLASSIFICATION OF THE THREAD

I. According to the surface where the thread is formed.

- *cylindrical* and *taper* threads;
- *external* and *internal* threads.

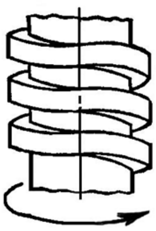


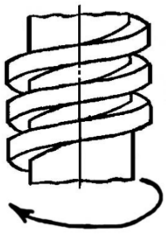


CLASSIFICATION OF THE THREAD

II. According to the direction of the thread

Left-hand thread *Right-hand thread*

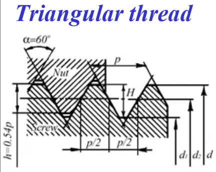




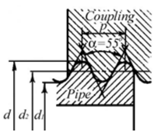
CLASSIFICATION OF THE THREAD

III. According to the thread profile

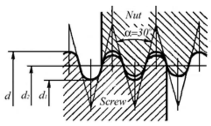
Triangular thread



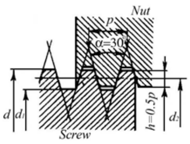
Pipe thread



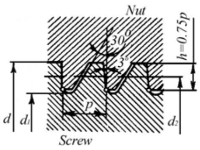
Round thread



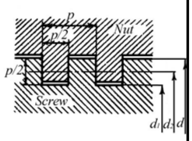
Trapezoidal thread



Buttress thread



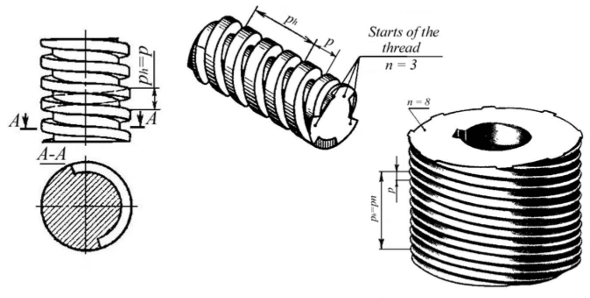
Square thread



CLASSIFICATION OF THE THREAD

IV. According to the number of starts

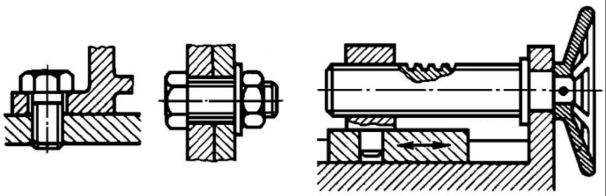
Single-start thread | *Multiple-start thread*



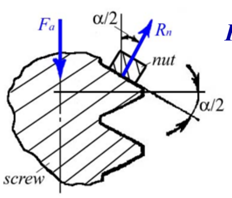
CLASSIFICATION OF THE THREAD

VI. According to the purpose of the thread

Fastening thread | *Translation thread*



FRICTION IN THE THREAD



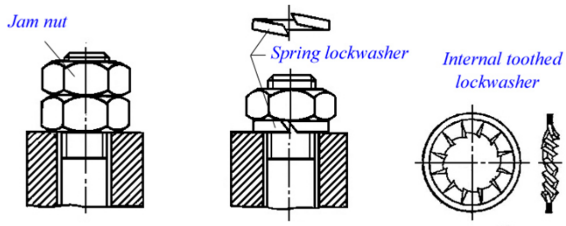
$$F_{fr} = f \cdot R_n = f \cdot \frac{F_a}{\cos\alpha/2} = f' \cdot F_a ,$$

where $f' = \dfrac{f}{\cos\alpha/2}$ – - effective friction factor.

- For triangular thread (α=60°) $f' = \dfrac{f}{\cos 30}{}^{\circ} = 1.15 \cdot f ;$
- For trapezoidal thread (α=30°) $f' = \dfrac{f}{\cos 15^{\circ}} = 1.03 \cdot f ;$
- For square thread (α=0°) $f' = \dfrac{f}{\cos 0^{\circ}} = f .$

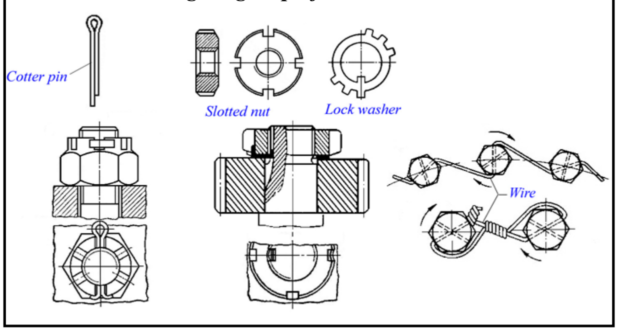
LOCKING THREADED JOINTS

I. Increasing friction in the thread by means of jam nuts, spring washers or toothed lock washers.



LOCKING THREADED JOINTS

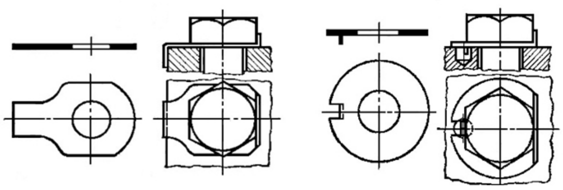
II. A nut is connected with a screw by means of cotter pins, lock washers or wire passed through a group of screws or nuts.



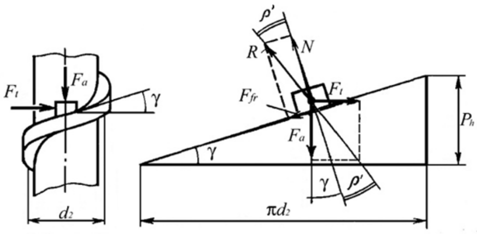
LOCKING THREADED JOINTS

III. A nut or screw head is connected with a machine element by means of flat metal locks.

Flat metal locks



FORCE ANALYSIS



$$F_{fr} = f' \cdot N;$$

$$f' = \frac{f}{\cos \alpha / 2}.$$

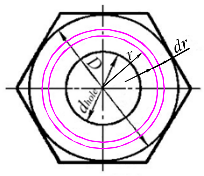
$$tg\rho' = f'.$$

$$F_t = F_a \cdot tg(\gamma + \rho').$$

Torque needed for overcoming frictional forces in the thread

$$T_{thread} = 0.5 \cdot F_t \cdot d_2 = 0.5 \cdot F_a \cdot d_2 \cdot tg(\gamma + \rho').$$

FORCE ANALYSIS



$$p = \frac{F_a}{A} = \frac{4 \cdot F_a}{\pi \cdot (D^2 - d_{hole}^2)};$$

$$A_{ring} = 2 \cdot \pi \cdot r \cdot dr;$$

$$F_{a\,ring} = p \cdot A_{ring} = p \cdot 2 \cdot \pi \cdot r \cdot dr;$$

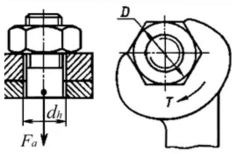
$$F_{fr} = F_{a\,ring} \cdot f; \qquad dT_{fr} = p \cdot f \cdot 2 \cdot \pi \cdot r^2 \cdot dr;$$

Torque produced by frictional forces on the nut face

$$T_{fr} = \int_{d_{hole}/2}^{D/2} p \cdot f \cdot 2 \cdot \pi \cdot r^2 dr = 2 \cdot \pi \cdot f \cdot p \cdot \left(\frac{D^3}{24} - \frac{d_{hole}^3}{24} \right) =$$

$$= 2 \cdot \pi \cdot f \cdot \frac{4 \cdot F_a}{\pi \cdot \left(D^2 - d_{hole}^2\right)} \cdot \frac{D^3 - d_{hole}^3}{24} = \frac{1}{3} \cdot f \cdot F_a \cdot \frac{D^3 - d_{hole}^3}{D^2 - d_{hole}^2}.$$

FORCE ANALYSIS



Total torque applied to the nut

$$T = T_{thread} + T_{fr}.$$

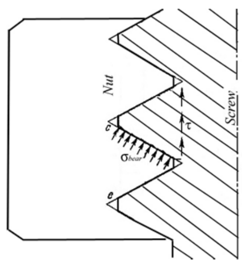
$$T_{thread} = 0.5 \cdot F_a \cdot d_2 \cdot tg(\gamma + \rho');$$

$$T_{fr} = \frac{1}{3} \cdot f \cdot F_a \cdot \frac{D^3 - d_{hole}^3}{D^2 - d_{hole}^2};$$

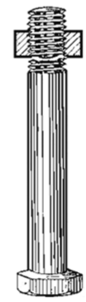
$$T = F_a \cdot \left[\frac{1}{3} \cdot f \cdot \frac{D^3 - d_{hole}^3}{D^2 - d_{hole}^2} + \frac{1}{2} \cdot d_2 \cdot tg(\gamma + \rho') \right].$$

MAIN MODES OF FAILURE OF THREADED JOINTS

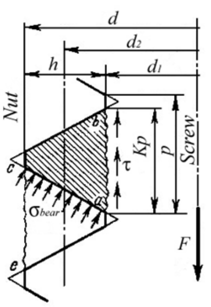
Failure of thread *due to shear, bearing and bending;*



Failure of bolt body



CALCULATION OF THREADS FOR STRENGTH



Strength condition of the thread in terms of shear

For screw $\tau = \dfrac{F}{\pi \cdot d_1 \cdot H \cdot K \cdot K_m} \le [\tau];$

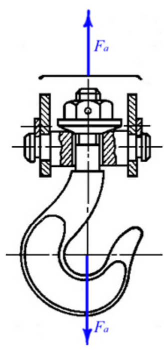
For nut $\tau = \dfrac{F}{\pi \cdot d \cdot H \cdot K \cdot K_m} \le [\tau];$

Strength condition of the thread in terms of bearing

$$\sigma_{bear} = \frac{4 \cdot F}{\pi \cdot (d^2 - d_1^2) \cdot K_m \cdot z} \le [\sigma_{bear}].$$

CALCULATION OF BOLTS FOR STRENGTH

I. Bolts are loaded by an axial force



$$\sigma = \frac{F_a}{A} \le [\sigma_{ten}]; \qquad A = \frac{\pi \cdot d_1^2}{4};$$

$$\sigma = \frac{4 \cdot F_a}{\pi \cdot d_1^2} \le [\sigma_{ten}].$$

$$[\sigma_{ten}] = \frac{\sigma_y}{S}; \qquad S = 1.2....2.5.$$

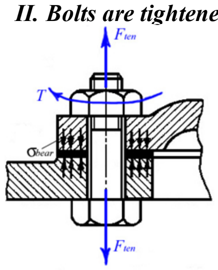
For bolts working for tension

$$[\sigma_{ten}] = 0.6 \cdot \sigma_y.$$

$$d_1 = \sqrt{\frac{4 \cdot F_a}{\pi \cdot [\sigma_{ten}]}}.$$

CALCULATION OF BOLTS FOR STRENGTH

II. Bolts are tightened without action of external forces



$$\sigma = \frac{F_{ten}}{\pi \cdot d_1^2 / 4}; \qquad \tau = \frac{T}{W_p},$$

where $W_p = \dfrac{\pi \cdot d_1^3}{16}$;

$$T = 0.5 \cdot F_{ten} \cdot d_2 \cdot tg(\gamma + \rho').$$

$$\sigma_{eq} = \sqrt{\sigma^2 + 3 \cdot \tau^2} \le [\sigma_{ten}].$$

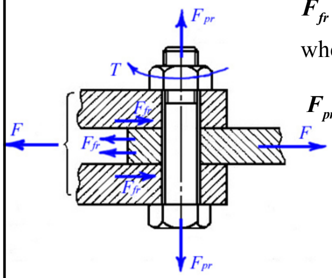
For standard metric threads

$$\sigma_{eq} \approx 1.3 \cdot \sigma = \frac{1.3 \cdot F_{ten}}{\pi \cdot d_1^2 / 4} \le [\sigma_{ten}]. \qquad d_1 = \sqrt{\frac{1.3 \cdot F_{ten}}{\pi \cdot [\sigma_{ten}] / 4}}.$$

CALCULATION OF BOLTS FOR STRENGTH

III. Bolts are loaded by shearing force

a) Bolt is fitted into hole with some play



$F_{fr} \ge F$ or $f \cdot i \cdot F_{pr} \ge F$,

whence $F_{pr} \ge \dfrac{F}{f \cdot i}$.

$F_{pr} = \dfrac{K \cdot F}{i \cdot f}$, where $K = 1.3....2.$

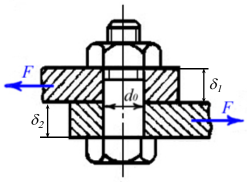
$$\sigma_{eq} = \frac{1.3 \cdot F_{pr}}{\pi \cdot d_1^2 / 4} \le [\sigma_{ten}].$$

$$d_1 = \sqrt{\frac{1.3 \cdot F_{pr}}{\pi \cdot [\sigma_{ten}] / 4}}.$$

CALCULATION OF BOLTS FOR STRENGTH

III. Bolts are loaded by shearing force

b) Bolt is fitted into hole with small interference



$$\tau = \frac{F}{\pi \cdot d_0^2 \cdot i / 4} \le [\tau].$$

$[\tau] \approx 0.4 \cdot \sigma_y$ - for constant load;

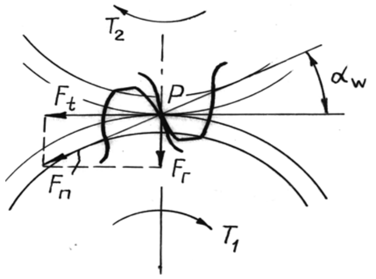
$[\tau] \approx 0.25 \cdot \sigma_y$ - for variable load;

$$d_0 = \sqrt{\frac{4 \cdot F}{\pi \cdot [\tau] \cdot i}}.$$

$$\sigma_{bear} = \frac{F}{d_0 \cdot \delta_{min}} \le [\sigma_{bear}], \qquad [\sigma_{bear}] \approx 0.8 \cdot \sigma_y.$$

CALCULATION OF STRAIGHT SPUR GEARS FOR STRENGTH

FORCES IN THE ENGAGEMENT OF STRAIGHT SPUR GEARS



- turning force

$$F_t = \frac{2 \cdot T_1}{d_1};$$

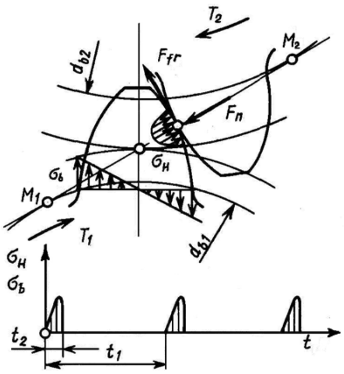
- radial force

$$F_r = F_t \cdot tg\ \alpha_w;$$

- normal force

$$F_n = \frac{F_t}{cos\ \alpha_w}.$$

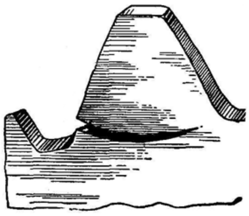
STRESSES IN SPUR GEAR TEETH

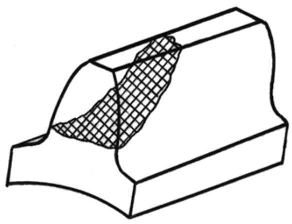


MODES OF TOOTH FAILURE

1. Failures due to bending stresses

Breakage

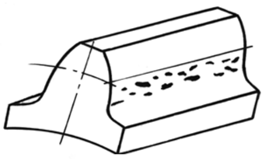


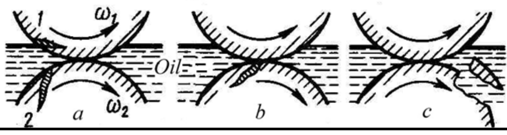


MODES OF TOOTH FAILURE

2. Failures due to contact stresses and frictional forces

A. Pitting

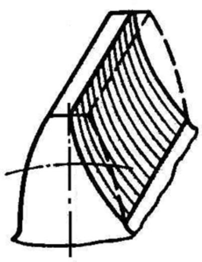




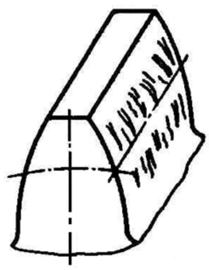
MODES OF TOOTH FAILURE

2. Failures due to contact stresses and frictional forces

B. Abrasive wear

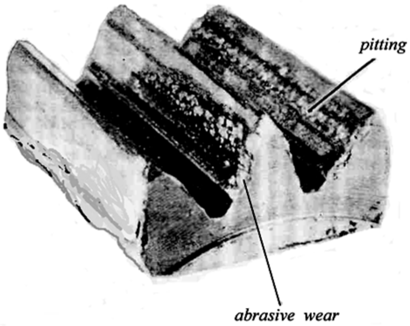


C. Seizure



MODES OF TOOTH FAILURE

2. Failures due to contact stresses and frictional forces



CHOICE OF MATERIALS OF TOOTHED WHEELS

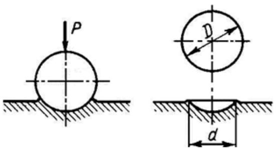
Possible materials of toothed wheels:

- Steel;
- Cast Iron;
- Plastics.

Steel is the basic material.

For gears medium carbon steels (0.40 C, 0.45 C or 0.50 C) and alloy steels (0.40 C-Cr, 0.40 C-Cr-Ni, 0.35 C-Cr-Mo, 0.45 C-Cr, etc.) are used.

DETERMINATION OF THE HARDNESS OF THE MATERIAL



$$H = \frac{P}{A}$$

- Steel ball – Brinell units BHN;
- Diamond cone – Rockwell units C;
- Diamond pyramid – Vickers units HV.

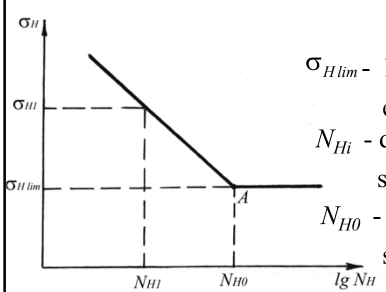
$1\,\mathrm{C} \approx 10\,\mathrm{BHN}$.

CLASSIFICATION OF STEEL GEARS

1. Gears with hardness $H \le 350$ BHN (heat treatment is normalizing or martempering);

2. Gears with hardness $H > 350$ BHN (heat treatment is full hardening, surface hardening, casehardening or nitriding).

CALCULATION OF ALLOWABLE STRESSES



σ_{Hi} - maximum contact stress before failure;

$\sigma_{H\lim}$ - limit of contact endurance;

N_{Hi} - design number of stress cycles;

N_{H0} - base number of stress cycles.

CALCULATION OF ALLOWABLE STRESSES

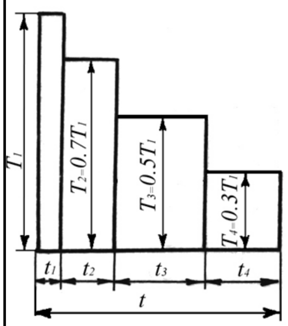
$$\sigma_{Hi}^{m} \cdot N_{Hi} = \sigma_{H\lim}^{m} \cdot N_{H0}.$$

$$\sigma_{Hi} = \sigma_{H\lim} \cdot \sqrt[6]{\frac{N_{H0}}{N_{Hi}}} = \sigma_{H\lim} \cdot K_{HL},$$

where

$$K_{HL} = \sqrt[6]{\frac{N_{H0}}{N_{Hi}}} \ge 1$$ is durability factor.

CALCULATION OF ALLOWABLE STRESSES



For variable load

$$K_{HL} = \sqrt[6]{\frac{N_{H0}}{N_{HE}}} \geq 1,$$

where

$$N_{HE} = N_{Hi} \cdot K_{HE},$$

$$K_{HE} = \sum_{i=1}^{n} \frac{t_i}{t} \left(\frac{T_i}{T_{max}} \right)^3 .$$

CALCULATION OF ALLOWABLE STRESSES

Allowable contact stress

$$[\sigma_H] = \frac{\sigma_{Hi}}{S_H} = \frac{\sigma_{H\,lim} \cdot K_{HL}}{S_H}.$$

Allowable bending stress

$$[\sigma_b] = \frac{\sigma_{b\,lim} \cdot K_{bL}}{S_b}.$$

CALCULATION OF ALLOWABLE STRESSES

$$K_{bL} = \sqrt[m]{\frac{N_{b0}}{N_{bE}}} \geq 1,$$

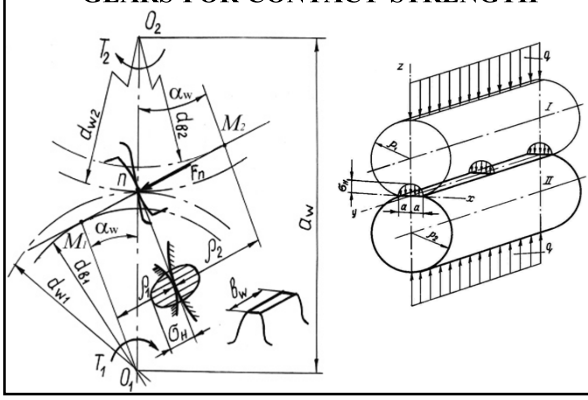
where $m = 6$ if $H \leq 350$ BHN,
$m = 9$ if $H > 350$ BHN.

$$N_{bE} = N_{bi} \cdot K_{bE},$$

$$K_{bE} = \sum_{i=1}^{n} \frac{t_i}{t} \left(\frac{T_i}{T_{max}} \right)^k .$$

where $k = 3$ if $H \leq 350$ BHN,
$k = 9$ if $H > 350$ BHN.

CALCULATION OF STRAIGHT SPUR GEARS FOR CONTACT STRENGTH



CALCULATION OF STRAIGHT SPUR GEARS FOR CONTACT STRENGTH

Hertz formula

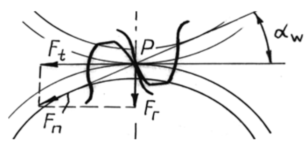
$$\sigma_H = 0.418 \cdot \sqrt{\frac{q \cdot E_{tr}}{\rho_{tr}}},$$

$q = \dfrac{F_n \cdot K_H}{l_\Sigma}$ is the specific design load;

$E_{tr} = \dfrac{2 \cdot E_1 \cdot E_2}{E_1 + E_2}$ is the transformed modulus of elasticity;

$\rho_{tr} = \dfrac{\rho_1 \cdot \rho_2}{\rho_2 \pm \rho_1}$ is the transformed radius of curvature.

CALCULATION OF STRAIGHT SPUR GEARS FOR CONTACT STRENGTH

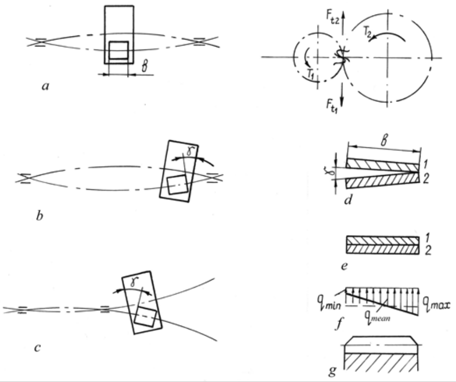


$$F_n = \frac{F_t}{\cos \alpha_w} = \frac{2 \cdot T_1}{d_1 \cdot \cos \alpha_w};$$

$$l_\Sigma = b;$$

$$K_H = K_{H\beta} \cdot K_{HV}.$$

CALCULATION OF STRAIGHT SPUR GEARS FOR CONTACT STRENGTH



CALCULATION OF STRAIGHT SPUR GEARS FOR CONTACT STRENGTH

$$\rho_1 = \frac{d_1}{2} \cdot \sin \alpha_w; \qquad \rho_2 = \frac{d_2}{2} \cdot \sin \alpha_w;$$

$$\frac{1}{\rho_{tr}} = \frac{1}{\rho_1} \pm \frac{1}{\rho_2} = \frac{2}{d_1 \cdot \sin \alpha_w} \pm \frac{2}{d_2 \cdot \sin \alpha_w} =$$

$$= \frac{2}{d_1 \cdot \sin \alpha_w} \cdot \left(\frac{u \pm 1}{u} \right).$$

CALCULATION OF STRAIGHT SPUR GEARS FOR CONTACT STRENGTH

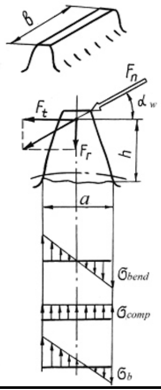
$$\sigma_H = 1.18 \cdot \sqrt{\frac{T_1 \cdot K_H \cdot E_{tr}}{d_1^2 \cdot b \cdot \sin 2\alpha_w} \cdot \left(\frac{u \pm 1}{u} \right)}.$$

$$T_1 \approx \frac{T_2}{u}; \qquad d_1 = \frac{2 \cdot a_w}{u \pm 1}; \qquad b = \psi_{ba} \cdot a_w;$$

$$\alpha_w = 20°; \qquad K_{HV} \approx 1.15.$$

$$a_w = 0.85 \cdot (u \pm 1) \cdot \sqrt[3]{\frac{T_2 \cdot K_{H\beta} \cdot E_{tr}}{[\sigma_H]^2 \cdot u^2 \cdot \psi_{ba}}}.$$

CALCULATION OF STRAIGHT SPUR GEARS FOR BENDING STRENGTH



$$\sigma_b = \sigma_{bend} - \sigma_{comp};$$

$$\sigma_{bend} = \frac{M_{bend}}{W} = \frac{6 \cdot F_t \cdot h}{b \cdot a^2};$$

$$\sigma_{comp} = \frac{F_r}{A} = \frac{F_t \cdot tg\, \alpha_w}{a \cdot b};$$

$$h = \gamma \cdot m; \qquad a = \beta \cdot m;$$

$$\sigma_b = \frac{F_t}{b \cdot m} \cdot \left(\frac{6 \cdot \gamma}{\beta^2} - \frac{tg\, \alpha_w}{\beta} \right).$$

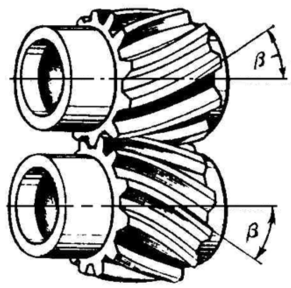
CALCULATION OF STRAIGHT SPUR GEARS FOR BENDING STRENGTH

$$\sigma_b = \frac{F_t \cdot K_b \cdot Y_b}{b \cdot m} \leq [\sigma_b]$$

$$m = \frac{2 \cdot T_2 \cdot Y_b \cdot K_b}{d_2 \cdot b \cdot [\sigma_b]}.$$

HELICAL SPUR GEARS

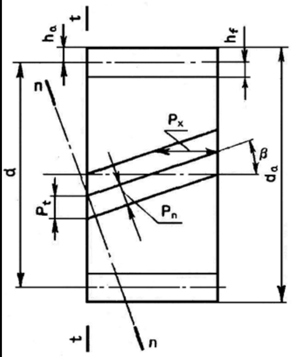
HELICAL SPUR GEARS



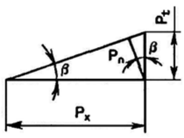
Helix angle

$\beta = 8...18°$

GEOMETRY OF HELICAL SPUR GEARS



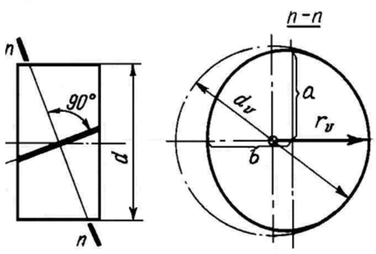
$$P_t = \frac{P_n}{\cos\beta}; \quad m_t = \frac{m_n}{\cos\beta}.$$



GEOMETRICAL PARAMETERS OF HELICAL SPUR GEARS

1. Addendum $h_a = m_n$;
2. Dedendum $h_f = 1.25 \cdot m_n$;
3. Nominal pitch circle diameter $d = m_t \cdot z = \dfrac{m_n}{\cos\beta} \cdot z$;
4. Addendum circle diameter $d_a = d + 2 \cdot m_n$;
5. Dedendum circle diameter $d_f = d - 2.5 \cdot m_n$;
6. Centre distance $a_w = \dfrac{m_n}{2 \cdot \cos\beta} \cdot (z_1 + z_2)$.

EQUIVALENT (VIRTUAL) STRAIGHT SPUR GEAR



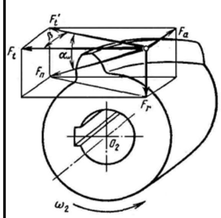
$$a = \frac{d}{2 \cdot \cos\beta};$$

$$b = \frac{d}{2}.$$

$$r_v = \frac{a^2}{b} = \frac{d}{2 \cdot \cos^2\beta}.$$

$$d_v = \frac{d}{\cos^2\beta}; \qquad z_v = \frac{z}{\cos^3\beta}.$$

FORCE ANALYSIS OF HELICAL GEARS



Turning force

$$F_t = \frac{2 \cdot T_1}{d_1};$$

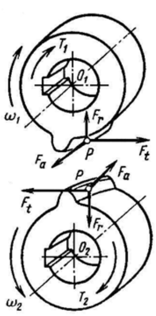
Radial force

$$F_r = \frac{F_t}{\cos\beta} \cdot tg\,\alpha_w;$$

Axial force

$$F_a = F_t \cdot tg\,\beta;$$

Normal force



$$F_n = \frac{F_t'}{\cos\alpha_w} = \frac{F_t}{\cos\alpha_w \cdot \cos\beta}.$$

CALCULATION OF HELICAL GEARS FOR CONTACT STRENGTH

$$\sigma_H = 0.418 \cdot \sqrt{\frac{q \cdot E_{tr}}{\rho_{tr}}},$$

$$q = \frac{F_n \cdot K_H \cdot K_{H\alpha}}{l_\Sigma} = \frac{2 \cdot T_1 \cdot K_H \cdot K_{H\alpha} \cdot \cos\beta}{d_1 \cdot \cos\alpha_w \cdot \cos\beta \cdot b \cdot \varepsilon_\alpha};$$

$$\frac{1}{\rho_{tr}} = \frac{1}{\rho_1} + \frac{1}{\rho_2} = \frac{2}{d_{v1} \cdot \sin\alpha_w} + \frac{2}{d_{v2} \cdot \sin\alpha_w} =$$

$$= \frac{2 \cdot \cos^2\beta}{d_1 \cdot \sin\alpha_w}\left(\frac{u+1}{u}\right).$$

CALCULATION OF HELICAL GEARS FOR CONTACT STRENGTH

$$\sigma_H = 1.18 \cdot Z_{H\beta} \cdot \sqrt{\frac{T_1 \cdot K_H \cdot E_{tr}}{d_1^2 \cdot b \cdot \sin 2\alpha_w} \cdot \left(\frac{u+1}{u}\right)},$$

where $$Z_{H\beta} = \sqrt{\frac{K_{H\alpha} \cdot \cos^2\beta}{\varepsilon_\alpha}}.$$

$$a_w = 0.75 \cdot (u+1) \cdot \sqrt[3]{\frac{T_2 \cdot K_{H\beta} \cdot E_{tr}}{[\sigma_H]^2 \cdot u^2 \cdot \psi_{ba}}}.$$

CALCULATION OF HELICAL GEARS FOR BENDING STRENGTH

$$\sigma_b = \frac{F_t \cdot K_b \cdot Y_b \cdot Z_{b\beta}}{b \cdot m_n} \le [\sigma_b],$$

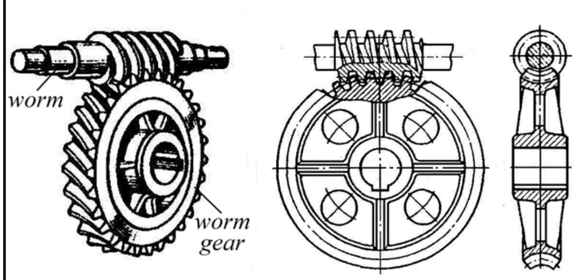
where

$$Z_{b\beta} = \frac{K_{b\alpha} \cdot Y_\beta}{\varepsilon_\alpha}.$$

$$m_n = \frac{2 \cdot T_2 \cdot Y_b \cdot K_b \cdot Z_{b\beta}}{d_2 \cdot b \cdot [\sigma_b]}.$$

WORM GEARING

WORM GEARING



MATERIALS OF THE WORM AND WORM GEAR

Worm: medium carbon and alloy steels (0.40 C – 0.50 C, 0.40 C-Cr, 0.40 C-Cr-Ni) heat-treated to hardness of Rockwell C 45-50.

Worm gear:

If $V_{sl} > 5$ m/sec – tin bronzes (Bronze 10 Sn-1P, Bronze 5 Sn – 5 Zn –5Pb);

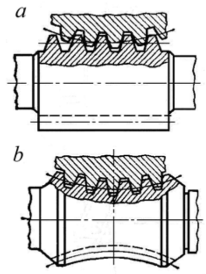
If $2 < V_{sl} < 5$ – tinless bronzes (Bronze 9Al-4Fe);

If $V_{sl} < 2$ m/sec –cast irons (Gray Cast Iron 15).

CLASSIFICATION OF THE WORM GEARING

According to the shape of the worm:

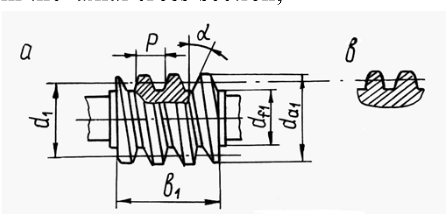
- worm gearing with <u>cylindrical worms</u> (Fig *a*);
- worm gearing with <u>globoid worms</u> (Fig *b*).



CLASSIFICATION OF THE WORM GEARING

According to the thread profile:

- worms with <u>rectilinear</u> profile of the thread in the axial cross-section;
- worms with <u>curvilinear</u> profile of the thread in the axial cross-section;

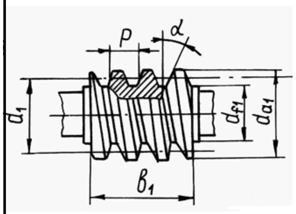


CLASSIFICATION OF THE WORM GEARING

According to the worm thread:

- Archimedes' worms;
- Convolute worms;
- Involute worms.

GEOMETRY OF THE WORM



Worm diameter factor

$$q = \frac{d_1}{m}; \quad q_{min} = 0.212 \cdot z_2;$$

Pitch circle diameter

$$d_1 = q \cdot m;$$

Pitch $P = \pi \cdot m;$

Addendum $h_a = m;$ Dedendum $h_f = 1.2 \cdot m;$

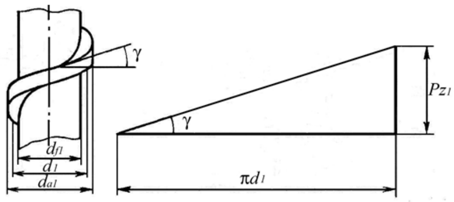
Major diameter $d_{a1} = d_1 + 2 \cdot m;$

Minor diameter $d_{f1} = d_1 - 2.4 \cdot m;$

Threaded length $b_1 \geq m \cdot (11 + 0.06 \cdot z2)$ for z_1=1; 2;

$b_1 \geq m \cdot (12.5 + 0.09 \cdot z2)$ for z_1=4.

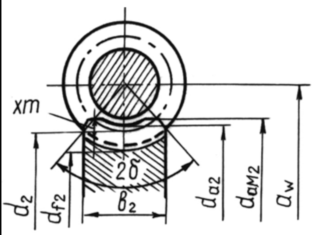
GEOMETRY OF THE WORM



Lead angle

$$tg\,\gamma = \frac{P \cdot z_1}{\pi \cdot d_1} = \frac{\pi \cdot m \cdot z_1}{\pi \cdot q \cdot m} = \frac{z_1}{q}.$$

GEOMETRY OF THE WORM GEAR



Pitch circle diameter

$$d_2 = m \cdot z_2;$$

Addendum circle diameter

$$d_{a2} = d_2 + 2 \cdot m;$$

Dedendum circle diameter

$$d_{f2} = d_2 - 2.4 \cdot m;$$

Maximum gear diameter $d_{aM2} = d_{a2} + \dfrac{6 \cdot m}{z_1 + 2};$

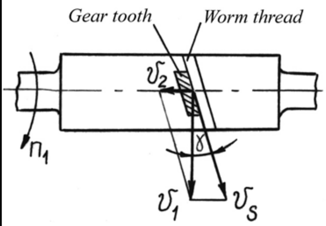
Face width $b_2 \leq 0.75 \cdot d_{a1}$ for z_1=1; 2;

$b_2 \leq 0.67 \cdot d_{a1}$ for z_1=4.

Centre distance $a_w = 0.5 \cdot (d_1 + d_2) = 0.5 \cdot m \cdot (q + z_2).$

SLIPPAGE IN THE CONTACT AREA

$$\bar{v}_s = \bar{v}_1 - \bar{v}_2;$$



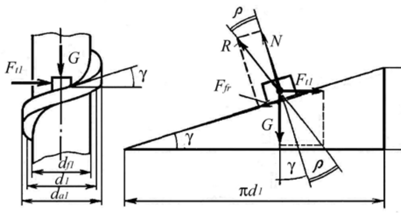
$$v_s = \sqrt{v_1^2 + v_2^2} = \frac{v_1}{cos\,\gamma}.$$

$$v_1 = \frac{\pi \cdot d_1 \cdot n_1}{60};$$

$$v_2 = \frac{\pi \cdot d_2 \cdot n_2}{60};$$

$$\frac{v_2}{v_1} = tg\,\gamma.$$

EFFICIENCY OF THE WORM GEARING

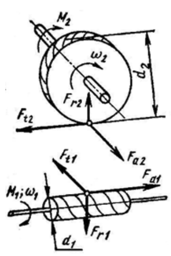


$$\eta = \frac{W_{out}}{W_{inp}} = \frac{G \cdot h}{F_t \cdot s \cdot cos\,\gamma};$$

$F_t = G \cdot tg(\gamma + \rho)$, where $\rho = arctg \dfrac{F_{fr}}{N} = arctg\ f$,

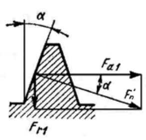
$$\eta = \frac{tg\,\gamma}{tg(\gamma + \rho)}.$$

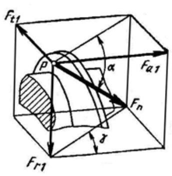
FORCE ANALYSIS OF THE WORM GEARING



$$F_{t1} = F_{a2} = \frac{2 \cdot T_1}{d_1}; \quad F_{a1} = F_{t2} = \frac{2 \cdot T_2}{d_2};$$

$$F_{r1} = F_{r2} = F_{t2} \cdot tg\,\alpha.$$





$$F_n = \frac{F_{t2}}{cos\,\alpha \cdot cos\,\gamma}.$$

CALCULATION FOR CONTACT STRENGTH OF THE WORM GEARING

$$\sigma_H = 0.418 \cdot \sqrt{\frac{q \cdot E_{tr}}{\nu_H \cdot \rho_{tr}}};$$

$$q = \frac{F_n \cdot K_H}{l_\Sigma} = \frac{2 \cdot T_2 \cdot K_H \cdot \cos\gamma}{d_2 \cdot d_1 \cdot \delta \cdot \varepsilon_\alpha \cdot \xi \cdot \cos\alpha \cdot \cos\gamma},$$

where $$l_\Sigma = \frac{d_1}{\cos\gamma} \cdot \delta \cdot \varepsilon_\alpha \cdot \xi;$$

$$\frac{1}{\rho_{tr}} = \frac{1}{\rho_2} = \frac{2 \cdot \cos^2\gamma}{d_2 \cdot \sin\alpha}.$$

CALCULATION OF THE WORM GEARING FOR CONTACT STRENGTH

$$\sigma_H = 1.18 \cdot \sqrt{\frac{T_2 \cdot E_{tr} \cdot K_H \cdot \cos^2\gamma}{d_2^2 \cdot d_1 \cdot \delta \cdot \varepsilon_\alpha \cdot \xi \cdot \sin 2\alpha}}.$$

$$d_1 = q \cdot m = q \cdot \frac{d_2}{z_2}; \quad d_2 = \frac{2 \cdot a_w}{\left(\frac{q}{z_2} + 1\right)}; \quad \varepsilon_\alpha \cdot \delta \cdot \xi \approx 1.3;$$

$$K_H \approx 1.1; \; \gamma \approx 10°; \; \alpha = 20°.$$

$$a_w = 0.625 \cdot \left(\frac{q}{z_2} + 1\right) \cdot \sqrt[3]{\frac{T_2 \cdot E_{tr}}{[\sigma_H]^2 \cdot (q/z_2)}}.$$

CALCULATION OF THE WORM GEARING FOR BENDING STRENGTH

$$\sigma_b = 0.7 \cdot Y_b \cdot \frac{F_{t2} \cdot K_b}{b_2 \cdot m_n},$$

where $$m_n = m \cdot \cos\gamma,$$

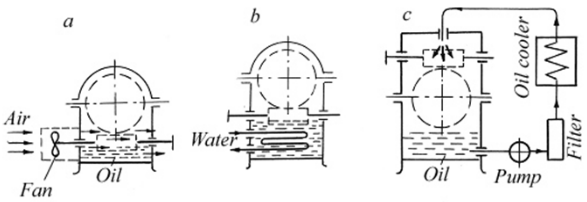
$$Z_{b\beta} = \frac{K_{b\alpha} \cdot Y_\beta}{\varepsilon_\alpha \cdot \xi} = \frac{1}{\varepsilon_\alpha \cdot \xi} = \frac{1}{1.9 \cdot 0.75} \approx 0.7.$$

HEAT REMOVAL ANALYSIS OF THE WORM GEARING

$$Q_{gen} = Q_{em};$$

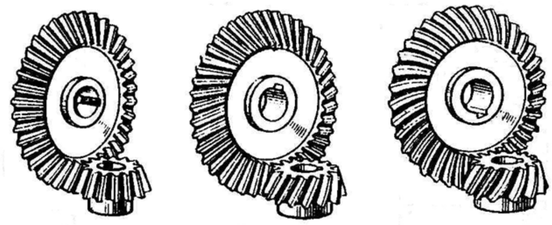
$$Q_{gen} = P_1 \cdot (1 - \eta); \qquad Q_{em} = K_t \cdot (t_{oil} - t_{air}) \cdot A;$$

$$t_{oil} = t_{air} + \frac{P_1 \cdot (1 - \eta)}{K_t \cdot A} \le [t_{oil}].$$

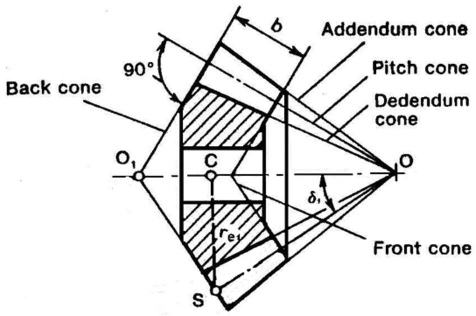


BEVEL GEARS

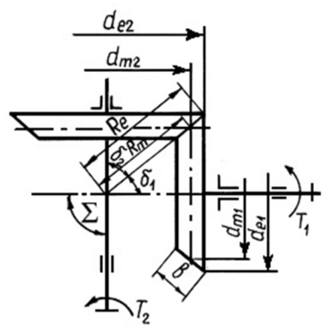
BEVEL GEARS



GEOMETRY OF BEVEL GEARS



GEOMETRY OF BEVEL GEARS



$$\Sigma = \delta_1 + \delta_2 = 90°.$$

$$\frac{d_e}{R_e} = \frac{d_m}{R_m};$$

$$R_e = R_m + 0.5 \cdot b.$$

GEOMETRICAL PARAMETERS OF BEVEL GEARS

- External pitch diameter $d_e = m_e \cdot z;$
- Addendum at the outer section $h_{ae} = m_e;$
- Dedendum at the outer section $h_{fe} = 1.2 \cdot m_e;$
- External addendum circle diameter $d_{ae} = d_e + 2 \cdot m_e \cdot \cos\delta;$
- External dedendum circle diameter $d_{fe} = d_e - 2.4 \cdot m_e \cdot \cos\delta;$
- Outer cone distance $R_e = 0.5 \cdot \sqrt{d_{e1}^2 + d_{e2}^2} =$

$$= \frac{d_{e1}}{2} \cdot \sqrt{u^2 + 1} = \frac{d_{e2}}{2 \cdot u} \cdot \sqrt{u^2 + 1}.$$

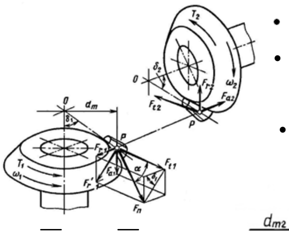
VELOCITY RATIO OF BEVEL GEARS

$$u = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{d_{e2}}{d_{e1}} = \frac{2 \cdot R_e \cdot \sin\delta_2}{2 \cdot R_e \cdot \sin\delta_1} = \frac{\sin\delta_2}{\sin\delta_1} = \frac{z_2}{z_1};$$

for $\Sigma = \delta_1 + \delta_2 = 90°$

$$u = \frac{\sin(90 - \delta_1)}{\sin\delta_1} = \frac{\cos\delta_1}{\sin\delta_1} = ctg\,\delta_1 = tg\,\delta_2.$$

FORCE ANALYSIS OF BEVEL GEARS



- Turning force $F_{t1} = \dfrac{2 \cdot T_1}{d_{m1}}$;
- Radial force

$$F_{r1} = F_{t1} \cdot \cos \delta_1 \cdot tg\ \alpha_w;$$

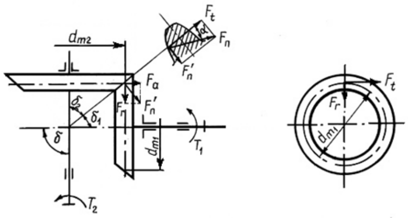
- Axial force

$$F_{a1} = F_{t1} \cdot \sin\delta_1 \cdot tg\ \alpha_w.$$

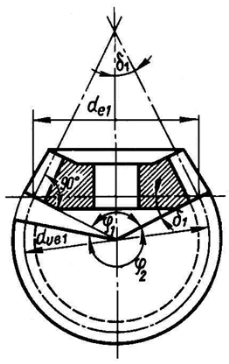
$$\overline{F}_{r1} = -\overline{F}_{a2};$$

$$\overline{F}_{a1} = -\overline{F}_{r2};$$

$$F_n = \frac{F_{t1}}{\cos \alpha_w}.$$



EQUIVALENT STRAIGHT SPUR GEAR



$$d_{ve} = \frac{d_e}{\cos \delta};$$

$$z_v = \frac{z}{\cos \delta}.$$

CALCULATION OF BEVEL GEARS FOR CONTACT STRENGTH

$$\sigma_H = 0.418 \cdot \sqrt{\frac{q \cdot E_{tr}}{v_H \cdot \rho_{tr}}};$$

$$q = \frac{F_n \cdot K_H}{l_\Sigma} = \frac{2 \cdot T_1 \cdot K_H}{d_{m1} \cdot b_2 \cdot \cos \alpha_w};$$

$$\frac{1}{\rho_{tr}} = \frac{1}{\rho_1} + \frac{1}{\rho_2} = \frac{2}{d_{v1} \cdot \sin \alpha_w} + \frac{2}{d_{v2} \cdot \sin \alpha_w} =$$

$$= \frac{2}{d_{m1} \sin \alpha_w}\left(\cos \delta_1 + \frac{\cos \delta_2}{u}\right) = \frac{2}{d_{m1} \sin \alpha_w}\left(\frac{\sqrt{u^2+1}}{u}\right).$$

CALCULATION OF BEVEL GEARS FOR CONTACT STRENGTH

$$\sigma_H = 1.18 \cdot \sqrt{\frac{T_1 \cdot K_H \cdot E_{tr}}{v_H \cdot d_{m1}^2 \cdot b \cdot \sin 2\alpha_w} \cdot \left(\frac{\sqrt{u^2+1}}{u}\right)}.$$

$$T_1 \approx \frac{T_2}{u}; \qquad b = \psi_{bR} \cdot R_e = 0.5 \cdot \psi_{bR} \cdot d_{e2} \cdot \frac{\sqrt{u^2+1}}{u};$$

$$d_{m1} = \frac{d_{m2}}{u} = \frac{d_{e2} \cdot (R_e - 0.5 \cdot b)}{R_e \cdot u} = \frac{d_{e2} \cdot (1 - 0.5 \cdot \psi_{bR})}{u};$$

$$d_{e2} = 1.7 \cdot \sqrt[3]{\frac{T_2 \cdot K_{H\beta} \cdot E_{tr} \cdot u}{v_H \cdot [\sigma H]^2 \cdot (1 - \psi_{bR}) \cdot \psi_{bR}}}.$$

CALCULATION OF BEVEL GEARS FOR BENDING STRENGTH

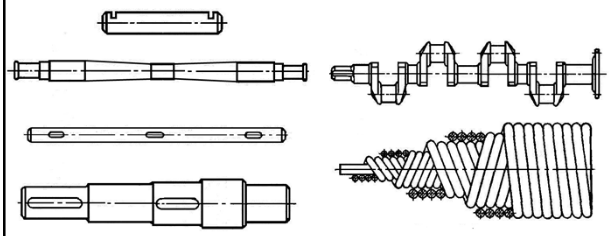
$$\sigma_b = \frac{F_t \cdot K_b \cdot Y_b}{v_b \cdot b \cdot m_m} \le [\sigma_b]$$

$$m_e = \frac{14 \cdot T_2 \cdot K_{b\beta}}{v_b \cdot d_{e2} \cdot b \cdot [\sigma_b]}.$$

AXLES AND SHAFTS

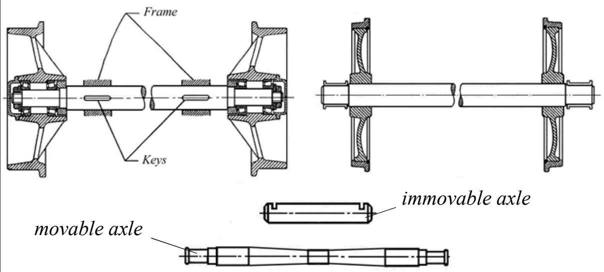
AXLES AND SHAFTS

Links intended to carry rotating elements (pulleys, sprockets, pinions, gears, half-couplings, etc.) are called as axles or shafts.



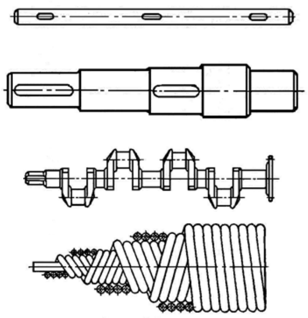
AXLES

Axles are intended to support rotating parts that do not transmit torques and are subjected to bending only.



SHAFTS

Shafts are designed to carry links which transmit torques and experience both bending and torsion.



CLASSIFICATION OF SHAFTS

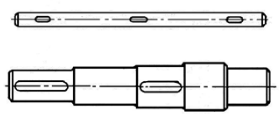
1. According to purpose

- Shafts of various drives (gear drives, belt drives, chain drives and so on);
- Main shafts of mechanisms and machines whose function is to carry not only drive elements but other elements that do not transmit torques such as rotors, fly-wheels, turbine disks, etc.

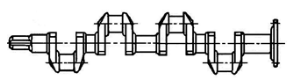
CLASSIFICATION OF SHAFTS

2. According to the shape

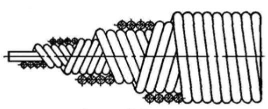
- Straight shafts;



- Cranked shafts;



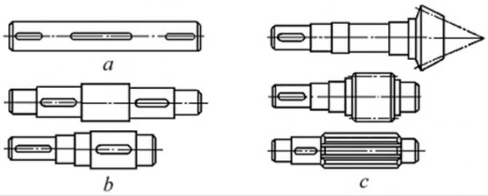
- Flexible shafts.



CLASSIFICATION OF SHAFTS

3. According to the construction

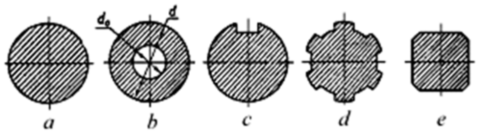
- Shafts of constant cross section (without steps);
- Shafts of variable cross section (of stepped configuration);
- Shafts made solid with gears or worms.



CLASSIFICATION OF SHAFTS

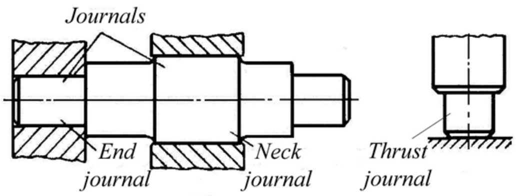
4. According to the shape of the cross section

- Shafts with solid circular cross section;
- Shafts with hollow circular cross section;
- Shafts with keyways;
- Shafts with splines;
- Shafts with rectangular cross section.



SHAFTS

Portion of the shaft which is in contact with a bearing is called ***journal***. We will distinguish between *end journal*, *neck journal* and *thrust journal*.



CALCULATION OF SHAFTS

Shafts may be calculated for:

- **Strength**;
- **Rigidity**;
- **Oscillations**.

CALCULATION OF SHAFTS FOR STRENGTH

Calculation of shafts for strength is divided into 3 stages:

1. Determination of the minimum diameter of the shaft;
2. Designing the shaft construction;
3. Strength analysis of the shaft.

DETERMINATION OF THE MINIMUM DIAMETER OF THE SHAFT

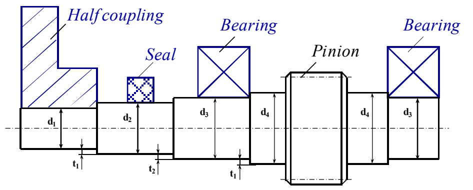
Minimum diameter of the shaft is determined taking into account torsion stresses only. In order to compensate neglect of bending stresses the allowable torsion stress is assumed as down rated ([τ]=20…40 MPa).

$$\tau = \frac{T}{W_p}; \qquad W_p = \frac{\pi \cdot d^3}{16}.$$

$$d_{min} = \sqrt[3]{\frac{T}{0.2 \cdot [\tau]}}.$$

DESIGNING THE SHAFT CONSTRUCTION

Input shaft



$d_1 = d_{min}$;

$d_2 = d_1 + 2 \cdot t_1$;

$d_3 = d_2 + 2 \cdot t_2$;

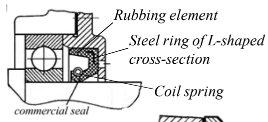
$d_4 = d_3 + 2 \cdot t_1$.

d, mm	20…50	55…120
t_1*, mm*	2; 2.5	5
t_2*, mm*	1; 1.5	2.5

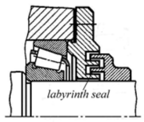
SEALS

Seals are divided into:

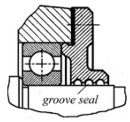
- Commercial seals;



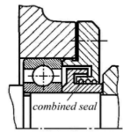
- Labyrinth seals;



- Groove seals;

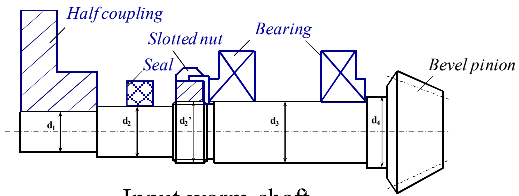


- Combined seals.

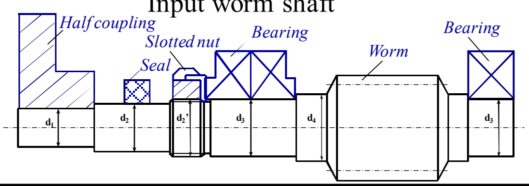


DESIGNING THE SHAFT CONSTRUCTION

Input bevel pinion shaft

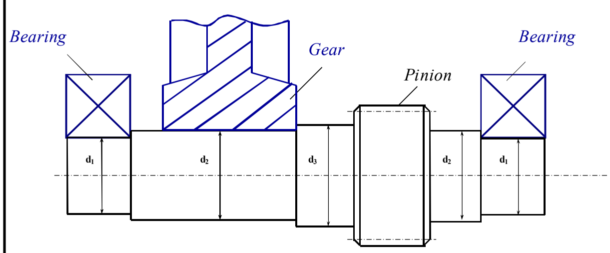


Input worm shaft



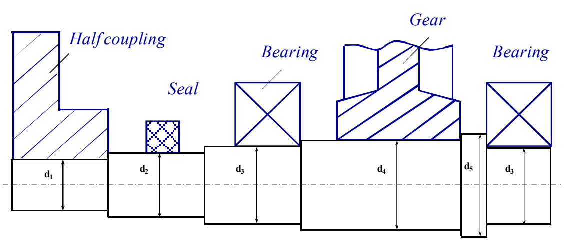
DESIGNING THE SHAFT CONSTRUCTION

Intermediate shaft



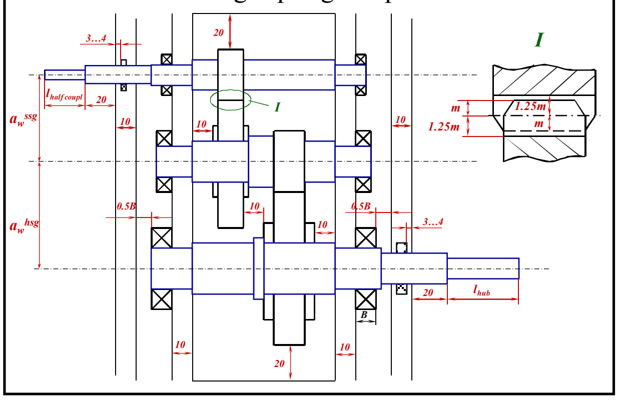
DESIGNING THE SHAFT CONSTRUCTION

Output shaft



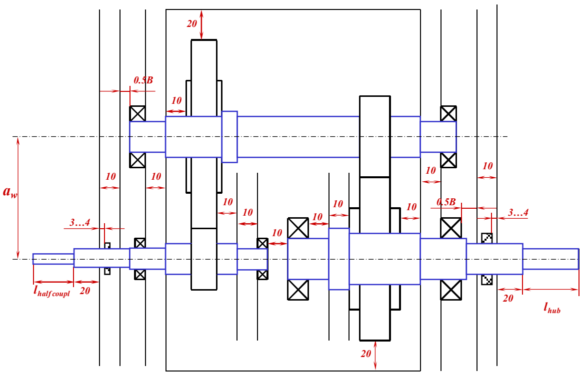
SKETCH LAYOUT

Double stage spur gear speed reducer



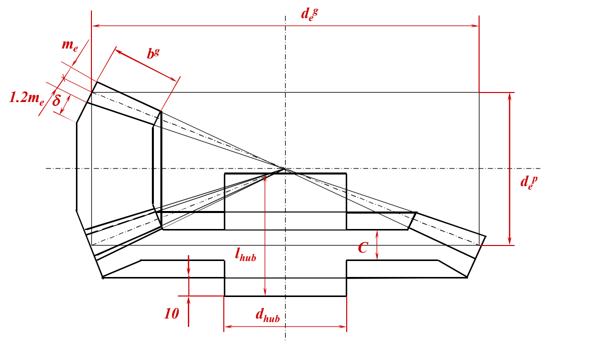
SKETCH LAYOUT

Double stage coaxial spur gear speed reducer



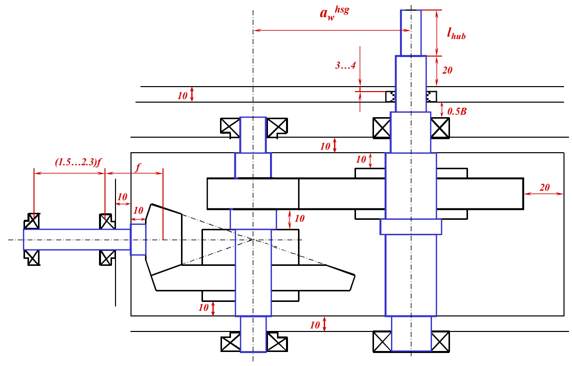
SKETCH LAYOUT

Bevel gears

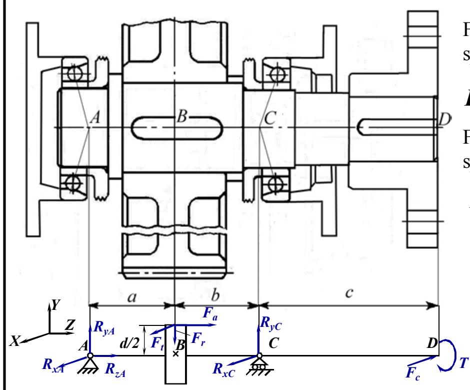


SKETCH LAYOUT

Double stage bevel and spur gear speed reducer



STRENGTH ANALYSIS OF THE SHAFT



For ***single*** stage speed reducers

$$F_c = 125\sqrt{T};$$

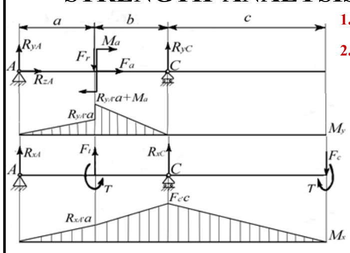
For ***double*** stage speed reducers

$$F_c = 250\sqrt{T}.$$

STRENGTH ANALYSIS OF THE SHAFT

1. Draw the analytical model in the vertical plane and transfer all forces to the shaft;
2. Determine vertical support reactions R_{yA} and R_{yC}. For this purpose we set up equations of moments relative to points A and C. For checking we will write equation of forces that are parallel to Y axis;
3. Plot the bending moment diagram in the vertical plane;
4. Draw the analytical model in the horizontal plane and transfer all forces to the shaft;
5. Determine horizontal support reactions R_{xA} and R_{xC}. For that we set up equations of moments relative to points A and C. For checking we write equation of forces that are parallel to X axis;
6. Plot the bending moment diagram in the horizontal plane;
7. Plot the total bending moment diagram $(M_\Sigma = \sqrt{M_x^2 + M_y^2})$;
8. Plot the twisting moment diagram;
9. Plot the reduced moment diagram $(M_{red} = \sqrt{M_t^2 + 0.75 \cdot T^2})$.

STRENGTH ANALYSIS OF THE SHAFT



1. $M_a = F_a \cdot \frac{d}{2}.$

2. $\sum M_A = 0: \; -F_r \cdot a - M_a + R_{yC} \cdot (a+b) = 0;$

$$R_{yC} = \frac{F_r \cdot a + M_a}{a+b};$$

$\sum M_c = 0: \; -R_{yA} \cdot (a+b) + F_r \cdot b - M_a = 0;$

$$R_{yA} = \frac{F_r \cdot b - M_a}{a+b};$$

Checking: $\sum F_{yi} = 0: \; R_{yA} - F_r + R_{yC} = 0.$

3. $0 \le x \le a; \quad M_y = R_{yA} \cdot x;$

$M_y(0) = 0; \quad M_y(a) = R_{yA} \cdot a;$

$a \le x \le a+b;$
$M_y = R_{yA} \cdot x + M_a - F_r \cdot (x-a);$

$M_y(a) = R_{yA} \cdot a + M_a; \; M_y(a+b) = 0.$

4. $T = F_t \cdot \frac{d}{2}.$

5. $\sum M_A = 0: \; F_t \cdot a + R_{xC} \cdot (a+b) - F_c \cdot (a+b+c) = 0;$

$$R_{xC} = \frac{-F_t \cdot a + F_c \cdot (a+b+c)}{a+b};$$

$\sum M_c = 0: \; -R_{xA} \cdot (a+b) - F_t \cdot b - F_c \cdot c = 0;$

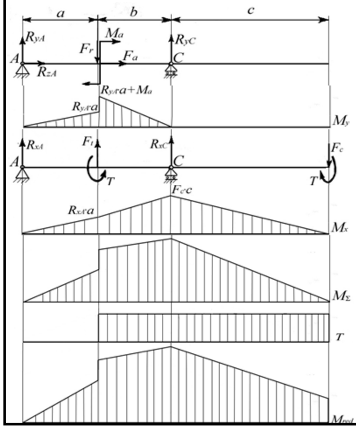
$$R_{xA} = \frac{-F_t \cdot b - F_c \cdot c}{a+b};$$

Checking: $\sum F_{xi} = 0: \; R_{xA} + F_t + R_{xC} - F_c = 0.$

6. $0 \le x \le a; \; M_x = R_{xA} \cdot x; \; M_x(0) = 0; \; M_x(a) = R_{xA} \cdot a;$

$0 \le x \le c; \; M_x = F_c \cdot x; \; M_x(0) = 0; \; M_x(c) = F_c \cdot c.$

STRENGTH ANALYSIS OF THE SHAFT



7. $M_{\Sigma} = \sqrt{M_x^2 + M_y^2}$;

8. T

9. $M_{red} = \sqrt{M_{\Sigma}^2 + 0.75 \cdot T^2}$;

Calculation for static strength

$$\sigma_b = \frac{M}{W} \le [\sigma_b];$$

$$M = M_{red\ max}; \quad W = \frac{\pi \cdot d^3}{32};$$

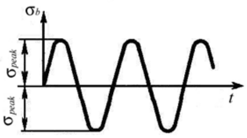
$$\sigma_b = \frac{M_{red\ max}}{0.1 \cdot d^3} \le [\sigma_b], \text{ where}$$

$M_{red\ max}$ is the reduced moment at the critical section;
d is diameter of the shaft at the critical section;
$[\sigma_b] = 100…120\,MPa$.

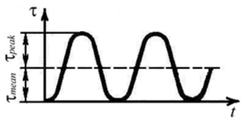
STRENGTH ANALYSIS OF THE SHAFT

Calculation of the shaft for fatigue strength

Changing of bending stresses



Changing of torsion stresses



Safety factor

$$S = \frac{S_\sigma \cdot S_\tau}{\sqrt{S_\sigma^2 + S_\tau^2}} \ge [S] = 1.5…2.5.$$

Safety factor for bending

$$S_\sigma = \frac{\sigma_{lim}}{\frac{K_\sigma}{K_d \cdot K_F} \cdot \sigma_{peak} + \psi_\sigma \cdot \sigma_{mean}};$$

Safety factor for torsion

$$S_\tau = \frac{\tau_{lim}}{\frac{K_\tau}{K_d \cdot K_F} \cdot \tau_{peak} + \psi_\tau \cdot \tau_{mean}}.$$

STRENGTH ANALYSIS OF THE SHAFT

Calculation of the shaft for fatigue strength

$$S_\sigma = \frac{\sigma_{lim}}{\frac{K_\sigma}{K_d \cdot K_F} \cdot \sigma_{peak} + \psi_\sigma \cdot \sigma_{mean}}; \quad S_\tau = \frac{\tau_{lim}}{\frac{K_\tau}{K_d \cdot K_F} \cdot \tau_{peak} + \psi_\tau \cdot \tau_{mean}}.$$

σ_{lim}, τ_{lim} – limit of endurance in bending and in torsion

$\sigma_{lim} = 0.43 \cdot \sigma_{ult}$ - for carbon steels;

$\sigma_{lim} = 0.35 \cdot \sigma_{ult} + 120$ - for alloy steels;

$\tau_{lim} = (0.2…0.3) \cdot \sigma_{ult}$.

σ_{peak}, τ_{peak} – variable (peak) components of bending and torsion stresses

$$\sigma_{peak} = \frac{\sigma_{max} - \sigma_{min}}{2} = \sigma_{max} = \frac{M_\Sigma}{W} = \frac{M_\Sigma}{0.1 \cdot d^3};$$

$$\tau_{peak} = \frac{\tau_{max} + \tau_{min}}{2} = \frac{\tau_{max}}{2} = 0.5 \cdot \frac{T}{W_p} = 0.5 \cdot \frac{T}{0.2 \cdot d^3}.$$

STRENGTH ANALYSIS OF THE SHAFT

Calculation of the shaft for fatigue strength

$$S_\sigma = \frac{\sigma_{lim}}{\frac{K_\sigma}{K_d \cdot K_F} \cdot \sigma_{peak} + \psi_\sigma \cdot \sigma_{mean}}; \quad S_\tau = \frac{\tau_{lim}}{\frac{K_\tau}{K_d \cdot K_F} \cdot \tau_{peak} + \psi_\tau \cdot \tau_{mean}}.$$

σ_{mean}, τ_{mean} – constant (mean) components of bending and torsion stresses

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = 0;$$

$$\tau_{mean} = \frac{\tau_{max} + \tau_{min}}{2} = \frac{\tau_{max}}{2} = 0.5 \cdot \frac{T}{W_p} = 0.5 \cdot \frac{T}{0.2 \cdot d^3}.$$

ψ_σ, ψ_τ – factors of constant components of bending and torsion stresses

$\psi_\sigma = 0.1$; $\psi_\tau = 0.05$ – for carbon steels;

$\psi_\sigma = 0.15$; $\psi_\tau = 0.1$ – for alloy steels.

K_σ, K_τ – effective stress concentration factors;

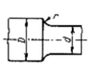
K_d – scale factor;

K_F - surface roughness factor.

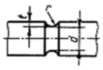
STRENGTH ANALYSIS OF THE SHAFT

The most typical stress concentrations of the shaft

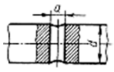
- Filleted transition regions;



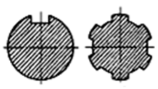
- Grooves;



- Radial holes;



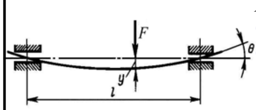
- Keyed and splined portions;



- Threaded portions;
- Interference fits.

RIGIDITY ANALYSIS OF THE SHAFT

Flexural rigidity



Basic criteria of flexural rigidity are:
- Maximum deflection (sag) y of the shaft;
- Angle of rotation θ of support sections.

Flexural rigidity conditions

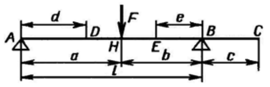
$$y \le [y]; \quad \theta \le [\theta],$$

where
[y] is the maximum safe sag; *[θ]* is the maximum safe angle of rotation.

[y]= 0.01m – for shafts of spur gears and worm gear drives;
[y]= 0.005m – for shafts of bevel gear, hypoid gear and hourglass worm gear drives;
[y]= (0.0002…0.0003)l – for general purpose shafts used in machine tools;
[θ]= 0.001 rad – for shafts mounted in sliding contact bearings;
[θ]= 0.005 rad – for shafts mounted in radial ball bearings.

RIGIDITY ANALYSIS OF THE SHAFT

Flexural rigidity



$$\theta_A = \frac{F \cdot a \cdot b \cdot (l+b)}{6 \cdot E \cdot J \cdot l};$$

$$\theta_B = \frac{F \cdot a \cdot b \cdot (l+a)}{6 \cdot E \cdot J \cdot l};$$

$$\theta_C = \theta_B; \qquad y_C = \frac{F \cdot a \cdot b \cdot c \cdot (l+a)}{6 \cdot E \cdot J \cdot l};$$

$$\theta_D = \frac{F \cdot b \cdot (l^2 - b^2 - 3d^2)}{6 \cdot E \cdot J \cdot l}; \qquad y_D = \frac{F \cdot b \cdot d \cdot (l^2 - b^2 - d^2)}{6 \cdot E \cdot J \cdot l};$$

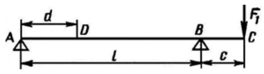
$$\theta_E = \frac{F \cdot a \cdot (l^2 - a^2 - 3e^2)}{6 \cdot E \cdot J \cdot l}; \qquad y_E = \frac{F \cdot a \cdot e \cdot (l^2 - a^2 - e^2)}{6 \cdot E \cdot J \cdot l};$$

$$\theta_H = \frac{F \cdot a \cdot b \cdot (b-a)}{3 \cdot E \cdot J \cdot l}. \qquad y_H = \frac{F \cdot a^2 \cdot b^2}{3 \cdot E \cdot J \cdot l}.$$

E is modulus of elasticity of the shaft material; ***J*** is centroidal moment of inertia.

RIGIDITY ANALYSIS OF THE SHAFT

Flexural rigidity



$$\theta_A = \frac{F_1 \cdot c \cdot l}{6 \cdot E \cdot J};$$

$$\theta_B = \frac{F_1 \cdot c \cdot l}{3 \cdot E \cdot J};$$

$$\theta_C = \frac{F_1 \cdot c \cdot (2 \cdot l + 3 \cdot c)}{6 \cdot E \cdot J}; \qquad y_C = \frac{F_1 \cdot c^2 \cdot (l+c)}{3 \cdot E \cdot J};$$

$$\theta_D = \frac{F_1 \cdot c \cdot (3 \cdot d^2 + l^2)}{6 \cdot E \cdot J \cdot l}; \qquad y_D = \frac{F_1 \cdot c \cdot d \cdot (l^2 + d^2)}{6 \cdot E \cdot J \cdot l}.$$

RIGIDITY ANALYSIS OF THE SHAFT

Torsional rigidity

Basic criterion of torsional rigidity is the angle of twist.

Torsional rigidity condition

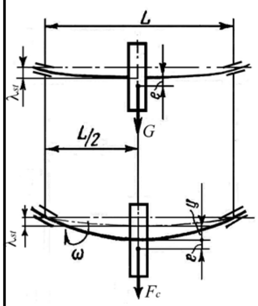
$$\varphi \le [\varphi],$$

where *[φ]* is the maximum safe angle of twist.

$$\varphi = \frac{T \cdot l}{G \cdot J_p},$$

where ***T*** is torque; ***l*** is length of the shaft; ***G*** is shear modulus; $J_p = \pi d^4/32$ is polar moment of inertia.

CALCULATION OF THE SHAFT FOR OSCILLATIONS



$$\lambda_{st} = \frac{G}{c}$$ - static deflection;

$$F_c = m \cdot \omega^2 \cdot (y+e);$$

$$F_{el} = c \cdot y;$$

$$F_c = F_{el};$$

$$m \cdot \omega^2 \cdot (y+e) = c \cdot y;$$

$$y = \frac{m \cdot \omega^2 \cdot e}{c - m \cdot \omega^2} = \frac{e}{\frac{c}{m \cdot \omega^2} - 1}$$ – dynamic deflection

$m \cdot \omega^2 = c$ - condition of resonance.

$$\omega_{cr} = \sqrt{\frac{c}{m}}$$ - *critical angular velocity*.

CALCULATION OF THE SHAFT FOR OSCILLATIONS

$$\omega = \frac{\pi \cdot n}{30} \Rightarrow n_{cr} = \frac{30}{\pi} \cdot \omega_{cr} = \frac{30}{\pi} \cdot \sqrt{\frac{c}{m}} = \frac{30}{\pi} \cdot \sqrt{\frac{c \cdot g}{m \cdot g}} = \frac{30}{\pi} \cdot \sqrt{\frac{g}{\lambda_{st}}};$$

$$n_{cr} = \frac{30}{\pi} \cdot \sqrt{\frac{g}{\lambda_{st}}}$$ - *critical rotational speed,*

where $g = 9.81\ m/sec^2$ - free fall acceleration;

$\lambda_{st} = \frac{G}{c}$ - static deflection;

$c = \frac{48 \cdot E \cdot J}{L^3}$ - rigidity of the shaft;

E - modulus of elasticity of the shaft material;

L - distance between shaft supports;

$J = \frac{\pi \cdot d^4}{64}$ - shaft moment of inertia.

CALCULATION OF THE SHAFT FOR OSCILLATIONS

if $n \le 0.7 \cdot n_{cr}$ *- rigid shafts;*

if $n \ge 1.2 \cdot n_{cr}$ *- flexible shafts.*

$$y = \frac{e}{\frac{c}{m \cdot \omega^2} - 1}.$$

$$\text{if } \omega \to \infty,$$

$$y \to -e.$$

In this case we deal with shaft self-centering.

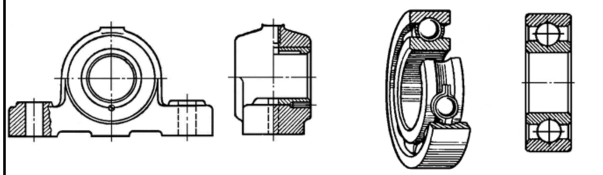
BEARINGS

BEARINGS

Bearings are machine elements intended to support axles and shafts.

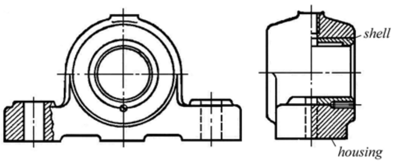
From the viewpoint of the friction experienced by bearing components they are divided into:

Sliding contact bearings and ***Rolling contact bearings***

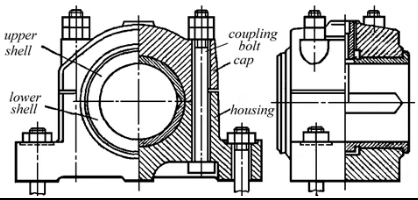


SLIDING CONTACT BEARINGS

Solid bearings

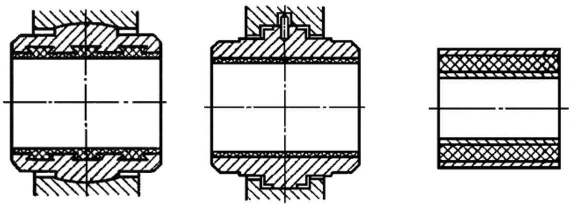


Split bearings



SLIDING CONTACT BEARINGS

Self-aligning bearings



Shell with spherical bearing surface

Shell with bearing surface formed by a narrow band

Shell with bearing surface consisting of an elastic cushion of oil-resistant rubber

SLIDING CONTACT BEARINGS

Advantages

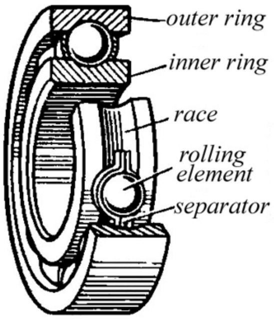
- Smaller dimensions in the radial direction;
- Can be split and mounted on any type of shaft;
- Can operate at high rotational speed (over 100000 rpm);
- Insensitive to impacts and vibrations;
- Can operate in water or any other corrosive medium;
- Permit radial clearance adjustment and, therefore, simplify shaft alignment

SLIDING CONTACT BEARINGS

Disadvantages

- High frictional losses and reduced efficiency;
- The need for elaborate lubrication systems and continuous lubricant feed control;
- Non-uniform wear of the mating surfaces;
- The need for expensive anti-friction materials;
- Relatively large axial dimensions.

ROLLING CONTACT BEARINGS



ROLLING CONTACT BEARINGS

Advantages

- Lower friction losses and higher efficiency;
- Generate much less heat;
- Develop an antitorque moment during start-up, which is 1/10 to 1/20 of that produced by sliding contact bearings;
- Do not require expensive non-ferrous metals;
- Have a small size in the axial direction;
- Permit easy maintenance and replacement;
- Use less oil;
- Have low cost due to mass production;
- Are highly interchangeable.

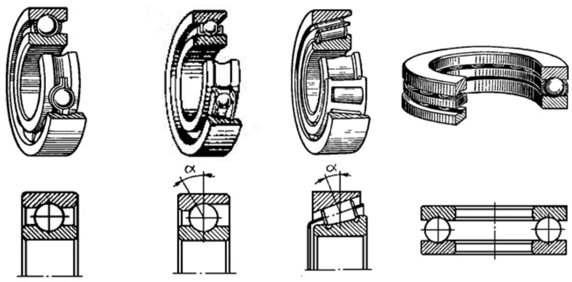
ROLLING CONTACT BEARINGS

Disadvantages

- Have limited application under heavy loads and at high angular speeds;
- Are unsuitable for operation under considerable impacts and vibration loads;
- Have a grater size in the radial direction.

CLASSIFICATION OF ROLLING CONTACT BEARINGS

I. According to the forces bearings can restrain

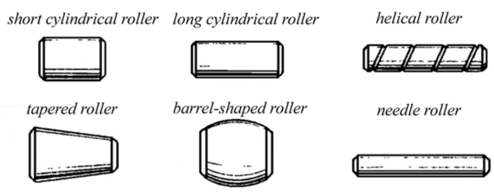


Radial bearings *Radial-thrust bearings* *Thrust bearings*

CLASSIFICATION OF ROLLING CONTACT BEARINGS

II. According to the shape of the rolling element

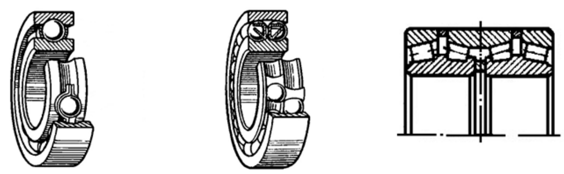
- *Ball bearings* 
- *Roller bearings*



CLASSIFICATION OF ROLLING CONTACT BEARINGS

III. According to the number of rows of the rolling elements

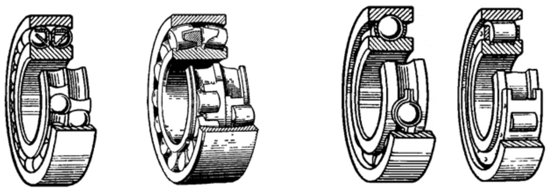
Single-row bearings ***Double-row bearings*** ***Quadruple-row bearings***



CLASSIFICATION OF ROLLING CONTACT BEARINGS

IV. According to the ability to compensate for shaft misalignment

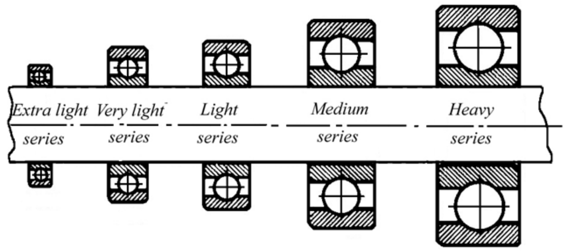
Self-aligning bearings | *Non-selfaligning bearings*



CLASSIFICATION OF ROLLING CONTACT BEARINGS

V. According to the load-carrying capacity

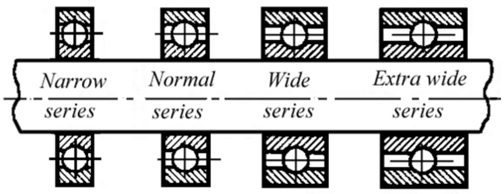
a) Depending upon sizes in the radial direction



CLASSIFICATION OF ROLLING CONTACT BEARINGS

V. According to the load-carrying capacity

b) Depending upon the width of the bearing



CLASSIFICATION OF ROLLING CONTACT BEARINGS

VI. According to the accuracy of manufacture

In the order of increasing accuracy

- *Bearings of 0 class*;
- *Bearings of 6 class;*
- *Bearings of 5class;*
- *Bearings of 4 class;*
- *Bearings of 2 class.*

DESIGNATION OF ROLLING CONTACT BEARINGS

5-36209 — Diameter of the bore

- 00 corresponds to d = 10 mm;
- 01 corresponds to d = 12 mm;
- 02 corresponds to d = 15 mm;
- 03 corresponds to d = 17 mm;
- from 04 to 99 corresponds to d = (04…99) × 5 = (20…495) mm.

DESIGNATION OF ROLLING CONTACT BEARINGS

Series of the bearing

5-36209

- 1 –Very light series;
- 2 – Light series;
- 3 – Medium series;
- 4 – Heavy series;
- 5 – Light wide series;
- 6 – Medium wide series.

DESIGNATION OF ROLLING CONTACT BEARINGS

Type of the bearing

5-36209

- 0 – Single-row radial ball bearings;
- 1 – Double-row self-aligning radial ball bearings;
- 2 – Radial bearings with short cylindrical rollers;
- 3 – Double-row self-aligning radial roller bearings;
- 4 – Needle or roller bearing with long cylindrical rollers;
- 5 – Radial bearings with helical rollers;
- 6 – Radial-thrust (angular-contact) ball bearings;
- 7 – Tapered roller bearings;
- 8 – Thrust ball bearings;
- 9 – Thrust roller bearings.

DESIGNATION OF ROLLING CONTACT BEARINGS

Structural features of the bearing

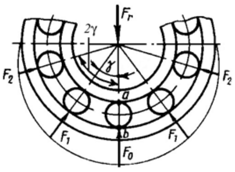
5-36209

Class of the bearing

In our case we deal with the angular-contact ball bearing (6) of light series (2) and 5th class (5) with an angle β = 12° (3) for the shaft of diameter d = 45 mm (09).

LOAD DISTRIBUTION AMONG ROLLING ELEMENTS

$$F_r = F_0 + 2F_1 \cos\gamma + 2F_2 \cos 2\gamma + ... + 2F_n \cos n\gamma,$$



where $\gamma = \dfrac{360°}{z}$.

$$F_1 = F_0 \cos^{3/2}\gamma, \; \dots, \; F_n = F_0 \cos^{3/2} n\gamma.$$

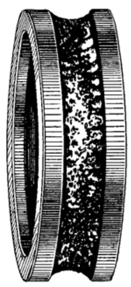
$$F_0 = \frac{F_r}{1 + 2\cos^{5/2}\gamma + ... + 2\cos^{5/2} n\gamma}.$$

Taking into account that $\dfrac{z}{1 + 2\cos^{5/2}\gamma + ... + 2\cos^{5/2} n\gamma} \approx 4.37$

we obtain $F_0 = 4.37 \cdot Fr / z.$

$$F_0 = \frac{5 \cdot F_r}{z}, \qquad F_n = \frac{5 \cdot F_r \cdot \cos^{3/2} n\gamma}{z}.$$

BASIC MODES OF FAILURE OF ROLLING CONTACT BEARINGS



- ***Fatigue pitting*** of the contact surfaces of the rolling elements and raceways due to *cyclic contact loading*. This failure occurs after long time operation and is accompanied by increased noisy and vibrations.

BASIC MODES OF FAILURE OF ROLLING CONTACT BEARINGS

- ***Permanent set*** which is characterized by appearance of *dents in the raceways*. This failure occurs at $n \le 1$ rpm under heavy and impact loads.

- ***Abrasive wear*** of the rubbing surfaces due to *insufficient protection against ingress of dust and dirt.* This failure is typical for bearings used in vehicles, tractors, and the like.

BASIC MODES OF FAILURE OF ROLLING CONTACT BEARINGS

- ***Breakdown of the rings and rolling elements*** due to *misalignment in assembly or heavy dynamic loads*. This failure seldom occurs in normal service.

- ***Breakdown of the separators,*** which is typical of high-speed bearings subjected to *appreciable centrifugal forces and pressure* exerted by the rolling elements.

CALCULATION OF ROLLING CONTACT BEARINGS

- ***Calculation for basic load rating*** to prevent *fatigue pitting;*
- ***Calculation for static load rating*** to prevent *permanent set.*

CALCULATION FOR BASIC LOAD RATING

This calculation is carried out for bearings whose inner rings rotate at n > 1 rpm (if n = 1 to 10 rpm, it is assumed for design purposes that n=10 rpm).

Basic condition of calculation

$$C_{req} \le C_{nom},$$

where

C_{req} is the required basic load rating in N or kN;

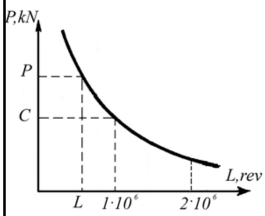
C_{nom} is the nominal basic load rating in N or kN.

CALCULATION FOR BASIC LOAD RATING

Nominal basic load rating C_{nom} is the constant radial load that 90 % of a group of identical bearings can withstand for *one million revolutions* of the inner ring without showing any signs of fatigue.

Rated life L is the number of revolutions or hours at a given constant speed that *90 % of a group of identical bearings* will withstand before the first evidence of fatigue develops.

CALCULATION FOR BASIC LOAD RATING



$$P^m \cdot L = C^m \cdot 1 \Rightarrow$$

$$C = P \cdot \sqrt[m]{L} \quad \text{or} \quad L = \left(\frac{C}{P}\right)^m,$$

where

m is a constant taken as ***3*** for ball bearings and as ***10/3*** for roller bearings;

C is the basic load rating in N or kN;

L is the rated life in million revolutions;

P is the equivalent load in N or kN.

CALCULATION FOR BASIC LOAD RATING

$$L = 60 \cdot n \cdot L_h \cdot 10^{-6},$$

where

n is the rotational speed of the inner ring;

L_h is the rated life in hours that depends upon a type of designing machine

- For one-shift operation machines working with underloading (electrical motors, general purpose speed reducers) $L_h \ge 12000$ hours;
- For one-shift operation machines working with full load (machines of general engineering, lift cranes, fans, distribution shafts) $L_h \ge 20000$ hours;
- For round-the-clock operation machines (compressors, pumps, mine hoists, stationary electric machines) $L_h \ge 40000$ hours.

CALCULATION FOR BASIC LOAD RATING

Equivalent load P is the constant stationary radial load which, if applied to a radial or radial-thrust bearing, would give the same life as that which the bearing will attain under the actual conditions of load and rotation.

$$P = (X \cdot V \cdot F_r + Y \cdot F_a) \cdot K_s \cdot K_t,$$

where

F_r and F_a are correspondingly the actual radial and axial loads acing on the bearing;

X and Y are the radial and axial force factors (specified by the manufacturer) ;

CALCULATION FOR BASIC LOAD RATING

$$P = (X \cdot V \cdot F_r + Y \cdot F_a) \cdot K_s \cdot K_t,$$

V takes into account which of the bearing rings is rotating (V=1 with the inner ring rotating and V=1.2 with the outer ring rotating);

K_s is the safety factor which takes care of the effect of the manner of loading on the rated life ($K_s = 0.01 \cdot W$, where W is the overload expressed in percentage; depending upon the manner of loading K_s may be ranged from 1 to 2.5);

CALCULATION FOR BASIC LOAD RATING

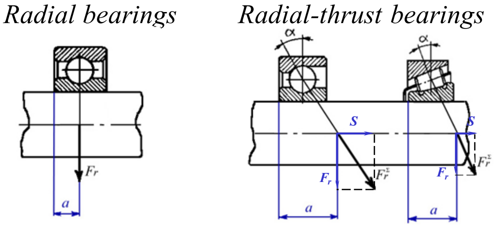
$$P = (X \cdot V \cdot F_r + Y \cdot F_a) \cdot K_s \cdot K_t,$$

K_t takes into account effect of temperature on the rated life:

t,C °	100	150	175	200	250
K_t	1.00	1.11	1.15	1.25	1.4

FEATURES OF CALCULATION OF RADIAL-THRUST BEARINGS FOR BASIC LOAD RATING

Radial bearings *Radial-thrust bearings*

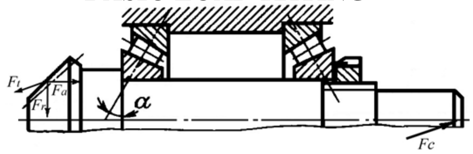


$$a = 0.5 \cdot B;$$

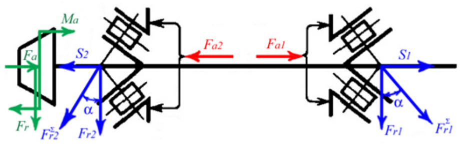
$$a = 0.5 \cdot \left(B + \frac{D+d}{2} \cdot tg\,\alpha \right)$$ - for ball bearings;

$$a = 0.5 \cdot \left(T + \frac{D+d}{3} \cdot e \right)$$ - for roller bearings.

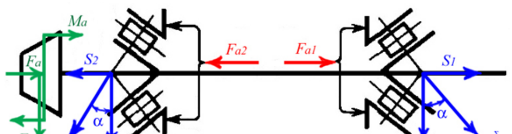
FEATURES OF CALCULATION OF RADIAL-THRUST BEARINGS FOR BASIC LOAD RATING



Analytical model



FEATURES OF CALCULATION OF RADIAL-THRUST BEARINGS FOR BASIC LOAD RATING



$$S_i = F_{ri} \cdot tg\,\alpha;$$

$$S_i = F_{ri} \cdot e',$$

where $e' = e$ – for ball bearings; $e' = 0.83 \cdot e$ – for roller bearings.

I. If $F_a + S_1 > S_2$, $F_{a1} = S_1$ and $F_{a2} = S_2 + (F_a + S_1 - S_2) = F_a + S_1$;

II. If $F_a + S_1 = S_2$, $F_{a1} = S_1$ and $F_{a2} = S_2$;

III. If $F_a + S_1 < S_2$, $F_{a1} = S_1 + (S_2 - (F_a + S_1)) = S_2 - F_a$ and $F_{a2} = S_2$.

CALCULATION OF ROLLING CONTACT BEARINGS FOR STATIC LOAD RATING

Calculation is carried out when $n < 1$ rpm.

Basic condition of calculation

$$P_0 \le C_0,$$

where

P_0 is the equivalent static load;

C_0 is the static load rating.

CALCULATION OF ROLLING CONTACT BEARINGS FOR STATIC LOAD RATING

Static load rating C_0 is the static load for which the total permanent set of the rolling elements and rings in the most loaded point of contact equals to ***0.0001·d***, where ***d*** is the diameter of the rolling element.

Equivalent static load P_0

$$P_0 = X_0 \cdot F_r + Y_0 \cdot F_a,$$

where

Fr and ***Fa*** are correspondingly the radial and axial loads;

X_0 and Y_0 are the radial and axial static load factors.

X_0=0.6, Y_0=0.5 for radial ball bearings;

X_0=0.5, Y_0=0.47…0.28 (α=12…36º) for angular contact bearings;

X_0=0.5, Y_0=0.22·tg α for tapered roller bearings.

JOINTS

CLASSIFICATION OF JOINTS

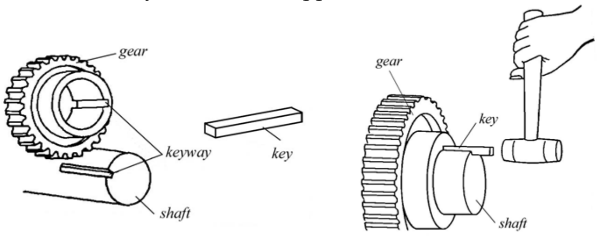
JOINTS

- Detachable joints
  - Keyed joints
  - Splined joints
  - Threaded joints
- Permanent joints
  - Welded joints
  - Riveted joints
  - Soldered joints
  - Adhesive joints

KEYED JOINTS

KEYED JOINTS

Keyed joints are used to fix elements (pulleys, gears, half-couplings, sprockets, etc.) on axles and shafts. These joints are formed by a ***key*** seated in matching slots that are made in a shaft and a gear hub so that two links may transmit the applied torque jointly.



ADVANTAGES AND DISADVANTAGES OF KEYED JOINTS

Advantages

- Keyed joints are simple and reliable in construction;
- Keyed joints are relatively inexpensive;
- Keyed joints are easy to assembly and disassembly.

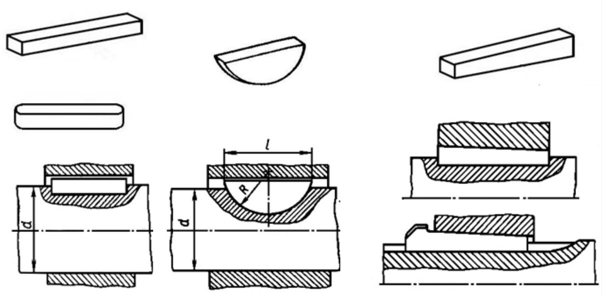
Disadvantages

- Keyed joints have reduced strength due to key slots in shafts and hubs;
- Keyed joints have inevitable stress concentrations;
- Keyed joints can transmit limited torque.

CLASSIFICATION OF KEYED JOINTS

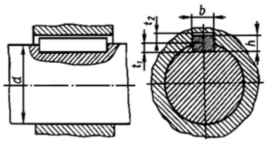
Unstrained keyed joints: Sunk keys, Woodruff keys

Strained keyed joints: Taper keys

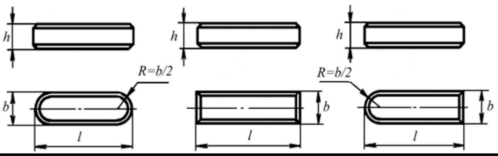


SUNK KEYS

Ordinary sunk keys



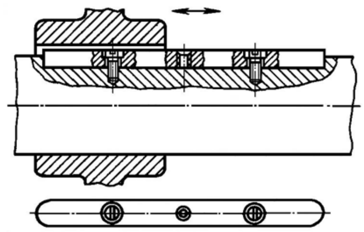
Key with round ends | *Key with square ends* | *Key with round and square ends*



SUNK KEYS

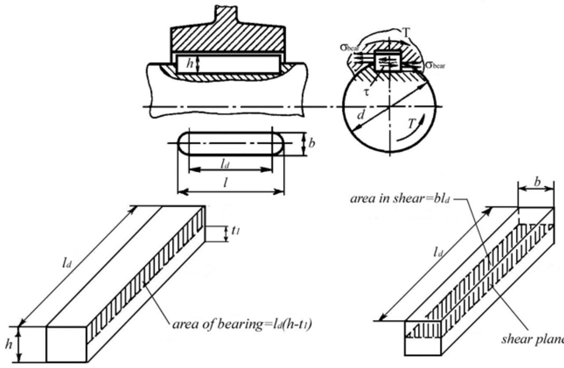
Feather keys

Feather keys are used when a gear hub has to move lengthwise. Being rather long, feather keys are fixed to the shaft by guide screws.

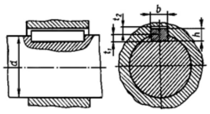


CALCULATION OF SUNK KEYS

Sunk keys work for **bearing** and **shearing**

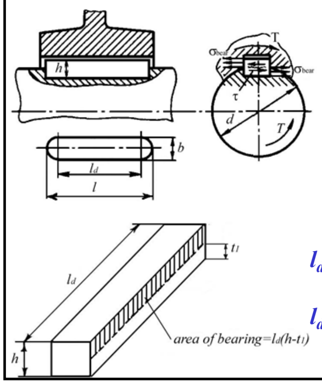


SELECTION OF SUNK KEYS



Shaft diameter d	Key cross section		Keyseat depth		Length l
	b	h	shaft, t_1	hub, t_2	
Over 17 to 22	6	6	3.5	2.8	Over 14 to 70
Over 22 to 30	8	7	4	3.3	Over 18 to 90
Over 30 to 38	10	8	5	3.3	Over 22 to 110
Over 38 to 44	12	8	5	3.3	Over 28 to 140
Over 44 to 50	14	9	5.5	3.8	Over 36 to 160
Over 50 to 58	16	10	6	4.3	Over 45 to 180
Over 58 to 65	18	11	7	4.4	Over 50 to 200
Over 65 to 75	20	12	7.5	4.9	Over 56 to 220
Over 75 to 85	22	14	9	5.4	Over 63 to 250
Over 85 to 95	25	14	9	5.4	Over 70 to 280
Over 95 to 110	28	16	10	6.4	Over 80 to 320
Over 110 to 130	32	18	11	7.4	Over 90 to 360

CALCULATION OF SUNK KEYS FOR BEARING



$$\sigma_{bear} = \frac{F_t}{A_{bear}} \le [\sigma_{bear}];$$

$$F_t = \frac{2 \cdot T}{d};$$

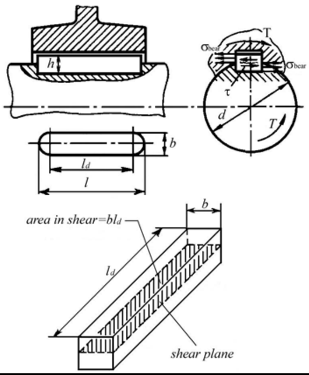
$$A_{bear} = l_d \cdot (h - t_1).$$

$$\sigma_{bear} = \frac{2 \cdot T}{d \cdot l_d (h - t_1)} \le [\sigma_{bear}]$$

$l_d = l$ – for keys with square ends;

$l_d = l - b$ – for keys with round ends.

CALCULATION OF SUNK KEYS FOR SHEARING



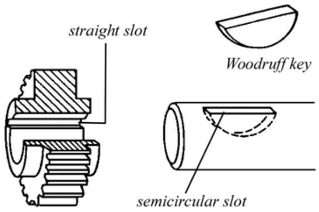
$$\tau_{shear} = \frac{F_t}{A_{shear}} \le [\tau_{shear}];$$

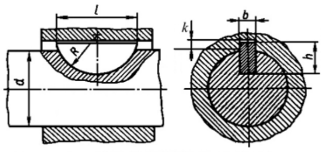
$$F_t = \frac{2 \cdot T}{d};$$

$$A_{shear} = b \cdot l_d.$$

$$\tau_{shear} = \frac{2 \cdot T}{d \cdot b \cdot l_d} \le [\tau_{shear}].$$

WOODRUFF KEYS





Advantages

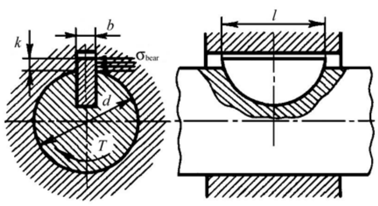
- Woodruff keys are seated deeper than ordinary sunk keys;
- Woodruff keys are more reliable in service;
- They better resist wrenching under load.

Disadvantages

- Deeper slot for Woodruff keys markedly weaken the shaft.

CALCULATION OF WOODRUFF KEYS

Woodruff keys are analyzed for **bearing**



$$\sigma_{bear} = \frac{F_t}{A_{bear}} \le [\sigma_{bear}];$$

$$F_t = \frac{2 \cdot T}{d};$$

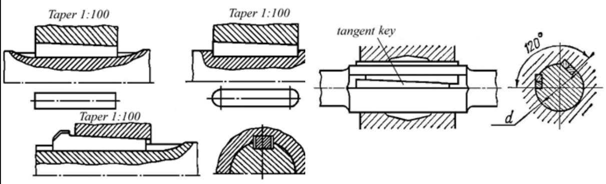
$$A_{bear} = k \cdot l;$$

$$\sigma_{bear} = \frac{2 \cdot T}{d \cdot k \cdot l} \le [\sigma_{bear}].$$

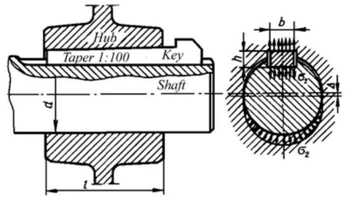
TAPER KEYS

Taper keys form **strained joints**

Draw keys *Tangent keys*



TAPER KEYS



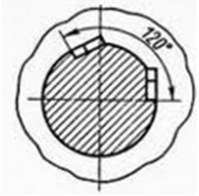
Advantages

- Taper keyed joint can transmit both a torque and an axial force due to frictional forces that develop on the key working surface.

Disadvantages

- Press-fitting a key gives rise to a certain misalignment Δ of the shaft and the hub.

<u>Tangent keys</u> are expensive but offer excellent service, because they decrease strength of shaft insignificantly and can transmit considerable loads. They may be used as a single or double key. When they are used as a single key the positioning depends on the direction of rotation of the shaft. For heavy load two keys can be used as shown in figure.

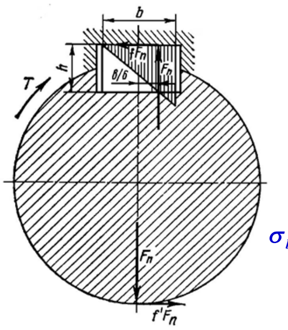


A **<u>flat key</u>** is used for light load because they depend entirely on friction for the grip. The sides of these keys are parallel but the

A **<u>saddle key</u>** is very similar to a flat key except that the bottom side is concave to fit the shaft surface. These keys also have friction grip and therefore cannot be used for heavy loads. Very little stress concentration occurs in the shaft in these cases.

CALCULATION OF TAPER KEYS

Tapered keyed joints are checked for **bearing strength**



$$\sigma_{bear} = \frac{2 \cdot F_n}{l \cdot b} \leq [\sigma_{bear}];$$

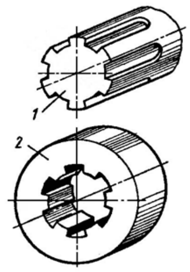
$$F_n = \frac{6 \cdot T}{b + 6 \cdot f \cdot d};$$

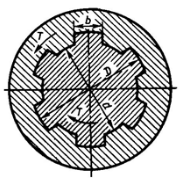
$$\sigma_{bear} = \frac{12 \cdot T}{l \cdot b \cdot (b + 6 \cdot f \cdot d)} \leq [\sigma_{bear}].$$

SPLINED JOINTS

SPLINED JOINTS

Splined (or *toothed*) *joints* are joints formed by projections on the shaft called as splines, which fit matching grooves in the hub.





SPLINED JOINTS

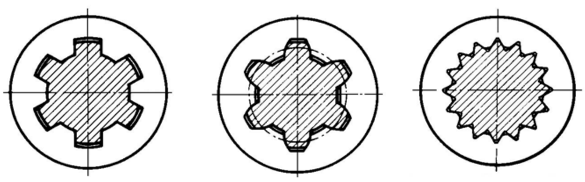
Advantages

- Splined joints offer a higher load-carrying capacity owing to an increased contact area;
- They permit better alignment of the mating parts;
- Stress concentration is less pronounced at the roots of the splines than in key slots;
- Splined joints are easier to manufacture;
- They can be made to a higher degree of accuracy.

CLASSIFICATION OF SPLINED JOINTS`

I. According to the shape of splines

Joint with straight-sided splines	*Joint with involute splines*	*Joint with triangular-tooth splines*

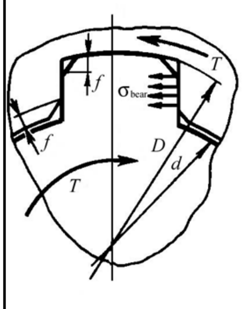


II. According to the depth and number of splines

- ***Light series;***
- ***Medium series;***
- ***Heavy series.***

CALCULATION OF SPLINED JOINTS

Splined joints are analyzed for **bearing.**



$$\sigma_{bear} = \frac{F_t}{A_{bear} \cdot K \cdot z} \leq [\sigma_{bear}],$$

$$F_t = \frac{2 \cdot T}{d_{mean}}; \quad A_{bear} = h \cdot l;$$

K = 0.7…0.9 takes into account non-uniform load distribution among splines;

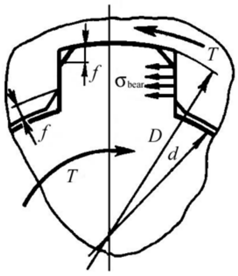
z is number of splines;

d_{mean} is mean diameter of the splined joint;

h is depth of splines;

l is effective length of the joint.

CALCULATION OF SPLINED JOINTS



$$\sigma_{bear} = \frac{2 \cdot T}{d_{mean} \cdot K \cdot h \cdot l \cdot z} \le [\sigma_{bear}].$$

For straight –sided splines

$$h = 0.5 \cdot (D - d) - 2 \cdot f;$$

$$d_{mean} = 0.5 \cdot (D + d).$$

For involute splines

$$h \approx m; \quad d_{mean} = m \cdot z,$$

where m is the module.