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THE ANALYTICAL CORE MODEL FORMATION OF THE NONLINEAR PROBLEM BOND ARMATURE WITH CONCRETE

It is proposed a nonlinear analytical model of the pivotal bond between the reinforcement and the concrete. It is established, that at the design of this process it is important to reach the level of engineering visibleness of the used dependences, dropping not essential features, while at the same time to get a reasonably accurate solution. In such search authors were stopped for the analytical model of nonlinear task of bond armature with a concrete, allowing to take into account elastic-plastic work of concrete, and also non-linearity of contact armature with a concrete. It is revealed that analytically the rod model of bond reinforced concrete bar is designed by the system, consisting of four equalizations, two from which are differential equalizations of first order. Nonlinear contact interaction of two materials is approximately examined as mutual relative displacements of armature and concrete. It is shown that the proposed model undoubtedly requires experimental confirmation by testing and numeral design more high level and degrees of working out in detail.

Keywords: *analytical model, bond armature with concrete, reinforced concrete constructions, physical nonlinearity, mutual relative displacement of reinforcement and concrete.*

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ПОБУДОВА АНАЛІТИЧНОЇ СТРИЖНЕВОЇ МОДЕЛІ НЕЛІНІЙНОЇ ЗАДАЧІ ЗЧЕПЛЕННЯ АРМАТУРИ З БЕТОНОМ

Запропоновано нелінійну аналітичну стрижневу модель зчеплення між арматурою та бетоном. Доведено, що при моделюванні цього процесу важливо вийти на рівень інженерної видимості використаних залежностей, відкинувши несуттєві ознаки, й у той же час отримати досить точне рішення. З'ясовано, що у такому випадку, треба зупинитися на аналітичній моделі нелінійної задачі зчеплення арматури з бетоном, що дозволила враховувати пружно-пластичну роботу бетону, а також нелінійність контакту арматури з бетоном. Виявлено, що аналітично стрижнева модель зчеплення залізобетонного стрижня формується системою, яка складається із чотирьох рівнянь, два з яких є диференціальними рівняннями першого порядку. З'ясовано, що така модель потребує експериментального підтвердження шляхом проведення випробувань і реалізації чисельного моделювання задач зчеплення більш високого рівня та ступеня деталізації.

Ключові слова: *аналітична модель, зчеплення арматури з бетоном, залізобетонні конструкції, фізична нелінійність, взаємні відносні зміщення арматури і бетону.*

Introduction. The problem of bond armature with concrete is fundamental in the resistance of reinforced concrete constructions.

Analysis of recent sources of research and publications. In order to solve the various problems associated with the bond armature and concrete in Ukraine and abroad conducted extensive experimental and theoretical studies [1, 3, 4, etc], including such eminent scientists as V.M. Bondarenko, A.B. Golyshev, A.A. Oatul, M. M. Kholmyanskii, G.N. Shorshnev, N.I. Karpenko, V.I. Kolchunov, G.N. Sudakova, E.M. Babich, B.A. Broms, L.I. Storozhenko, H. Goto, Y.A. Ivanchenko, V.A. Kuukosk, S.M. Mirza, Y. Haud, P.P. Nazarenko, G. Rem, S.M. Skorobogatov and A.V. Trofimov. However, analysis of research in the field of bond armature with concrete evidence of the ambiguity of approaches to solving this problem, and the lack of a unified theory-based calculation method [1, 3 – 6].

A good bond-strength between any reinforcement type and concrete is one of the main prerequisites for a reliable static function of steel-concrete constructions. The bond-strength is affected by many factors [7]: the adhesive forces between concrete and reinforcement, the friction forces caused by the surface inhomogeneities of the flat parts of the reinforcement, the surface geometry (ribs, imprints and corrugations), the composition and mechanical properties of the used concrete, its processing, curing time and also the position of the reinforcement in the concrete.

The development of the nonlinear bond stress-slip model of fiber reinforced plastics sheet-concrete interfaces with a simple method is represented in [8]. It should be noted work [9] on the finite element modelling of reinforcment with bond.

Reducing the bond armature with concrete leads disclosure to excessive cracking, reduced rigidity of and reduced bearing capacity of construction [1]. Detection of regularities contact concrete and reinforcement is one of the most important reinforced concrete tasks [3, 4] in the transmission of tensile forces through the armature in cracking conditions.

Identification of general problem parts unsolved before. The regularities contact concrete and reinforcement are shown in the study of the problem pulling reinforcement bar from the concrete block (prism), which is put in recent years by the majority of researchers in the framework of formation estimated bond model [5].

The estimation of resistance reinforced concrete structural elements can be made in the presence of macro-cracks [5, 6] on the basis of solving this problem. The case of one of the central reinforcing rod deserves special attention when pulling it out of the concrete matrix and more due to the fact that to it exactly or approximately reduced most types of reinforcement prismatic elements of longitudinal rods system [3, 4].

Formulation of the problem. The process of connections destruction bond armature with a concrete is a difficult sequential process at pulling up an armature bar from the concrete matrix. It is accompanied by the presence of multistage process of a heterogeneous and inelastic deformation, violation of adhesive bonds, emergence and development of cracks which have different form and orientation, by the presence of changing areas of contact and unstudied phenomena. For the correct decision of this task, it is necessary to use analytical models, describing the interaction concrete with armature which is characterized by bond forces [3, 4, 5].

Basic material and results. The analytical cored model of nonlinear problem of bond armature with concrete is worked out and analyzed in this article by authors.

1. Creation of an analytical model.

Analytically rod bond reinforced concrete model is simulated by calculation scheme (fig. 1 and 4), and by the system consisting of the four equations, two of which are differential equations (DE) of first order (the boundary value problem Cauchy).

The model examines the reinforced concrete element with a single central reinforcement. The left end is rigidly fixed element of any movement, the right end – free.

By reinforcing bar tensile force $N_s(x)$ is applied, the armature is pulled out of the concrete matrix. This effort causes a displacement and deformation of concrete reinforcement bar $\varepsilon_s(x)$ and concrete of reinforced concrete element $\varepsilon_c(x)$ over the entire length of of the squared beam. In the concrete matrix occurs due to force $N_c(x)$ interaction with armature in concrete beam contact region and the relative displacements of mutual reinforcement and concrete $\varepsilon_g(x)$. The contact interaction of the two materials is approximately regarded as a mutual relative displacement of reinforcement and concrete. The force $\tau(x)$ per unit length is acting on the direction of the current load in concrete along the entire length of contact shear, but in an armature, – oppositely directed (Fig. 1).

The dependence of the tangential bond stress τ_{bond} from mutual relative displacement of reinforcement and concrete $\varepsilon_g(x)$, characterizes the bond of material is a bi-linear in nature (Fig. 2) and is presented below:

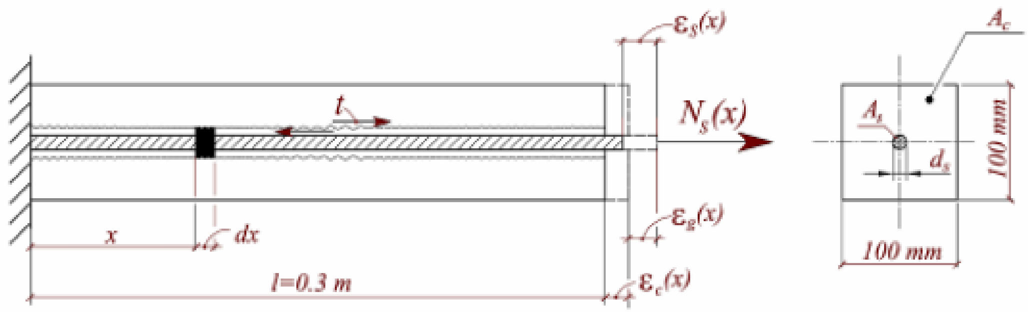

Figure 1 – The calculation scheme of the reinforced concrete element

$$\tau_{bond} = k \cdot \varepsilon_g(x) = 0{,}4 \cdot E_{cm} \cdot [\varepsilon_s(x) - \varepsilon_c(x)]; \tag{1}$$

$$\text{if } \varepsilon_g(x) = [\varepsilon_s(x) - \varepsilon_c(x)] \le \varepsilon_g^*(x) = 4{,}95 \cdot \frac{f_{ctm}}{E_{cm}};$$

$$\tau_{bond} = 0{,}0232 \cdot E_{cm} \cdot [\varepsilon_s(x) - \varepsilon_c(x)] + 1{,}866 \cdot f_{ctm}, \tag{2}$$

$$\text{if } \varepsilon_g(x) = [\varepsilon_s(x) - \varepsilon_c(x)] > \varepsilon_g^*(x) = 4{,}95 \cdot \frac{f_{ctm}}{E_{cm}},$$

where $\varepsilon_g^*(x)$ – the boundary relative mutual displacement of the concrete and reinforcement, corresponding to the end point of the first section bond diagram (Fig. 2);

f_{ctm} – the average tensile strength of concrete in tension, Table 1 and Norms [2];

E_{cm} – the average initial modulus of elasticity of concrete, Table 1 and Norms [2];

The bilinear diagram σ_c–ε_c is shown on Fig. 3. It also describes the concrete work on this model. The expressions for each of the sections of the diagram included in the system of equations. It describes the deformation of concrete by next equations:

$$\varepsilon_c(x) = \begin{cases} \dfrac{N_c(x)}{E_{cm} \cdot A_c}, & \text{if } \dfrac{N_c(x)}{A_c} \le 0{,}9 \cdot f_{ctm} \\ \dfrac{18 \cdot N_c(x)}{E_{cm} \cdot A_c} - 15{,}3 \cdot \dfrac{f_{ctm}}{E_{cm}}, & \text{if } \dfrac{N_c(x)}{A_c} > f_{ctm}. \end{cases} \tag{3}$$

Using the equilibrium condition of the concrete and armature rods get the following two differential equations that relate the forces in the rods and shear bond stresses (Fig. 4):

– for armature (reinforcment):

$$-N_s + N_s + dN_s - t \cdot dx = 0; \tag{4}$$

– for concrete:

$$-N_c + N_c + dN_c + t \cdot dx = 0. \tag{5}$$

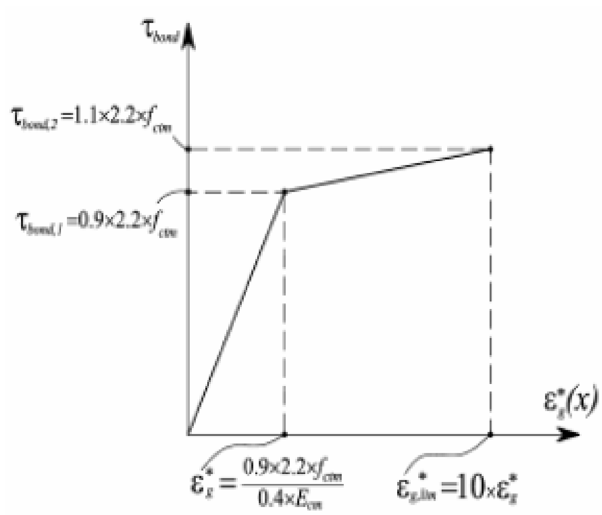

Figure 2 – The dependence of bond stress τ_{bond} on the relative mutual displacement $\varepsilon_g^*(x)$

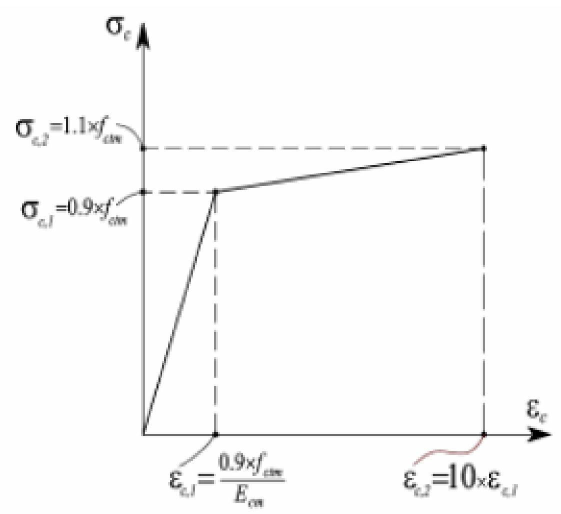

Figure 3 – The concrete deformation diagram $\sigma_c - \varepsilon_c$ in the analytical model

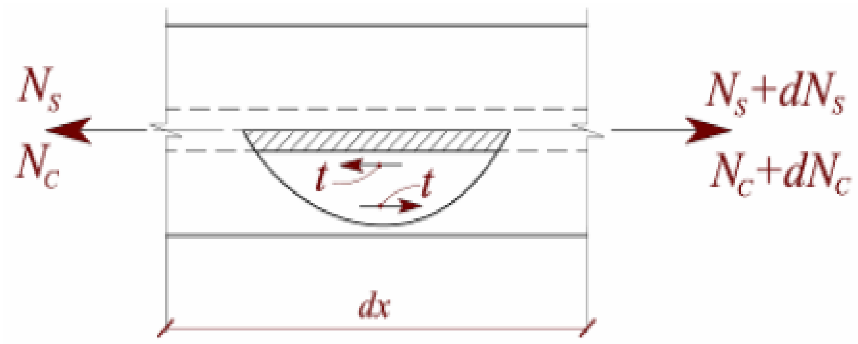

Figure 4 – Working reinforcement in concrete

After appropriate algebraic transformations, we have:

– for armature (reinforcement):

$$\frac{dN_s}{dx} = \tau_{bond} \cdot \pi d_s \,; \tag{6}$$

– for concrete:

$$\frac{dN_c}{dx} = -\tau_{bond} \cdot \pi d_s \,. \tag{7}$$

In the equations (4) and (5), $t = \tau_{bond} \cdot \pi \cdot d_s$, N_s – efforts in reinforcement, N_c – efforts in concrete.

For armature Hooke's law is valid.

$$\sigma_s = E_s \ \ \varepsilon_s; \Rightarrow \varepsilon_s = \frac{\sigma_s}{E_s} = \frac{N_s(x)}{E_s \cdot A_s}. \tag{8}$$

Thus, a nonlinear boundary value problem is received. It consists of four equations, two of which control the first order, which is as follows:

$$\begin{cases} \varepsilon_s(x) = \dfrac{1}{E_s \cdot A_S} \cdot N_s(x); \\ \varepsilon_c(x) = \begin{cases} \dfrac{N_c(x)}{E_{cm} \cdot A_c}, & if \quad \dfrac{N_c(x)}{A_c} \le 0{,}9 \cdot f_{ctm}, \\ \dfrac{18 \cdot N_c(x)}{E_{cm} \cdot A_c} - 15{,}3 \cdot \dfrac{f_{ctm}}{E_{cm}}, & if \quad \dfrac{N_c(x)}{A_c} > f_{ctm}; \end{cases} \\ \dfrac{dN_s(x)}{dx} = \begin{cases} \pi \cdot d_s \cdot 0{,}4 \cdot E_{cm} \cdot [\varepsilon_s(x) - \varepsilon_c(x)], & if \quad \varepsilon_g(x) \le \varepsilon_g^*(x) = 4{,}95 \cdot \dfrac{f_{ctm}}{E_{cm}}, \\ \pi \cdot d_s \cdot \{0{,}0232 \cdot E_{cm} \cdot [\varepsilon_s(x) - \varepsilon_c(x)] + 1{,}866 \cdot f_{ctm}\}, & if \quad \varepsilon_g(x) > \varepsilon_g^*(x) = 4{,}95 \cdot \dfrac{f_{ctm}}{E_{cm}} \end{cases} \\ \dfrac{dN_c(x)}{dx} = \begin{cases} -\pi \cdot d_s \cdot 0{,}4 \cdot E_{cm} \cdot [\varepsilon_s(x) - \varepsilon_c(x)], & if \quad \varepsilon_g(x) \le \varepsilon_g^*(x) = 4{,}95 \cdot \dfrac{f_{ctm}}{E_{cm}}, \\ -\{0{,}0232 \cdot E_{cm} \cdot [\varepsilon_s(x) - \varepsilon_c(x)] + 1{,}866 \cdot f_{ctm}\}, & if \quad \varepsilon_g(x) > \varepsilon_g^*(x) = 4{,}95 \cdot \dfrac{f_{ctm}}{E_{cm}}. \end{cases} \end{cases} \tag{9}$$

The boundary conditions of the problem:

$$N_c(x=1) = 0, \quad N_S(x=1) = C. \tag{10}$$

This problem is solved with the use of numerical methods in the application of computer algebra package Wolfram Mathematica system.

2. Analysis of the results of the analytical model of the bond.

Varying the values efforts N_s at the end of a reinforcing bar distribution graphs were obtained of the desired functions $\varepsilon_s(x)$, $\varepsilon_c(x)$, $N_s(x)$, $N_c(x)$ along the length of the rod (Fig. 5, 6).

The initial data are presented at the Table. 1.

Table 1 – The initial data for the dependency formation of the unknown functions with the effort at the end of the rod

The main materials characteristic	Value
E_s	$2{,}04 \cdot 10^5$ MPa
A_s	$7{,}854 \cdot 10^{-5}$ m^2
E_{cm}	$2{,}7 \cdot 10^4$ MPa
A_c	10^{-2} m^2
d_s (armature class A400C)	10^{-2} m
f_{ctm} (concrete class C16/20)	1,9 MPa

The limiting factors in this case were:

– tensile strength reinforcement in tension f_t;

– the average tensile strength of concrete in tension f_{ctm}, which is determined by the stage of the work of the concrete section of the element;

– limiting the relative displacement of the armature with respect to the concrete at the end $\varepsilon_{g,lim} = 10 \times \varepsilon_g^*$, in which there is a failure of reinforcement with concrete ties, according to the adopted work depending reinforcement contact with concrete (Fig. 2);

– ultimate tensile strain of concrete in tension $\varepsilon_{c,2} = 10 \times \varepsilon_{c,1}$, according to the accepted depending on the concrete deformation (Fig. 3).

It is observed a proportional increase in the length of the rod according to the current force values of the unknown functions with increasing efforts at the end of reinforcement bar in displayed graphs. The value of efforts N_s at the input corresponds to the force in the valve output (Fig. 5, *a*), which is one of the criteria for the correctness of the building model. The effort N_c in the concrete on the right side is zero (Fig. 6, *a*), which fully corresponds to the initial conditions and the physical side of the problem. For values N_s=0.025…0.03 *MN* of stresses in the concrete exceed the tensile strength of concrete in tension $0{,}9 \cdot f_{ctm}$ a nd concrete starts to work on the second branch of the stress-strain diagram. The tensile load application area there is a sharp, abrupt increase N_s and U_s and decrease N_c and U_c (begins at a distance of 0,3 m from the left edge and extends to the right edge).

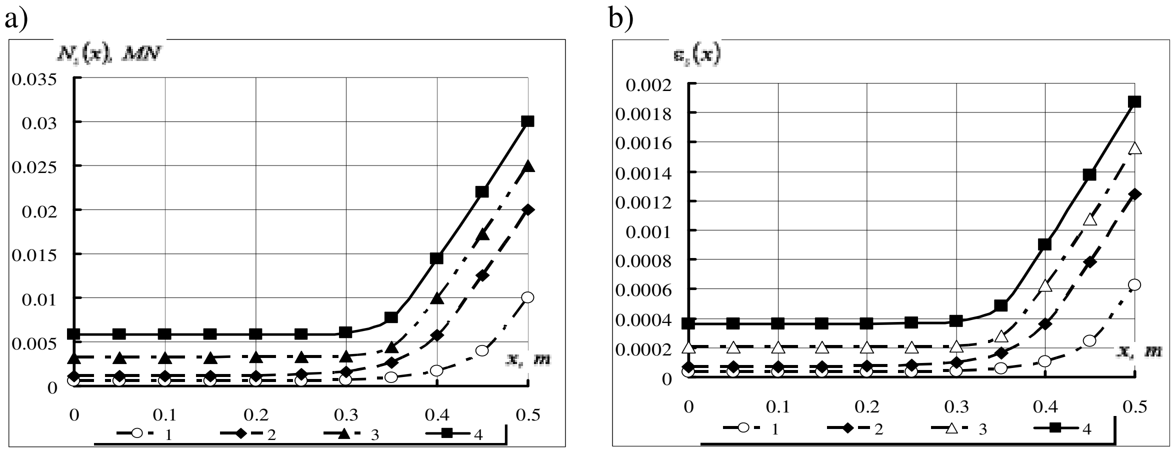

Figure 5 – The distribution graph of the longitudinal effort in the armature $N_s(x)$ **(a) and relative longitudinal strains in the reinforcement** $\varepsilon_s(x)$ **(b) along the length of the rod, depending on the efforts acting on the end:**
1 – for tensile effort at the end equal to 0.01 MN; 2 – the same, equal to 0.02 MN; 3 – the same, equal to 0.025 MN; 4 – the same, equal to 0.03 MN

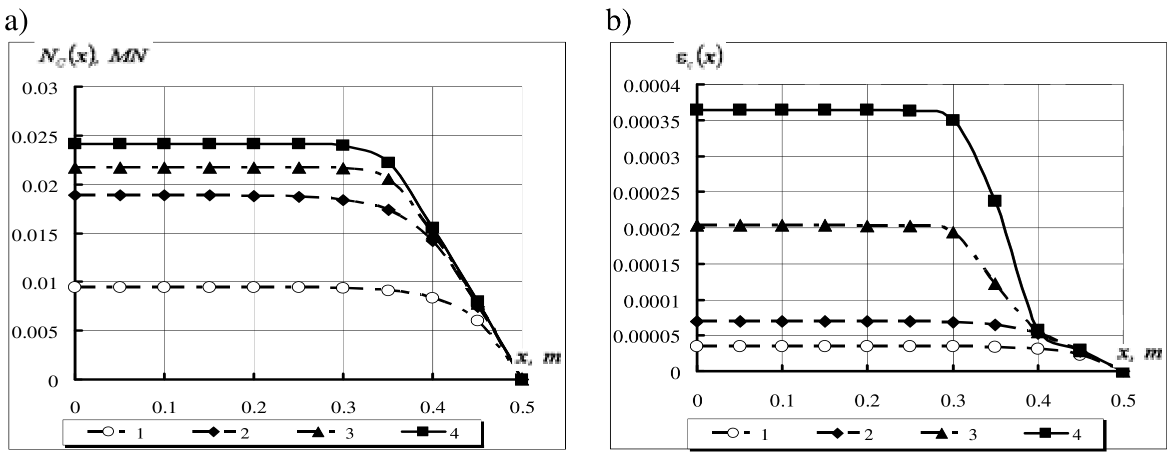

Figure 6 –The distribution graph of the longitudinal effort in the concrete $N_c(x)$ **(a) and longitudinal strains in concrete** $\varepsilon_c(x)$ **(b) along the length of the rod, depending on the efforts acting on the end:**
1 – for tensile effort at the end equal to 0.01 MN; 2 – the same, equal to 0.02 MN; 3 – the same, equal to 0.025 MN; 4 – the same, equal to 0.03 MN

Also, the results of the calculation of the analytical models were built unknown functions $\varepsilon_s(x)$, $\varepsilon_c(x)$, $N_s(x)$, $N_c(x)$ distribution graphs for different diameters of rebar from the concrete to pull out (Fig. 7, 8). The initial data are presented at the Table 2.

Table 2 – The initial data for the dependency formation of the unknown functions along the length of the rod for the different diameters of armature

The main materials characteristic	Value
E_s	$2{,}04 \cdot 10^5$ MPa
E_{cm}	$2{,}7 \cdot 10^{-5}$ m^2
A_c	10^{-2} m^2
f_{ctm} (concrete class C16/20)	1,9 MPa
N_s	0.022 MPa
d_s (armature class A400C)	Ø10; Ø12; Ø16 (10^{-2} m, $1{,}2 \cdot 10^{-2}$ m, $1{,}6 \cdot 10^{-2}$ m)

a) b)

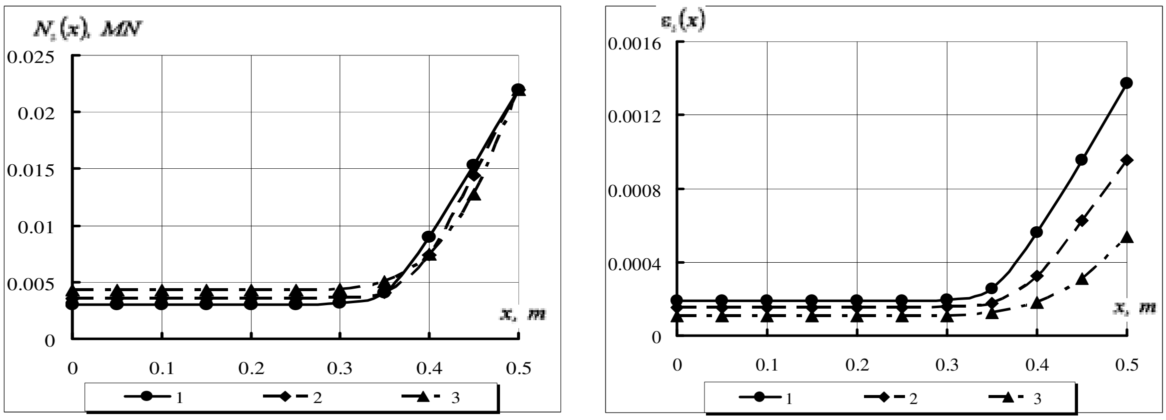

Figure 7 – The distribution graph of the longitudinal effort in the armature *$N_s(x)$ (a)* and relative longitudinal strains in the reinforcement *$\varepsilon_s(x)$ (b)* along the length of the rod, depending on the diameter of armature:
1 – armature Ø 10 mm; 2 – armature Ø 12 mm; 3 – armature Ø 16 mm

a) b)

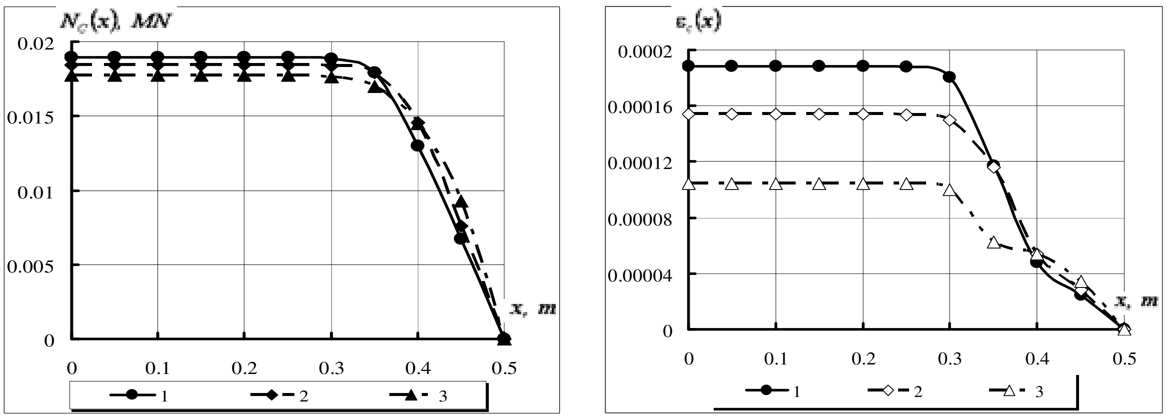

Figure 8 – The distribution graph of the longitudinal effort in concrete *$N_c(x)$ (a)* and relative longitudinal strains in concrete in the reinforcement *$\varepsilon_c(x)$ (b)* along the length of the rod, depending on the diameter of armature:
1 – armature Ø 10 mm; 2 – armature Ø 12 mm; 3 – armature Ø 16 mm

There is a slight increase in the acting force at increasing the armature diameter (Fig. 7, *a*) in the graph depending $N_s(x)$ on the length of the rod; plots $\varepsilon_s(x)$, $\varepsilon_c(x)$, $N_s(x)$, $N_c(x)$ show an increase in the values of these functions while reducing the diameter of armature (Fig. 7, 8), which also corresponds to the physical picture of the phenomenon.

Conclusions. Analytical model of bond armature with concrete is a fairly accurate and informative model. This model takes into account the elastic-plastic concrete work, as well as the non-linearity of armature in contact with the concrete, and allows you to efforts distribution of graphics and concrete strains and reinforcment along the length of the rod.

The proposed model is relatively simple, although it is not without drawbacks, and therefore requires experimental confirmation by testing and implementation of numerical simulation bond problems and a higher level of granularity.

References

1. *Белов В. В. Диахронная модель деформирования коррозионно-поврежденных железобетонных элементов с трещинами / В. В. Белов, С. Е. Никитин // Вестник гражданских инженеров. – 2011. – №4. – С. 18 – 25.*
   Belov V. V. Diahronnaya model deformirovaniya korrozionno-povrezhdennyh zhelezobetonnyh elementov s treshchinami / V. V. Belov, S. E. Nikitin // Vestnik grazhdanskih inzhenerov. – 2011. – №4. – S. 18 – 25.
2. *ДБН В.2.6-98:2009. Бетонні та залізобетонні конструкції. Основні положення. [Чинний з 2011-07-01]. – К. : Мінгеріонбуд України, ДП «Укрархбудінформ», 2011. – 71 с.*
   DBN V.2.6-98:2009. Betonni ta zalizobetonni konstruktsiyi. Osnovni polozhennya. [Chinniy z 2011-07-01]. – K. : Mingerionbud Ukrayini, DP «Ukrarhbudinform», 2011. – 71 s.
3. *Бондаренко В. М. Расчетные модели силового сопротивления железобетона: монография / В. М. Бондаренко, В. И. Колчунов. – М. : Изд-во АСВ, 2004. – 472 с.*
   Bondarenko V. M. Raschetnye modeli silovogo soprotivleniya zhelezobetona: monografiya / V. M. Bondarenko, V. I. Kolchunov. – M. : Izd-vo ASV, 2004. – 472 s.
4. *Карпенко Н. И. Общие модели механики железобетона / Н. И. Карпенко. – М. : Стройиздат, 1996. – 416 с.*
   Karpenko N. I. Obshchie modeli mehaniki zhelezobetona / N. I. Karpenko. – M. : Stroyizdat, 1996. – 416 s.
5. *Левин В. М. Математическое моделирование совместного деформирования арматурного стержня и бетона в окрестности изолированной трещины (применительно к модели дискретных трещин) / В. М. Левин, Н. Ю. Рогожин // Вестник Донбасской национальной академии строительства и архитектуры. – 2015. – № 3(113). – С. 5 – 7.*
   Levin V. M. Matematicheskoe modelirovanie sovmestnogo deformirovaniya armaturnogo sterzhnya i betona v okrestnosti izolirovannoy treshchiny (primenitelno k modeli diskretnyh treshchin) / V. M. Levin, N. Yu. Rogozhin // Vestnik Donbasskoy natsionalnoy akademii stroitelstva i arhitektury. – 2015. – № 3(113). – S. 5 – 7.
6. *Холмянский М. М. Контакт арматуры с бетоном / М. М. Холмянский. – М. : Стройиздат, 1981. – 184 с.*
   Holmyanskiy M. M. Kontakt armatury s betonom / M. M. Holmyanskiy. – M. : Stroyizdat, 1981. – 184 s.
7. *Problems and normative evaluation of bond-strength tests for coated reinforcment and concrete / P. Pokorný, M. Kouŕil , J. Stoutil, P. Bouśka, P. Simon, P. Juránek // Materials and technology. – 2015. – № 49(6). – P. 847 – 856.*
8. *Dai J. Development of the nonlinear bond stress-slip model of fiber reinforced plastics sheet-concrete interfaces with a simple method / J. Dai, T. Ueda, Y. Sato // J. of Composites for Construction. – 2005. – № 9(1). – P. 52 – 62.*
9. *Jendele L. Finite element modelling of reinforcment with bond / L. Jendele, J. Cervenka // Computers and Structures. – 2006. – № 84(28). – P. 1780 – 1791.*

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