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SUBJECTIVE ENTROPY OF PREFERENCES

Subjective analysis

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International Standard Book Number 978-83-63539-08-5

Publisher: Institute of Aviation Scientific Publications
Al. Krakowska 110/114, 02-256 Warsaw
Phone (48 22) 846 00 11 int. 391
Fax (48 22) 846 44 32

Printer: ALKOR
05-070 Sulejowek, Krucza 4.

CONTENTS

APPRECIATION

I would like to express my deepest gratitude to my colleagues for their helpful advice and discussion of different ideas and issues related to the themes of the book and will undoubtedly improve its content. First of all, my greatest appreciation turns to professors: V.A. Ignatov, V.V. Kulish, V.V. Pavlov, O.V. Petrenko, G.A. Suslova.

I am also grateful to dr A.V. Goncharenko which greatly helped in the composition of e-book. Provide substantial assistance in the work on the book scientist's dr eng. Krzysztof Szafran and Tadeusz Korsak who put a lot of effort to translate the book into English.

I`m obliged to thank the management of the Institute of Aviation in Warsaw, in particular, the Director Institute dr hab. eng. W. Wiśniowski will provide an opportunity of edition. Also I am gratefully to the National Aviation University in Kiev, in the person of the Rector, professor N.S. Kulik, and Director of the Aerospace Institute of the University professor V.N. Shmarov, the head of department of mechanics professor V. V. Astanin who created the necessary conditions for writing the book.

INTRODUCTION

Studies „from the frontiers of sciences" possess large attractive force in spite of the high degree of risk for those, who go along this way. There is always a danger of becoming an object of critical „shelling" from the professional's broadside in each of the adjacent unitary regions.

Roger Penrose, the chairman of the Department of Mathematics of Oxford University mentions about these dangers in his remarkable book, dedicated to the consideration of the fundamental possibility of designing of an artificial intelligence [135].

The result of border research can be both the amazing new ideas and theories, and useless rehash of already well known.

High risk of interdisciplinary research not only stops, but, as can be seen even encourages outstanding scientists and draws them into the border area. The „victims" of this passion, in a good sense, have become many well-known mathematicians, mechanics, physics: N. Moiseev, S. Kapica, I. Prigogine, J. Neumann, H. Haken, Malinetskii, Y. Klimontovich and many others.

To work on the junction of different sciences is productive and interesting. It is only necessary to feel as a student in each of the border fields and to understand that you are in the vicinity of scientific bifurcation.

Representatives of „exact science", including mathematicians are drawn into this place of formalized region as by a magnet force, region which is still „untouched by the mathematician's foot".

Benefit from the advent of mathematicians is mutual. Into the nonformalized sciences the mathematician introduces not only and not so much the possibility to calculate anything, as mathematical order in the reflections, the formulations, the terminology - as a whole - increases discipline of scientific search in such regions as philosophy, psychology, sociology, history and, even, culture. From the other side these latter mentioned regions give to the dry, removed from context mathematical formulas „sensual" painting, and sometimes generate new mathematical theories. Specifically, for example, the fuzzy mathematics appeared as a result of this synthesis. One can say that mathematics gives „corporality" to the non-formalized sciences and, in turn, obtains „spirituality" from them.

The reasoning's given above to a certain degree justify the author, whose basic specialties are - mechanics and mathematics, in the attempt to engage into studies in the synthetic region, which is here designated by term „Subjective analysis".

The starting point of these studies was the work, connected with the optimization of training process, which the author carried out in the 70's together with his college professor V.A. Ignatov. Then the so-called problematic- resource approach, which proved to be sufficiently universal [1, 2, 3], was proposed.

Subsequently for the author it was possible to enlarge the conceptual framework. It became obvious, that from a formal point of view this approach can be interpreted in the terms of the theory of binary relations, although in the meaningful sense it preserves a number of original positions. This relates, in particular, to understanding of categories „problem" and „purpose" and interrelation between them.

The author's first book on this theme was published in 2003 with the title „The elements of subjective analysis". This work reflects the tendency to develop synthetic concept and the schematic of analysis of active systems, which is rested on the problematic - resource method, the theory of binary relations, the theory of usefulness, the theory of categories, some ideas and the positions of synergetics, information theory. At first it seemed that author will succeed in constructing ordered noncontradictory theory; however, it was sufficiently rapidly explained that in the selected direction this cannot be carried out.

Therefore the book contains in a certain sense, the intermediate results, so to speak, „sprinkled" with the collection of attempts and alternative models, connected together with the small number of cardinal ideas: „built-in" optimality of the functioning of psyche with the solution of the problem of selection, the strictly individual nature of any problems and the required presence there of individual „carrier", a vital difference in the active systems from the passive, the use of subjective entropy and subjective information as the fundamental measures, which define state and dynamics of active systems, view on the usefulness as to the objective characteristic of situation and on the preferences as to its subjective characteristic.

To me the insufficient strictness and theoretical validity of some conclusions is obvious; reproach in the superfluous theorization, the absence of the experimental confirmation of a number of assumptions and ideas would be justified. Generally, in view of the absence in many instances of experimental material is selected such way, when the specific a priori principles, for example the principle of subjective optimality *at first are formulated, conclusions and consequences of them and then are analyzed. And, if they correspond to general empirical ideas, then to say „to the common sense", then they are recognized as those having the right to exist and subsequently serve as basis for planning and organizing the experimental studies: what to measure? for what purpose? what are measurement requirements? how to plan an experiment? what are the endogenous and exogenous parameters? As a whole, it is possible to say that the presence of a priori models „spiritualizes" experiment, it makes it possible to leave from the of the so-called „black box" schematic - task is reduced to the parametric, or structural identification of the models, whose class is determined a priori by the* *principles adopted. In the present work such a priori principle is the „principle of subjective optimality", in formulation of which the „subjective entropy" plays the basic role.*

I will foresee criticism from the side of those, who consider that it is exclusive empirical experience, is the source of theories. Closer to the truth is the assertion, that the „practice" (experiment) is the criterion of theory truthfulness, when the theory is born first, and experiment follows the theory and is formed with theory. This point of view pleases me more. Let us give on this occasion quote from the book of J. Neumann [124]: „Usually there are many ways to construct a mathematical model, so the question arises as to which of them select. In some cases, the adequacy of the specific model can be validated empirically. In other cases, verification of such kind can be very difficult, and we must rely on our intuitive sense suggests to us that the mechanism postulated in the model satisfactorily corresponds to the phenomenon under study".

The issue under consideration of priority and primacy: the „theory or experiment?" is akin to the philosophical problem of what is primary *word* or *thought*. This issue is discussed in the already mentioned paper by Roger Penrose and resolved in favor of the primacy of *thought*.

Actually, it is possible to give many examples, when thought is clearly primary, and word serves only as the formulation of thought, moreover as the nonsingular method. What is born in the head of architect or the artist, the artistic means of future creation, or his verbal portrait? What is born in the head of composer, the melody or its verbal interpretation (can it be the score record?) In these examples the thought is primary, but word comes out as the „notary evidence" of thought, not always perfected and comprehensive. The theoretical concept and experimental facts can be located in the same relation. There is, of course, and an inverse relationship, when a set of experimental data gives life to the model, which simulates them in the same or a different physical space.

In the first case (the theory superiority) any experiment in view of the fundamental limitations cannot with certainty confirm the theory, but it can disprove it, and, conversely, in the second case, the theory explaining the experiment, must *„go beyond the experiment"*, otherwise it is meaningless.

An important question about the adequacy of the models is related to Gödel's theorem, which requires an external add-on. According to this theorem, „the Hamilton program" to create a consistent and closed mathematical axiomatic is not feasible. More of this, locked and non-contradictory theoretical concept cannot be, according to this meaning, developed in this work. It seems to me that a selected theme has not been profoundly analyzed yet and, by this way, it\`s possible to expect astonishing ideas and results, applications in different subject areas. It does not confuse also that it\`s something what that is written in the book is already made by others, moreover, for sure so it is. But I`m convinced that, even though there is a well known results described earlier , in the book, there are a lot of new aspects that are to be analyzed and examined. That`s why this publication is justified - to widen the knowledge on concerned matter.

It is also clear, that the planned building still is found in the construction stage and the „scaffolding is not taken yet".

They over shade facade and they make the architectural concept not completely determined.

Introduction - this is such chapter, which let us allow wider view on the problem, constituting basic contents of the book, the consideration of adjacent concepts and ideas in the freer form, in order to create the background, on which, the further basic action is developed.

In this sense let us examine a number of general ideas and views, including the author's position, and from the other side, closely related to the fundamental assumptions.

One of such questions is a question about the possibility of designing of artificial intelligence, which is discussed, in particular, in the book of Roger Penrose [135], and where the answer proves to be negative. Here also essential role plays Gödel's theorem.

*It`s possible to create a computer able to think, feel, act, create, but all that within margins of a model of „behaviorism", developed by psychologists, that treats a human as cybernetic system, which is determined by environment conditions - a concept of "*homo economics*" in contrast to "*homo sapiens*". This computer will probably pass any of Turing test, but it will not become „*silicon person*". Roger Penrose sees a fundamental impossibility of creating an adequate artificial mind with ability of spontaneous manifestations of creation, including scientific, fantasy. Basis of these attributes lie deep on the quantum level. Completely fantastic and improbable captivating appears to be an idea* of large-scale quantum coherence*, expressed by Penrose. Human's brain can be a quantum laser system.*

One of the used basic categories - a „problem" is understood as a „desire" on a background of alternative possibilities of its satisfaction, therefore, a concept of the „problem" is to the same degree universal as universal the „desire" is. There is a point of view, that the moment of „desire appearance" is the same moment of „ life appearance". The thesis about the fact that „life arose there and then, where and when arose desire" is being discussed. A comparison of „living" with „desire" is imagined to be regular, but immediately a question arises about superiority: which is primary the „life" or the „desire". The answer to this question proposes synergetic - a branch of science that emerged in the second half of past century, and which studies processes of self-organizing in animate and inanimate nature from the new positions. Recently a series of books of synergetic founders in Russian language have been published: H.

Haken, I. Prigogine, G. Nikolis, their followers W. Ebeling, A. Engel, R. Faystel [8, 9, 14, 55, 132, 133, 151, 153], and also of Russian and Ukrainian authors S. Kapitsa, S. Kurdyumov, G. Malinetskiy, D. Trubetskov, R. Barantsev, A. Loskutov, I. Adrianov, L. Manevich, A. Nazaretyan, L. Melnik, O. Chaly and other [84, 101, 123, 126, 127, 147, 158].

An overall meaning of an answer lies in the fact that self-organizing, which leads to forms of living matter and is, after all, a consequence of the fundamental openness and dissipation of natural systems as the reaction of matter to the presence of physical heterogeneities and gradients. A prototype of the „desire" is an orientation of dipole when the gradient of electric potential is present, and the „retraction" of it is the confluence of force lines.

Let's look, at biblical allegories. God created Adam. He was deprived of emotions and desires, he was indifferent to surrounding. Eve's creation of („heterogeneity") gave birth to the desire - the „desire of knowledge" - a first scientific research interest. Adam and Eva were a first scientific research workers, an entire remaining universe - a scientific laboratory of that created and financed by God, and the first scientific theme was called as follows: „To determine that there is good and that there is evil".

In any case, man appears as a tool of knowledge, also, in this, apparently consists the main, original sense of his existence. His intelligence arose and developed as a result of presence of information traffic. Creating man God wanted to solve the problem, which Gödel realized and represented theorems in the form: the problem of an external addition. He divided world as an object of knowledge and a subject of knowledge - human- researcher.

After a certain time the man destroyed the limitations established for him: he began to think about the fact what he is itself and, moreover, what the God is. Thus the problem of external addition was deepened and aggravated.

We are the part of the universe and we are intelligent creatures, therefore, *the universe is the living being*, which possesses the reason, most likely, not only ours. From the aforesaid it follows, that the possible main sense of human existence consists of the knowledge of the surrounding world and itself.

This anthropomorphous concept is an impressing one. From it follows, that the „masters of life" are not God's „goals" on Earth, but only the „tools".

The „aims" of God are those „super flashes of the Nova stars" that once in a century illuminate the world with new knowledge: Copernicus, Newton, Einstein, Michelangelo, Raphael, Beethoven, Chopin,... These selected by God (read by Nature) are called to the Sinai mountain in order to tell them the revelations, which they communicate to people as the biblical prophets. Not Gödel devised the „fundamental Gödel's theorem", but God. Gödel only reported it to the rest of the people, like the Prophet. By this theorem, „the mathematics" proved to be the same open, dissipative system like any other and, specifically, because of its openness, capable of development.

What does indicate the term „subjective analysis", mentioned in the text and carried out into the title of the book?

This term is connected with the „*active system*" concept, i.e. such system, the nucleus, the core of which is the main element –human – *subject* („person, who makes decisions"). Relying on the irreducibility of the human consciousness, the psyche to any „computer simulation model", to the impossibility of creation absolutely adequate artificial intelligence over any time perspective, taken as the initial postulate, we conduct the fundamental boundary between „active" and „passive" systems.

Active systems are in this work the object of attention.

„*Subjective analysis*" - it is the analysis of the active systems functioning. It is only „subjective" in that sense, that all solutions are taken as active system subjects and therefore they are „subjective".

Existence of active systems, existence and subjects activity - is the objective fact.

It is again necessary to return to the explanation of the sense principle mentioned above.

The „artificial intelligence" will be most likely created, which in different relations will be considerably more perfect and more effective than human intelligence. Even now, as the „calculator" the computer is many orders of magnitude more perfect than man. Computer's gigantic memory, speed of information transmission, storage medias, branched systems of solution support is already unattainable for the human. Nevertheless, as yet the computer is similar to primitive rock axe, it is merely „the extension" of human intelligence. Principally can be created the computer, which will „live" its own life, may leave out of human's control, it will be self-reproducing and self-improving, but it will not be human. First of all because human is endowed with the capacity for spontaneous creativity (not random, but spontaneous) and also because it seems, one never will succeed to give the computer the ability to „believe". Computer's intellect always and exclusively will be based only on the knowledge, while human intelligence is based on the organic combination of knowledge and faith. *Quite obvious is, that the* knowledge *does not exist without the* faith, *just as* faith *does not exist without the* knowledge. *Specifically, this organic combination ensures the continuous process of learning – this is, so to speak, technological principle. Between* the domain of knowledge *and* the domain of unknown *lies* „the boundary layer" *of faith. This* faith *not only in God, but the* faith, *understood more broadly,* belief, *that permeates permanently all the pores of human existence, along with* knowledge. *The* faith *and the* knowledge *are mixed and are the nourishing concentrate for the brain.*

Thus, the most rigorous mathematical theories are basically a system of axioms that is an inevitable element of *faith*. All further: lemmas, theorems, equations… represent *knowledge*, but as we see the basis, at the foundation, lies the faith.

What we have said gives grounds to conduct fundamental division between artificial and natural intelligence; between the active and passive systems. What basic assumptions, principles do determine the content of this book? Most common of these has already been mentioned above and they were provided with the appropriate commentaries. Let us pause in for more detail at the working postulates.

The main and most important is the principle of subjective optimality. *In its formulation the main role is given to* subjective entropy. *Therefore, we can speak of a* „variational entropy principle", *which allows one to receive and examine different models of preference functions. In this part this work is tightly connected with the works of Jensen [204, 205], and also Haken [151, 152], Neumann and Morgenstern [125], Stratonovich [137]. Variational principle is the unique boundary, which separates, and at the same time establishes connection between the endogenous processes, i.e. the psyche deep processes, and exogenous processes in the „external environment", which in one way or another are affecting subject's interests. However, the internal work of psyche is manifested for the external „observer" as the system of preferences formed by the subject on the set of alternatives. Basic assumption is that into subject's psyche the specific principle of the forming the distribution of preferences is „inscribed", build-in, and that this principle appears* variational*, that makes each time this distribution* optimum *in the sense of certain functional. Then the next task is to specify, which this functional is. Here, the author relied on the already mentioned works, as well as on one of the basic assumptions of the categories theory.*

We know that each next generation of scientists „stands on the giants arms". We have reason to trust their insights, which have accumulated previous experience. For me the starting point was the idea of Leonhard Euler that „*nothing at all takes place in the universe in which some rule of the maximum or minimum does not appear*". This assertion sounds like a biblical truth and, in this case, it is an element of faith, extended on the psychical processes.

Carl Siegel writes: "according to Leibnitz our world is the best of all possible worlds, and therefore the laws, which control them, can be described by extreme principles". *We know, that in mechanics and physics, leastwise, in the case of conservative systems the dynamics is described by the equations, following from the variational principles, such as the Hamilton's least-action principle. The author of this work was able to show that for certain classes of dissipative systems the variational principles exist [66, 68].*

This is, however, insufficient in order to „impose" the extreme principles of human psyche functioning. Even reference to the theory of categories does not resolve this issue to the end.

We do not have the necessary statistical data, and the distributions of preferences are „poorly observable" objects. The results of direct observation and measurement of preferences by testing, interviews, submitted questionnaires are unreliable and insufficient to confirm the assumption about *„embedded" optimality of the psyche*.

There is, however, another circumstance, which is testifying in favor of the assumption made. In some works (e.g. [133, 134]), subjective entropy is closely associated with the internal freedom. In a number of the philosophical systems, including theological, it is considered that human is endowed with the *„freedom of will"*, that is the *„freedom of choice"* (and, consequently, – with responsibility for the selection made). Then, if the principle of the maximum of *subjective entropy* is taken, which acts independently of the external circumstances, thus we are forced to postulate the organic property of psyche to function so that the *„internal freedom"* is at maximum each time. Below, in the present work it is asserted, that *„freedom"* is a dynamic category and, consequently, extreme principle requires aspiration of the isolated active system toward maximum *„internal freedom"*.

The presence of „will freedom" requires a method and algorithm of its implementation and the algorithm may not be optimal. This reasoning - heuristic argument for the thesis of the subjective optimality principle, „integrated" into the psyche.

If there is no algorithm for the implementation of „freedom of will", then the latter is transformed into the „anarchy of will".

The second essential element of the theoretical foundations is type I and type II preferences: *object preferences* and *rating preferences*. The cardinal model, which historically has been a precursor to the ordinal model, is accepted.

The author suggests that in depths of psyche the preferences are formed as cardinal, but their external fixation (for example, in parliamentary elections) is ordinal in nature. There may exist a similar relationship between „internal" radical and „external" ordinal preferences as that, which exists between the *„thought"* and *„word"* (in the Penrose's sense).

For each distribution of preferences the *subjective entropy* is introduced as a measure of uncertainty and *subjective information* as a measure of entropy variability. In the formulation of the variational principle significant is that to each is assigned its individual, „personal" functional. Common here is their additive structure for all subjects – the „carriers" and presence of the „subjective entropy" in each of them as the main term.

It may seem, that in connection with the fact, that to each distribution of utilities (resources) the variational principle associates well-defined distribution of preferences, no „freedom of will" exists there. This is not so. The subject chooses the set of alternatives, the „endogenous" parameters of the distributions are at his „disposal", finally, the so-called entropy thresholds and selection itself, which not compulsorily corresponds to maximum preference, but considers „ethical component", they remain in subject „conduct" and are outside the competence of the „variational principle".

The third element of foundation is the *problem-resource approach*, which has already been discussed above. In connection with this approach, the resolution of each problem is represented as a transformation or transfer of resources, a certain, sufficiently general classification of resources is proposed. The most significant from an ideological point of view, in this classification is the separation of active and passive resources.

One of the circumstances that prompted the authors to study issues presented in this book, was the lack of active systems safety and aeronautical safety in particular. Aviation transport system, its structural components, flying vehicle together with its crew, are examples of the active systems, where the „human factor" role is manifested especially vividly. It is known that about 70% of all aircraft accidents are due to the „human factor" action and, in particular, incorrect, or made at the wrong time decisions.

In this sense the description of the pilot role in the form of the „pilot-operator" models, as it is done in the majority of works, for example, in the publication [206], proves to be insufficient. In such studies the component of mental activity in the decision making multi-alternative situation, the conflict situations between the participants, the presence of entropy thresholds and other important circumstances are ignored.

From the standpoint of flight safety issues (Chapter 6) the content of Chapters 1-5 can be regarded as the preparation of the theoretical foundation. In this case, the possibility of effective application of theory developed here in a number of other areas: in the theory of economics, the theory of conflicts, the social dynamics and others becomes apparent.

The work comprises seven chapters.

Chapters 1-4 contain the materials, which form the concept of the „subjective analysis".

Chapter 1 addresses basic condition of the problem-resource approach, a brief excursus into the theory of binary relations is given.

Chapter 2 deals with the type I individual preferences of different types (the „subject preference"), provides information on utility theory and the elements of category theory, introduces the entropy of the type I subjective preferences and the subjective information, a variant of the algebra of alternatives. A definition is proposed for the entropy of ordinal distributions of preferences.

Chapter 3 deals with the formation of the basic variational principle - the „principle of subjective optimality" for the type I preferences. For different forms of efficiency function the distributions of preferences, which are conditionally named „canonical" are obtained.

In chapter 4 the groups of the interacting and interdependent subjects and the new type of preferences - rating preferences are examined. The assumption is done, that they are also formed as the optimal on the basis of variational principle. Information from the theory of collective utility and collective selection is given. The concept of the mutual utility, through which integral and differential ratings are expressed is introduced. Along with the ratings the ranks as „organizational alternatives" are considered and the concept of „organizational challenge" is formed. The attempt is made to consider influence on the distribution of the type I ethics preferences - ethical imperatives, the defined formalized schematic of the classification of ethical systems from the point of view of relationship with alternatives sets is proposed, and also the range of the preferences distribution models, in which both utilitarian and ethical component is reflected. Author attempted to take into account the impact on the distribution of I type ethics preferences - the ethical imperatives that offered some formalized scheme of ethical systems classification in terms of relations with the sets of alternatives, as well as several models of the distribution of preferences, which is reflected as a component of the utilitarian and ethical. In connection with this in the chapter material the brief survey of the ethics systems is included. Chapter concludes with a section that reflects an attempt to develop a model of „virtual subject" (or „collective intelligence").

Chapter 5 is entirely devoted to studying the dynamics of preferences. The concepts of exogenous and endogenous dynamics and the matching systems of differential equations are introduced. As the possible application of subjective analysis its application to the economic dynamics is examined. The Walras-Leontief system of equations is supplemented with the equations, which describe the dynamics of preferences, which in turn depend on economic factors. As a means of the simulation of some mental processes associated with economic dynamics, the attractors usage is encouraged (Lorenz's attractor, Brusselator). The presented quantitative calculations results show, that the inclusion of the preferences dynamics significantly affects the nature of changes in economic parameters. The possibility of determining the value of subjective entropy together with the assumption about existence of the entropy thresholds of decision making, makes it possible to modify the models of economic dynamics and to investigate directly the influence of psychological factors on the economy. Definite place in the chapter allocated elasticity and stiffness analysis of subjective preferences, which the author provisionally called the elasticity and stiffness of the psyche, and that may be useful in developing methods for selection of pilots, flight crew, teaching methods, sociodynamics problems.

Chapter 6 deals with the problem of active systems safety. It is formulated as independent promising trend. The role of the „human factor" is understood differently here, in comparison with those studies on the systems safety, which include human, where it's role is regarded as the operator. As in all other cases in the present work, the problem of safety of active systems is formulated and is solved as the task of subjective analysis. Proposed is the theoretical diagram, in which is realized the synthesis of probabilistic description with the components of subjective analysis, in particular, the modified Feller equations, in which figure the distributions of preferences together with the probabilistic distributions. Examples from the region of aviation safety are examined. Outlined is the approach taking into consideration of the effect of a priori stable imperatives, or rules, defined by normative documents, for the distribution of preferences and, immediately, on the period of decision making.

Finally, Chapter 7 *is dedicated to the application of subjective analysis to the theory of conflicts. Within the framework of this approach it proves to be possible to give a certain classification of types of conflicts in the active systems. In this case the conflict of any type is considered as the conflict of the distributions of preferences. Within the framework of entropy approach the so-called „entropy maps" are examined, which reflect the topology of the „reign of freedom" and the „reigns of need". Situation dynamics provides for the passages of active system from one „reign" into another.*

A brief survey is included in chapter, in the historical sequence, philosophical treatments and the definitions, connected with the freedom category. As in the other cases author assumes, that the view on the theory of conflicts from the subjective analysis point of view, represented in this chapter, it will be useful with the analysis of flight safety issues, in the theory of instruction, in social dynamics.

In order to exclude misunderstandings about this work, it is necessary to determine, for what it does not pretend to be. *First of all, the author would want to say with the complete certainty: this work is not a psychology study. It does not investigate the mental phenomena as such in the traditional sense. Similarly, the author does not seek to improve the philosophical bases of ethics, the theory of conflicts, and the theory of economic dynamics. In each case discussion deals with the interpretation of various phenomena and processes in the terms of „subjective analysis", which is intended to give researcher in each of the named regions the quantitative analysis tool in the form of the models of canonical distributions of preferences.*

Thus, in each case the specific mathematical formalism is proposed, more or less consonant with the matter essence. Certainly, it is possible that some ideas, expressed in the work, can prove to be useful in these subject areas in the meaningful sense. This is, however, a matter of the future.

Must be said about what author would like to make, but that for various reasons could not do. Among the reasons are the work time frame and the book size physical limitations.

First of all, the work is deliberately not presenting the statistical methods for analyzing the subjective preferences - completely natural and effective method for identifying the canonical distribution of preferences, containing necessary for this parametric, and in some cases, structural arbitrariness. A study in this area can be extremely productive and timely extension in the theoretical and applied senses.

The author excluded the probabilistic interpretation of preferences, and also suggests the use of fuzzy mathematics, where the „point" representation of preferences could be replaced by fuzzy sets [91].

These natural continuations would be undesirable complication in the stage of primary molding of concept and, they would most likely, add few fundamental moments.

To a question about sense in which this work is related with synergetics, one can answer that this work is not about synergetics as the self-organizing science, but adjoins it closely. An entire series of the excellent last time publications about synergetics of strictly scientific and popular type (mentioned above) served for the author as powerful irritant and stimulus.

One can say that with this book the author prepared soil for strictly synergetic studies of self-organizing active systems, especially in the group of subjects - the effects of clustering, cumulative effects, effects of coherent behavior and other important from the synergetics point of view phenomena, within the framework of subjective analysis ideas, naturally. As a whole, the continuation of the dynamics of preferences research for the different models of canonical distributions is imperative... Especially promising is a study in relation to the groups of subjects.

Very small number of papers reflected the problem of hierarchical systems subjective analysis [95, 110, 111, and 112].

One of the very promising regions of the subjective analysis application is didactics, processes of education and training of all levels and types. Here asserts itself the obvious gradation of tasks and analogies:

- *a change in distributing the preferences by the transmission of utilitarian training information - influence on the structure of information resources, the creation of the knowledge reserve;*
- *a change in the base properties of intellect, endogenous characteristics - memory, the speed of the information perception, ability to make analytical actions;*
- *a change in the ethical ideas, training, personality molding.*

Application of the education problems to the subjective analysis devoted one chapter in my first book on the subjective analysis [64]. Apropos of this book, I must say that it was necessary for me to refuse from something later, but much more, including that, what was not entered into this work, has important significance.

In particular, the problem of the relation between the „shadow" and „light" economies, which have been formulated in terms of subjective analysis, and then the numerical results were obtained and proved to be very close to reality.

It should be noted that Chapter 4 shows how relying on entropy optimization principle it is possible to theoretically obtain dependences of supply and demand on prices very close to dependences well known in the macroeconomics, what can be considered as one of the evidences in favor of the realism of the above principle.

One additional question, which is necessary to answer in the introduction: „for whom this book is written and how one should use it?"

I must immediately confess that, first of all, I wrote this book for myself because I was simply interested in doing this. Wide, anything and anybody unlimited space of ideas, approaches, fantasies and inventions. Sensation of complete freedom after the decades of work on the „economic agreements", difficult interrelations with the customers, with the ministerial officials, with university research sector and research parts, with tens of visas on the reports, „knockout" of states and financing.

I acknowledge that, working at the book, I completely did not think about its „practical value", but when the work in essence was made, it became obvious, that it has much greater practical value, than many of my previous works.

Certainly, the work is intended not only for the author himself. At the university where I work, there is a so-called a „New-technology Institute", in which the „piecework" is organized, preparing future scientists according to the individual programs. I assume that the book will be useful for the students of this institute and for graduate students.

Therefore it is simultaneously both the scientific monograph and the teaching aid.

I think that it will find readers among those interested in the synergy and, in general research in frontier areas of science, in one way or another connected with the need to consider the human role and the subjective factors.

The question of how many readers the book will have is absolutely not clear.

At the same Penrose book [135] I read this statement: „Every formula reduces the number of readers by half". If this is true, then will be no readers of my book. It contains too many formulas.

It is mostly difficult to create concept, the more easy is to construct theory on the base of concept and still more easily (certainly, in the ideological sense) to write pile of equations and formulas.

Since, as I think, there is a certain similarity of concept and theory and many formulas in this book, it is possible to suggest to the reader not immediately attempt to dive deeply into the dense forest of mathematical formulas and to go to the breach. The effect can be surprising: order emerging from the woods in torn clothes, bruises *and abrasions, such a reader may discover to his disappointment that in the „forest" there is nothing original, exciting for him, - only the „alder or aspen tree".*

Therefore as the advice, it is possible to propose to the reader to move at first along the openings, going around brushwood formulas and to try to grasp the sense of the developed approach and, then, in proportion to the appearance of an interest, refer to the formulas.

Completing introduction I would want to say that the directions of further studies are more or less obvious, just as essential theoretical difficulties and, in a number of cases, the imperfection of the materials presented in the book. Is also obvious, that the theme is not exhausted and not completed, and in proportion to the advance forward, on the spot of each solved task, the two new tasks appear.

1. ACTIVE SYSTEMS

1.1. Active systems - introductory remarks

In different subject areas we encounter a need of a qualitative or a quantitative level of evaluation, forecast, and description of man- subject participation.

Practically each kind of activity can be represented as a functioning of a certain system that is more or less individualized, in center of which a subject – an active element of the system stands. Its role is varied and will be gradually analyzed and refined further on. *The system, in center of which the subject is situated, that to a considerable extent determines the system functioning, is called an active in contrast to passive system, for example, purely technical, natural, which doesn't include active elements?*

The said in a strict sense is not the definition of an active system yet. This definition will be formed subsequently. The revelation between objective and subjective factors, which characterize system and establishment of the boundary between these features, is one of primary tasks of active systems studying.

In the work [64] the concept of „active system" was discussed in „*the first approximation*". It was assumed that, by main property, the distinctive special feature of the active system is its ability *to generate its own problems*. Functioning of an active system in this meaning is considered as the permanent activity, directed to a solution of its own problems. It is assumed that each active system at each moment is situated in a certain problem- *resource situation*. Any alternation of situation is the objects of „situation dynamics".

By the *subject* we understand both individual and certain group of individuals of those connected with of general problems and by consolidated resources. In the latter case active system can be subjected to decomposition on subsystems which are in „vertical" (hierarchical), or „horizontal" relations. Such systems have their own internal structure. Structural changes are caused both by a change in external factors and „spontaneous" changes, which also could be considered a distinguishing feature of active systems a designated by term „self-organizing"? In this connection active system can be considered as an object of synergetic.

We have to work out such methods of analysis and synthesis which would consider in an explicit form subjective factors, connect with an activity of a „subject" – „decision making person" accomplishing the activity, directed to realization of the accepted solutions. It's namely such a sense we have in mind speaking about „subjective analysis".

Methods, being the subject of this discussion, have to enable carrying out analysis, processing of statistical data with a obviously designating purpose and in the specific terms, the identification of such systems quantitative models, their synthesis, prediction and control.

One of tools of description and study of active systems is *the problem- resource method* [47, 64, and 69], consonant with the sufficiently detailed elaborated utility *theory* [27, 38, 53, 61, 113, 118, 119, 125, 163, 165, 171, 184, 194, 197, 198, and 199]. The analogy of simplest problem in the utility theorem [149] is a transitive binary relation of preference ρ determined on the set alternatives S_σ, quantitative measure of which is a utility function.

The utility theory arises in works of the 18th Century economists. Later as a quantitative measure of preferences a utility function begins to use. By important stake in a development of the quantitative theory of the utility became the work of Neumann and Morgenstern [125]. Subsequent development stages of theory are described in monograph [149]. We will refer to this work later on.

Subsequently productive proved to be a synthesis of the utility theory with the new direction of studies, which today is designated as „synergetic".

By the founders of synergetics are considered H. Haken and I. Prigogine [128, 132, 133, 151, 152, and 153]. Term „synergetics" belongs to H. Haken a specialist in region of quantum mechanics, theory of coherent emission, non-equilibrium phase conversion.

Studies in the field of physics of the non-equilibrium self-organizing systems led to understanding of the fact that effects of self-organizing on the macroscopic level can have an actuality in wider spectrum of realization, and the synergetics as scientific discipline go far out of the scope of physics. Effects of self-organizing in biology, economy [9, 11, 12, 22, 56, 84, 86, 99, 100, by 101, 121, 126, 134, 155], in social systems and structures were investigated. As an example from the range of biology the Belousov-Zabotinsky reaction, stage of development of a fungus and other, more complex phenomena could be mentioned. Ideas of synergetics penetrated in psychology [127, 153] and such extremely complicated region as processes of development and formation of cultures, political and civilization processes - generation, development and the loss of civilizations [30, 41, 55, 60, 84, 86, 121, 122, 133].

Synergetics cannot pretend to be on exceptional rights of description and explanation of objects mentioned above. However, it is an effective tool of studies, including quantitative, badly formalized processes and phenomena.

1.2. Notions and criteria of the problem- resource analysis

In this division basic concepts and categories of the problem - resource analysis are discussed. At first we carry out concepts of *„active system"*, *„subject"*, *„problem"*, *„purpose"*, *„resources"* and some others.

We will also discuss a connection between problem - resource analysis and other known theories and methods, including the theory of binary relations [149].

In the previous division we began a discussion about a concept of an *„active system"*. Let's continue this consideration. It is obvious that any active system is placed in a certain environment and it is assumed that there is a possibility to individualize system, i.e., to indicate its boundaries in hyperspace of factors or characteristics which are selected for describing of the system.

The studied system interacts with *„an environment"* and in this meaning it is *open*. Environment can be natural, or one including other active systems. It is understandable that in the number of characteristic of the active system a description of methods and „channels" of interaction with the environment should be included.

As it was already said, *subject* is a central component of an active system. Therefore it is natural to call analysis of active system subjective analysis. The active system is grouped „around" the subject similarly to the components of a living cell that are grouped around its nucleus. If there is no subject - there is no active system. Technogenic or natural systems are not an object of this theory. A presence *of subject* assumes a presence *of the object* of his activity. Resources (including - other active systems) are objects of activity in the majority of the cases.

Generally, functioning of the active system with a known imagination can be interpreted as conversion and displacement (translation) of resources. The conditionality of this interpretation we will discuss below.

Because of this we are forced to study category „resources", classification of resources, processes of their conversion and translation. Resources are *the object* in active system, which the activity *of subject* is inverted to.

First of all the system is characterized by a collection of possible states.

Let's designate the certain state as σ_i. Then $\sigma_i \in S_\sigma$. The state σ_i is understood as the complex characteristic - „vector", comprised with particular characteristics (qualitative or quantitative). *Possible* will be count such a state upon transfer in which system remains *„itself"* (it does not lose its individuality). It means that, first of all, *the subject* remains *the same*. Secondly the certain collection of technologies of resources conversion and translation, kept constant. Also, the certain collection of basic links with other systems remains.

It is clear that this definition is not strict and exhaustive, but an absolute strictness - „the closure" of definition here from the point of view is impossible of the author and even it can be harmful brake in the development of the theory.

What, nevertheless, does the expression: „System does remain still itself" or „subject does remain the same" indicate?

Within the framework of the subjective analysis, object of which are active systems and their functioning, we must try in any manner to define concretely the concept *of the active system*, to indicate such a *„constant"* - *invariant* and, in the same time, essential, which makes it possible to operate with this concept, to build quantitative and qualitative models, to speak about interaction of systems.

It is intuitively clear that the active system, as for as it is substantially connected with its nucleus - subject (man), group of subjects, it must reflected in its „formalized" determination the subject basic properties. Let's note that first of all we are, of course, interested in the mental properties and, to a considerably smaller degree - physiology.

Physiology can be essential only in the sense that different periods in man's life are characterized by different collections of possible states, collections of alternatives and preferences of alternatives, the fact that the physical life of subject is final and therefore, „the life" of the active system, connected with individual is final too. With each individual elementary, individual active system is connected. Each social, which consists of *M* individuals, generates *M* elementary active systems. However, within the limits of each social occur a dynamic processes of composition and decomposition of active systems, a formation and disintegration of coalitions (unions), consolidation and deconsolidation of resources, aggregation and a disaggregation of individual and groups of alternatives, a change in the type of preference relations.

So let's accept the following compromise definition, which is at the same time the assumption: active system remains itself, is individualized as long as there is „a subject" of system (physically or legally), who has the non-empty set of possible alternatives S_σ, who establishes on this set (strictly speaking – on the product $S_\sigma \; x \; S_\sigma$) preference relation $\rho : \precsim$ and have available resources (passive and active). Important from the point of view of this definition is concept of „active resources", since operating by passive resources each time requires the use (expenditures) of active resources. Thus, the exhaustion of active resources is equivalent to the curtailment of the functioning of active system. It is necessary to assume that the structure and the scales of alternatives set are connected in a defined manner with the presence of active resources.

„Tightening" of alternatives set S_σ to an empty set or an appearance of an indistinguishability of alternatives from the given set, if this state is steady one, indicates the curtailment of „external" functioning of the active system. It becomes indistinguishable from without. This state can be treated as „*entropy*" death (not physical), while striving for to zero of active resources is equivalent to „physical *death*". More accurately, apparently *physical death* is such state, when expenditures of passive resources, necessary for a renewal of the active resources of subject become infinite. This not medical (not physiological) determination of death is clearly understood. This is the definition, which makes it possible to individualize the active system within the framework of the subjective analysis.

The given reasoning's are „leading" and debatable. In this stage they give a certain right to speak about the active system as about the chosen, individualized and identified subject of a study. In this case by identifiability we imply a possibility to determine „*the boundary*" of this active system, which separates it from other active systems in „*an environment*" (Fig. 1.1).

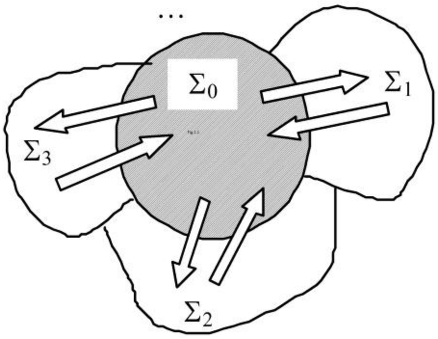

Fig. 1.1

It is understandable that, first of all we can demarcate resources, which this system manages from resources belonged to the other systems. Then we should describe connections, existing between this system and an environment. It seems that these connections are realized in the form of mutual transfer of resources (material, energy, information). In a character of information resources finances, directives, advices, recommendations, any economic, political, technical, military and other information can come out. With the transfer of resources they change „owner". That mental boundary on which this occurs can be treated as the boundary of active system. Is it possible to transmit active resources? It is seemed that „*active resources*" - this is such a specific form of resources, which cannot be transmitted, from one individual to another. These assertions can be considered with the determination of what are „*active resources*". Thus „*active resources*" are not transferred between the individuals, but with an interaction of individuals in the group „cumulative effect" of a growth of active resources, has place due to an intensive information exchange. This can occur in the process of instruction, in the command sport games and so forth.

In social or in groups the processes of self-organizing which particularly are object of synergists training occur as a result of interaction of individual „active systems".

In the work [55] as the active system is considered those capable to export entropy (in our case subjective entropy). As it will be seen in future active systems could decrease it's entropy by itself.

Let now certain part of states of set S_σ is studying by the subject from the point of view of their priority for a realization on next step. A collection of such comparable states we will designate through $S_a \subset S_\sigma$, and the corresponding states through σ_i:

$$\sigma_i \in S_a \subseteq S_\sigma.$$

It is convenient to call states σ_i in this case alternatives. The selection of set S_a is a result of subjective analysis. Thus, set S_a is the important characteristic of subject. In the concept *of alternative* can be included not only states from S_σ, but also the admissible strategies of reaching these states. Let U_σ set admissible strategies of passages in the states $\sigma_i \in S_\sigma$. In this case it is, of course, assumed that the system is in a certain initial state σ_0. The permissible set is always locked and determined each time by resource limitations.

Let $U_a \subseteq U_\sigma$ be set of strategies of those studying by subject. Let's designate the Cartesian product of sets S_a and U_a trough W_a:

$$W_a = S_a \times U_a.$$

We will write $\sigma_i \in W_a$ bearing in mind that σ_i can be both a desired state and a strategy of reaching a desired state.

In *the utility theory* it is assumed that subject forms on the set W_σ his preferences. *The utility function*, which is a quantitative measure of preferences in the majority of cases, is not calibrated and serves for determining a relative value of alternatives ,σ_i.

Subsequently for simplification of designations we will instead W_σ (or W_a) also write S_σ (or S_a) specifying each time what is an intention: state or strategy.

We must consider the important circumstance that occurs regardless of the fact, which we do examine as a system a small particular enterprise or a history of the development of the entire peoples: *any active system has a limited „sizes" in space and the final „lifetime"*.

Hence it follows in particular, that functions of preference, a distribution of preferences on S_a as well as set alternatives S_a vary with time not only as a result of a change in external (exogenous) conditions and train of selected problem solution, but also „it is spontaneous", i.e., it vary as a result the action of the original properties of psyche and physiological processes, in other words - by an action of endogenous factors.

1.2.1. Problem

Problem- resource analysis is first of all based on a concept „*of problem*".

Problem is understood as a realized nonconformity between an existing state of active system and its desired state. In other words, problem is the realized desire of subject, or the realized preference - consequence of the desire.

In this definition as minimum two states present σ_e is exist, and σ_d is desired, and also „*the carrier*" of this „realized desire" - *subject*. Therefore the problem does not exist apart from its carrier - subject. It is assumed that *the desires* of subject are distributed on a certain set S_a (or perhaps W_a,). Depending on the fact what „physically" the states σ_i are set S_a can be countable (including - finite), or continuum.

In the indicated sense the simplest problem can be interpreted as an ordered pair of the symbols:

$$P: \sigma_e \langle \sigma_d,$$

or

$$P: \sigma_e \langle \forall\sigma_d \in S_d \subset S_a.$$

In the second case each state of the subset S_d of set S_a is better (preferred) then initial state σ_e (existing state). It is possible to say, that each state from a certain subset $S_1 \subset S_a$ is better than each state from $S_2 \subset S_a$:

$$\forall\sigma_i \in S_1 \langle \forall\sigma_j \in S_2 \ (S_1 \subset S_a \text{ and } S_2 \subset S_a). \qquad (1.2)$$

A problem is attached to *the existing* state σ_e, while set of problems is attached to the set $S_a \subset S_\sigma$. Selection S_a from S_σ is a subject's prerogative and therefore, it is subjective.

The set of problems, assigned on S_a x S_a differ from the set of alternatives S_a regarding the fact that the first it is always connected with the existing (achieved at the given moment) state $\sigma_e \in S_a$, $\sigma_0 \overline{\in} S_a$. Preference relation can be strict $\rho : \langle$, or lax $\rho : \underset{\sim}{\langle}$, allowing equivalence. „Problem" we will consider a strict relation. The problem can represent desire to preserve existing state σ_e. This corresponds to the case, when from the point of view of subject all states $\sigma_i \in S_a$ ($i \neq e$) are less preferable than the state σ_e, subject himself is found in at the given moment.

Number of binary relations ρ in S_a are equal N^N . In the general case preference relation can be realized in σ - algebra A_σ of subsets S_a:

$$A\rho B \Leftrightarrow A \underset{\sim}{\rangle} B\,, \quad A, B \in A_\sigma\,. \tag{1.3}$$

As it has already been mentioned, the simplest problem can be interpreted according to the symbolism accepted in the theory of binary relations as a binary relation ρ:

$$P\!: \sigma_e \langle\, \sigma_d \Leftrightarrow \sigma_d\ \rho\ \sigma_e.$$

However, there are special features in a formulation of the basic concepts that make it possible to consider problem- resource analysis as an original theory that uses the theory of binary relations and the utility theory.

In particular special features are in the fact that active systems are an object of problem- resource analysis; in the fact that the problem of subjective and objective factors separation is solved, in the fact that, as it will be seen from the following, the sufficiently definite and clear classification of resources is conducted, and resources are also considered as a subjective category.

Problems are subdivided into *qualitative* and *the quantitative*. Qualitative are such a problem, the result of solution of which has only two values: „yes" or „no" (respectively two numerical values: 0 and 1). For example, student either passed test or didn't (if he is interested just positive mark in). Meissner subjugated Everest or didn't, Amundsen reached the South Pole or no, and so on.

If S_σ is set of the permissible states of system, S_a is set of alternatives and A_σ is σ-algebra of subsets S_σ, then it is possible to determine preferences in the form

$$S_{a1} \langle\, S_{a2};\ \ S_{a1} \in A_\sigma;\ \ \ S_{a2} \in A_\sigma.$$

1.2.2. Resources

Reasoning's in this and following paragraphs bear deterministic nature, although it is obvious that the chance essentially interferes behavior, conditions and results of an activity of active systems. The author base himself on the fact that accounting of randomness in this stage would complicate an account of basic sense and could be important only to obtain quantitative results.

Resources after the *problem* are the second most important category of problem-resource analysis.

Let's name resources any means and factors, which the subject consciously uses or will intend to use for the solution of its problems, or consider them as an expected result of the problems solution.

It would be tempting to present any *problem as the desired operation with resources: a conversion of some resources in others, a displacement (translation) of resources. A question consists in the following: is it possible to give to any problem this interpretation. A solution of any problem requires expenditures of resources. However, apparently there are problems, such that a result of solution of which is* *not new resources. For example, problems of enjoyment pleasures obtaining. Moreover, in certain cases, the solution of subject problem leads to decrease of its available resources. Indeed smoking, use of drugs reduces lifetime, i.e., decreases* „a quantity" *of available time of subject.*

In any case we can isolate a class of problems, *whose sense consists of* processing, converting *resources, or* displacement *of resources. At the same time there are problems, whose sense does not consists in obtaining* of new resources, *but of satisfaction of some important based from the point of view of the subject needs, moreover this result, whatever it is, cannot be subsequently used as the initial resources for the solution of the subsequent problems.*

A following reasoning is connected with the question distressing above. Let's assume that we deal with respect to the problem of second kind, when with the result of its solution is pleasure, enjoyment, generally, a satisfaction of personal needs of subject over a certain reasonable minimum, sufficient for maintenance and reproduction of his personal working and creative possibilities - personal potential. In this case it would be also possible to indicate, that satisfaction of such is an increased needs serves after stimulus for further activity, maintaining the will to further „navigation in the problem-resource ocean".

Because of that, we don't always identify the concept of state $\sigma_i \in S_a$ *of system with available resources.*

Examine definition of resources given above, that will be designated by symbol R. This definition connects resources with problems and with the subject. Since the problem (or alternative) is the subjective category, which exists in the consciousness of subject, than „resources" don't exist separately from the subject, and are also a subjective category. We want to say that some real objects, energy, information, labor, money become „resources", as soon as they in the consciousness of subject are connected with certain problem. For another subject they might not be resources, or to be the same, but in connection with completely different problem.

Thus, concept „resources" makes dual sense: on one side resources identify with ideas of subject about the fact that some objectively existing means and factors can be used for the solution of his problems and, from the other side - these are real means and factors (Fig. 1.2).

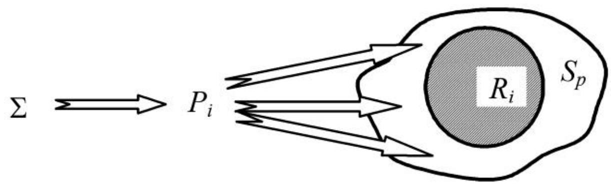

Fig. 1.2

In a Fig 1.2 we want to show that the subject at first realizes problem P_i, and then basing on the set S_p of accessible means and factors selects some which necessary for solution of the problem P_i , then calls this resources R_i. If at the point of any reason he rejects solution of the problem P_i, then at the same moment selected means and factors cease to be service lives.

„Reserves" are the variety of resources, intended for the solution of problems of the certain type in the future. Let's correlate reserves with set of states alternatives S_a. Reserves let's designate S_p (Supply).

Consequently reserves, being the variety of resources, have a dual nature as well: as a subjective category and as the actually existing means and factors. We will suppose that studying reserves and set S_a (or W_a), the subject create a distribution of preferences. In this case „reserves" are transformed in resources, correlated to concrete problems.

For further analysis a classification of resources on a number of signs should be conducted.

Passive and active resources

With the solution of each problem, corresponding resources are used: finances, material, energy, information, which is available for the subject. In order to use these resources a subject is forced to act personally, i.e., to use his intellectual, physical service lives, his own time, which is for each subject the most important form of resources. The first of the mentioned forms of resources - „external" with respect to the subject - we will call passive resources R_p. The second type of resources (intellectual, physical, time of subject himself) - „internal" (endogenous) resources of subject name active resources R_a.

It is natured to presuppose that. The use of passive resources in the course of problem solution requires from the subject with the necessity to spend of certain part of his own active resources.

Thus, if R_{p1}^{req} - required passive resources, then required active resources R_a^{req} are determined by amount of passive resources, which subject uses for solution of his problem:

$$R_a^{req} = F\left(R_{p1}^{req} \ldots\right). \quad (1.4)$$

In some tasks the relationship between "speeds" of resources conversion is used.

Let $$v_a^{req} = \frac{dR_a^{req}}{dt}; \qquad v_p^{req} = \frac{dR_{p1}^{req}}{dt},$$

Then $$v_a^{req} = f' v_{p1}^{req},$$

where: *f'* is the operator of the resources conversion.

In the simplest case: $$f = \frac{\partial f\left(R_p^{req}\right)}{\partial R_p^{req}}.$$

In its turn, a capability of subject for active work in the course of his problems solution is ensured by the presence of sufficient active resources $R_a^{disp} \geq R_a^{req}$, where

R_a^{disp} - available active resources. Their reproduction requires expenditures of certain quantity of passive resources. In other words, there is dependence

$$R_{p2}^{req} = g\left(R_a^{req},\ldots\right). \tag{1.5}$$

Equation (1.4) would be written in the more correct form

$$R_a^{req} = f\left(R_{p1}^{req} + R_{p2}^{req},\ldots\right). \tag{1.6}$$

The relations between „speeds" of a change in the resources can be examined analogously.

Relations (1.4) - (1.6) have only a symbolic sense and just explain existing connections of active and passive resources. Functions f and g are not determined and in the general case they're unknown; however, in certain cases, they take a concrete form.

The following indication of the resources classification is from the accounting of the multiplicity of their use. We will distinguish resources of single use (for example, tooth paste) and repeated use (for example, toothbrush). Subsequently of the resources of first type we will for the brevity call simply resources, and of the second type resources technologies.

Thus, **technology** - means resources of repeated usage that ensure the conversion of single use resources as the result of the problem solution. We will designate technologies through f.

By particular technology f_α^β , it is meant a technology of the type α resources conversion on the „entrance" of the problem solution process for the type β resource on the „output" from this process, i.e.:

$$f_\alpha^\beta\left(R_a\right) \to R_\beta .$$

In more general meaning we will talk about the technology of the resources conversion as the result of the problem solution: $R^{\exp}$ (in contrast to resources designating through R^{req}, R^{disp}).

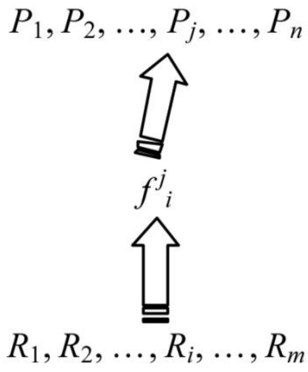

Fig. 1.3

Let there be m different types of the resources: R_1, R_2,..., R_m , and n different problems to solve: P_1, P_2,..., P_n. Figure 1.3 illustrates the concept of the elementary technology f_i^j as the means of resources conversion of *i-th* kind to the result of the *j - th* of problem solution.

Fig. 1.4 shows schematically, what is implied by the packaged technology of the problem P_j solution, or simply by technology for the problem P_j:

$$F_j = \left(f_s^j,\ldots,f_i^j,\ldots,f_r^j\right). \tag{1.7}$$

It is obvious that F_j is not the sum of elementary technologies. Generally, it is doubtful that it is always possible „*to decompose*" the packaged technology on the elementary ones. Actually, the transformation of metallic billet (R_1) into the finished part requires the expenditures of electric power (R_2) and, therefore, in this case it is difficult to decompose the technology of components production on the elementary technologies. In certain cases this succeeds in making, if we present the process of the problem solution in the form of hierarchical procedure. This means that in this case it is necessary to disaggregate problem P to the hierarchically connected sub-problems. Let us designate as F_m^n the rectangular matrix $m\times n$ of the elementary technologies:

$$F_m^n = \begin{bmatrix} f_1^1 & f_1^2 & \cdots & f_1^n \\ f_2^1 & f_2^2 & \cdots & f_2^n \\ \cdots & \cdots & \cdots & \cdots \\ f_m^1 & f_m^2 & \cdots & f_m^n \end{bmatrix}. \tag{1.8}$$

Technologies, being the resources of repeated use, possess the property of physical and moral antiquating.

The separate class of problems can be described as the problems of creation of technologies, i.e., a creation of the resources of repeated use. In the economic applications production functions are examined.

We will use subsequently term technological functions *or* technological operators *for the designation of the procedures of the conversion of resources into the results of the permission of problems solutions.*

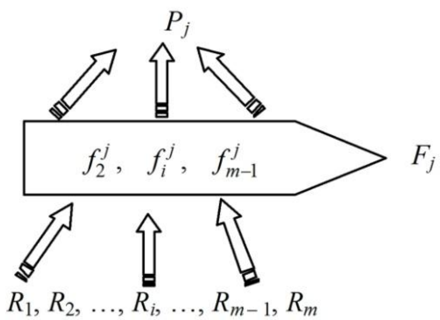

Fig. 1.4

In the practical tasks the need for defining concretely the forms of resources appears. The roughest classification is the division of resources into material R_m, energy R_e and information R_{inf}. Generally speaking, each of these forms of resources are not encountered in the pure form. They are always mixed in a certain proportion. Thus, information resources have a material carrier, and their translation requires expenditures of energy.

In some applications, in Kobb - Douglas production function, for example, labor resources R_L (or L) and capital R_C (or C), are examined. Labor resources of hired workers, who are not a subject of this active system, are conveniently placed in the category of passive resources. We will carry labor resources of the system subject to the kind of active resources.

The most important kind of resources is money which in many cases appears universal. Precious metals and other values, that have features of money, are another form of universal resources.

Discussion the theories of money are not our task. Let's make a note of only three important observations.

1. Not only money or treasures can posse's universality. In the first years of passage to market relations in the countries of the CIS, the role of money was practically brought to zero, and barter transactions, when to one extent or another the properties of money were appropriated to different goods, were widely used.

The theory of money is presented, for example, in a classical book of Harris [154]. The role of money as information resources is examined in the book of Chernavskiy [159], including analysis of some dynamic models of changes in a monetary stock.

Any form of resources can be characterized with degree of their universality. If on the set S_a (or W_a) set of problems P_a , is assigned P_a^* is the subset of $P_a : P_a^* \subset P_a$ and some kind of resources can be used for solution of the problems $P \in P_a^*$, but it cannot be used for of the problems solution P $\overline{\in}$ P_a^* i.e. - on set the $P_a \backslash P_a^*$, then the degree of universality of such resources can be determined by any criterion, which characterizes relationships of sets P_a^* and P_a. In the simplest case it can be relation N^* / N , where N^* is number of different problems is P_a^*, and N - number of problems in P_a. It seems that the utility of resources of certain kind is determined not only by their consumer properties, but also by a degree of their universality.

2. Money in the sense of the classification given above can be attributed to information resources, since they contain information about previous activity of subject and about his potential possibilities in future in the concentrated impersonal form.
3. Not all problems can be solved, having available only money. Moreover, it is possible to give an infinite number of examples, when money, are not resources, they cannot be used, moreover not only in the past, but also due to conditions of today's peace. There are set of problems of this kind appearing before commanders during the war, different kind of creative problems, solved by scientists and other.

In connection with this it is possible to refer Paul S. Bragg's statement, made on another occasion, but the well illustrating position, formulated above: „At the point of money it is possible to purchase bed, but not sleep; food, but not appetite; medicine, but not health; building, but not domestic center; book, but not mind; adornment, but not beauty; luxury, but not culture…"

As we can see, in each of combination „left side" - these are real values, acquired at the point of money, i.e., something, objectively existing out of the subject and; „right side" - constitute subjective perceptions and sensation himself and the surrounding world.

Time and space as resources

Time is the most important form of lives service. We will distinguish an *astronomical* time and an *operating* time. The astronomical times generally are not resources, and it will be used as an independent variable in the dynamic tasks. The operating time will be considered as service lives (quantity of training hours, an account of this discipline, quantity of hours of trainings of athlete, a time of flight operations of aircraft, so forth). The operating time R_t is a function of the astronomical time t:

$$R_t = R_t(t, \ldots).$$

Rate of an expense of operating time (temporal resources) (Fig. 1.5):

$$v_t = \frac{dR_t}{dt} = \begin{cases} 0; \\ 1. \end{cases}$$

Similarly we shall distinguish an astronomical space and an operating space effective area of shop, store, area for a sport game, volume of a camera of refrigerator and so forth. The operating space „is inscribed" in the astronomical space, its properties are determined not only by geometric dimensions, but also by the nature of the problem, for solution of which it is used.

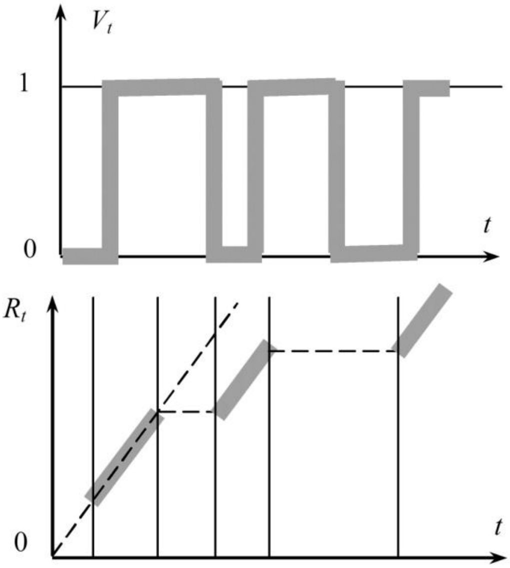

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Fig. 1.5

Available, required and expected resources

A classification of resources referred to their relation to the sense of the solved problem and sequence of actions of subject will be required for further constructions. Following concepts will be used:

1. Available resources: $R^{disp} = R^{d}$. These resources at the given moment of time are in subject's disposal, and he has a capability to manage by them according to his discretion. If subject studies set P_a^* of problems, then he leans against „*his*" available resources. If these resources are universal for entire subset P_a^*, then we will consider that one and the same quantity of resources R^{disp} places as basis of an analysis of the solvability of all problems $P(\sigma_i)$. If resources R^{disp} are not universal, then a part of them are separated for each of problems $P(\sigma_i)$ from a total amount of available resources

$$R^{disp}(\sigma_i) \subseteq R^{disp},$$

which can be used for the solution of this specific problem?

2. To each specific problem $P(\sigma_i) \in P_a$ *required resources* $R^{req}(\sigma_i)$ are confronted. The determination *of required resources* is a task of forecast and is carried out by different methods (a statistical analysis of retrospective information, a determined calculation of expenditures, an expert estimation, ...).

We further assume that a confrontation of required and available resources makes it possible to form set of alternatives S_a (or W_a). In this case it is assumed that both forms of resources allow measuring them in comparable units. Then the problem $P(\sigma_i)$, for which

$$R^{req}(\sigma_i) < R^{disp}(\sigma_i), \tag{1.9}$$

must be included by the subject in the set S_a (or W_a), which in the theory of control is called the attainable set. Subsequently we will talk about an interpretation of properties of attainability and controllability in connection with problem- resource situations again.

3. Expected resources $R^{exp} = R^{e}$ are those, which the subject expects to obtain as a result of the problem solutions.

If a problem consists of a conversion of resources then, as it seems to us, a subject must exclude an examination of a problems, for which a condition $R^{exp}(\sigma_i) \leq R^{req}(\sigma_i)$ is fulfilled and to examine such alternatives, for which

$$R^{exp}(\sigma_i) > R^{req}(\sigma_i). \tag{1.10}$$

Thus, the conditions for the including of a problem in the set P_a (alternatives - in the set S_a or W_a) are inequalities (1.9) and (1.10).

It is natural from the point of view of a comparison of the resources R^{disp}, R^{req} and R^{exp} to divide possible problems in two classes:

- the one of problems, which compose the set P_a^{α} in the case, if only first inequality (1.9) is fulfilled;
- the other of problems, which compose the set P_a^{β} if they carry out both inequalities (1.9) and (1.10). Set P_a^{α} is wider than set P_a^{β}:

$$P_a^{\beta} \subseteq P_a^{\alpha}.$$

In the set P_a^{α} such problems are contained, the result of solution of which is not new resources $R^{exp}(\sigma)$, but satisfaction of individual needs, when, let's say, it cannot be considered that active resources of subject adhere.

When the set of problems are not limited with inequalities (1.9) and (1.10) another view on the problem is possible, and it covers all accessible to an attention of the subject hypothetically possible states (alternative), which in principle can be realized, may be with a very small probability.

If we take a probabilistic point of view during an estimation of a reliability of alternatives, then set of alternative- possibilities studied by the subject would be substantially enlarged. Accordingly, the set of problems would be enlarged, if problem was formulated as follows: *„I am at the present time in state σ_a, but state σ_i, pleases me more*, *and in this case I don't think if, my resources will be sufficient, in order to reach state* σ_i, but *theoretically there is a possibility that under specific conditions in future, such resources can appear*".

The set of such problems $P_a^{(c)}$ is wider than the set of problems P_a, limited by inequalities (1.9) and (1.10):

$$P_a \subseteq P_a^{(c)}.$$

The set $P_a^{(c)}$ can reflect a wide range of a priori preferences of a subject, his tastes, from results of training, cultural level, ethnic, religious, political preferences.

A determination of a real resource basis of preferences separates pragmatic set of problems P_a and makes it possible „to include" quantitative methods for analysis of problem- resource situations.

1.2.3. Purposes

The next major category of problem-resource analysis is the category of „purpose". [105]. This category, in a first approximation is described in [64], where an appropriate definition of purpose, corresponding to what is understood in this case is given. We will not repeat the reasoning, given in already [64], just note the fundamental positions from our point of view.

The purpose is the intention, the decision to act in accordance with one of the existing alternatives in the problem-resource situation.

As you can see, the purpose - is a subjective category. The purpose should have a carrier - a subject. Target selection often is being outsourced to machine (rocket selects target, computer plays chess with world champion and wins, etc.) However, it is clear that the algorithm for selecting targets and to meet the criteria is transferred to the machine, the primary carrier of which is the subject.

In logical and temporary sense the *purpose* is a subordinated category. It can appear only since the *problem* is realized. Certainly, it could be imagine a certain chain of problems and purposes, when one problem, for example, P_0 generates the purpose A_0, in the process of motion to the purpose A_0 appear new intermediate problems P_1, P_2, ..., P_n and, correspondingly, new purposes A_1, A_2, ... However, in each individual case a „*problem*" is considered first, and the purpose - second.

The set of problems P_a would be possible to name *set of potential purposes*, and the selected problem for realization - *urgent purpose*. This, however, is a question of terminological agreement, but not a clue of the matter.

The selection of purpose is achieved as a result of analysis *of problem-resource situation*, i.e., the set of alternatives S_a (or W_a), available and required resources $R^{disp}(\sigma_i)$, $R^{req}(\sigma_i)$, and also expected new resources $R^{exp}(\sigma_i)$, and the distribution of preferences on S_a (W_a) or their Cartesian products.

The more detailed determination of the problem-resource situation within the framework of subjective analysis is proposed at p.1.3.

Purpose is represented as „an *operator*", the starting mechanism of reaching a selected state σ_i - conversion of resources.

The scheme proposed here puts problem in the first place, and assign to purpose the subordinate place.

1.3. Elements of theory of individual utility

1.3.1. Binary relations, ordering

It was already said, that the utility theory can be used as the basis, while studying the active systems. The simplest problem is defined as preference relation on the set of alternatives. The utility theory gives the model of the preferences distribution forming. The utility problem- resource analysis is not the precise copy of the theory. The latter, is built within the framework of ordinal approach; while the subjective analysis of problem- resource situations does not exclude cardinal approach, calculation of ethical factors and others.

It is known that early marginal's (followers of the margined utility theory) considered that subjective tastes and preferences on set specific collections of goods is determined *by the cardinal* utility, which can take any numerical values from the given set. This function is considered as the cardinal measure of the satisfactoriness of user. It is implied that the user can assign to this measure arbitrary numerical values and, thus, determine his preferences [199].

At the point of the change *to cardinalism, the* alien another point of view, was taken. It is from the persuasion, that the consumer at best, evaluating the utility of the different collections of goods, can determine their order, i.e., speaking in modern language, to establish binary relation ρ: 〉, 〈, ~.

The corresponding direction was called *ordinalism*, and the preferences distribution - ordinal distribution. One of the arguments in favor of *ordinalism* is the absence of the reliable means of measurement and, which especially important, prediction of subjective preferences. It is supposed that each user can say: „This collection of goods, pleases to me more (less, equivalent) than another collection of goods". However, passage to the *ordinalism* gave birth to the large number of logical and mathematical difficulties, especially, when it goes on about collective preferences. The getting over these difficulties is frequently represented in the form „the theorems about impossibility". There are reasons for the criticism of *ordinals*, as well as, *cardinalism*, but from the different positions.

The theory, which as particular cases contains both cardinal and ordinal versions, is preferable.

The postulation *of the variational principle*, in correspondence with which the preferences are formed and, therefore, - the property of the optimality of the corresponding mental processes (chapter 3) is essential in the present work.

Variational principle, and its modifications, leads to the so-called *canonical distributions of the preferences*, which depend on such quantitative characteristics as resources of different types, the quantitatively determined utility and harmfulness.

The canonical distributions give the analyst the promising and flexible apparatus of quantitative analysis. A deficiency in the experimental data is completed by the postulation of qualitative principle. The concepts *of subjective entropy* and *subjective information* that play exceptional role are introduced.

Thus, return to the *cardinal* position on qualitatively new level gives a number of advantages. Preferences are subdivided as the rationalistic, connected with the utilitarian interests (utility, harmfulness, resources,...) and irrationals (stable imperatives, ethical, religious, political,...). In this case the assumption is done that if the rationalistic preferences, are object of optimization („in the depths of psyche") on the basis of variational principle mentioned above, then stable imperatives are assigned by a -

priori and cannot be the object of the selection, including of optimum. They are the result of past accumulated experience.

One of the arguments in favor of the use of cardinalism in the contemporal conditions is the presence of powerful „assistant" for decision making, such as computer technologies are - different systems of decision making support, in principle capable with no matter how small „step" of discrediting alternatives with respect to the characteristics of their effectiveness.

Truly immense literature is dedicated to the theory of utility. The axiomatic method of this theory constructing adapts by Ramsay, Neumann and Morgenshteyn, by Sevitsh, Debra at al. We give the very selective and to a considerable extent random list of some works in this region. For future reference it will be useful to give the brief enumeration of basic determinations and assertions. We will use for this purpose the summary-type reporting following the book Fishbourn [149] and work [27].

The theory of utility is from the mathematical theory of binary relations, as a basis of which the concept of the preferences lies and which in turn rests on the more general theory of categories [96].

There are, at least, two variations of the utility theory. In the first version probabilistic ideas about the utility are not used, in the second the uncertainty, expressed through the probability, is considered. The corresponding theory bears the name of the expected utility theory.

Subsequently we will not repeat references each time. Retreats from the mentioned sources will be specified. In the account we omit the proofs of theorems, and partially change designations, taking into account the use of various symbols in this book for other purposes. In particular binary relation is designated by letter ρ, since through R everywhere subsequently we will designate resources. Set of alternatives we will designate through S_a, alternatives - through σ, φ, ξ,... or σ_i, σ_j, σ_k,...

Arbitrary binary relation is designated through ρ, and the relation of ordering by symbol „$\langle$". Symbol „$\Rightarrow$" indicates „it draws". Symbol „$\Leftrightarrow$" indicates „then and only then".

Relation $\sigma \langle \eta$ is called weak relative ordering, if

$$\sigma \langle \eta \Leftrightarrow U(\sigma) < U(\eta), \tag{1.11}$$

where U (.) - the function of utility.
Relation $\sigma \langle \eta$ is strict partial ordering if

$$\sigma \langle \eta \Rightarrow U(\sigma) < U(\eta). \tag{1.12}$$

Let's enumerate properties, which the binary relations can possess (symbol «$\forall$» is read „for all", symbol «$\exists$» indicates „there exists").

1. Relation ρ is reflexive, if $\sigma\rho\sigma$ for $\forall\sigma \in S_a$.
2. Relation ρ is non-reflexive, if $\sigma\bar{\rho}\sigma$ for $\forall\sigma \in S_a$.
3. Relation ρ is symmetrical, if $\sigma\rho\eta \Rightarrow \eta\rho\sigma$ for $\forall\sigma, \eta \in S$.

For example, if „σ is a brother η", then „η is a brother σ ".

4. Relation ρ is asymmetric, if $\sigma\rho\eta \Rightarrow \eta\bar{\rho}\sigma$ for $\forall\sigma,\eta\in S_a$.

Here „ $\bar{\rho}$ " indicates „not to be available in this sense".

In this case, for example, if „I prefer η in comparison with σ, then σ is not preferable with respect to η":

$$\sigma \langle \eta \Rightarrow \eta \bar{\langle} \sigma . \tag{1.13}$$

5. Relation ρ is antisymmetric, if $(\sigma\rho\eta, \eta\rho\sigma) \Rightarrow \sigma = \eta$ for $\forall\sigma, \eta \in S$.
6. Relation ρ is transitive, if $(\sigma\rho\eta, \eta\rho\xi) \Rightarrow \sigma\rho\xi$ for $\forall\sigma, \eta, \xi \in S_a$.
7. Relation ρ is negatively transitive, if $\left(\eta\bar{\rho}\sigma, \eta\rho\xi\right) \Rightarrow \sigma\rho\xi$ for $\forall\sigma, \eta, \xi \in S_a$.
8. Relation ρ is called connected or complete, if for $\forall(\sigma, \eta) \in S_a$ occurs either $\eta\rho\sigma$ or $\sigma\rho\eta$.
9. Relation ρ is called weakly connected, if for $\forall\sigma, \eta \in S_a$ and $\sigma \neq \eta$ occurs either $\eta\rho\sigma$ or $\sigma\rho\eta$.

Binary relation ρ introduces in S_a *the weak ordering*, when it is asymmetric and it is negatively transitive.

Relation ρ introduces is S_a *a strict ordering*, $\Leftrightarrow$ if it is weakly connected (see p.9) and weakly ordered.

Relation ρ is relation of equivalence (~) $\Leftrightarrow$, if it *is reflexive* (p.1), *transitive* (p.6) and *symmetrical* (p.3).

Relation of equivalence $\rho = \sim$ assigns the partition of set S on the classes of equivalence $S(\sigma)$. Symbol „~" is defined as the relation of indifference. Class $S(\sigma)$ catches by element σ, $\forall\sigma \in S_a$, has its class of equivalence, an in particular can be consisting of one element - very σ, the set of all classes of equivalence we'll designate through $S_\sim$.

The following table in compact form characterizes forms of ordering described above.

<u>ρ introduces in S_a weak ordering $\Leftrightarrow$ when it</u>

1) is asymmetric: $\sigma\rho\eta \Rightarrow \eta\bar{\rho}\sigma$ for $\forall(\sigma_1\eta) \in S_a$;

2) is negatively transitive: $\left(\eta\bar{\rho}\sigma,\eta\rho\xi\right) \Rightarrow \sigma\rho\xi$.

<u>ρ introduces in S_a a strict ordering $\Leftrightarrow$ when</u>

1) between the elements S_a is a weak connection: for $\forall\eta, \sigma\in S_a, \eta\neq\sigma \Rightarrow \sigma\rho\eta$ or $\eta\rho\sigma$.

2) occurs weak ordering $\Rightarrow$

$$\Rightarrow \begin{cases} \text{asymmetry;} \\ \text{negative transivity.} \end{cases}$$

<u>ρ is equivalence relation (~) $\Leftrightarrow$ when it</u>

1) is reflexive: $\eta\rho\eta$, $\forall\eta \in S_a$;
2) is symmetrical: $\eta\rho\sigma$, $\forall\sigma, \eta \in S_a$;
3) is transitive: $(\eta\rho\sigma, \xi\rho\eta) \Rightarrow \xi\rho\sigma$; $\forall\sigma, \eta, \xi \in S_a$.

<u>ρ introduces in S_a a strict partial ordering $\Leftrightarrow$ it</u>

1) is non-reflexive: $\sigma\bar{\rho}\sigma$, $\forall\sigma \in S_a$;

2) is transitive.

The example of negative tranistivity are the relation, installed by assertion „who to us it not enemy, then he is our friend". The relation of seniority is an example of asymmetric relation. If $\eta\rho\sigma$ means that „σ is a son", and „η is the father", then must be $\eta\bar{\rho}\sigma$ („η is not son for σ ").

From the set of all possible binary relations preference relation will interest us, for which the symbol $\langle$ is used:

$$\rho = \langle .$$

Below we will study the properties of the binary preferences relation. Let's give some theorems and definitions.

Theorem 1

Let $\langle$ - weak ordering on S_a, then:
1) for $\forall\sigma, \eta \in S_a$ occurs at least one of the relationships:

$$\eta \ \langle \ \sigma; \sigma \ \langle \ \eta; \sigma \sim \eta;$$

2) relation ~ is equivalence, i.e., reflexive, symmetrical and transitive;
3) relation $\langle$ is transitive and connected;
4) from $(\sigma \langle \eta, \eta \sim \xi) \Rightarrow \sigma \langle \xi$;
5) let $S_\sim$ is set of the classes of equivalence on S_a, and $\langle_\sim$ - preference relation on the set of classes $S_\sim$. Relation $\langle_\sim$ is determined by the condition:

$$S_\sim(\sigma) \langle_\sim S_\sim(\eta) \Leftrightarrow$$

when can be found elements $\sigma \in S_\sim(\sigma)$ and $\eta \in S_\sim(\eta)$ such, that $\sigma \langle \eta$. Then relation $\langle_\sim$ on $S_\sim$ is a strict ordering.

Theorem 2

If relation $\langle$ on S_a is weak ordering, $S_\sim$ is countable set, *then there is a real function U*, S_a, for which:

$$\sigma \langle \eta \Leftrightarrow U(\sigma) < U(\eta); \forall\sigma, \eta \in S_a.$$

Relation $\sigma \approx \eta$ is introduced, which is carried out when and only when condition ($\sigma \sim \xi$ $\eta \Leftrightarrow \sim \xi$) are satisfied for $\forall\xi \in S_a$. Here equivalence is determined by the comparison of elements σ and η with the third element ξ, which must exist at S_a.

Theorem 3

If $\langle$ on S_a is strict partial ordering, then
1) for $\forall\sigma, \eta \in S_a$ one of the relations $\sigma \langle \eta$; $\eta \langle \sigma$; $\sigma \approx \eta$ $(\sigma \sim \eta \wedge \overline{\eta \approx \sigma})$ is fulfilled;
2) relation $\approx$ is relation of equivalence;
3) $\sigma \approx \eta \Leftrightarrow (\sigma \langle \xi \Leftrightarrow \eta \langle \xi$ and $\xi \langle \sigma \Leftrightarrow \xi \langle \eta)$ for $\forall\xi \in S_a$;

4) ($\sigma \langle \eta$; $\eta \approx \xi$) $\Rightarrow \sigma \langle \xi$ and ($\sigma \approx \eta$; $\eta \langle \xi$) $\Rightarrow \sigma \langle \xi$;
5) let $\langle^*$ - preference relation on the set of the classes of equivalence $S_{\approx}$ (where $\approx$ is relation of equivalence, defined above) such, that for $\forall S_1$, $S_2 \subset S_{\approx}$, $S_1 \langle^* S_2 \Leftrightarrow$will be found such elements $\sigma \in S_1$ and $\eta \in S_2$, that $\sigma \langle \eta$. In this case $\langle^*$ is strict partial ordering on $S_{\approx}$.

In the utility theory essential role plays Zorn's lemma:

Let ρ is strict partial ordering on S_a and for $\forall Z \subset S_a$, on which ρ - strict ordering, there is an element $\sigma \in S_a$ such, that for $\forall \xi \in Z$ the condition $\xi\rho\sigma$ or $\sigma = \xi$ is satisfied. Then such element $\eta^* \in S_a$, for which $\eta^*\rho\xi$ doesn't carry out for any $\xi \in S_a$ can be found.

1.3.2. Determinate utility

The following theorem introduces the function of utility $U(\sigma)$ if relation $\langle$ on S_a is strict partial ordering, and the set of the classes of equivalence $S_{\approx}$ is countable . Then, according to theorem, there is a real function U on S_a such, that for $\forall \sigma, \eta \in S_a$

$$\sigma \langle \eta \Rightarrow U(\sigma) < U(\eta); \tag{1.14}$$

$$\sigma \approx \eta \Rightarrow U(\sigma) = U(\eta). \tag{1.15}$$

In connection with the consideration of preference relation the question about the distingishability of alternatives naturally arises.

Theorem 2 bases existence of the utility function, when relation $\langle$ makes an weak ordering on S_a. The following theorem makes it possible to conduct the utility function in the case, when relation $\langle$ corresponds to a strict partial ordering on S_a.

Theorem 4

If relation $\langle$ is strict partial ordering on S_a, set of subsets of equivalence $S_{\approx}$ are countable, then there is a real function $U(\sigma)$, assigned on S_a such, that for $\forall \sigma, \eta \in S_a$ (1.14) and (1.15) are carried out. The important expansion of the theory given above is the case of interval ordering. The regulated indifference intervals do not coincide with the classes of equivalence and reflect the properties of the psyche of subject that for him there are zones of the indistinguishability of alternatives.

Let's examine the additional conditions, which introduce alternative ordering, but in this case they preserve the property of the intransitivity of the relation of indifference (~):

$$\left(\eta \sim \sigma; \xi \sim \eta\right), \text{but } \overline{\sigma \sim \xi}.$$

Let's add to the conditions of theorem 3 following conditions:

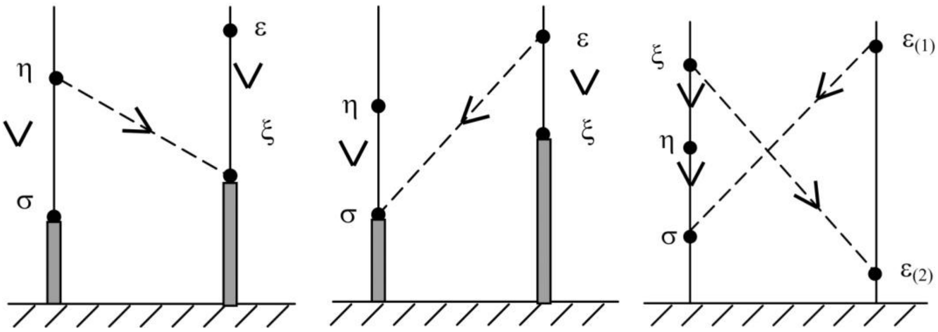
$$\left(\sigma \langle \eta; \xi \langle \varepsilon\right) \Rightarrow \left(\sigma \langle \varepsilon \text{ or } \xi \langle \eta\right) \text{ for } \forall \sigma, \eta, \xi, \varepsilon \in S_{\sigma}; \tag{1.16}$$

$$(\sigma \langle \eta; \eta \langle \xi) \Rightarrow (\sigma \langle \varepsilon \text{ or } \varepsilon \langle \xi) \text{ for } \forall \sigma, \eta, \xi, \varepsilon \in S_\sigma; \quad (1.17)$$

The sense of these conditions can be illustrated graphically if we assume that elements σ, η, ξ, ε - real numbers (Fig 1.6).

It is evident that, if relation 〈 is non-reflexive and, furthermore, two conditions, recorded above are satisfied, then relation 〈 is transitive.

Relation 〈 is called *interval ordering*; if it is nonreflexive and possesses property (1.16). But if additionally property (1.17), is carried out, then it is called *interval semi-ordering*.



a b c

Fig. 1.6

If relation 〈 on S_a is interval ordering, and $S_\approx$ is countable, then there are real functions $U(\sigma)$ and $\delta(\sigma)$, assigned on S_a such, that $\delta(\sigma) > 0$ for $\forall\sigma \in S_a$, and

$$\sigma \langle \eta \Leftrightarrow U(\sigma) + \delta(\sigma) < U(\eta) \text{ for } \forall \sigma_1 \eta \in S_a. \quad (1.18)$$

This theorem introduces *function of uncertainty* δ(σ), therefore the retention of the intransitivity of the indifference relation «~» is possible. Indifference interval:

$$I(\sigma) = [U(\sigma), U(\sigma) + \delta(s)]. \quad (1.19)$$

The indifference interval $I(\sigma)$ lies entirely to the left of the interval $I(\eta)$ if, and only if σ 〈 η.

If the intervals $I(\sigma)$ and $I(\eta)$ intersect, correspondent elements $\sigma, \eta \in S_a$ are located with respect to indifference.

The following theorem establishes indifference intervals for the case of semi-ordering on S_a.

Theorem 6

Let 〈 is semi-ordering on S_a and set of subsets of equivalence $S_\approx$ is finite.

Then on S_a *there* is a real function $U(\sigma)$ such, that

$$\sigma \langle \eta \Leftrightarrow U(\sigma) + 1 < U(h) \text{ for } \forall \sigma, \eta \in S_a. \quad (1.20)$$

Instead of one there can be any finite number. As it is evident in this case indifference intervals can have identical length.

The theory of utility on the non-countable sets is based on the concept of the density of alternatives set relatively ordered.

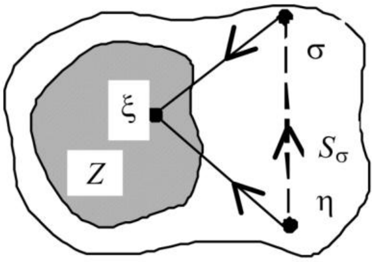


Fig. 1.7

Definition

Let ρ is the binary relation on S_a, moreover S_a - non-countable set. Set $Z \subseteq S_a$ is called ρ - dense in S_a, if $\forall\sigma,\ \eta \in S_a$, not belonging at the same time to Z: ($\sigma \,\overline{\in}\, Z; \eta \,\overline{\in}\, Z$) and for which σρη such $\xi \in Z$, that (σρ ξ, ξρη) can be found (Fig.1.7). Assume that the relation $\langle_{\sim}$ is determined on the set of the classes of equivalence $S_{\sim}$.

Theorem 7

There exist such real function U(σ) on S_a, that the equivalence $\sigma \langle \eta \Leftrightarrow U(\sigma) < < U(\eta)$ for $\forall\sigma,\ \eta \in S_a$ occurs then and only then, when preference ⟨ on S_a introduces weak ordering and set $S_{\sim}$ separable relative to the ordering $\langle_{\sim}$.

Set is called separable, if there is everywhere dense countable set in it [149].

Set *A* is called everywhere dense, in the set *B* if $\bar{A}$ coincides with the set *B*.

If preference relation ⟨ is introduces on S_a a strict partial ordering, then the following theorem occurs.

Theorem 8

Let ⟨ on S_a is strict partial ordering, and $\langle_{\approx}$ separable subset of the set of the equivalence classes $S_{\approx}$ exists.

Then there is an assigned on S_a real function *U*(σ) such, that

$$\left.\begin{array}{l}\sigma \langle \eta \Rightarrow U(\sigma) < U(\eta) \\ \sigma \approx \eta \Rightarrow U(\sigma) = U(\eta)\end{array}\right\} \forall \sigma, \eta \in S_a .$$

The condition of separability is here sufficient, but not necessary.

Existence of the utility function *U*(σ) in the case of the increasing preferences and weak ordering is established by the following theorem.

Theorem 9

Let S_a is „rectangle", the Cartesian product of the sets:

$$S_a = S_{a1} \times S_{a2} \times \ldots \quad \times S_{an}$$

in the space R^n, quantitative sense to symbols σ, η,… is assigned and for ∀σ ∈ S_a the conditions are satisfied:
1. Relation 〈 on S_a is weak ordering.
2. σ < η ⇒ σ 〈 η.
3. (σ 〈 η; η 〈 ξ) ⇒ there are such α, β ∈ (0, 1), that ασ + (1 – α)ξ 〈 η; η 〈 βσ + (1 – β)ξ.
Then there is a real function *U*(σ) on S_a, which satisfies the condition:

$$\sigma \langle \eta \Leftrightarrow U(\sigma) < U(\eta) \text{ for } \forall \sigma, \eta \in S_a.$$

Condition 2 is called the condition of monotonicity or non-saturation and means that the preferences grow with any increase in the quantity. Condition 3 is called Archimedes condition and is used while establishing of separable subset relative to ordering 〈 is made.

It follows from the previous theorem that, if σ, η, ξ ∈ S_a and σ < η < ξ, then there is exactly one such α ∈ (1, 0), that

$$\eta \sim \alpha\sigma + (1-\alpha)\xi .$$

In the case of the non-decreasing preferences and strict partial ordering the existence of function U(σ) is established by the following theorem.

Theorem 10

Let S_a is non-negative octant in R^n and on S_a the following conditions are satisfied:
1. Relation 〈 on S_a - strict partial ordering.
2. [(σ < η; η 〈 ξ) or (σ 〈 η; η < ξ)] ⇒ σ 〈 ξ.
3. σ 〈 η ⇒ ξ < η for ∀ξ , that σ < ξ.
Then there is a real function *U*(σ) on S_a, which satisfies the condition:

$$\sigma \langle \eta \Rightarrow U(\sigma) < U(\sigma) \text{ for } \forall \sigma, \eta \in S_a.$$

With these conditions $\sigma < \eta \Rightarrow \eta \langle \sigma$, which indicates, that an increase in the quantity does not decrease preference.

In [149] lexicographical orderings with respect to the preferences are examined when density condition is disrupted, and in the case of the strictly partial ordering the separability is sufficient, but not necessary for existence of real function of utility.

Additive utilities on the finite sets are determined by the relation:

$$\sigma \langle \eta \Leftrightarrow U_1(\sigma_1) \; + \ldots \quad + U_n(\sigma_n) < U_1(\eta_1) \; + \ldots + U_n(\eta_n) , \tag{1.21}$$

where each alternative $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ is the „vector" - the element of the subset of the Cartesian product:

$$S_a = \prod S_{ai} = S_{a1} \times S_{a2} \times \ldots \quad \times S_{an} \ .$$

The previous relation assumes the absence of correlation of the factors - components of vector σ.
Preferences on the Cartesian degrees of the sets are examined:

$$S_a^{(n)} = \underbrace{S_a \times S_a \times S_a \times \ldots \times S_a}_{n},$$

where n – is the number of time periods.

The introduction of time factor makes it possible to examine the dynamic processes of a change of the utilities with time. The concepts formulated in the temporal aspect are studied: „persistence", „impatience", „discounting". Investigation in this direction belongs to Kushlen, Diamond, Williamson and others [149].

1.3.3. Expected utility

The important division of the utility theory relates to the so-called *expected utilities*, connected with the orderings on the sets of probability measures. The expected utilities were studied by Neumann and Morgenstern and, somewhat previously by Ramsay. Further modifications of theory belong to Friedman, Sewich, Kersten, Milnor , Cramer, Reiff, Blackwell, Girschik, Lewis.
Bibliographical references to the work of the authors mentioned above can be found in Fishburn [149].
Let's examine briefly some concepts of the theory *of the expected utilities*.
Simple probability measure P on the set X is assigned by conditions:
1. $P(A) \geq 0$ for $\forall A \subseteq X$. (1.22)
2. $P(X) = 1$.
3. $P(A \cup B) = P(A) + P(B)$, if A, $B \subseteq X$ and $A \cap B = \varnothing$.
4. $P(\varnothing) = 0$.
The mathematical expectation of function f for the countable or final set X is determined by the formula

$$E(f,P) = \sum_{x \in X} f(x) P(x), \tag{1.23}$$

where: $x \in X$ is elements of the set X. Sum is apply on all elements of set and the following theorem occurs.

Theorem 11

If P is simple probability measure, then $P(x) = 0$ for $\forall x_i \in X$, besides of the finite number of values x_i and

$$P(A) = \sum_{x_i \in A} P(x_i) \text{ for } \forall A \subseteq X \,. \tag{1.24}$$

If measure is convex, then the condition:

$$E\left[f, \alpha P + (1-\alpha) Q\right] = \alpha E(f, P) + (1-a) E(f, Q), \tag{1.25}$$

is satisfied;
where $\alpha \in (0, 1)$, P, Q are simple probability measures.

Theorem 12

Let P_a is the set of all simple probability measures on S_a and $\langle$ is the binary relation of preference on set P_σ. Them for existing a real function $U(\sigma)$ on S_a and feasibility of condition:

$$P \langle Q \Leftrightarrow E(U, P) < E(U, Q) \text{ for } \forall P, Q \in P_a \tag{1.26}$$

it is necessary and sufficient, that for $\forall P, Q, R \in P_a$:
1. relation $\langle$ was weak ordering on P_a.
2. $(P \langle Q, 0 < \alpha < 1) \Rightarrow \alpha P + (1-\alpha)R \langle \alpha Q + (1-\alpha)R$.
3. $(P \langle Q, Q \langle R) \Rightarrow (\alpha P + (1-\alpha)R \langle Q$ and $Q \langle \beta P + (1-\beta)R)$ for some $\alpha, \beta \in [0, 1]$.

Positive function $U(\sigma)$, assigned on S_a, is called the utility function and is unique with an accuracy to the positive linear transformation. This means that the function
$V(\sigma) = a\, U(\sigma) + b$, where $a > 0$, satisfies the condition:

$$P \langle Q \Leftrightarrow E(v, P) < E(v, Q) \text{ for } \forall P, Q \in P_a, \forall \sigma \in S_a. \tag{1.27}$$

In the case of the additive expected utility, if P is a collection of measures P_i and Q is collection of measures $Q_i\left(i \in \overline{1, n}\right)$, the condition is satisfied

$$P \langle Q \Leftrightarrow \sum_{i=1}^{n} E(U_i, P_i) < \sum_{i=1}^{n} E(U_i, Q_i), \tag{1.28}$$

where: $U = (U_1, U_2, \ldots, U_n)$.

The interdependent expectations are studied. Important development is the theory of expected utility.

The survey of some facts of the utility theory, including a number of the theorems, which base existence of the utility functions, was given above following [149].

1.4. Further analysis of the fundamental notions.

1.4.1. Problem- resource situation

The brief account of the theory of binary relations and elements of the utility theory and also some concepts of problem- resource approach given above makes it possible to carry out further concrete definition of concept „active system", defining its distinctive properties.

1. Active system can exists being isolated, and to evolve thus far the available resources will not be exhausted; however, it tends to the interaction with „the environment", organizes this interaction, creating the flows of substance, energy, information by forming different gradients.
2. Assigned external effect on system causes the ambiguous reaction (response) of system. This most likely is manifested in the fact that

a) the distribution of preferences on the set of alternatives S_a, formed with system on this set changes;

b) set of alternatives S_a, changes, new alternatives appear, either the alternatives existed previously disappear or on a and another occurs at the same time.

3. Interacting with „the environment", system itself selects strategy. This strategy is not correlated unambiguously with the state of environment, and contains „remnant's" or spontaneous component.
4. System has „its" individual („build it") criterion of the optimality (better to say „rationality" or „effectiveness") - „sewn" in the consciousness of subject - in „the central control of a system". Most frequently this criterion bears the nature of Paretto criterion, when the solution is selected based on the unimprovable set of the values of vector criterion.
5. Any external action system converts as a certain set of „*its*" own problems. But most important is the fact that the active system „spontaneously" generates „*its*" problems. Some problems are solute, part is not.

Active system manifests effusiveness on a set of the admissible states SA, and forms set of alternatives S_a by itself. Thus, the required and formal attributes of active system are:

- the set of principally possible (admissible) states *SA* is the objective characteristic of system (for example, man cannot exists at a temperature of the body 45° C); for each realized state σ_0 at the given instant *t* - set of the attainable states $S_{att}|\sigma_0$ and set of alternatives $S_a|\sigma_0$. This latter is subjective characteristic of active system. $S_a|\sigma_0$. can coincide with $S_{att}|\sigma_0$;
- the presence on the set $S_a|\sigma_0$ preference relation ρ;
- the presence of set of problems $P_a|\sigma_0$ in each initial state.

All problems are divided as those solvable and insoluble. We will assume for the purpose of simplification in the theory that the insoluble problems are rejected immediately at first stages of analysis and the selection of the most preferable problem is achieved among the solvable problems.

The characterization of initial „state" σ_0 includes the available (available) resources R_0^{disp}, the required resources $R^{req}\left(\sigma_i\right)=R_i^{req}$ and the expected effect, i.e., the new resources as a result of the solution of the problem $R^{exp}\left(\sigma_i\right)=R_i^{exp}$. Alternative assumes the forthcoming action: passage $\sigma_0 \rightarrow \sigma_i$.

„Event" s is characterized as a „state" plus moment of time t: s: = (σ_0, t). „Situation" is characterized as an „event" plus set of alternatives $S_a|\sigma_0$. In the characterization of situation the resources R^{disp}, R^{req}, R^{exp} are included. Into the number of resources the available time t^{disp} and the required time t^{req} have to be included. In our case we will speak about „problem-resource situation"

$$m := \left[\sigma_0, t, S_a \mid \sigma_0, \rho, R^{disp}, R^{req} (S_a \mid \sigma_0)\right],$$

where R^{req} is set of the values of required resources on $S_a|\sigma_0$, ρ preference relation.

The subject of on active system is situated continuously in a problem- resource situation (PRS). Alternation of PRS is defined as a „situation dynamics" (SD).

Set of problems $P_a|\sigma_0$ is introduced as correspondence $(\sigma_0 \rightarrow \sigma_i)\,\tilde{\rho}\,(R_i|\sigma_0)$ between the alternatives and the resources. Here by resources vectors $R_i|\sigma_0$: = $(R^{disp}, R^{req}(\sigma_i), R^{exp}(\sigma_i))$ are implied. If alternative makes purely economic sense, then the determination „directed" preference is connected with the comparison of the expenditures $R^{req}(\sigma_i)$ and the expected effect $R^{exp}(\sigma_i)$.

Problem $P_i|\sigma_0$ is considered solvable, if the conditions $R^{req}(\sigma_i) < R_0^{disp}$ is satisfied and insoluble, if $R^{req}(\sigma_i) \geq R^{disp}$ are satisfied. Among the solvable problems the part of the problems are effective. These are such problems, for which the inequality $R^{req}(\sigma_i) < R^{exp}(\sigma_i)$ is fulfilled.

For the ineffective problems $R^{req}(\sigma_i) \geq R^{exp}(\sigma_i)$.

1.4.2. Situational dynamics

By the „situational dynamics" from one side we understood the process of changing the problem-resource situation, from another side - the mathematical description of this process.

All elements of system undergo chronological variation. The realized state of system changes together with time, the set of alternatives $S_a|\sigma_0$, preference relation $\rho(S_a|\sigma_0)$, the available resources $R^{disp}(\sigma_0)$, and also the required resources, assigned for the change of alternatives $\sigma_i \in S_a|\sigma_0$. Let's note that as „the limitations" in the course of alternatives selection can come out not only the integral available resources, but also the maximum speeds of their use (expense, conversion).

To describe dynamics PRS m — it means to determine the system of the mathematical relationships, which describe chronological variation of all elements, which are contained in the formal description of situation.

„Being moved" in the time, system changes its actual state („point of view") σ_0. Simultaneously in connection with a change of „point of view" and a change in the available resources $R^{disp}(\sigma_0)$, modification of the set of alternatives $S_a|\sigma_0$ occurs. In view of the action of internal and external factors (for example, change in the economic situation, laws, prices and others) a change in the required resources $R^{req}(\sigma_i)$, $\sigma_i \in S_a|\sigma_0$ occurs.

Finally taking into account entire aforesaid, the distribution of preferences on the set $S_a|\sigma_0$ changes. Subsequently to each preference relation quantitative characteristic - distribution of preferences $\pi(\sigma_i|\sigma_0)$ is placed in the correspondence. Cardinal approach to the forming of preferences adapts.

We will examine the cases, when there is a correspondence,

$$\rho(S_a \mid \sigma_0) \Leftrightarrow \pi(S_a \mid \sigma_0), \tag{1.29}$$

and from (1.29) it follows that

$$\sigma_i \precsim \sigma_j \mid_{\sigma_0} \Leftrightarrow \pi\left(\sigma_i \mid \sigma_0\right) \leq \pi\left(\sigma_j \mid \sigma_0\right). \tag{1.30}$$

In the case, when $\sigma_0 \overline{\in} S_a \mid_{\sigma_0}$, the function of preferences we will designate $\pi(\sigma_i)$ and call „absolute" or „non- conditional". To assign the description of situation dynamics for the active system - means to assign conversion (morphism):

$$m(t_2) = F\left[m\left(t_1\right)\right], \tag{1.31}$$

or

$$M\left(t_2\right) = F\left[M\left(t_1\right)\right], \tag{1.32}$$

where $M(t_1) \in M_a(t_1)$ and $M(t_2) \in M_a(t_2)$, $M_a(t)$ is set of alternative PRS, and M(t) is subset PRS. Depending on the accepted model of description morphism (1.32) can be assigned in a various forms: as the relation, probabilistic or illegible relation. In the course of time the function of preferences $\pi(\sigma_i|\sigma_0)$ is modified. Since by hypothesis that lifetime of each active system is finite, than $(t_1, t_2) \in [t_0, t_0 + t_f]$, where t_f —time „of life" of system.

In the simplest case transformation (1.31) is represented as the system of ordinary differential equations, or as the system of ordinary differential equations and equations in partial derivatives.

If in a change of the situation population processes, transmitting of imperious authorities, or processes of instruction and some other specific processes, which possess „memory", participate, then mathematical model includes integro-differential equations.

The description of situation m includes $S_a|\sigma_0$, $R^{req}(S_a|\sigma_0)$, $R^{disp}(S_a|\sigma_0)$,..., therefore the model of dynamics must reflect the dynamics of a change in these sets. This requires the attraction of the corresponding methods.

1.4.3. Dynamics of sets and derivative with respect to measure

One of the possible approaches is the use of a concept of set's function [16].

Let E is the set, $E' \subset E$, and U(E) is set of subsets E'. Assume that to $\forall E' \in U(E)$ the number $x_{E'} \in R$ is placed in the correspondence, where R is set of real numbers.
Let's designate this correspondence *F*.

$$F : U(E) \to R . \tag{1.33}$$

It is assumed that the conditions are satisfied:

$$\forall A, B \in U(E) \Rightarrow A \cup B \in U(E)\,; \tag{1.34}$$

$$\forall A, B \in U\left(E\right) \Rightarrow A \setminus B \in U\left(E\right). \tag{1.35}$$

The domain of mapping *F* definition — *U*(*E*) is ring. $\varnothing \in U(E)$. Symmetrical difference

$$A\Delta B = (A \setminus B) \cup (B \setminus A) \in U(E)\,. \tag{1.36}$$

In [16] „a variation" of the set is used. A variation of the set *E* with the aid of the set *B* is called set $E' = E\Delta B$. Let further $\mu(E')$ be a measure - positive, additive, uniform function. The three (E, $U(E)$, μ) is the measurable space.
The function of set *F* (*E'*) is continuous, if for $\forall\varepsilon > 0\ \exists\delta(\varepsilon, E') > 0$, such, that the condition:

$$|\,\mu(E'\Delta B) - \mu\left(E'\right)| < \delta(\varepsilon, E') \Rightarrow |\,F(E'\Delta B) - F\left(E'\right)| < \varepsilon \tag{1.37}$$

is satisfied.

Derivative *F*(*E'*) with respect to the measure $\mu(E)$ is determined. Let the sequence of sets $\{B_n\}$, $n \in \overline{1,\infty}$ and set *B* have such property, that for $\forall\varepsilon > 0\ \exists N(\varepsilon, \{B_n\})$, for which following condition is satisfied:

$$n > N\left(e, \left\{B_n\right\}\right) \Rightarrow |\,\mu\left(B_n\right) - \mu\left(B\right)| < \varepsilon\,. \tag{1.38}$$

Then *B* is a limit of sequence $\{B_n\}$ on the measure μ:

$$B\,|_{\mu} = \lim_{n\to\infty} B_n \quad . \tag{1.39}$$

Derivative *F*(*E'*) with respect to the measure μ is determined by the following formula:

$$\left.\frac{dF\left(E'\right)}{d\mu\left(E'\right)}\right|_{\left\{B_n\right\}} := \lim_{B_n\to B} \frac{F\left(E'\Delta B_n\right) - F\left(E'\right)}{\mu\left(E'\Delta B_n\right) - \mu\left(E'\right)} \quad . \tag{1.40}$$

If set *E'* change with a change in a certain parameter *t*, which can be interpreted as a time $t \in [t_1, t_2]$ and for $\forall t\ F' \in U(E)$, then the derivative $\frac{dF}{d\mu}$, that corresponding „to moment" *t* is determined by the relationship:

$$\left.\frac{dF}{d\mu}\right|_{t} := \lim_{B_n\to F'_t} \frac{F\left(E'_t\Delta B_n\right) - F\left(E'_t\right)}{\mu\left(E'_t\Delta B_n\right) - \mu\left(E'_t\right)} \quad . \tag{1.41}$$

1.4.4. Dynamics of sets and moments problem

The approach to the description of the dynamics of a change in the sets, given in the mentioned work [16], does not exhaust all possibilities. There are approaches based on other determinations „of distance" between the sets (depending on the nature of the studied sets): D -statistic, Manhattan distance, the distance of Minkowski, etc). It is seemed that the more detailed description

It seats that the more detailed description of the set alteration in the course of time) can be built on the basis of the approach consonant to the Hamburger moments problem (or its modifications) [4].

Let each state $\sigma_i \in S_a|\sigma_0$ can be parameterized and characterized by the real parameter $r_i \geq 0$ (for example, by some resources), and $\pi(S_a|\sigma_0)$ is the distribution of preferences on $S_a|\sigma_0$. Let's compose the sequence of sums (supposing at first that N = const):

$$l_k = \sum_{i=1}^{N} r_i^k \pi\left(r_i\right) \geq 0 \quad . \tag{1.42}$$

Hamburger proved theorem for the case of real variable $r \in (-\infty, +\infty)$. Necessary and sufficient condition such that there exist the non-decreasing function $\mu(r)$, having the finite number of points of increase and such, that

$$\int_{-\infty}^{+\infty} r^k d\mu\left(r\right) = l_k \ ; \quad \left(k \in \overline{0,\infty}\right) \quad , \tag{1.43}$$

appears the positivity of sequence $\{l_k\}$.

Positivity means that all quadratic forms for all m and any collections of real values $(x_1, x_2, \ldots, x_m)$ are non-negatively determined:

$$\sum_{i=0}^{m} \sum_{j=0}^{m} l_{i+j} \ x_i \ x_j \geq 0 \quad . \tag{1.44}$$

In our case the role of measure $\mu(r)$ plays the function of preference $\pi(r_i)$. Task simplifies by the fact that distribution $\pi(r_i)$ or, in the general case, $\pi(S_a|\sigma_0)$ is considered the given one, and the number of alternatives is finite (or countable). In any event in practice it is possible to limit only to the finite number of moments l_k; $k_{\max} = K$. Moments l_k $k \in \overline{0,K}$ form (K +1) dimensional vector: $\left(l_0, l_1, l_2 \ldots, l_K\right) = \vec{l}_K \left(S_a \mid \sigma_0\right)$ of real positive numbers, which represents set $S_a|\sigma_0$ with the assigned on it distribution of preferences.

Now a change in the set $S_a|\sigma_0$ (with constant number of alternatives N) can be characterized as a change in the vector $\vec{l}_k$, in particular, to determine the derivative $\left.\frac{dS_a \mid \sigma_0}{dt}\right|_{N=\text{const}}$ as $\left.\frac{d\vec{l}_K}{dt}\right|_{N=\text{const}}$:

$$\left.\frac{dS_a \mid \sigma_0}{dt}\right|_{N=\text{const}} = \left.\frac{d\vec{l}_K}{dt}\right|_{N=\text{const}} \quad . \tag{1.45}$$

Let's name $\vec{l}_k$ *the k- image* of set $S_a|\sigma_0$ with the preferences. Each component of the vector $\vec{l}_k$ it is the function N of arguments r_i ($i \in$ 1, N). If we introduce now the vector space L with the basis $\left(\vec{e}_0, \vec{e}_1, ..., \vec{e}_k\right)$, then

$$\vec{l}_K = \sum_{k=0}^{K} \vec{e}_k \; l_k. \tag{1.46}$$

Derivative

$$\frac{\partial \vec{l}_K}{\partial r_i} = \sum_{k=0}^{K} \vec{e}_k \frac{\partial l_k}{\partial r_i} = \sum_{k=0}^{K} \vec{e}_k \left(k\pi\left(r_0 \mid \sigma_0\right) + r_i \frac{\partial \pi\left(r_i \mid \sigma_0\right)}{\partial r_i} \right) r_i^{k-1} \quad . \tag{1.47}$$

If $r_0 = \sigma_0$, then

$$\frac{\partial \vec{l}_K}{\partial \sigma_0} = \sum_{k=0}^{K} r_i^k \frac{\partial \pi\left(r_i \mid \sigma_0\right)}{\partial \sigma_0} \vec{e}_k \quad .$$

Further it is possible to examine the multidimensional space of resources r_i $\left(i \in \overline{1,N}\right)$ as *N-th* vector space with the basis $\left(\vec{\varepsilon}_1, \vec{\varepsilon}_2, ..., \vec{\varepsilon}_N\right)$, then the gradient of function l_k in the space of the resources

$$\overrightarrow{grad}_R \, l_k = \sum_{i=1}^{N} \vec{\varepsilon}_i \frac{\partial l_k}{\partial r_i} .$$

If we designate

$$\frac{d\vec{r}}{dt} = \sum_{i=1}^{N} \vec{\varepsilon}_i \frac{dr_i}{dt}. \tag{1.48}$$

and introduce the scalar product

$$\frac{dl_k}{dt} = \overrightarrow{grad}_R \, l_k \frac{d\vec{r}}{dt} = \sum_{i=1}^{n} \frac{\partial l_k}{\partial r_i} \frac{dr_i}{dt},$$

then

$$\left. \frac{d\vec{l}_k}{dt} \right|_{N=\text{const}} = \sum_{k=0}^{K} \vec{l}_k \frac{dl_k}{dt} = \sum_{k=0}^{K} \vec{l}_k \sum_{i=1}^{N} \frac{\partial l_k}{\partial r_i} \frac{dr_i}{dt}. \tag{1.49}$$

The outlined approach to the description of dynamics of the sets change has some advantages in comparison with the approach, presented in the work [16].

1.4.5. Different sets of states

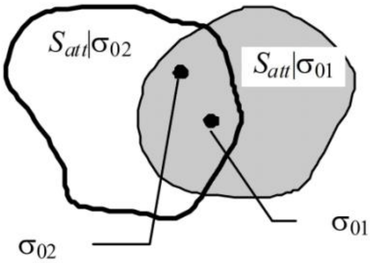


Fig. 1.8

At each moment of time the system is found in one of the states, let this state is σ_0. Having the available specific resources, the subject of system can principally convert system to any state, which belongs to a certain subset $S_{att}|\sigma_0$ of attainable states.

Set $S_{att}|\sigma_0$ *includes all states* σ_i *of „*arrival*" from the state* σ_0 *of „*departure*" such, that the corresponding required resources for the realization of passage* $\sigma_0 \to \sigma_i$ $R^{req}(\sigma_i) \leq R^{disp}(\sigma_i)$*. Configuration of* $S_{att}|\sigma_0$ *depends on initial position* σ_0*:*

$$S_{att} \,|\, \sigma_{01}\sigma_{02} = S_{att} \,|\, \sigma_{01} \cap S_{att} \,|\, \sigma_{02} \,.$$

Name „boundary" state σ_{ib} such state, which corresponds to the optimal strategy of the resources use in the course of realization of the passage based on σ_0. In a general sense the corresponding variation problem relates to the class of the tasks of optimal control with the limited resources (for example [36,148]).
It is obvious that $S_{att}|\sigma_0$ is a subset of set *SA*:

$$S_{att} \,|\, \sigma_0 \subseteq S_\sigma \,.$$

Not all states, which belong to $S_{att}|\sigma_0$, are considered by the subject as alternatives. Being located in „the position" σ_0, subject studies the part of the states of set $S_{att}|\sigma_0$ and evaluates them from the point of view of their preferability with respect to σ_0. In other words, subject introduces the relation „of the preference" ρ on the certain subset $S_a|\sigma_0 \subseteq S_{att}|\sigma_0$.Relation ρ separate from $S_{att}|\sigma_0$ subset of elements $\sigma_i \in S_a|\sigma_0$ connected with this relation. In this sense ρ is identified with the set $\rho = S_a|\sigma_0 \times S_a|\sigma_0$, where the sign „$\times$ "indicates the Cartesian product of sets. Relation ρ is a subset of the ordered pairs (σ_i, σ_j).

It is necessary to assume that all or the part of the states $\sigma_i \in S_{att}$ are distinguishible and comparable between themselves. This means that between the elements can be determined the binary relation of preference ρ. Set of states $\sigma_i \in S_{att}|\sigma_0$, between which the subject establishes the preference relation ρ: $\rangle$ (or $\underset{\sim}{\langle}$) there is set of alternatives $S_a|\sigma_0$. The states, distinguishable and comparable in ρ, that interest subject, are called alternatives.
There exist two possibilities:

1. $\sigma_0 \in S_a \,|\, \sigma_0$;
2. $\sigma_0 \,\overline{\in}\, S_a \,|\, \sigma_0$.

In the first case the subject studies set of alternatives $S_a|\sigma_0$ „from without" (Fig. 1.9, a). This means that the subject cannot remain in the initial state σ_0. This state is not one of the alternatives.

In the second case the subject „looks" on the set $S_a|\sigma_0$ „from within", and $\sigma_0 \in S_a|\sigma_0$ is one of the alternatives, i.e., subject can, in particular, select as most preferable against the background of existing alternatives the state, in which he has already been located, i.e., σ_0.

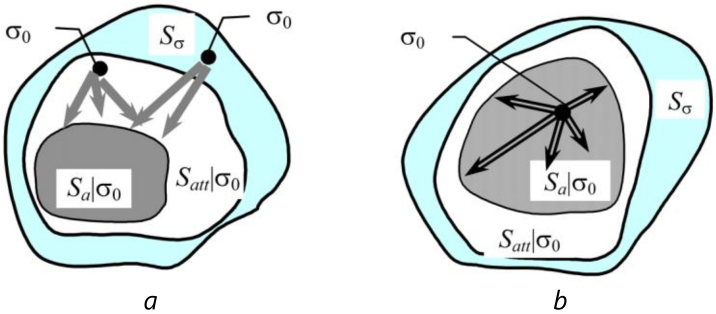


Fig. 1.9

„System" must be individualized, at least, during a certain finite time interval. In other words, it would be desirable to assume that during this interval there is a certain invariant, which characterizes system (in the sense of the theory of categories). Invariant is a function of cardinal.

If it is limited by the finite number of states either alternatives or problems, then invariant is expressed as this quantity.

We already said above that the number of alternatives in the set $S_a^{N_a}|\sigma_0$ can be changed in the course of time and, therefore, „invariant" connected with the number of alternatives is „relative". Let's name „absolute" invariant the invariant, assigned on the set $S_\sigma^{N_\sigma}$ - cardinal number („Card"):

$$I_{S_a} = \text{Card}\, H\left(S_\sigma, S_\sigma\right). \tag{1.50}$$

Here $H(S_\sigma, S_\sigma)$ is dimorphism's set $S_\sigma \to S_\sigma$. In practice we frequently deal with respect to the systems, the number of permissible states of which can be changed in the course of time. This situation, for example, occurs when in the system any „failures" occur.

Invariant I_{S_σ} could be considered as an objective system characteristic.

„*Relative*" or subjective invariant

$$I_{S_a} = \text{Card}\, H\left(S_a\,|\,\sigma_0, S_a\,|\,\sigma_0\right) \tag{1.51}$$

is connected with the quantity of alternatives N_a, which, obviously, also changes (in the general case) in the course of time.

At first in the simplified version the systems for which N_σ = const and N_a = const, at least, during the period of time spent on the solution of the selected problem will be examined. The numbers N_σ and N_a can be changed upon transfer to the following cycle. It is possible to use other invariants, for example:

$$\text{Card}\, H\left(S_\sigma, S_\sigma\right) > \text{Card}\, H\left(S_a \mid \sigma_0, S_a \mid \sigma_0\right),$$

$$I_{S_a,R_a} = \text{Card}H\left(S_a, \mathfrak{R}_a^{req}, \mathfrak{R}_a^{\exp}\right), \tag{1.52}$$

where $H\left(S_a, \mathfrak{R}_a^{req}, \mathfrak{R}_a^{\exp}\right)$ - the set of morphisms $S_a \to \mathfrak{R}_a^{req}, \mathfrak{R}_a^{\exp}$, and $\mathfrak{R}^{req}$, $\mathfrak{R}^{\exp}$ are sets of resources.

Using the concepts introduced earlier it is possible to propose the following determination of „*intellectual catastrophe*".

Let S_σ is set of the states of the system (it can be finite, countable, continuous); $S_a|\sigma_0$ is set of alternatives, generated by subject. Cardinal numbers for S_σ and $S_a|\sigma_0$ are correlated as follows

$$\text{Card}\, H\left(S_\sigma, S_\sigma\right) > \text{Card}\, H\left(S_a \mid \sigma_0, S_a \mid \sigma_0\right) \quad ,$$

following possibilities can occur:

1. $S_{att}|\sigma_0 \subseteq S_\sigma$ and $S_a|\sigma_0 \cap S_{att}|\sigma_0 = S_a$.

Subject desires some permissible and accessible (Fig. 1.10, *a)*.

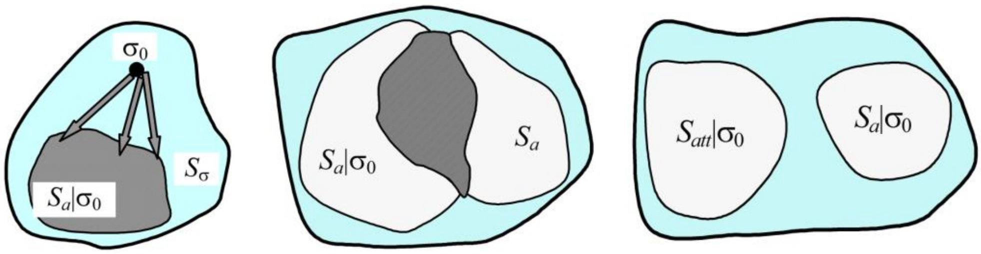
2. $S_{att}|\sigma_0 \subseteq S_\sigma$ and $S_a|\sigma_0 \setminus S_{att}|\sigma_0 \neq \varnothing$, $S_{att}|\sigma_0 \subseteq S_\sigma$.

Subject desires some permissible, but not all is accessible (Fig. 1.10, *b)*.

3. $S_{atta}| \subseteq S_\sigma$ and $S_a|\sigma_0 \cap S_{att}|\sigma_0 = \varnothing$.

Subject desires some permissible $S_a|\sigma_0 \subseteq S_\sigma$, but not accessible from σ_0 (Fig. 1.10, *c)*.

4. $S_a|\sigma_0 \setminus S_\sigma \neq \varnothing$.



a *b* *c*

Fig. 1.10

Subject desires some inadmissible (not all is possible).

If $S_a|\sigma_0 \cap S_\sigma = \varnothing$, subject desires are impossible.

Finally, set of alternatives can be empty $S_a|\sigma_0 = \varnothing$. In the latter case, and also in the case 4 it is possible conditionally to speak about „*intellectual catastrophe*".

Subject does not examine any alternatives or he examines the ephemeral alternatives, which foresee reaching states not compatible with existence of this system.

If only alternative is examined, then the entropy, which corresponds to relative invariant is equal to zero (singular case), more precisely: $H_\pi = 0$, if $S_a|\sigma_0 = \sigma_1$.

„Resource catastrophe" appears, if for all $\sigma_i \in S_a|\sigma_0$ the following condition is satisfied

$$R^{\text{req}}(\sigma_i) \geq R^{\text{disp}} \quad ,$$

i.e., problems *are insolvable*.

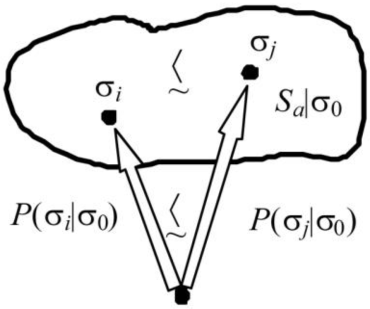


Fig. 1.11

What is difference between the concepts *„problem"* and *„alternative"*? Set of alternatives $S_a|\sigma_0$are a set of the states, considered by the subject as permissible, accessible and preferable with respect to the initial state σ_0 (that occurs at the given moment). States accessible but less preferable than σ_0: $\sigma_i \precsim \sigma_0$ either are not included in $S_a|\sigma_0$ or can be included with zero utility by the subject as permissible. On the formed set $S_a|\sigma_0$ the binary relations of preference between all states are established. In this case there is a homomorphism

$$\sigma_i \precsim \sigma_j \Leftarrow \; P\left(\sigma_i \,|\, \sigma_0\right) \lesssim P\left(\sigma_j \,|\, \sigma_0\right) \quad . \tag{1.53}$$

The right side of this relationship means that the solution of problem $P(\sigma_i|\sigma_0)$ is more preferable than the solution of problem $P(\sigma_i|\sigma_0)$. Correspondence is homomorphism since the relation $\sigma_i \lesssim \sigma_j$ can occur also for other problems and another initial state σ_0.

Correspondence (1.53) shows the difference between the concepts „problem" and „alternative", and also the preference relation ρ on $S_a|\sigma_0$.

Continuing to discuss the concept of an *„active system"* we now can indicate that this is the system, which possesses the enumerated distinctive properties. In contrast to the active system, passive system, having a set of possible states S_σ, does not form set of alternatives, $S_a|\sigma_0$, it does not establish in this set system of preferences, and it does not generate „its problems".

Examples of the active systems: nuclear plant + personnel, automobile + driver, flight vehicle + crew, research laboratory + scientific leader; instructor + student + real - technical resources of instruction, computer + researcher…

Examples of the passive systems: atomic nucleus, computer, automobile, the solar system, the atmosphere, ocean,…

By analogy with „thermodynamic death" let's name „*entropy death*" of active system the situation, when: (a) resource (including - information) exchange with „the environment" is absent and for $\forall\sigma_i \in S_\sigma$ the condition $R^{\text{req}}(\sigma_i) = idem(i)$ is satisfied, (*b)* entropy $H_\pi = H_{\pi\max}$ and this situation is steady (definition of subjective entropy see in chapter 3 of this works).

A change of system state occurs as a result

- of the goal-directed activity, connected with the expense of the available resources;
- of the changes, proceeding in „the environment", not depending on the system, but changing its state;
- „spontaneous" change in the system as a result „internal" processes, proceeding in the system.

It would be tempting to interpret all these changes as changes *in the available resources*. This method of formalization makes the theory more laconic and makes it possible to significantly unify mathematical description. This however does not always correspond to the essence of the matter, especially, if we consider influence on the distribution of the preferences of ethical factors.

1.4.6. Properties of simple alternatives

Let's return to the properties of alternatives. Alternatives σ_i and σ_j are incompatible, if they cannot be realized simultaneously. For example, subject cannot simultaneously be located in two different places; it is not possible on the competitions to occupy simultaneously the first and second places for one athlete. Let's designate this circumstance by the symbol

$$\sigma_i \wedge \sigma_j = \varnothing,$$

where $\wedge$ is the composition of alternatives. Such alternatives can be achieved consecutively; in particular they can be trajectory elements (track):

$$\ldots \to \sigma_i \to s_j \to \ldots$$

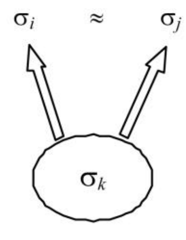


Fig 1.12

Together with „the composition" $\sigma_i \wedge \sigma_j$ (binary composition) the binary „disposition" $\sigma_i \vee \sigma_j = \sigma_s$ can be used alternative consisting in the fact that one of three possibilities: σ_i either σ_j or simultaneously and σ_i and σ_j (i.e., composition) will be realized.

Two alternatives are independent, if two ways Tr_{ij}: $\sigma_i \to \sigma_j$ and Tr_{ji}: $\sigma_j \to \sigma_i$ are equivalent. This means that the realization of each of the alternatives does not assume the obligatory realization of another: $Tr_{ij} \sim Tr_{ji}$. Two alternatives σ_i and σ_j are equivalent conditionally relative to the third alternative σ_k, if the way Tr_{kj} and Tr_{ki} are equivalent.

The classes (subsets) of equivalence have already been discussed above. Each element $\sigma_i \in S_a$ has its class of equivalence, which contains as minimum one element σ_i. Designate through $S_{ai} \subseteq S_a$ the class of equivalence of element σ_i .. Set S_a thus is separated on the nonintersecting classes of equivalence. If two elements σ_i and σ_j are equivalent: $\sigma_i \sim \sigma_j$, then their classes of equivalence coincide $S_{ai} = S_{aj}$. The function of preferences is constant on any class of equivalence (this circumstance has already been used above):

$$S_{ai}\ \pi(\sigma_i) \to idem(i); \quad \forall \sigma_i \in S_{ai}$$

If on S_a preference relation $\rho: \rangle$ is assigned, and S_{ai} is a class of the equivalence of element σ_i (as this element any element $\sigma_i \in S_{ai}$ can be selected), then the remaining part of the complete set S_a: $S_a \backslash S_{ai}$ is divided an two subsets S^+_{ai} and S^-_{ai} — more preferable than σ_i and less preferable elements than σ_i . If σ is the parameter with the values on real axis, then on the basis the theorem about the fact that the open set on real axis is a sum of the finite or countable number of mutually disjoined intervals (open subsets), we will consider also for *the n-dimensional* case that the classes of equivalence are the open subsets. If set S_a is finite or countable, then „boundary" between S^+_{ai} and S^-_{ai} resembles the boundary between the states, surrounded as on two sides „boundary posts" - by the alternatives, which belong to the co-sets of equivalence.

Assume that the following binary relations $\sigma_i \rangle \sigma_k$; $\sigma_j \rangle \sigma_k$, are now determined, moreover it is previously known that from the realization of problem P: $(\sigma_k \to \sigma_i)$ follows the realization of problem P: $(\sigma_k \to \sigma_j)$. In this case, we will indicate that the alternative σ_i " *absorbs*" alternative σ_j: $\sigma_i \rangle\rangle \sigma_j | \sigma_k$. This means that „the alternative" σ_i absorbs alternative σ_j with the condition that, the initial and directly previous state is state σ_k. The property of absorption, does not coincide with the properties of inclusion or belonging ($\subset$, $\in$). Each element σ_i has its absorbed set, which in particular case contains only one element σ_i. In the case of absorbing the alternatives we will say that problem P: $(\sigma_k \to \sigma_i)$ *absorbs* problem P: $(\sigma_k \to \sigma_j)$. This does not mean that the alternative σ_j can be excluded from the examination.

For example, if available resources are insufficient for the solution of the first problem, then it is necessary to solve second problem. The property of absorption is transitive, reflexive and asymmetric.

States σ_i and σ_j are incompatible, if there does not exist such a state $\sigma_k \in S_a$, which would absorb simultaneously σ_i and σ_j with the initial state σ_k and they are compatible, if such a state exists.

In the first case the state $\sigma_s \sim \sigma_i \wedge \sigma_j$ is a not realized state in S_a: $\sigma_s \,\overline{\in}\, S_a$, in the second case $\sigma_s \sim \sigma_i \wedge \sigma_j \in S_a$. Recalling the determination of the states composition, name problem $P_s \sim P_i \wedge P_j$ *„the product"* of problems P_i and P_j, if P_s: $(\sigma_k \rightarrow \sigma_s)$, where $\sigma_s \sim \sigma_i \wedge \sigma_j \backslash \sigma_k$. Problem P_s *„is solvable"* in S_a, if $\sigma_s \in S_a$ and *„is insoluble"* in S_a if $\sigma_s \,\overline{\in}\, S_a$.

This, the problem, which provides as a desired alternative the composition of alternatives, is the product of problems. Let's designate, through $P_s = P_i \vee P_j$ *„sum"* of problems P_i and P_j — the problem, which supposes reaching at least one of the alternative states: σ_i or σ_j. from the initial state σ_k. Problem P_s is solvable, if at least one of the problems P_i and P_j is solvable.

The set of the elements, for each of which the subset of absorption consists of this one element, name *fundamental*, on the contrary, if all states are built in a row of the mutually-absorbing states

$$\sigma_1 \langle\langle \sigma_2 \langle\langle \sigma_3 \langle\langle \ldots \langle\langle \sigma_{N-1} \langle\langle \sigma_N ,$$

corresponded set name universal or *universally- connected*.

Such a situation can occur, if all elements are characterized by one universal index (for example, by cost equivalent), and subject possesses the necessary quantity of units of this equivalent in order to realize alternative σ_{i+1}. In this case alternative σ_i will be realized and also all *„young"* alternatives σ_{i-1}, σ_{i-2},...

As an example can serve collection as the alternatives of points, consecutive on the track (Fig. 1.13). Reaching of A_i point includes reaching of all preceding points.

Let's give some additional definitions, which are concerned category *„problem"*:

1. *„Simplest problem"* $P^{(1)}$: $(\sigma_0 - \sigma_i)$ — the problem, whose solution is the one-act action of passage from the initial state σ_0 to the another, „simple" state σ_i.

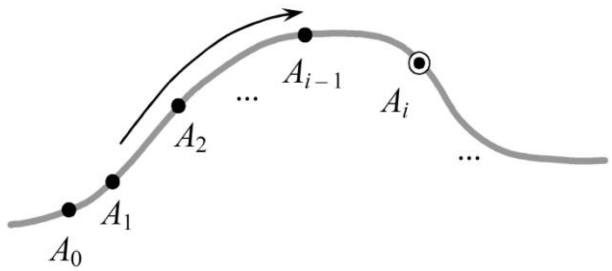


Fig.1.13

2. *„Complex problem"* $P^{(k)}$:$(P_1, P_2, \ldots, P_k) \sim \{(\sigma_0 \rightarrow \sigma_1), (\sigma_0 \rightarrow \sigma_2), \ldots, (\sigma_0 \rightarrow \sigma_k)\}$ is the collection of the simple problems, based on which in the final analysis a certain composition (or product) of order $n \leq k$ is selected. Complex problem can consist not only of simple problems, but also include different compositions. Thus, if S_a is countable, then set of all complex problems have a power of continuum.
3. *„Vector problem"* $\vec{P}_s$ - corresponds to that case, when the desired state σ_i is represented in the form the collection of components and can be considered as the vector

$$\vec{\sigma}_i = \left\{\sigma_i^1, \sigma_i^2, ..., \sigma_i^s\right\}.$$

Then problem, also is represented as the vector $\vec{P}_s : \left\{P_i^1, P_i^2, ..., P_i^s\right\}$. This means that there is a certain linear transformation in the *s-dimensional* space, components of which they are variable $\sigma_i^r \left(r \in \overline{1,s}\right)$.

4. „*Hierarchical problem*" - is the problem, formed as the collection of the simple (or complex) problems, deciding in the assigned sequence. Moreover so that the solution of each subsequent problem is possible only after the solution of previous one, there is a resource conditionality of this sequence.
5. „*The subordination*" of problems - is relationship between the problems, when a certain sequence of the solution of problems is established, but there is no resource conditionality of this sequence.

2. INDIVIDUAL OBJECT PREFERENCES

2.1. Individual functions of preference, state, composition, strategy

In order to supply a set of alternatives S_a (and a set of problems P_a) with a quantitative measure, reflecting binary relations in this set, besides of the function of utility $U(\sigma...)$ already examined above, let's introduce the function of preference $\pi(\sigma...)$.

The difference between the concepts „function of utility" and „function of preferences" is in the fact that to the utility function $U(\sigma)$ an objective sense (quantity of calories in the foodstuffs, a probability of recovery with use of a certain method of treatment, any probability of success, achieving a desired result, determined on basis of objective data) is assigned. A set of alternatives S_a, supplied with a distribution of utilities' $U(\sigma)$ is alike the „menu". However, a selection from this „menu" achieves subject on basis of subjective preferences, depending on more or less know utilities' and also on the specific ethical norms.

In order to elaborate quantity description of this subjective process, and obtain a possibility to simulate it and, then, to make forecasts of subject behavior, along with functions the utility of the function of preference $\pi(\sigma...)$ have been introduced.

The distribution of utility on S_a $U(\sigma_i)$ cannot coincide with a distribution of preferences. To be more precise, ordering on utilities' cannot coincide with a rank ordering on preferences.

For example, a certain subject can accept the following relations

σ_2 „sport"	σ_2 „smoking"	σ_3 „alcoholism"	
$U(\sigma_1)$	$> U(\sigma_2)$	$> U(\sigma_3)$	(2.1)
$\pi(\sigma_1)$	$< \pi(\sigma_2)$	$< \pi(\sigma_3)$	(2.2)

Then we can see that for this subject, classifying in according with utilities is opposite to classifying according to subjective preferences. It is possible to say that „utilities'" distribute „God", and preference „devil".

Interrelations between *utilities* and *preferences* and both in a general theoretical sense and in a formally mathematical level present an independent problem.

Function $\pi(\sigma...)$ is always positive and limited for $\forall\sigma \in S_a$. In the majority of cases, the distribution of preferences on the set S_a depends on that kind of position, in which at the given moment the system is found. Let's designate this actual state as σ_0. It is possible to assume, that there are such alternatives, whose preferability either depends weakly on actual state σ_0 or it does not depend at all. In this case the state σ_0 („the point of view" of subject) either belongs to set alternatives S_a $(\sigma_0 \in S_a)$, or it does not belong $(\sigma_0 \overline{\in} S_a)$.

Let's name the distribution of preferences and the function of preferences, *non conditional* (or *absolute*), if $\pi(\sigma\ldots)$ does not depend on σ_0, or $\sigma_0 \overline{\in} S_a$. Designate this function of preference through $\pi(\sigma)$. If the distribution of preferences and the function of preferences depend on that, in which state (σ_0) the system is situated, then we will talk about a conditional (or relative) distribution.

The composition of alternatives c^k of order k, is a complex alternative that foresees „simultaneous" action of a system on the realization of all „simple" alternatives composing the composition; in other words, directing of available resources simultaneously on k directions. Because of the fact that alternatives in a set of composition c^k can be dependant, realization of composition can prove to be cheaper or more expensive than realization of all k of alternatives separately. Extended set of alternatives' includes all compositions of orders $m \in \overline{1, K}$, $k \leq N$, including „simple" alternatives (k=1). Cardinal number of this set (or cardinal number) is

$$Card\ S_c^k = \sum_{m=1}^{N} \frac{N!}{m!\left(N-m\right)!}. \tag{2.3}$$

If a set of simple alternatives is countable, then set compositions have power of continuum

$$Card(S_a \mid \sigma_0) < Card(S_c \mid \sigma_0). \tag{2.4}$$

Utilities in this case are non-additive. Introduction together with „simple" alternatives of compositions enlarge s possibilities of subjective analysis.

On extended set of alternatives the function of preference $\pi(c^k)$ can be assigned, $c^k \in S_c^k$. Alternatives don't always have a quantitative nature. Frequently they are defined as an event, which either occurs or not: „to construct a house", „to reach the Moon", „to win a match", „to learn Newton's law". It is not possible, to learn Newton's law half or in 30%. It is not possible to build a house without covering it with a roof. However, it is possible to talk about an output of a necessary quantity of gas or oil, about a deduction of a certain profit, income…

For example, when a composition is more profitable than separately resolved problems, that correspond to simple alternatives, entering the composition, an adjusting production of two automobiles models appears, if there are standardized identical completion units and components. Collective actions under war time give advantages. Military strategies and the organization of armies are based on this.

Nothing hinders to count a set of compositions $S_c|\sigma_0$ as a new set of „simple" alternatives by renumbering of compositions. It is obvious that $S_a \subset S_c$. Among all possible compositions there are separated those, that have providing with available resources and only such static compositions are included into the set S_c of renumbering. Another point of view states that, with a dynamic approach set S_c (or S_a) can include compositions (alternative) unattainable with those available resources, that are available at the given moment, but they can perspectively be achieved after solving some number of intermediate problems. Sometimes we deal with desires extending over one of the limits of our today's possibilities.

We will not introduce new designation and, if special circumstances are not stipulated the designation S_c shall be subsequently used. The examination of compositions as an independent concept and a subject of studies are convenient when dynamics of a change in the problem - resource situation is studied. As it has already been mentioned, generally the distribution of preferences changes, the set of attainability S_{att}, the set of alternatives S_a, „*the point of view*" of the subject, i.e., current state σ_0, and finally, a „physical" content of alternatives modified. The use of compositions makes it possible to formalize the concept of problem - resource strategy.

One of the versions of an idea of a *problem - resource strategy* S_{tr} performance looks like following:

dynamics of situation is considered as a sequence of the compositions changing each other $c_s\, t$, t_s^i, t_s^f, V_s^r,..., where s - number of a composition: $s \in \overline{1,N_c}$; t_s^i, t_s^f - initial and final points „of composition existence" c_s, V_s^r are rate of conversion and distribution of the available resources inside the composition c_s.

Let's name the sequence of passages from one composition to another „*trajectories*", or a „problem - *resource trajectory*".

A formal shape of strategy $S_{tr}(c_s, t)$ is an aggregate of sequences c_s, t_s^i, t_s^f, $V_s(t, t_s^i, t_s^f)$.

We may say that a strategy is a join of compositions set, and sets of trajectories. Each step along the trajectory is a replacement of one composition by another; this is a morphism of a certain type from those, which are discussed in paragraph 2.9.

$$\left.\begin{aligned} &c_s\!: \; c_1, c_2, \ldots, c_m, \ldots \\ &t_s^i\!: \; t_1^i, t_2^i, \ldots, t_m^i, \ldots \\ &t_s^f\!: \; t_1^f, t_2^f, \ldots, t_m^f, \ldots \\ &V_s(t)\!: \; V_1(t), V_2(t), \ldots, V_m(t), \ldots \end{aligned}\right\} Str. \tag{2.5}$$

Here c_s are compositions of alternatives changing each other; $V^r_s(t)$ are particular strategies of an expense of available resources on *the s-th* stage. Of course, the expense of resources is accomplished with in limitations of subject and with the analysis of „convexity" or „concavity" of a set of alternatives; expendable resources play the role of a parameter due to conditions of the convexity (concavity) of functions.

Generalization of a concept of „convexity" („concavity") for the distribution of preferences ($\langle$, $\underset{\sim}{\langle}$) on a set of alternatives S_a is important from the point of view of strategies' *Str* effectiveness. On a set of strategies, it is also possible to introduce a binary relation of preferences.

Let's note that set of compositions of any order $k \in 1, N$ is divided into separate subsets: C_+^k of such compositions of order *k*, for which the condition of convexity („profitable") is fulfilled, C_-^k of such compositions of order *k*, for which the condition of concavity („unprofitable") is fulfilled, and $C_\sim^k$ of such compositions, whose use does not change resource's proportions and it does not influence the result of problem- resource process. Corresponding subsets of simple alternatives S_{a+}, S_{a-} and $S_{a\sim}$ in this case have non-empty intersections.

It is possible to assert that a certain strategy, which uses compositions $c^k \in C_+^k$ is better than strategies, that use $c^k \in C_\sim^k$, and the last is better than strategy, comprises compositions $c^k \in C_-^k$. Thus, there is an order of preferences $Str(C_t^k) \rangle Str(C_\sim^k) \rangle Str(C_-^k)$. These preferences have an objective nature and can be ordered with the use of the correspondent utility function $U_i\,(Str_i) \in U_+(Str)$ (or $U_-(Str)$, $U_\sim(Str)$). Thus the object of studies of preferences distribution together with the set of alternatives S_a are set of strategies of corresponding problems $P(\sigma_0 \to S'_a \subset S_a)$. On the set of strategies, if is possible under specific conditions to introduce a subjective quantitative measure of preferences $\pi(Str_i)$ is the function of preferences on the set of permissible (alternative) strategies.

The subset $U_+(Str)$, can exist as a set of alternative strategies. However, in certain cases the subject is forced to select strategy of behavior objectively not advantageous (for example, economically unfavorable) as a result of presence of non-economic circumstances, „external" actions of individual tastes, prehistory, an influence of ethical imperatives.

The circumstance that certain strategy Str_i, uses only compositions $c^k \in C_+^k$ does not guarantee, that strategy as a whole - the trajectory of compositions will be „more advantageous" than strategy, which does not use them. The selection of the best strategy from the set $U(Str)$ is an object of optimal planning of problem - resource strategies. *Situation dynamics* is the tool of *problem- resource processes* study.

Conditions for existence of the subjective quantitative measure of preference are given by theorems from chapter 1.3 and depend on what type of ordering to preference relations, installed by subject in the set of alternatives S_a correspond. As it will be shows further the function of preferences $\pi(\sigma)$ can be both monotonic and non-monotonic. Theorems and properties given above for the utility function can be related also to the function of preferences.

If preference relation $\langle$ on S_a is weak order then, according to theorem 2 (p.1.3) there is a real function $\pi(\sigma)$ such, that

$$\sigma \langle \eta \Leftrightarrow \pi(\sigma) \langle \pi(\eta) ;\ldots \ \forall \sigma, \eta \in S_a \tag{2.6}$$

Properties of weak ordering are described by conditions of theorem 1. It is also required that the quotient set - set of subsets of equivalence would be countable.
If a relation $\langle$ on S_a is strict partial ordered, then in accordance with theorem 4 (p. 1.3) there is a real function $\pi(\sigma)$ such, that for $\forall\sigma, \eta \in S_a$ relationships:

$$\sigma \langle \eta \Rightarrow \pi(\sigma) < \pi(\eta); \tag{2.7}$$

$$\sigma \approx \eta \Rightarrow \pi(\sigma) = \pi(\eta). \tag{2.8}$$

occurs.

Definition of $\approx$ relation is given above in p. 1.3. In this case, the function of preference can be non-monotonic.

It is necessary to emphasize once more that within frames of proposed scheme, a difference between the utility an preference is visible especially in the fact that, generally, the condition $U(\sigma) < U(\eta)$ is not followed by $\pi(\sigma) < \pi(\eta)$. It means that ordering of alternatives from S_a based on the comparison of utilities, generally does not coincide with the ordering of the same alternatives, based on the comparison of preferences.

2.2. Preference functions at the set of simple alternatives and their normalization

We will assume that all functions of preference are normalized. These condition is not only question of mathematical convenience, but also reflects the specific feature of subject's psyche, which consists of the fact that, if in a certain limited collection of alternatives S_a the „interest" of subject in one of the alternatives grows, then the „interest" in other alternatives must decrease.

In some research works preferences are not normalized, or they are normalized in another method, for example, in the work of Wilson [33], where alternatives of trips of city inhabitants are studying, and also variation principles very close to that shown in this book are examined. Let's assume that if S_a is finite or countable, then

$$\sum_{i=1}^{N,\infty} \pi(\sigma_i) = 1;\ \pi(\sigma_i) \geq 0\ ;(\forall \sigma_i \in S_a). \tag{2.9}$$

If S_a is a continuum, and σ symbolizes a certain measurable parameter, then $\pi(\sigma)$ is *„the density of preferences distribution "*, and the integral

$$\int_{(S'_a \in S_a)} \pi(\sigma)d\sigma = \pi(S'_a);\ (\forall\sigma_i \in S_a) \tag{2.10}$$

is a value of preference of the event S'_a that of state σ, belonging to the subset $S'_a \subset S_a$. Assume that following condition of normalization have place:

$$\int_{(S_a)} \pi(\sigma)d\sigma = 1;\ \ \pi(\sigma) \geq 0\ ;\ (\forall\sigma \in S_a). \tag{2.11}$$

A set S_a can be limited or unlimited.

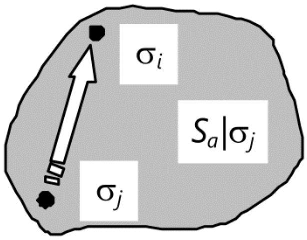


Fig. 2.1

Let $\pi(\sigma_l|\sigma_j)$, is the conditional function of preferences, σ_j is actual state (a state in which the system is located at the given instant). State σ_j is components of the set of alternatives $\sigma_j \in S_a|\sigma_j$ (designation σ_j is here used instead of σ_0). This means that the retention of the existing state is one of the alternatives. For each σ_j the following condition of the normalization occurs.

$$\sum_{i=1}^{N(\infty)} \pi(\sigma_i \mid \sigma_j) = 1;\ \pi(\sigma) \geq 0\quad \pi(\sigma_i|\sigma_j) > 0\ ;\ (\forall\sigma_i,\sigma_j \in S_a\,\sigma_i|\sigma_j). \tag{2.12}$$

If in particular to assume that composition $S_a|\sigma_j$ does not change with the change of „the points of view" σ_j, then it is possible to use a designation S_a (without the indication of actual state).

The presence of the preferences distribution makes it possible to determine *„subjective distance*" between two alternatives. Let's designate the subjective distance through ρ_s and assume that:

$$\rho_s(\sigma_i,\sigma_j) = |\,\pi(\sigma_i) - \rho(\sigma_j)\,|, \tag{2.13}$$

analogously to (2.13) for the conditional preferences distribution

$$\rho_s(\sigma_i,\ \sigma_j|\sigma_k) = |\pi(\sigma_i|\sigma_k) - \pi(\sigma_j|\sigma_k)|. \tag{2.14}$$

It can be verified that the distance $\rho_s(\sigma_i, \sigma_j)$ and $\rho_s(\sigma_i, \sigma_j|\sigma_k)$ fulfills three requirements, which are claimed for distances:

$$\left.\begin{array}{l}\rho_s\left(\sigma_i,\sigma_i\right)=0;\\ \rho_s\left(\sigma_i,\sigma_j\right)=\rho_s\left(\sigma_j,\sigma_i\right);\\ \rho_s\left(\sigma_i,\sigma_k\right)\le\rho_s\left(\sigma_i,\sigma_j\right)+\rho_s\left(\sigma_j,\sigma_k\right)\end{array}\right\}. \tag{2.15}$$

The distance ρ_s was named subjective because it is expressed through subjective preference function.

Let's introduce now an *„objective distance"*, expressing it through the function of utility , if last is determined. Let's assume that:

$$\rho_0(\sigma_i,\sigma_j)=|\,U(\sigma_i)-U(\sigma_j)\,|. \tag{2.16}$$

A distance defined these trajectories, as well as distance ρ_s, fulfills three necessary conditions.

A subjective distance ρ_s has a relation to the task about „reproduction" and „death" of alternatives. It is possible to assume that each subject has certain level of „resolution" ρ^*_s such, that, if $\rho_s \le \rho^*_s$, alternative σ_i and σ_j become indistinguishable and are imagined as an equivalent.

2.3. Normalization of complex alternatives preferences

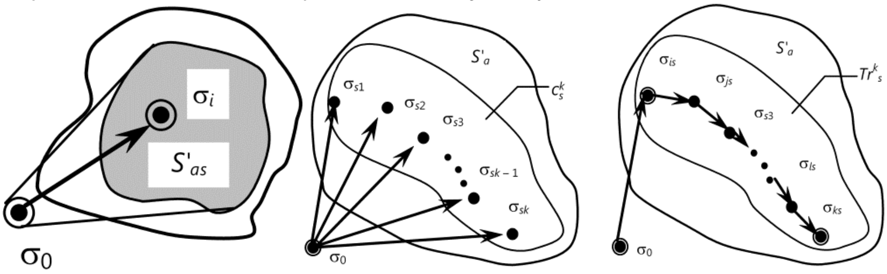
The function of absolute preferences („single-point") can be treated as the function of a priori preferences.

We already introduced concepts of *alternatives subset* $S'_a \subset S_a$, which is considered as an alternative consisting of the fact that as a result of problem solution the system will pass from the initial state σ_0 to one of states $\sigma_i \in S'_a$, and also concept of composition c^k of order k, understood as a collection of states, which are realized simultaneously (in parallel), and available resources are distributed in several directions („they are pulverized").

The sense of an introduction of compositions consists, particularly in the fact that simple alternatives being a part of a composition require different volume of resources, different time for the realization and individual required resources can be changed, particularly decrease as a result of a simple alternative including in composition.

It was also mentioned about additional concept - *strategy* $Str|_a$. A strategy is interpreted as a sequence of steps - passages from one simple state to another (or from one composition to another) with a simultaneous indication of *a time structure* of these passages (trajectory). If we forget about the time structure and examine (compare) only sequences of states σ_j (either the sequence of compositions, including compositions of the 1-st order (k = 1), i.e., simple states), then we come to the concept of alternative „*ways*" or „*trajectories*" (*Tr*).

The Fig. 2.2, *a, b, c* reflects schematically difference between concepts of a subset of simple alternatives S'_a, composition c^k_s, trajectory *Tr*.



a *b* *c*

Fig. 2.2

A complete strategy is assigned on a set of alternative trajectories by adding information about the time structure of passages and the distribution of resources between stages of trajectories and inside the composition in each stage. For each trajectories „*initial*" and „*final*" state are determined.

Being located in a certain state σ_i, subject has a possibility to compare *simple alternatives*, *subsets of alternatives*, *compositions*, *trajectories* and *strategies*.

Consequently, in each case conditions of normalization should be provided. If $\{S'^p_a\}$ is a set of subsets S^N_a of power $1 \le p < N$, then the number of such subsets $C_N^p = \dfrac{N!}{p!(N-p)!}$. Consequently, if we compare only subsets of the same power p, the normalization conditions take the form:

$$\sum_{i=1}^{C_N^p} \pi\left(S'^p_{ai}\right) = 1; \quad S'^p_{ai} \in \left\{S'^p_a\right\}, \tag{2.17}$$

If all subsets of power from 1 to $p = N - 1$ are examined simultaneously as alternatives, and moreover each simple alternative is encountered in each S'^p_{ai} only one time, then normalization takes the form:

$$\sum_{p=1}^{N-1} \sum_{i=1}^{Z} \pi\left(S'^p_{ai}\right) = 1, \tag{2.18}$$

$$S'^p_{ai} \in \left\{S'^1_{ai}, S'^2_{ai}, \ldots, S'^{N-1}_{ai}\right\}; \; Z = C_N{}^p.$$

Let's examine conditions of trajectories normalization. It is possible to imagine a situation, when versions of possible strategies are compared, moreover one-step, two-step, three step,..., n- *step* trajectories are examined. Technological feasibility and economical expediency are basis for this comparison and selection of the best alternative. Different trajectories can be regarded alternatives, if they *are technologically feasible*, but *not mutually conditioned* (they do not compose *technological chain*).

The process of achieving the final state begins either from the fixed state σ_0 („beginning point"), which is not considered while normalization is mind, or from the alternative state, which subject selects as an *„initial"* σ_i. If simple alternatives come out stages of *„wandering"*, then the corresponding preference function s for *p-1* trajectories shall be designated as follows:

$$\left.\begin{aligned} &\pi\left(\sigma_i,\sigma_j\right); \qquad p-1=1, p=2;\\ &\pi\left(\sigma_i,\sigma_j,\sigma_k\right); \quad p-1=2, p=3;\\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ &\pi\underbrace{\left(\sigma_i,\sigma_j,\dots,\sigma_s\right)}_{N-1}; \; p-1=N-2, p=N-1;\\ &\pi\underbrace{\left(\sigma_i,\sigma_j,\dots,\sigma_t\right)}_{N}; \; p-1=N-1, p=N. \end{aligned}\right\} \tag{2.19}$$

For each *p* a number of all trajectories is equal N^p. An appropriate function of preference could be named „p- *point function*". Let for example *N* = 3, i.e., S_a: (σ_1, σ_2, σ_3). Number of possible one-step trajectories (*r* = 2) $N = 3^2 = 9$, namely

$\sigma_1 \to \sigma_2$; $\sigma_1 \to \sigma_3$; $\sigma_2 \to \sigma_3$; $\sigma_3 \to \sigma_1$; $\sigma_2 \to \sigma_1$; $\sigma_3 \to \sigma_2$; $\sigma_1 \to \sigma_1$; $\sigma_2 \to \sigma_2$; $\sigma_3 \to \sigma_3$.

Last three versions correspond to the case, when system remains in the initial position. (As an result of a battle with the enemy, the ending position keeps; but this retention requires an expense of resources.)

If as the alternatives two - step trajectories (*r* = 3), i.e., passages of the type $\sigma_l \to \sigma_j \to \sigma_k$ are examined, the number of versions equals to $3^3 = 27$. Among versions of trajectories with $p > 2$ there are those, that states can be repeated and trajectories with a great number of steps has all the passages „sections" with smaller number of steps, if we identify all trajectories of the form

$$\underbrace{\sigma_i \to \sigma_i \to \dots \to \sigma_i}_{s} \to \underbrace{\sigma_j \to \sigma_j \to \dots \to \sigma_j}_{p-s}$$

with the trajectories $\sigma_i \to \sigma_j$ with $1 \le s \le (p-1)$, i.e., to consider them equivalent. „*Trajectory*" or „*way*" is a component to a „*strategy*" determination. While studying „ways" temporal aspect is not considered. The subject has an ability to establish preference relations on the set „*of trajectories*". To the whole totality of equivalent „*trajectories* " a total „weight" will be assigned - the value of the preference function of only alternative (in the given example - alternative trajectories $\sigma_i \to \sigma_j$).

Conditions of normalizing of the *p - point* function of preference in this case take the form:

$$\underbrace{\sum_{i=1}^{N}\sum_{j=1}^{N}\cdots\sum_{t=1}^{N}}_{p} \pi\left(\sigma_i, \sigma_j, \ldots, \sigma_t\right) = 1. \tag{2.20}$$

Quantity of different indexes *i, j,…, t* is equal *to p*. In special cases *p*=2 and p=3 we have:

$$\left.\begin{aligned} &\sum_{i=1}^{N}\sum_{j=1}^{N}\pi\left(\sigma_i, \sigma_j\right) = 1; \\ &\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{n=1}^{N} \pi\left(\sigma_i, \sigma_j, \sigma_k\right) = 1. \end{aligned}\right\}. \tag{2.21}$$

Arguments of the function $\pi\left(\sigma_i, \sigma_j, \ldots, \sigma_t\right)$ are arranged in the order, in which a motion along the trajectories occurs. Indexes *i*, *j*, *k*, …, *t* designate sequential positions along the trajectories, and simultaneously the number of an alternative state. If set S_a is a continuum in the sense that σ_i ($i \in \overline{1,N}$) is a real variable which takes values in the region of S_a *N-dimensional* real space and integral is determined on this set, then the analog of normalizing conditions of the preference function density $\pi\left(\sigma_i, \sigma_j, \ldots, \sigma_t\right)$ is

$$\underbrace{\int_{(S_a)}\int_{(S_a)}\cdots\int_{(S_a)} \pi\underbrace{\left(\sigma_i, \sigma_j, \ldots, \sigma_t\right)}_{p}}_{p} d\sigma_i\, d\sigma_j \ldots d\sigma_t = 1 \quad . \tag{2.22}$$

This should be understood in such a way those steps along „*the way*" are served discretely but each time a new state is selected from the continuous set.

At the beginning it is possible to accept different conditions of normalization, for example, to exclude a repetition of states „*along the trajectories*". Then after each step the number of alternative states (cardinal number of set S_a) shall decrease by one. The „journey" will occur in sets of decreasing size:

$$S_a^N \to S_a^{N-1} \to S_a^{N-2} \ldots \to S_a^{N-p}.$$

The corresponding conditions of normalization take the form:

$$\sum_{i=1}^{N}\sum_{j=1}^{N-1}\ldots\sum_{t=1}^{N-(p-1)} \pi\underbrace{\left(\sigma_i,\sigma_j,\ldots,\sigma_t\right)}_{p} = 1 \quad . \tag{2.23}$$

In the particular case, for p = 2 and p = 3:

$$\left.\begin{aligned} &\sum_{i=1}^{N}\sum_{j=1}^{N-1}\pi\left(\sigma_i,\sigma_j\right) = 1; \\ &\sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{k=1}^{N-2}\pi\left(\sigma_i,\sigma_j,\sigma_k\right) = 1. \end{aligned}\right\} \quad . \tag{2.24}$$

It is possible to imagine the case, when a length of a path p is greater than a number of simple alternatives in S_a: $p > N$, then obviously, that some states will be repeated. Thus, if $N = 2$, and path length $p = 3$, then the number of different variants (alternative trajectories) $N^p = 2^3 = 8$.

2.4. Hypothesis of factorization

In the probability theory of a following statement comes out from axioms adopted. It is their consequence. In the stated theory of preferences, it starts as hypothesis and plays the role of particular axiom, which either can or cannot be valid.

Let's assume, that the function of preferences of a one-step trajectories $T_r^{(1)} = \left(\sigma_i,\sigma_j\right)$ can be presented in the form of a product

$$\pi\left(\sigma_i,\sigma_j\right) = \pi\left(\sigma_i\right)\pi\left(\sigma_j \mid \sigma_i\right), \tag{2.25}$$

where $\pi(\sigma_i)$ is the function of absolute preferences, and $\pi(\sigma_j|\sigma_i)$ is the function of conditional preferences, appearing after the system turn into the state σ_i. For the function of preference of two step trajectories $T_r^{(2)}$ (p = 3) following property is postulated:

$$\pi\left(\sigma_i,\sigma_j,\sigma_k\right) = \pi\left(\sigma_i\right)\pi\left(\sigma_j \mid \sigma_i\right)\pi\left(\sigma_k \mid \sigma_i,\sigma_j\right) \quad . \tag{2.26}$$

Generally, for the trajectories of length p (T_r^{p-1}) with „visit" p – 1 intermediate state we will consider a following formula as a proper one:

$$\begin{aligned}&\pi\left(\sigma_{s1},\sigma_{s2},\sigma_{s3},\ldots,\sigma_{s(p-1)},\sigma_{sp}\right)=\\&=\pi\left(\sigma_{s1}\right)\pi\left(\sigma_{s2}\mid\sigma_{s1}\right)\pi\left(\sigma_{s3}\mid\sigma_{s1},\sigma_{s2}\right)\ldots\pi\left(\sigma_{sp}\mid\sigma_{s1},\sigma_{s2}\ldots\sigma_{s(p-1)}\right)\quad.\end{aligned}\tag{2.27}$$

The sense the factorization hypotheses means that after „the visit" of the sequential state the preferences are redistributed (more precisely they can be redistributed) depending on already passed steps.

Among all factorized trajectories the class of „*Markov's trajectories"* is separated and the factorization is called „*The Markov's factorization*". It means that the conditional distribution of preferences on the next step depends on only from the state, in which the system is located at the given moment and it does not depend on prehistory at all - the preference of each alternative on next step along the trajectories is determined only by the last realized alternative (by state, in which the system is located before an accomplishment of the next step) and it does not depend on already passed states. This fact is expressed by the following relation:

$$\pi\left(\sigma_{sk}\mid\sigma_{s1},\sigma_{s2},\ldots,\sigma_{sk-1}\right)=\pi\left(\sigma_{sk}\mid\sigma_{sk-1}\right).\tag{2.28}$$

Here σ_{sk} is the state, reached after $k-1$ step „along" the trajectories. Then for step $(p-1)$ of way, we obtain:

$$\pi\left(\sigma_{s1},\sigma_{s2},\ldots,\sigma_{sk-1},\sigma_{sp}\right)=\pi\left(\sigma_{s1}\right)\pi\left(\sigma_{s2}\mid\sigma_{s1}\right)\pi\left(\sigma_{s3}\mid\sigma_{s2}\right)\ldots\pi\left(\sigma_{sp}\mid\sigma_{sp-1}\right).\tag{2.29}$$

The assumption, expressed by the formula (2.29) is practically the assumption of specific nature of subject psyche - method of his analysis „*without looking back*". Figuratively speaking, this subject „*looks only forward*" and doesn't take into account prehistory.

In accordance with the definition of the concept of trajectory, the motion along the trajectories occurs only in one direction:

$$\sigma_{s1}\to\sigma_{s2}\to\sigma_{s3}\to\ldots,$$

Generally speaking, the transposition of arguments in *the p- point* function of preference changes the function. Consequently, for example for the 2-point function $\pi(\sigma_i, \sigma_j)$ in general case, we have

$$\pi(\sigma_i,\sigma_j)\neq\pi(\sigma_j,\sigma_i).\tag{2.30}$$

However, in particular case, with the validity of the hypothesis of factorization, equality $\pi(\sigma_i, \sigma_j) = \pi(\sigma_j, \sigma_i)$ can occur, then

$$\pi(\sigma_i)\,\pi(\sigma_j|\,\sigma_i)=\pi(\sigma_j)\,\pi(\sigma_i|\,\sigma_j).\tag{2.31}$$

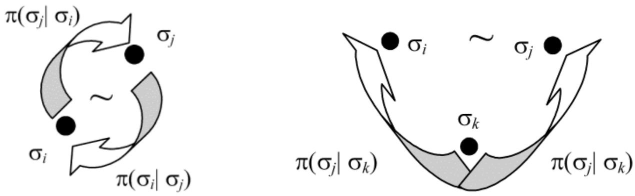
This means that trajectories $\sigma_i \to \sigma_j$ and $\sigma_j \to \sigma_i$ are equivalent. The equivalence of trajectories does not coincide with the equivalence of states, since trajectories are assigned on the Cartesian product $\underbrace{S_a \times S_a \times \ldots S_a}_{p}$.

We can see that in the case of the factorization hypotheses fulfillment two states σ_i and σ_j are absolutely equally preferable if and only the condition $\pi(\sigma_i | \sigma_j) = \pi(\sigma_j | \sigma_i)$ is fulfilled. In this case $\pi(\sigma_i) = \pi(\sigma_j)$. Together with absolute equivalence (~) it is possible to examine conditional equivalence. We will indicate that two states σ_i and σ_j are conditionally mutually equivalent (Fig. 2.3, *a)*, if

$$\pi(\sigma_i | \sigma_j) = \pi(\sigma_j | \sigma_i) \tag{2.32}$$

and is conditionally equivalent relatively to the third state σ_k (Fig. 2.3, *b)*, if

$$\pi(\sigma_i | \sigma_k) = \pi(\sigma_j | \sigma_k).$$



a *b*

Fig. 2.3.

These definitions can be easily applied to other conditional distributions. Let's examine normalizations of conditional preference function.

For each collection of pairs $(\sigma_i, \sigma_j) \in S_a \times S_a$ the normalizing condition is satisfied

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \pi\left(\sigma_i, \sigma_j\right) = 1$$

and let

$$\sum_{j=1}^{N} \pi\left(\sigma_i, \sigma_j\right) = \pi\left(\sigma_i\right) \quad . \tag{2.33}$$

If the hypothesis of factorization is true, the previous equality can be written in the form

$$\sum_{j=1}^{N} \pi\left(\sigma_i\right) \pi\left(\sigma_j \mid \sigma_i\right) = \pi\left(\sigma_i\right) \sum_{j=1}^{N} \pi\left(\sigma_j \mid \sigma_i\right) = \pi\left(\sigma_i\right),$$

since by the value of formula (2.7) $\sum_{j=1}^{N} \pi\left(\sigma_j \mid \sigma_i\right) = 1.$

Let's designate

$$\sum_{j=1}^{N} \pi\left(\sigma_i,\sigma_j\right) = \pi\left(\sigma_j\right),$$

then with condition (2.31)

$$\sum_{i=1}^{N} \pi\left(\sigma_i\right)\pi\left(\sigma_j \mid \sigma_i\right) = \pi\left(\sigma_j\right) \quad . \tag{2.34}$$

It is possible to interpret that $\pi(\sigma_i)$ is a priori distribution of absolute preferences before the passage $\sigma_i \to \sigma_j$, and $\pi(\sigma_j)$ is a posterior distribution of absolute preferences after passage $\sigma_i \to \sigma_j$.
Let, for example for four alternatives, σ_1, σ_2, σ_3, $\sigma_4 \in S_a$ a priori absolute distribution of preferences appears as following one:

i	1	2	3	4
$\pi(\sigma_i)$	0,1	0.2	0,3	0,4

and conditional distribution $\pi(\sigma_j | \sigma_i)$ is assigned by the tables 2.1

Table 2.1

	№№	σ_i			
		1	2	3	4
σ_j	1	0,1	0.2	0,3	0,4
	2	0.2	0	0,1	0,3
	3	0,3	0.2	0	0,4
	4	0,4	0,4	0,4	0
Σ		1,0	1,0	1,0	1,0

Posteriori preferences in accordance with formula (2.34) are following:

j	1	2	3	4	Σ
$\pi(\sigma_j)$	0,35	0,17	0,23	0,25	1,0

Conditional functions of the preference of more general have form:

$$\pi(\underbrace{\sigma_i,\sigma_j,\ldots,\sigma_k}_{p_2} \mid \underbrace{\sigma_1,\sigma_2,\ldots,\sigma_{k-1}}_{p_1}) = \pi\left(Tr^{p2} \mid Tr^{p1}\right), \tag{2.35}$$

that determines the preferability of future route segment Tr^{p2} depending on the already passed route segment Tr^{p1}.

Let's assume that the number of alternatives, studied by the subject, changes after each step. For the Markov's trajectories with the length p a following dependence occurs:

$$\begin{array}{cccccc} \sigma_{s1} \rightarrow & \sigma_{s2} \rightarrow & \sigma_{s3} \rightarrow & \ldots & \rightarrow \sigma_{s1(p-1)} & \rightarrow \sigma_p \\ \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow \\ N_1 & N_2 & N_3 & \ldots & N_{p-1} & N_p \end{array}$$

The initial state is selected from N_1 alternatives $\sigma_{s1} \in S^{N1}{}_a$, state σ_{s2} is selected from N_2 alternatives $\sigma_{s2} \in S^{N2}{}_a$, ... then normalizing condition takes the following form:

$$\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}\ldots\sum_{i_p=1}^{N_p}\pi(\sigma_{i1})\pi(\sigma_{i2}\mid\sigma_{i1})\pi(\sigma_{i3}\mid\sigma_{i2})\ldots \quad (2.36)$$

$$\ldots\pi\left(\sigma_{i(p-1)}\mid\sigma_{i(p-2)}\right)\pi\left(\sigma_{ip}\mid\sigma_{i(p-1)}\right)=$$

$$=\sum_{i_1=1}^{N_1}\pi(\sigma_{i1})\sum_{i_2=1}^{N_2}\pi(\sigma_{i2}\mid\sigma_{i1})\sum_{i_3=1}^{N_3}\pi(\sigma_{i3}\mid\sigma_{i2})\ldots\sum_{i_p=1}^{N_p}\pi\left(\sigma_{ip}\mid\sigma_{i(p-1)}\right)=1 \quad .$$

This condition is fulfilled since each of sums is individually equal 1. The concept of the composition of alternatives, in particular *binary composition* $\sigma_i \wedge \sigma_j$, which corresponds to the complex alternative, that consists of the realization of simultaneously both alternatives was already discussed in previous chapter 1.4 and 2.6. Together with the binary composition the binary disposition $\sigma_i \vee \sigma_j$, which was equivalent to the complex alternative, which foresees three possibilities: realization only σ_i, realization only σ_j, or realization σ_i and σ_j (i.e., composition), was examined.

However, in this case, all possible binary compositions must be included in product $S_a \times S_a$ with an appropriate enumeration and be taken under consideration with the normalization.

2.5. Analogy to distributions of probabilities. About the algebra of alternatives.

It was already mentioned that distributions of subjective preferences have a lot of common properties with probabilistic distributions. Many constructions are analogical; however, this analogy bears formal nature, since the sense of preferences and probabilities is various, and distributions on one of the same set alternatives can radically differ, moreover, - one of them cannot exist, whereas another one exists.

In this paragraph, we will examine some formal analogies and similarities with simultaneous consideration of differences. Furthermore, introduce additional important concepts shell.

Let's take S_a is set of alternatives (property of this set we do not define concretely), A, are subset S_a, name alternatives (or complex alternatives) in contrast to simple alternatives and the compositions, we considered before:

$$A \subset S_a.$$

A complete or universal alternative is U = S_a. There is an alternative $\bar{A}$ additional to A, such, as the sum $A \vee \bar{A} = S_a$ or $S_a \setminus A = \bar{A}$.

Assumption

1. Alternatives *are divided*: they contain subsets, which are alternatives.
2. There is an „*impossible*" alternative $\varnothing$ (or *V*).
3. There is a sum of alternatives $A \vee B$, which consists in the fact that at least one of them can be realized. For example, $A \vee \bar{A} = S_a$.

A difference of *examined complex alternatives* from previously introduced *compositions* consist of the fact that in the first case reaching of all *complex alternative* states is not assumed, but at least one of $\sigma \in A \vee B$. In this respect the definition of complex alternative is analogical to the definition of event in the probability theory.

4. There is a subjective „*product*" *C* of alternatives *A* and *B* that represents the general part „of cofactors", if *A* and *B* are divisible: $A \wedge B$.

Alternatives *A* and *B*, are subjectively *incompatible*, if $A \wedge B = V$ — impossible alternative.

Elementary alternatives $E_i \subset S_a$ compose a finite set of subsets S_E and satisfy conditions: $E_i \wedge E_j = V = \varnothing$ for $\forall E_i, E_j \in S_E$ ($\varnothing_s$ - „*subjective zero*") and $\bigvee_{i=1}^{N} E_i = U = S_a$.

Non-empty set S_a is such, that:

1. From $A \subset S_a \Rightarrow \bar{A} \subset S_a$; (2.37)
2. From $\forall A, B \subset S_a \Rightarrow A \vee B \subset S_a$.

S_a contains an impossible alternative *V*.

The algebra of a set of alternatives S_A is determined by conditions: let S_A is a set of subsets S_E, where S_E is global alternative and following conditions are satisfied

1. $S_E = U \in A_A$; (2.38)
2. $\varnothing = V \in A_A$;

3. $A \in S_A \Leftrightarrow \bar{A} \in S_A$, were $\bar{A} = S_a \setminus A$;

4. From $A \in A_A$; $B \in A_A \Rightarrow (A \vee B \in A_A; A \wedge B \in A_A)$.

It is necessary to keep in mind, that operations „$\wedge$" and „$\vee$", a concept „compatibility" and „incompatibility" are determined on a subjective level. Generally, all categories in this case have dual nature. It concerns concepts of „compatibility" and „incompatibility". Two alternatives can be compatible in „physical" or more precise, in objective sense, what must be reflected through an objective characteristics - a function of utility , being incompatible in the subjective sense at the same time (for example, for ethical reasons and for the concrete subject). The latter one must be reflected in the preferences distribution of alternatives.

Let's note that set of subjective alternatives S_a cannot coincide with a set of objective alternatives, O_a. For example, before Russia's baptism of prince Vladimir there were many subjective alternatives: orthodoxy, Judaism, Islam, which did not have direct objective analogs.

In this work, we will not investigate problems connected with consequences, which follows from the non-coincidence of dimensionality and semantic content of sets S_a and O_a. We will subsequently consider that power of both sets coincide and coincide „names" of particular alternatives coincide as well. However, together with subjective operations „$\wedge$ "and „$\vee$ ", we will use objective operations „$\cup$" and „$\cap$" for the designation of objective addition (for example, resources, areas, ...) and objective multiplication.

Alternatives $A, B \in S_A$ are objectively incompatible, if $A \cap B = \varnothing$ and are compatible, if $A \cap B \neq \varnothing$. If as the function of utility $U(\sigma)$ (or $U(A)$; $A \in A_\sigma$) probability will be used, then entire further terminology and axiomatic completely coincide with the probabilistic.

In the majority of cases, the function of utility is not normalized, and in this case, also differences do appear.

We assume that sets A_σ and O_σ coincide: $A_\sigma \equiv O_\sigma$.

In connection with the aforesaid let's introduce the additional assumptions:

5. Necessary condition of the subjective „*impossibility*" of alternative is its objective „*impossibility*":

$$V = A \cap \bar{A} = \varnothing_0 \text{ (objective zero).}$$

6. Necessary condition of the subjective *incompatibility* of alternatives *A* and *B* is objective *incompatibility* ($\varnothing_0$). Thus

$$A \cap B = \varnothing_0 \Rightarrow A \wedge B = \varnothing_s. \tag{2.39}$$

Properties of compatibility and incompatibility can have one-sided nature both in an objective and in a subjective sense, for example, - temporary irreversibility of events (irreversibility in time) as well as substantive irreversibility of a sense temporal some forms of resources in others in principle - bread cannot be converted as wheat).

It is possible to imagine another situation, when subject, in spite of „physical" objectively no feasibility of some alternative in view of let's say insufficient information, considers it attainable and assigns to it correlation nontrivial preference.

Together with properties of practicability (reliability) and compatibility (respectively – no feasibility, incompatibility) it is necessary to discuss a concept of dependence and independence of alternatives again on an axiomatic level. A circumstance that under specific conditions on the set S_a (or A_σ) some order can be established with help of preference relation, it doesn't mean yet that particular alternatives are dependent on each other.

We will indicate that two alternatives A, B $\in S_a$ are dependent on each other, if realization of one of them influences a value of the preference of another and none of alternatives is impossible. In this case, it is certainly assumed that conditions that ensure a possibility of an introduction of a quantitative scale of preferences are fulfilled. As we know, it's not always possible.

We will use following designations:

$A \rceil B$: «*A* does not depend on *B*» $\Rightarrow \pi(A|B) = \pi(A)$;
$\underline{A} \lceil B$: «*A* depend on *B*» $\Rightarrow \pi(A|B) \neq \pi(A)$;
$A \lfloor B$: «*A* and *B* are mutually independent» $\Rightarrow \pi(A|B) = \pi(A)$; $\pi(B|A) = \pi(B)$;
$\underline{A} \lceil B$: «*A* and *B* are mutually dependent» $\Rightarrow \pi(A|B) \neq \pi(A)$; $\pi(B|A) \neq \pi(B)$.

We will distinguish a *temporary* and a *substantive* dependence.

The temporally dependence of alternatives $A, B \in S_A$ (or $\sigma, \eta \in S_a$) means that they can be carried out only in a determined order.

Example: alternative η - study of aerodynamics, alternative σ - study of mathematics $\Rightarrow \eta \lceil \sigma$ in time.

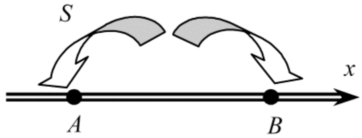
The substantive dependence of alternatives $A, B \in S_A$ (or $\sigma, \eta \in S_a$) means that *A* and *B* (σ, η) depend on general resources, prices, they have three-dimensional dependence.

Example: alternative is in acquisition of product *A*, alternative *B* is an acquisition of product *B*. There is a condition that if the product *A* will be acquired, then with a purchase of product *B* a reduction will be made. This means that $B \lceil A$ — a substantive dependence.

An incompatibility of alternatives $A, B \in S_A$ draws their *independence*, but *independence* not necessarily draws *incompatibility*.

Let's give some examples.

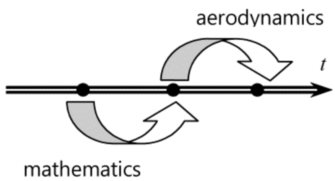
1. Subject simultaneously cannot be in a point *A* (alternative *A*) and in a point *B* (alternative *B*).



Example 1

Alternatives *A* and B are substantively inconsistent and independent.

2. An athlete on certain competitions wins the 1-st prize - alternative *A*, and also wins the 2nd prize - alternative *B*. *A* and *B* in a temporary sense are inconsistent and independent.
3. A student intends to study a *subject A* (alternative *A*) and a *subject B* (alternative *B*). If subjects are not logically connected and resources of time are sufficient, then alternatives *A* and *B* are compatible, but they are not connected (*A* ⎣*B)*.
4. An alternative *A* is mathematics, an alternative *B* is aerodynamics. It is clear that studying of aerodynamics requires preliminary acquaintance with mathematics. *A and B* are compatible and, in the temporally sense, one-side dependant *B*⎡*A*.



Example 2

5. *A* is alternative to build a house; a B is alternative to get a piece of land. *A and B* are compatible and in the time one-sidedly connected. Analogously it is not possible to construct a roof of a house before building walls.
6. An alternative *A* is to organize a production of an article *A*, an alternative B is a production *of article B*. If articles *A* and B have common parts, their joint production can prove to be more advantageous than a separate one.

2.6. Axiomatic approach to preference functions

An axiomatic construction of theories of preference functions apparently can be executed by different methods.

If we do not previously assume an existence of ideas about the collection of elementary (incompatible and independent) alternatives in subject's mind, which certainly reduces level of requirements at the point of to the possibilities of analytical abilities of subject, then the preferences function could be introduced by following axioms:

1. A subject can form set of alternatives S_A (not compulsorily elementary).
2. On this set he can determine a binary relation of preferences for each pair of alternatives:

$$\forall A, B \in S_A: A \langle B,$$

having features provided by theorem 2 (or 4), paragraph 1.3.

3. There exists a function π (.) such, that

$$\left.\begin{array}{c} 1 \geq \pi(A) \geq 0 \text{ for } \forall A \in S_A \quad , \\ \pi(U) = \pi(S_A) = 1 \quad , \\ \pi(V) = 0 \quad . \end{array}\right\} \tag{2.40}$$

If follows from these axioms, that:

$$\left.\begin{array}{c} 1. \text{ If } A \wedge B = \varnothing : \forall A, B \in S_A, \text{ then } \pi(A \wedge B) = 0; \\ 2.\ \pi(\bar{A}) = 1 - \pi(A); \\ 3. \text{ For } \forall A, B \in S_A : \pi(A \vee B) \leq \pi(A) + \pi(B). \end{array}\right\} \tag{2.41}$$

Let's introduce some algebraic relationships for absolute preference function. In this case, these relationships, in contrast to the theory of probability, are postulated (when the presence of complete set of elementary alternatives it is not required).

A preference of a sum of two alternatives

$$\pi(A \vee B) = \pi(A) + \pi(B) - \pi(A \wedge B). \tag{2.42}$$

Let d is an income (or harvest) obtained from this „*field*". *A and B* respectively two „*fields*". Then it is possible to write

$$d(A \vee B) = d(A) + d(B) - d(A \wedge B),$$

which explains a sense of a formula „of addition" or preferences for the sum $A \vee B$.

Examine appearance and treatment of conditional preference function.

Let $A, B \in S_A$. Determine a preference „of product" of alternatives by the formula

$$\pi(A \wedge B) = \pi(A)\,\pi(B \mid A), \text{ if } B \lceil A \text{ and } \pi(A) \neq 0 \qquad (2.43)$$

or

$$\pi(A \wedge B) = \pi(B)\,\pi(A \mid B), \text{ if } A \lceil B \text{ and } \pi(B) \neq 0.$$

These relationships, just as in the general case, are postulated. We see that since $\pi(B| A) \leq 1$, then

$$\pi(A \wedge B) \leq \pi(A) \Rightarrow A \wedge B \precsim A, \quad (2.44)$$

i.e., *a preferability „of product" of two alternatives is always not more than a preferability of each alternatives separately - cofactors individually*.

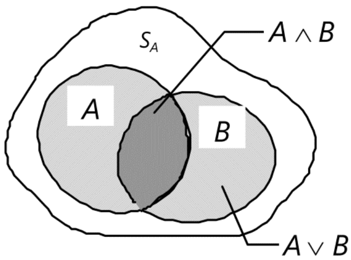


Fig. 2.4

Relationships can be treated as formulas for the preference of one-step trajectories $A \rightarrow B$ (or $B \rightarrow A$).

The function of conditional preferences in formula (2.43) determines preference *„to be located at least in a certain part B without leaving A"*.

If $A \wedge B = \varnothing = V$ (A and B are incompatible) $\pi(A \wedge B) = 0$. Since with this $\pi(A) \neq 0$, it leads to the condition

$\pi(B| A) = 0$, i.e., the preferability of alternative B when alternative A it must be realized (or is already realized), is equal to zero.

A simultaneous validity of both relationships (2.43) is a particular assumption, from which it follows that

$$\pi(A)\pi(B| A) = \pi(B)\pi(A| B). \qquad (2.45)$$

A condition $A \lceil B$ corresponds to it. From (2.45) we can find

$$\frac{\pi(A)}{\pi(B)} = \frac{\pi(A \mid B)}{\pi(B \mid A)} \quad .$$

It is evident that from inequality

$$\pi(A) < \pi(B)$$

it follows that $\pi(B|\ A) > \pi(A|\ B)$, hence, in its turn preference relation of two *„one-step"* routes (trajectories) follows

$$T_{AB}^{(1)}:\left(A\to B\right)\rangle T_{BA}^{(1)}:\left(B\to A\right).$$

The preference of product $\pi(A \wedge B)$ is being treated as the preference of realization of part *A*, either part B, it is common for *A* and B consecutively or simultaneously . Because it is natural to consider that, the preference „of a part" is always less than the preference of a whole, then

$$\pi(A \wedge B) \le \pi(A) \quad \text{and} \quad \pi(A \wedge B) \le \pi(B).$$

Thus we obtain

$$A \wedge B \langle A; \quad A \wedge B \langle B.$$

Reasoning is given above are easily transferred to the case of bigger number of alternatives than 2: $N > 2$.

2.7. About preference functions for a discrete set of alternatives S_a

Let S_a consists of finite set of alternatives $\sigma_1, \sigma_2, \ldots, \sigma_N$, moreover, on S_a a binary relation of preference ρ: $\langle$ is assigned so that following order (indexes correspond to an ascending order of preferences) can be established:

$$\sigma_1 \langle \sigma_2 \langle \sigma_3 \langle \ldots \langle \sigma_{N-1} \langle \sigma_N \tag{2.46}$$

moreover, let an alternative are not „physically" mutually stipulated. In this case, in principle any passages both „forward" and „back" along a sequence are permitted.

Alternatives $\sigma_i \in S_a$ can be compatible in pairs, or they can be incompatible. Alternatives σ_i we will name „simple". All other alternatives, which one way or another are formed with the help of simple alternatives, are complex (or composite) ones.

Let's examine some versions of complex alternatives.

1. Alternative *A* means that one simple alternative σ_i of a subset $S_a^K \subset S_a^N$. Will be fulfilled. In this case, it is unessential which of belonging to these subset simple alternatives.
2. Alternative A means that at least one of alternatives of subset $S_a^K \subset S_a^N$ ($K \in \overline{2, N-1}$) is fulfilled.
3. Alternative c^K consisting of the fact that, a group of *K* alternatives will be simultaneously realized. c^K is *composition of alternatives*.
4. Alternative $T_r^{(K)}$ consisting of the fact that certain chain of simple alternatives of length *K* will be consecutively fulfilled, $T_r^{(K)}$ - *trajectory alternatives* (*trajectories*).

5. Alternative $S_t^{(K)}$ consisting of the fact that a trajectory „by the length" K (containing K steps), at each step of which a realization of a composition of a certain dimensionality (this case also was discussed above) is assumed, will be fulfilled.

A composition assumes a realization of all alternatives included. This situation is more preferable than a realization of only part of them. However, as it was already said, not all compositions were accessible, i.e., were provided by located resources; therefore, not all alternatives are included in an appropriate set of alternatives. Due to conditions of mutual incompatibility $\sigma_i \in S_a$

$$\pi\left(c_s^K\right) = \pi(\sigma_{s1} \vee \sigma_{s2} \vee \ldots \vee \sigma_{sk}) = \pi\left(\sigma_{s1}\right) + \pi\left(\sigma_{s2}\right) + \ldots + \pi\left(\sigma_{sk}\right). \qquad (2.47)$$

In the other case, when the discussion deals with a trajectory $T_r^{(K)}$, frequently it is possible to assert that, the longer trajectories the less it is preferable. Under conditions of an incompatibility of simple alternatives, following a formula corresponds to this heuristic preference:

$$\pi(Tr^K) = \pi(\sigma_{s1} \wedge \sigma_{s2} \wedge \ldots \wedge \sigma_{sk}) = \pi(\sigma_{s1})\pi(\sigma_{s2}) \cdot \ldots \cdot \pi(\sigma_{sk}), \qquad (2.48)$$

since for $\forall S_i \in \overline{1,K}$ π (σ_{Si}) < 1. If is evident that a preferability of trajectories $T^{(K)}$ will be the higher, the higher is a preferability „of intermediate", passable states (alternatives) during motion to the final alternative σ_{sk},. In particular, if the preferability of at least one intermediate alternative is equal zero, then preferability of the entire trajectories is equal zero too.

A quantity of trajectories of a complete order Tr^N (K = N without repetitions and returns) is equal M = N!, if all passages both „forward" and „back" are permitted.

If passages are permitted only to one side, then there is only one trajectory.

If trajectories of the length $K \in \overline{2, N-1}$, are possible, but only „forward", i.e., in a direction of an increasing preferability (without „delays"), a number of versions is:

$$M = C_N^K = \frac{N!}{K!(N-K)!}.$$

If there is isolated „purpose", i.e., final element $\sigma_f \in S_a$ is fixed, then the number of trajectories with the length K of those leading to the element σ_f is equal

$$M = \frac{(N-1)!}{K!(N-1-K)!}.$$

A quantity of compositions of order K: c^k is equal

$$M = C_N^K = \frac{N!}{K!\left(N-K\right)!}.$$

General case is alternatives that include both trajectories and compositions. We conditionally call such alternatives strategies

$$S_t : (\underbrace{c^{k1} \to c^{k2} \to \ldots \to c^{kL}}_{L}).$$

Then

$$\pi\ (S_{tr}) = \pi\ (c^{k1} \wedge c^{k2} \wedge \ldots \wedge\ c^{kL}).$$

During a detailed interpretation of this formula, both formulas „of multiplication" and a formulas „of addition" of preferences are used.

The subjectivity of given concepts, consists, in particular, in the fact that different subjects have individual features: different resolution - maximum number of simultaneously studied alternatives, ability „to decompose" alternatives - to differentiate preferences. In groups of subject, these differences must play role during aggregation of preferences, utilities, resources, selection of corporate problems.

2.8. Elements of theory of categories and its application to an analysis of active systems

2.8.1. Introductory remarks

With formulation of variational problems models of preference functions for search a question about a substantiation of the variation principle raises, - what is the source of ideas about structure of optimizable criteria.

One of the possibilities to raise to higher level of generalization lies at the theory of categories, where the principle of optimality is postulated, from which, as special cases, variational problems of information theory [137,138] can be obtained.

An assumption, that following Euler, each time in surrounding environment and in us ourselves in that number, a sense of some maximum or minimum is visible makes it possible to use this circumstance as tool for models construct, obtaining conclusions, and so-called „laws of conservation".

A classical mechanics gives examples for it. Thus, mechanics of conservative systems follows from the principle of Hamilton action minimum. It is possible to show that it is correct for some dissipation systems, if we modify an expression for action. Variation principles occur in a quantum mechanics, a relativistic quantum electrodynamics, in space theories. In an information theory, a number of variational problems [137] are examined, as it was already mentioned. In biology a principle of an optimal structure of organisms, „the principle of maximal reproductive potential", is used.

In conditions of insufficient actual information, experimental data or an impossibility to carry out a straight experiment, we can sometimes go along way of a priori postulation of some general principle, to obtain with its help quantitative models of an interesting phenomenon and to compare results of simulation in a final analysis with already known data.

However, an inverse problem occurs. It consists of the fact that a mathematical model of phenomenon on basis of processing experimental data is built, and then a functional, for which a mentioned model is a steady-state solution [115] is restored.
The principle of optimality, postulated in the theory of categories, is more general and „primary" with according to variation principles in particular fields of sciences, which can be interpreted in terms of the categories theory. This principle says, *that those states of systems are achieved, whose [ecvistrukture] is extreme, in other words*: systems with an extreme invariant are preferable or, which is equivalent, with an extreme (maximum) number of conversions (morphism), changing a state, but preserve a structure of a system.

In this chapter, some basic concepts of the categories theory are presented, following publication [96].

2.8.2. The definition of category

A category K is a totality of two classes: a class of objects (Obj j) and a class of morphism (conversions Mor K), for which are following states:
1. To each pair of objects A, $B \in$ Obj K corresponds a certain set of morphisms $H_k(A, B) \in$ Mor K.
2. Each morphism belongs to one and only one of sets $H_k(A, B)$.
3. In one class of morphism Mor K a partial binary internal law of the composition is introduced: product $\beta \wedge \alpha$, where $\alpha \in H_k(A, B)$ and $\beta \in H_k(C, D)$, is determined then and only then, when an object B coincides with an object C. In this case the morphism is:

$$\alpha\beta = \alpha \wedge \beta \in \mathrm{H_k(A, D)}.$$

The composition of morphism is associative.

4. In each set of morphism $\mathrm{H_k(A, A)}$ an identical morphism $\mathrm{I_A}$ of an object A of such, that $\alpha\mathrm{I_A} = \alpha$, $\mathrm{I_A}\beta = \beta$ for $\forall\alpha \in \mathrm{H_k(X, A)}$ and $\forall\beta \in \mathrm{H_k(A, Y)}$, where A, X, Y $\in$ Obj K, is contained.
A subcategory L of a category K is called the region, for which
1. Obj $L \subset$ Obj K.
2. $H_L(A, B) \subset H_k(A, B)$.

3. Composition of morphism, which belongs to L is a composition of the same morphism from K.
4. The only morphism from L is the only morphism from K.

The categories theory examines structured sets and conversions of these sets, is „the force" of structures of structured sets is defined, cardinal structures of Boolean - marking sets [6], a concept of a quantity as the property of sets conversions, invariants of mathematical structures .

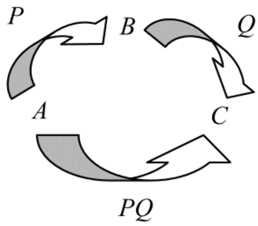
2.8.3. Correspondences and mapping

Following we considered, sins publication, relation between two sets A and B, used in a theory of categories, we will examine below.

Let's define the Cartesian product of sets A and B as totality of all ordered pairs (a, b); $a \in A$; $b \in B$ and let's designate it AxB.

A subset of product AxB is called *a correspondence S* from *A in B*.

A composition of correspondences P: $A \to B$ and Q: $B \to C$ let's designate PQ: $A \to C$.

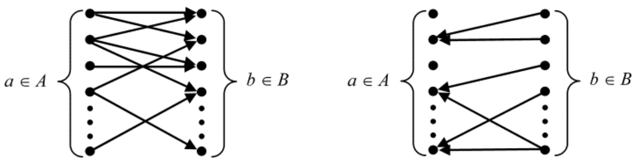


Correspondence S^*: $B \to A$ is combined S: $A \to B$ then and only then, when the following condition is fulfilled

$$(a, b) \in S \Leftrightarrow (b, a) \in S^*.$$

Correspondences can be of different type and can have following canonical properties:

1. S: $A \to B$ *is everywhere determined*, if $\forall a \in A$ has non- empty reflection $b \in B$. $\forall a \in A$ can have more than one reflection, but in A there are no elements (prototypes) that don't have reflections in B.
2. S: $A \to B$ *is surjective*, if $\forall b \in B$ has a prototype in S $a \in A$ (at least one).



a *b*

Fig. 2.5

3. *S*: *A* → *B is functional*, if ∀*a* ∈ *A* either has or hasn't the only reflection in *S b* ∈ *B*.

Figure show evident that some of the elements in *A* does not have reflection in B, rest *a* ∈ *A* have the only reflection. In this case one and the same element *b* ∈ *B* can be a reflection of several elements *a* ∈ *A*.

4. *S*: *A* → *B is injective*, if ∀*b* ∈ *B* either does not have prototypes or has the only prototype in A.

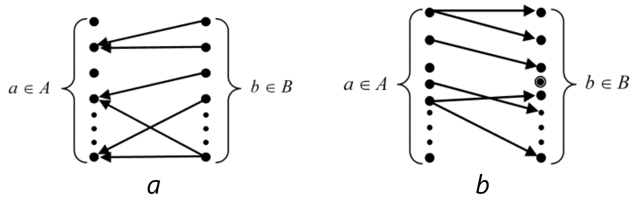


Fig. 2.6

The figure we can see that some elements *B*, including one of elements *B* (*), don't have prototypes in *A*; remaining elements *B* have only one prototype.

The following important concept is „*mappings*". Mappings are correspondences, allotted with more than one of properties enumerated above.

1. *a completely defined* and *functional correspondence is called „function*" or single-valued mapping.
2. a completely defined, functional and surjective correspondence is called „*surjective*" mapping or *surjection*.
3. a completely defined, functional and injective correspondence is called „*injective*" mapping or *injection*.
4. a mapping that has all described properties is called „ *bijective*" mapping or one-to-one mapping - *bijection*.

The concept of relation and analysis of binary relations are given in the chapter, dedicated to an account of the theory of utility based on a monograph of Fischbourn [149]. Relations do not assume conversion - morphism.

As it was already mentioned, diverse variants of ordering on set alternatives are examined.

Let's examine concepts „of tolerance" and „equivalences" of relations.

A tolerance T on *A* is called a reflexive and symmetrical relation. A subset *K* ⊂ *A* that has properties, is indicated us a class of tolerance.

A equivalence is defined as a reflexive symmetrical and transitive relation *E* (or ~) on *A*. If *E*- equivalence on *A*, then class *K* ⊂ *A* of equivalence on *E* consists of all elements σ ∈ *A E-equivalent*. A coating $K_{E\alpha}$ (α ∈ *R*) is a division *A* on classes of equivalence, if subsets $K_{E\alpha}$do not have general elements. The latter one is connected with a condition of transitivity of relation *E*.

Let's limit to the case, when A is finite, or countable. In this case, a quantity of classes of equivalence is either finite or countable. Division of A on classes of equivalence on E is designated also through A_E or $A_{\sim}$.

Assuming that the mapping f is assigned: $A \to B$. a relation E_1 on A is an equivalence, if from $f(\eta) = f(\xi)$, η, $\xi \in A$ belonging $(\eta, \xi) \in E_f$ follows. The set of all elements E_f is called the nucleus of mapping f.

2.8.4. Cardinal structure of sets, cardinal numbers, invariants

Boolean is a set of A degrees, *the* totality of all subsets A – P(A) = 2^N.

Set A and B are equivalent on definition if there exists a biection f: $A \leftrightarrow B$. If P(U) is Boolean of universe U - set of sets, a relation ρ on P(U): „a *set A* is *equivalent to a set B*" is an equivalence since it is reflexive, symmetrical and transitive: $P = \sim (E)$. An equivalence ~ (or *E) „sets A and B are equivalent*" is determined on Boolean $P(U)$ by a factor- set $P/\rho = P/\sim$.

Elements of a factor- set of Boolean $P(U)$ with accordance to „equivalence" are called *cardinal numbers*.

A cardinal number of a set A — Card A is called class from the factor- set $P/\sim$ of Boolean $P(U)$ with respect to an equivalence, $A \subset U$.

A cardinal number is equal to power $\mu(A)$ for sets with a define number of elements N

$$\text{Card } A = N.$$

Set A is infinite, if there is a subset $A' \subset A$ different from A and heaving the same cardinality $A_N = N$.

Theorem 1

Let *the* set and $P(A)$ is a set of all subsets A. Sets A and $P(A)$ are nonequivalent:

$$\text{Card } P(A) \neq \text{Card } A .$$

Depending on the type of mappings an ordering of cardinal numbers is achieved. Thus, if there is an injection from A in B, then $\mu(A) \leq \mu(B)$, where μ — power, and Card $A \leq$ Card B.

Let's examine types and features of morphisms. In the theory of categories *a monomorphism* and *epimorphism* are determined.

Morphism U: $A \to B$ of a category K is called monomorphism, if for $\forall X \in$ Obj K and $\forall \alpha, \beta \in$ Mor K achieving a mapping $X \to A$, a following condition is fulfilled: from an equality of compositions $\alpha U = \beta U$ it follows $\alpha = \beta$.

A meaning of this definition will become clear, if we recall a definition of a composition of morphism. We can see that, if compositions of morphism αU and βU lead to the same result, then morphism α and β must be identical.

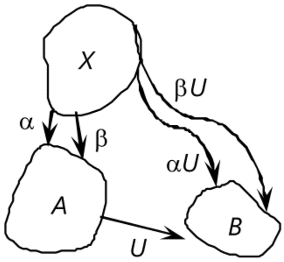


Fig. 2.7

Morphism U: $A \to B$ of the category K is called an epimorphism, if for $\forall Y \in \text{Obj } K$ and $\forall \alpha, \beta \in \text{Mor } K$ achieving a mapping $B \to Y$, a following condition is satisfied: from an equality of compositions $U\alpha = U\beta$ an equality of morphism α and $\beta \Rightarrow \alpha = \beta$ follows.

Morphism U is called biomorfizm, if it is simultaneously a monomorphism and an epimorphism.

Morphism U: $A \to B$ of the category K is called isomorphism if there is a morphism V: $B \to A$ such, that a composition $VU = I_B$ and $UV = I_A$, where I_B and I_A identical morphism of sets in themselves, i.e. such, when $\forall a \in A$ are mapped into a, and $\forall b \in B$ are mapped into b.

Following assertions, whose proofs would be omitted, are true:

1. An injection is a monomorphism, suriection is an epimorphism.
2. Category isomorphism are its bimorphism.
3. If composition UV is monomorphism, then morphism U is a monomorphism.
4. A class of all objects and a class of all morphism of arbitrary category K compose subcategory.

Morphism U: $X \to A$ majorizes the morphism V: $Y \to A$, if there is a morphism ξ: $Y \to X$ such, that composition $\xi U = V$.

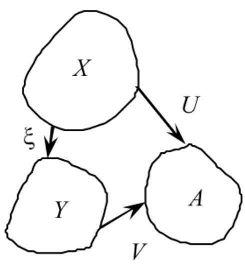


Fig. 2.8

Monomorphisms U and V, each of which majorizes another morphism, are called equivalent. Among equivalent monomorphisms U_i: $X_i \to A$ a sole representative $U: X \to A$ can be selected, which is called a canonical one. A region „of sending" of monomorphism U is called a sub-object A.

Functions are mappings, which are applied to a category, i.e., to a complex {Obj K, Mor K}.

„One-place" covariant functor from the category K to the category L is called mapping $F: K \to L$, for which following conditions are fulfilled:

1. For$\forall\alpha$: $A \to B$, where A, $B \in$ Obj K we have $F(A) \in$ Obj L.
2. For$\forall\alpha$: $A \to B$, where A, $B \in$ Obj K a condition $F(\alpha) \in H_L(F(A), F(B))$ is fulfilled.
3. For any „unit" $I_A \in K$ a relationship $F(I_A) = I_{F(A)}$ is fulfilled.
4. If a morphism $\alpha \in H_K(A, B)$ and a morphism $\beta \in H_K(B, C)$, then $F(\alpha\beta) = F(\alpha)F(\beta)$, i.e., a functor of a composition („product") of morphisms is composition of functors from „coefficients" - morphisms α and β.

Invariants of structures are an important concept of the theory of categories. They can be considered as one of methods of generalizing the concept of a quantity.

The role of invariants on conversions set was determined, particularly, in the work Kolmogorov A.N. [75].

A detailed and deep study of invariants of morphisms in connection with a concept of entropy as measures of a priori uncertainty of an assumed experiment is shows in the monograph [149].

Basic theoretical complexities are here connected with very general assumptions about properties of alternatives set, particularly about their power and power of factor- sets. It is clear, that in an application to the subjective analysis, such high generality can turn out to be excessive in a view of the finite „resolution" of subject - quantity of simultaneously studied alternatives.

2.8.5. Invariants and variation principle in the theory of categories

An invariant $I(A|\, X)$ of object A relative to object X is called a cardinal number of the set of all morphisms from X in A. in other words, the invariant $I(A|\, X)$ is equal to the cardinal number of morphisms set on a correspondence S from X in A:

$$I(A|\, X) = \text{Card}\, H_S(X, A). \tag{2.49}$$

Let $\mu(A)$ is power of factor- set E_A (i.e. set of subsets of equivalence). μ *(a)* characterizes a number of such conversions (morphisms) of the system, which leave its structure constant, or actual possibilities of system, caused by its structure.

Complexity of a system C(A) is defined as a logarithm from μ *(a)*, i.e.:

$$C(A) = \log \mu(A). \tag{2.50}$$

In [96] a following example is examined.

Let $M = (a_1, a_2, \ldots, a_N)$ is a set of elements a_i, on which equivalent relation E_M (~) is assigned. Objects of the theory of categories are sets $(a_{i1}, a_{i2}, \ldots, a_{iN})$, while some of elements can be repeated. Number of times of appearances of an element $a_j \in A$ is $\varphi(a_j)$, and then measure $\mu(a_j)|_A$ can be described by a relationship

$$\mu\left(a_j\right)|_A = \frac{\varphi\left(a_j\right)}{N},$$

i.e. as frequency. Let $(K_1, K_2, \ldots, K_m)$ be a partition of M an classes of equivalence $(K_s \in M,\ s \in \overline{1,m})$ with respect relation E_M (~). Then $\varphi(K_s) = N\mu(K_s)$ — morphism.

Morphism U: $A \to A$ enters into a group of morphism G_A then and only then, when $\forall x \in A$ $x\ E_M U(x)$.

Between any systems of dimensionality N there is $N!$ morphism. Complexity in this case is determined by a formula

$$C\left(A\right) = \log\prod_{j=1}^{m}\left(N\mu\left(K_j\right)\right)! \tag{2.51}$$

Let number of structures be X, then

$$C\left(A\right) = \log XN! - \sum_{j=1}^{m}\log\left(N\mu\left(K_j\right)\right)! \tag{2.52}$$

After using the Stirling formula:

$$\log Q! \approx Q\left(\log Q - 1\right) + \ln\sqrt{2\pi Q}, \tag{2.53}$$

we will obtain:

$$C\left(A\right) \approx \log X - N\sum_{j=1}^{m}\mu\left(K_j\right)\log\mu\left(K_j\right). \tag{2.54}$$

With $X = 1$ we get on information, according to Rachewsky, if an equivalent relation E_M is assigned with a help of a graph or by an algebraic model on M. If, in this case, each element from M is equivalent only to itself, then a complexity of a model corresponds Shenon's information.

If U is a morphism: $A \to B$, then an *information* is connected with a morphism

$$\inf(U) = I(U) = C(B) - C(X), \tag{2.55}$$

where X is invariant set relatively to morphism U. Together with an information co-information is introduced

$$\mathrm{coinf}(U) = CI(U) = C(A) - C(X). \tag{2.56}$$

A following theorem occurs: if morphism U: $A \to B$ - co-information, then $C(A) \geq C(B)$, if U: $A \to B$ is information, then $C(A) \leq C(B)$.

As we shall see further on, in certain cases decrease of located resources leads to decrease of entropy of preferences, also because of decrease of alternatives number. The corresponding morphism can be treated as co-information. An increase of required resources can lead to the same effect. Resource changes in an opposite direction can be treated as an *information*.

Precision of information I_A, associated to a certain system, is called a logarithm of an invariant of this state structure , i.e., a logarithm of morphism number from an object A into itself (automorphism). In other words - this is the number of conversions (automorphism), which keeps the structure of the system

$$I_A = \log \mathrm{Card} H_s(A_0 A). \qquad (2.57)$$

A following optimization principle is postulated: precision of information of real states of natural systems is extrem.

This principle, although appears more general in comparison with the principle of a maximum of entropy in information theory [137] or in thermodynamics, nevertheless it doesn't bring much new from an ideological point of view. It does immediately bring a question about basis (theoretical or empiric). Furthermore, some certain limitedness of this principle is visible. For example a case of systems with a variable structure (in the case of finite-dimensional objects, when CardHs = n — systems with n variables) falls out. An appearance of structures – self-organizing (which is a subject of synergetic) and also an appearance (growth) of information is characteristic for open dissipation systems.

We will try to examine below these and other questions, connected with the principle of optimality in the course of discussion.

Great theoretical complications appear, if objects in question, are denumerable sets and sets, which have greater power (aleph zero, χ_0, χ_i, $i > 0$). The corresponding theory, whose basis was laid by Cantor and Zermelo, Goedel, Frenkel, etc, is based on a so-called choice axiom, the Zermelo theorem and a number of other fundamental assertions.

In our case, we will go around these difficulties. We will at least restrict with finite objects. We have some certain foundation for doing so, since it is low probable that the subject of an active system – the main hero of our „western" – could simultaneously examine the infinite number of alternatives and, moreover – the simply great number of alternatives. Even with the use of a computer number of studied alternatives is finite. Even set alternatives, which at first glance look like continuous (for example, „the greater, the better"), actually they can always be discretized and represented by a certain finite set. In the case of the finite object *A*, which has *n* elements?

$$\mathrm{Card}A = n.$$

There are 16 types of invariants depending on which correspondences possible morphism suit.

There were introduced four types of dependences above „complete certainty", „surjective", „injective", „functionality", which are designated below with indices *p*, *s*, *i*, *f*, respectively. If we designate by the index „0" case of set arbitrary morphism, then they take place of following 16 versions of combinations of indices:

0, *p*, *s*, *i*, *f*, *ps*, *pi*, *pf*, *si*, *sf*, *if*, *psi*, *psf*, *pif*, *sif*, *psif*.

Mappings, which correspond to combination indices *pf* are one-to-one correspondence and are called „*functions*", combinations *psf* suit „*surjection*", combinations *pif* - „*injection*" and finally combinations *psif* ,- one-to-one correspondences or „*bijection*".

In the case of finite set *A* set of all subsets of *A*, including empty subset has dimensionality (power or cardinal number)

$$\mu(A) = 2^n .$$

Particularly, power of all subsets of denumerable set is equal to the power of continuum C).

Let's give values of invariants for some of *N* possible types of morphism from the set *X* to the set *A* mentioned above. Subscript designates the type of morphism.

In the case of arbitrary morphism („0"):

$$\Im_0(A|\, X) = 2^{\mathrm{Card}A\,\mathrm{Card}X} . \tag{2.58}$$

For other morphism, the invariants occur:

$$\left.\begin{array}{l}
\Im_p(A|X)=\left(2^{\mathrm{Card}A}+1\right)^{\mathrm{Card}X};\\
\Im_f(A|X)=\left(\mathrm{Card}A+1\right)^{\mathrm{Card}X};\\
\Im_s(A|X)=\left(2^{\mathrm{Card}X}-1\right)^{\mathrm{Card}A};\\
\Im_i(A|X)=\left(\mathrm{Card}X+1\right)^{\mathrm{Card}A};\\
\Im_{pf}(A|X)=\left(\mathrm{Card}A\right)^{\mathrm{Card}X};\\
\Im_{pi}(A|X)=\sum\limits_{k=0}^{\mathrm{Card}A}\binom{\mathrm{Card}A}{k}I_{pfs}\left(\mathrm{Card}A\right);\\
\Im_{si}(A|X)=\left(\mathrm{Card}X\right)^{\mathrm{Card}A};\\
\Im_{pfsi}(A|X)=\left(\mathrm{Card}X\right)!;\\
\Im_{fsi}(A|X)=\dfrac{\left(\mathrm{Card}X\right)!}{\left(\mathrm{Card}X+\mathrm{Card}A\right)!}.
\end{array}\right\} \qquad (2.59)$$

Let's examine now the case of the finite set A, which contains n elements of those distributed in $k \le n$ classes. Let's designate through $\Im_i$ a particular invariant, which corresponds to class i. For the arbitrary automorphism $A_i \to A_i$: $\Im_{0i}=2^{n_i^2}$. For an entire set

$$\Im_0=\prod_{i=1}^{k}\Im_{0i}=\prod_{i=1}^{k}2^{n_i^2}=2^{\sum_{i=1}^{K}n_i^2}.$$

Corresponding information

$$I_0=\log\Im_0=\left(\sum_{i=1}^{k}n_i^2\right)\log 2. \qquad (2.60)$$

If base of logarithm are 2, then

$$I_0=\sum_{i=1}^{k}n_i^2 .$$

Here $\sum\limits_{i=1}^{k}n_i=n$. Value of $S=\sum\limits_{i=1}^{k}\left(\dfrac{n_i}{n}\right)^2$ is called the variety index of Simpson, Margalef, Pielou.

Then information $I_0 = n^2S$. If we designate $\dfrac{n_i}{n}=p_i$, then $\sum\limits_{i=1}^{k}p_i=1$. In the case, when the volumes of all classes are identical: $u:=\dfrac{n}{k}$, then $p_i=\dfrac{1}{k}$ and, therefore, the variety index $S=\sum\limits_{i=1}^{k}p_i^2=\sum\limits_{i=1}^{k}\dfrac{1}{k^2}=\dfrac{1}{k}$, and maximum information

$$I_{0\max} = \frac{n^2}{k}. \tag{2.61}$$

In the case of injection and surjection in the presence of k classes, the invariant

$$\Im_{pfs} = \Im_{psi} = \prod_{i=1}^{k} n_i^{n_i}. \tag{2.62}$$

For bijection

$$\Im_{pfsi} = \prod_{i=1}^{k} n_i ! \tag{2.63}$$

Let's determine the information in the last two cases:

$$I_{pfs} = I_{psi} = \log\left(\prod_{i=1}^{K} n_i^{n_i}\right) = \sum_{i=1}^{K} n_i \log n_i. \tag{2.64}$$

In the case of equivalent classes ($n_i = \frac{n}{k}$) we have

$$I_{pfs} = I_{psi} = n \log \frac{n}{k}. \tag{2.65}$$

For bijection (automorphism)

$$I_{psi} = \log \prod_{i=1}^{K} (n_i)! \tag{2.66}$$

If $n_i = \frac{n}{k}$, then

$$I_{psi} = k \log\left(\frac{n}{k}\right)! \tag{2.67}$$

Similar to the variation principle discussed above following optimization task for example have place: to find the „specific structure" function of population containing N individuals separated to the K classes of powers n_a each, which brings extremum to the functional.

$$I_{psf(pfi)} = \prod_{a=1}^{K} n_a^{n_a} \tag{2.68}$$

with the constraints:

$$\sum_{a=1}^{K} n_a = N;$$

$$\sum_{a=1}^{K} R_a n_a = R^{req} = R^{disp},$$

where R_a – resources, consumed by one individual of class and, R^{req} – total required resources are equal to available. Certainly, can be examined the case, when

$$\sum_{a=1}^{K} R_a n_a \le R^{disp}.$$

Let's write the functional of the extended task:

$$\Phi^* = -\log \Im_{ps(pfi)} - \alpha \sum_{a=1}^{K} R_a n_a + \beta \sum_{a=1}^{K} n_a. \tag{2.69}$$

We will obtain after obvious transformations

$$\Phi^* = NH_k - N\log N - \alpha \sum_{a=1}^{K} R_a p_a + \beta \sum_{a=1}^{K} p_a, \tag{2.70}$$

where $H_k = -\sum_{a=1}^{K} p_a \log p_a$ - the entropy of structure, $p_a = \frac{n_a}{N}$ - the weight of class, moreover

$$\sum_{a=1}^{K} p_a = 1.$$

The second constant term in the formula for Φ^* (with constant N) does not influence on the result of the variational problem solution and can be rejected.

Let's find distribution of the classes weight p_a in the population from the condition $\frac{\partial \Phi^*}{\partial p_a} = 0$, determined as function of the required resources:

$$p_a = \frac{e^{-\alpha Ra}}{\sum_{j=1}^{K} e^{-\alpha Rj}}. \tag{2.71}$$

The physically acceptable result is obtained: the greater required resources, the less of corresponding class. It is obvious that this result is only idealized model and bears demonstration nature. Subsequently we will examine the diverse variants of functional and different solutions of variation problems. In expressions for the entropies, we will use natural logarithms.

2.9. Jensen inequality for ordered sequences

Suppose that there exist ordered sequence the alternatives

$$\sigma_1 \langle\ \sigma_2 \langle \sigma_3 \langle \ldots \langle \sigma_N$$

and let π is the sign, in accordance with which the ordering have been achieved.

In paragraph 1.4 we already spoke about the binary composition. Let $(\sigma_i \wedge \sigma_j)$ is binary composition on the denumerable (finite) set of alternatives. Composition $c^2(\sigma_i, \sigma_j)$ is an alternative $\sigma_{ij} = c^2(\sigma_i, \sigma_j)$.

It could means, for example, that „Solution about simultaneously realization of two alternatives is accepted: the solution of complex problem P_{ij}: $c^2(P(\sigma_i| \sigma_0); P(\sigma_j| \sigma_0))$ with initial state σ_0, not belonging sequence (2.71)" and with separation of the resources R^{disp} for two problems.

Suppose that this version in respect to the expenses is more effective than the separate solution of problems $P(\sigma_i| \sigma_0)$; and $P(\sigma_j| \sigma_0)$ in the sense that

$$R^{req}(C_2(\sigma_i, \sigma_j)| \sigma_0) \leq R^{req}(\sigma_i| \sigma_0) + R^{req}(\sigma_j| \sigma_0). \tag{2.72}$$

In the probabilistic sense, it is possible to assume that under specific conditions

$$p(c^2(\sigma_i, \sigma_j)| \sigma_0) \geq p(\sigma_i| \sigma_0) + p(\sigma_j| \sigma_0). \tag{2.73}$$

We will call the ordered sequence $S_a| \sigma_0$ the convex one, if for $\forall \sigma_i, \sigma_j \in S_a| \sigma_0$, the condition for preference:

$$c^2_{ij} = \sigma_{ij} \underset{\sim}{\rangle} (\sigma_i, \sigma_j);\ \forall ij \in \overline{1,N}. \tag{2.74}$$

fulfills.

Let on $S_a| \sigma_0$ the distribution of probabilities $p(\sigma_i)$, $\sigma_i \in S_a| \sigma_0$ is given and $S_a| \sigma_0$ is complete set of simple alternatives.

Not all outcomes σ_i are independent between each other and let the following relationship be correct

$$c^2_{ij} = \sigma_{ij} \underset{\sim}{\rangle} (\sigma_i, \sigma_j) \Leftrightarrow p(\sigma_{ij}) \geq p(\sigma_i) + p(\sigma_j). \tag{2.75}$$

Here it is intended to compare the version of the simultaneous solution of two interconnected problems, with the appropriate possible savings of resources, with the version of their separate, isolated solutions.

As it will be evident further, the formation of one paired composition leads to the withdrawal from initial set of alternatives S^N_a one alternative and replacement it with another, newly formed composition. As a result new set of alternatives have N-1 alternative: S_a^{N-1}.

Let us name as *K- composition* c^K , the composition of K alternatives, not necessarily „adjacent" in the ordered sequence of preferences.

$$c^K = \bigvee_{i=1}^{K} \sigma_i;\ \forall \sigma_i \in S_a| \sigma_0;\ K \leq N. \tag{2.76}$$

If the probability p_i = $p(\sigma_i)$ corresponds to each simple alternatives, and $\sum_{i=1}^{N} p_i = 1$, and if we carry out the numeration of alternatives with the help of numbers of the natural series section (1,2, 3,..., N) and take the number of alternatives as its weights, then the mathematical expectation of preferences $\pi_1(\sigma_i) = \pi_1(i) = \overline{i} = 2i\,N^{-1}(N+1)$.

$$E\left(\pi_1(\sigma_i) \mid S_a \mid_{\sigma_0}\right) = \sum_{i=1}^{N} ip(\sigma_i) = \frac{1}{N}, \tag{2.77}$$

if all $p(\sigma_i)$ are identical and equal to $\frac{1}{N}$. If $\pi(\sigma_i)$ is indication of the preference of alternative σ_i, and $\pi\left(c^K(\sigma)\right) = \pi\left(\bigvee_{i=1}^{K} \sigma_i\right)$ is the preference of composition, the mathematical expectation of the simple alternative effectiveness

$$\overline{\pi}_1 = E\left(\pi_1 \mid p_s\left(S_a \mid \sigma_0\right)\right), \tag{2.78}$$

where $p_s(S_a| \sigma_0) \in \mathrm{P}(S_a| \sigma_0)$ is the original distribution of probabilities assigned to $(S_a| \sigma_0)$

$$\overline{\pi}_1 = E\left(\pi_1 \mid p_s\left(S_a \mid \sigma_0\right)\right), E\left(\pi_1\left(S_a \mid \sigma_0\right)\right) = \sum_{i=1}^{N} \pi_1(\sigma_i) p_s(\sigma_i \mid \sigma_0). \tag{2.79}$$

The stronger condition is

$$E\left(\pi\left(c^K\right) \mid p_s\left(S_a \mid \sigma_0\right)\right) \geq \sum_{i=1}^{N} \pi_1(\sigma_i) p_s(\sigma_i \mid \sigma_0), \tag{2.80}$$

where $K \in \overline{2,N}$ - the order of the alternatives composition. If (2.79) is correct for any $K \in \overline{2,N}$ then we deal with the absolute convexity of function of two variable $\pi(K, \sigma_i)$. Condition (2.80) can be considered as a certain modification of the Jensen inequality.

It is the peculiar case, when as an indication of effectiveness the corresponding probability is selected:

$$\pi_K = p_s(c^K);\; K \in \overline{1,N}. \tag{2.81}$$

For K= 1 mathematical expectation of effectiveness is equal

$$\overline{\pi} = \sum_{i=1}^{N} p_s^2(\sigma_i \mid \sigma_0) \text{ with condition } \sum_{i=1}^{N} p_{si} = 1. \tag{2.82}$$

We have an example for two alternatives:

$$\begin{array}{c|c} \sigma_1 & p_1 \\ \hline \sigma_2 & p_2 \end{array}; \; p_1 + p_2 = 1 \Rightarrow p_2 = 1 - p_1;$$

$$\bar{\pi} = p_1^2 + \left(1 - p_1\right)^2 = p_1^2 + 1 - 2p_1 + p_1^2 = 2p_1^2 - 2p_1 + 1;$$

$$\max \bar{\pi} \,|\, p_1 \Rightarrow \frac{d}{dp_1}\left(2p_1^2 - 2p_1 + 1\right) = 4p_1 - 2 = 0 \quad \Rightarrow p_1 = \;0{,}5 \quad .$$

For three alternatives

$$\bar{\pi} = p_1^2 + p_2^2 + \left(1 - p_1 - p_2\right)^2;$$

$$\frac{\partial \bar{\pi}}{\partial p_1} = 2p_1 - 2\left(1 - p_1 - p_2\right) = 0;$$

$$\frac{\partial \bar{\pi}}{\partial p_2} = 2p_2 - 2\left(1 - p_1 - p_2\right) = 0;$$

$$\begin{array}{l} 2p_1 - 1 + p_2 = 0; \\ 2p_2 - 1 + p_1 = 0, \end{array} \Rightarrow \; p_1 = 1 - 2p_2;$$

$$1 - 3p_2 = 0 \Rightarrow p_2 = \frac{1}{3},$$

Consequently, $p_i = \frac{1}{3}$ for $\forall i \in \overline{1,3}$ – identical probabilities.

Let's examine in more detail way the concept of the „convexity" („concavity") of the preferences distribution on S_a.

Hypothesis of the technological separability of the alternatives: any two alternatives $\forall \sigma, \eta \in S_a$ are technologically separated or are independent.

This means that σ and η with presence of the corresponding resources can be realized in *any sequence*. If two states σ_i and σ_j are technologically interdependent (dependant) so there is such a way, for instance, that the state σ_i cannot be realized, if σ_i is not realized jet. Then they form „the component" of technological chain and subject in this case does not have complete freedom of choice of one of them as preferable.

It is repeated, that *the set of states with the determined on it preference relation we call the set of alternatives* $S_a | \sigma_0$.

The set of the states, connected with each other as a technological sequence could be called *„technological chain" - Ch*.

Thus, the sequence

$$\sigma_1 \langle \sigma_2 \langle \sigma_3 \langle \ldots \langle \sigma_{k-1} \langle \sigma_k \langle \sigma_{k+1} \langle \ldots \langle \sigma_N \tag{2.83}$$

is set of alternatives S_a, the sequence

$$\sigma_s \to \sigma_{s+1} \to \ldots \to \sigma_{p-1} \to \sigma_p \to \sigma_{p+1} \to \ldots \to \sigma_L \qquad (2.84)$$

is the technological chain Ch_s^L. If in a certain set of states there are technological chains, they must be withdrawn from S_σ in such a way, that in S_σ would remain only last (finite) states of chain (σ_L). These states can be alternatives.

Version I

Let's name set of alternatives S_a^N - strictly convex relative to the paired compositions of alternatives, if for $\forall k \in 2, N-1$ following distribution of preferences have place:

$$\sigma_1 \langle \sigma_2 \langle \sigma_3 \langle \ldots \langle \sigma_{k-2} \langle \sigma_k \langle c^2_{k-1,\,k+1} \langle \sigma_{k+2} \langle \ldots \langle \sigma_N, \qquad (2.85)$$

where $c^2_{k-1,\,k+1} = \{\sigma_{k-1}, \sigma_{k+1}\}$ — paired composition of the elements of those bordering element σ_k. The relation occurs:

$$c^2_{k-1,\,k+1} \rangle \sigma_k$$

and in view of the transitivity $c^2_{k-1,\,k+1} \rangle \sigma_s < k$.

In other words formation of the paired composition $c^2_{k-1,\,k+1} = \{\sigma_{k+1}, \sigma_{k-1}\}$ corresponds to the replacement of two alternatives with one more preferable and to the transposition of alternatives. If the new alternative $c^2_{k-1,\,k+1}$ proves to be more preferable of a certain number of alternatives, then we obtain the new sequence

$$\begin{gathered}\sigma_1 \langle \sigma_2 \langle \sigma_3 \langle \ldots \langle \sigma_{k-1} \langle \sigma_{k+1} \langle \ldots \\ \ldots \langle \sigma_{\sigma-1} \langle c^2_{k-1,k+1} \langle \sigma_{\sigma+1} \langle \ldots \sigma_{N-1} \langle \sigma_N, s \geq k.\end{gathered} \qquad (2.86)$$

Cardinal number of the set S_a^N in this case decrease by 1 and are equal $N-1$. The conditions of convexity are satisfied in three cases:

$$\left.\begin{array}{ll} 1. & \sigma_s = c^2_{k-1,k+1} \rangle \sigma_k, \quad \text{but } \sigma_s \langle \sigma_{k+1}, \\ 2. & \sigma_s \rangle \sigma_k, \quad \text{but } \sigma_s \sim \sigma_{k+1}, \\ 3. & \sigma_{k+1} \langle \sigma_{s-1} \langle \sigma_s \langle \sigma_{s+1}. \end{array}\right\} \qquad (2.87)$$

Let's note that new set of alternatives S_a^{N-1} could be not *convex*.

Graphic interpretation of the first method of the convexity determination of the set S_a^N is shows in Fig. 2.9.

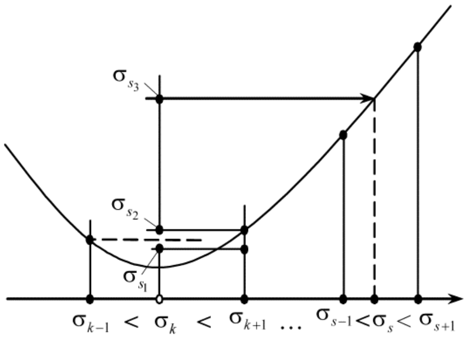
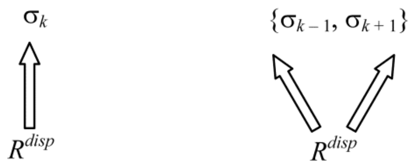


Fig. 2.9

The sense of composition forming from the point of view of the available resources using method can be explained as follows: in the case of the convexity S_a^N with $\forall k \in \overline{2, N-1}$ the expense of the resources R^{disp} for unitary problems $P(\sigma_k|\sigma_0)$ separate solution is always less profitable than the expense of the resources R^{disp} for simultaneous solution of composition of two problems:

$$\{P(\sigma_{k+1}|\sigma_0), P(\sigma_{k-1}|\sigma_0)\} = P_{ck}(c^2_{k-1,\,k+1}|\sigma_0) \quad . \tag{2.88}$$



Version II of the set S_a^N convexity determination.

Composition is formed based on the adjacent alternatives σ_{k-1}, σ_k :
$c^2_{k-1,\,k} = \{\sigma_{k-1}, \sigma_k\}$ are equivalent to the simultaneous solution of two problems, i.e.: complex problem = P_{C2}: $\sigma_0 \rightarrow \{\sigma_{k-1}, \sigma_k\}$.

Let the condition be satisfied:

$$c^2_{k-1,\,k} \rangle \sigma_k$$

and therefore, in view of the transitivity

$$c^2_{k-1,\,k} \rangle \sigma_{k-1} \quad .$$

New set S_a^{N-1} contains $N-1$ alternative; alternatives σ_{k-1} and σ_k are excluded and substituted with the alternative $c^2_{k-1,\,k}$:

$$\underbrace{\sigma_1 \langle \sigma_2 \langle \sigma_3 \langle \ldots \langle \sigma_{k-2} \langle c^2_{k-1,k} \langle \sigma_{k+1} \langle \ldots \langle \sigma_W}_{N-1 \text{ elements}} \quad , \tag{2.89}$$

As in the previous case, the new set S_a^{N-1} is not convex with inevitability.

Further reasoning about of the convexity of the set $S_a^N \mid \sigma_0$.

Let $c^2_{k-1,k} = \{\sigma_{k-1}, \sigma_k\} = \sigma^2_{k-1,k}$. The following methods of the conversions of alternatives set could be examined:

a) the power of S_a^N increases by 1. $\sigma^2_{k-1,k}$ is added as new alternative, afterward σ_k since $\sigma^2_{k-1,k} \rangle \sigma_k$.In this case $S_a^N \mid \sigma_0 \to S_a^{N+1} \mid \sigma_0$. The distribution of preferences undergoes change, since

$$\pi(\sigma^2_{k-1}) \geq \pi(\sigma_{k-1}) + \pi(\sigma_k) \quad . \tag{2.90}$$

Since $\sum_{i=1}^{N+1} \pi\left(\sigma_i\right) = 1$, then the sum of the preferences of remaining alternatives must decrease.

b) the power of S_a^N does not change and alternative σ_k is substituted by the alternative $\sigma^2_{k-1,k}$, new set $S_a'^N$ again is ordered on the preferences.

c) the power of S_a^{N-1} decreases by 1, alternatives σ_{k-1} and σ_k are excluded, and new alternative - composition $\sigma^2_{k-1,k}$ is introduced. Ordering has been produced again.

Concerning new alternative - composition $\sigma^2_{k-1,k}$ location in the new ordering we have to note the following: let $\sigma^2_{k-1,k} \rangle \sigma_s$, where $s \geq k$ and $\sigma^2_{k-1,k} \langle \sigma_{s+1}$:

$$\langle \sigma_s \langle \sigma^2_{k-1,k} \langle \sigma_{s+1} \langle \ldots,$$

then in new ordering the new alternative $\sigma^2_{k-1,}$ occupies a position between σ_s and σ_{s+1}. Alternatives should be renumbered.

If this situation occurs for all $k \in \overline{2, N-1}$, the set $S_a^N \mid_{\sigma_0}$ is called *„strictly convex to the left on the paired preferences"*.

If the inverse condition occurs

$$\sigma^2_{k-1,k} \langle \sigma_s, \quad \text{there} \quad s \leq k-1 \quad ,$$

the set $S_a^N \mid_{\sigma_0}$ is called *„strictly concave to the left on the paired preferences"*. This corresponds to the fact, that any paired composition of adjacent alternatives is less preferable than left one less preferable then two alternatives σ_{k-1} and σ_k, formed composition.

In the case (b), the exclusion of alternative σ_k is based by the fact that the composition $\sigma^2_{k-1,k}$ are more preferable than σ_k and in the course of implementation $\sigma^2_{k-1,k}$ alternative σ_k will be nevertheless realized. In the same way, it is possible to base the case of (c): $N \to N-1$.

Some generalizations:

Suppose that the composition includes two arbitrary alternatives from $S_a^N|_{\sigma_0}$:

$$c^2_{k,\,m}: = \{\sigma_k,\, \sigma_m\} = \sigma^2_{k,\,m},\ k,m \in \overline{1,N},\ m > k \qquad (2.91)$$

$\sigma^2_{k,m}$ is considered as new unitary alternative such, that

$$\sigma^2_{k,\,m} \rangle\, \sigma_s;\ s \geq m$$

$$\sigma_1 \langle\, \sigma_2 \langle\, \sigma_3 \langle \ldots \langle\, \sigma_k \langle \ldots \langle\, \sigma_m \langle \ldots \langle\, \sigma_s \langle\, \sigma_{s+1} \langle \ldots \langle\, \sigma_N$$

$$\ldots \langle\, \sigma_s \langle\, \sigma^2_{m,\,k} \langle\, \sigma_{s+1} \langle \ldots \qquad (2.92)$$

In the new set of alternatives one alternative σ_m or both σ_k, σ_m is excluded.

Which of the versions - (a), (b) or (c) - is selected, it depends on the nature of the utility distribution on the alternatives.

If the utility distribution coincides (qualitatively) with the distribution of preferences, i.e., the order of preferences coincides with the order of utilities (example of the non coincidence: sport training is absolutely more useful than smoked of several cigarettes,

sport $\rangle$ smoking,

but subject gives preference to smoking, i.e., $\pi_1 < \pi_2$), then the convexity of $S_a^N|_{\sigma_0}$ on the preferences implies the convexity of $S_a^N|_{\sigma_0}$ on the utilities and vice versa. If such coincidence of distributions is absent, then the coincidence of the convexity or concavity properties is absent too.

If in the given above rank orderings for the composition replace a strict preference by the lax $\rangle \rightarrow \underset{\sim}{\rangle}$, then we will speak on „not strictly convex" or „not strictly concave" set of simple alternatives.

It is natural that the compositions of higher order ($p > 2$) can be formed, where p the order of the composition:

$$c^3_{k,\,m,\,n} = \{\sigma_k,\, \sigma_m,\, \sigma_n\} \quad ;$$

$$\sigma_k \langle\, \sigma_m \langle\, \sigma_n \qquad \text{and} \qquad (2.93)$$

$$c^3_{k,\,m,\,n} \rangle\, \sigma_s;\ s \geq n \quad .$$

Several versions of the new set of alternatives formation here also appear. Exception in the cases of (a) and (b) of alternatives σ_k can be explained by the fact that reaching of state σ_k in the composition of the alternative $\sigma^2_{k,m}$ can cost less than simple alternative and, naturally, subject will not examine σ_k individually. At the same time realization σ_{k-1} as a parts of composition can „cost more" than it will be cost if

σ_{k-1} come out as simple alternative and therefore it should be preserved in the set $S_a'^N|_{\sigma_0}$.

Let $g(\sigma)$ is real scalar function assigned on $S_a^N|_{\sigma_0}$ in the general case not standardized. Let's determine the mean value of function $g(\sigma)$ on the preferences $\pi(\sigma_i)$ distributed on $S_a^N|_{\sigma_0}$

$$E_\pi\left(g(\sigma)\right)=\sum_{i=1}^{N}g(\sigma_i)\pi(\sigma_i) \quad . \tag{2.94}$$

We will say that the distribution of preferences $\pi_1(\sigma)$ on $S_a^N|_{\sigma_0}$ is more preferable in average than $\pi_2(\sigma_i)$ relative to $g(\sigma)$ if

$$E_{\pi_1}\left(g(\sigma)\right)>E_{\pi2}\left(g(\sigma)\right)\Rightarrow\pi_1\rangle\pi_2 \quad . \tag{2.95}$$

and not less preferable, if

$$E_{\pi_1}\left(g(\sigma)\right)\geq E_{\pi2}\left(g(\sigma)\right)\Rightarrow\pi_1 \underset{\sim}{\rangle}\pi_2 \quad . \tag{2.96}$$

Thus, we introduced preference relation for the class (set) of the preferences distributions $\prod\left(S_a^N|_{\sigma_0}|\,g(\sigma)\right)$ on $S_a^N|_{\sigma_0}$ relative to real function $g(\sigma)$. Define concretely σ as the certain real parameter r: $\sigma_i\to r_i$.

We have

$$E_\pi\left(g(r)\right)=\sum_{i=1}^{N}g(r_i)\pi(\sigma_i) \quad . \tag{2.97}$$

If function $g(r)$ is convex, then the Jensen inequality on the preferences occurs

$$E_\pi\left(g(r)\right)\geq g\left(E_\pi(r)\right) \quad , \tag{2.98}$$

where $$E_\pi(r)=\sum_{i=1}^{N}r_i\pi(\sigma_i) \quad .$$

Let $S_a^L|_{\sigma_0}\subset S_a^N|_{\sigma_0}$ ($L < N$) is the subset $S_a^N|_{\sigma_0}$ and the function of preference is normalized on $S_a^N|_{\sigma_0}$. Then it is possible to speak about the preferability of function $\pi_1(\sigma)$ against the function $\pi_2(\sigma)$ relative to the subset $S_a^L|_{\sigma_0}$

$$\sum_{S_a^L}\pi_1(\sigma_i)>\sum_{S_a^L}\pi_2(\sigma_i)\Rightarrow\pi_1\rangle\pi_2\,|\,S_a^L \quad . \tag{2.99}$$

On the contrary, with the assigned function of preferences $\pi(\sigma)$ it is possible to speak about preferability of the subsets $S_{a_1}^{L_1}$ and $S_{a_2}^{L_2}$ relative to the distribution $\pi(\sigma)$

$$S_{a_1}^{L_1} \rangle S_{a_2}^{L_2} \Leftrightarrow \sum_{S_{a_1}^{L_1}} \pi(\sigma_i) > \sum_{S_{a_2}^{L_2}} \pi(\sigma_i) \quad . \tag{2.100}$$

In the more general case, if real function with the values on $S_a^N|_{\sigma_0}$, is assigned then it is possible to use the following relation of the group preferability of two subsets:

$$S_{a_1}^{L_1} \rangle S_{a_2}^{L_2} \Big|_{g(\sigma)} \Leftrightarrow \sum_{S_{a_1}^{L_1}} g(\sigma_i)\pi(\sigma_i) > \sum_{S_{a_2}^{L_2}} g(\sigma_i)\pi(\sigma_i) \quad . \tag{2.101}$$

If in the cases (a), (b), (c) the given conditions for the paired compositions with $\forall k,m \in \overline{1,N}$, are satisfied the corresponding set $S_a^N|_{\sigma_0}$, is called „convex to the left with respect to the preference relation".

Let now the distribution of probabilities $p_s(\sigma) \in P\ (S_a^N|_{\sigma_0},)$ from a certain set of distributions on $S_a^N|_{\sigma_0}$, come out as a function $g(\sigma)$. For all $p_s(\sigma)$

$$\sum_{i=1}^{N} p_s(\sigma_i) = 1 \quad . \tag{2.102}$$

Determine the value

$$E_\pi(\pi \,|\, p_s) = \sum_{i=1}^{N} \pi(\sigma_i) p_s(\sigma_i) \quad . \tag{2.103}$$

and name it the mean value or expected preferability. From the other side, in view of the symmetry of right side (2.103) relative to distributions $\pi(\sigma)$ and $p_s(\sigma)$, this value can be treated as mean value of subjective probability on the set of alternatives $S_a^N|_{\sigma_0}$.

Suppose that to each state σ_i corresponds quantitative characteristic - real number r_i and, furthermore, $\pi_t(\sigma)$ and $p_s(\sigma)$ are convex distributions on $S_a^N|_{\sigma_0}$ in the sense defined above, and let r_{km} is composition of variables r_k and r_m such, that $r_{km} \neq r_k + r_m$, then

$$\pi_t(r_{km}) \geq \pi(r_k) + \pi(r_m) \quad ; \tag{2.104}$$

$$p_s(r_{km}) \geq p(r_k) + p(r_m) \quad . \tag{2.105}$$

Determining the values

$$\pi\left(\sum_{i=1}^{N} r_i p_s\left(r_i\right)\right) = \pi\left(E_{P_s}\left(r\right)\right) \quad ; \tag{2.106}$$

$$p\left(\sum_{i=1}^{N} r_i \pi\left(r_i\right)\right) = p_s\left(E_\pi\left(r\right)\right) \quad , \tag{2.107}$$

we can write two inequalities

$$E_\pi\left(p_s\left(r\right)\right) \geq p_s\left(E_\pi\left(r\right)\right) \quad ; \tag{2.108}$$

$$E_{P_s}\left(\pi\left(r\right)\right) \geq \pi\left(E_{P_s}\left(r\right)\right) \quad . \tag{2.109}$$

Let's examine the special case, when function $\pi(\sigma)$ comes out as function $g(\sigma)$. Then

$$E_\pi\left(\pi\left(\sigma\right)\right) = \sum_{i=1}^{N} \pi^2\left(\sigma_i\right) \quad . \tag{2.110}$$

If distribution is uniform, i.e., all $\pi(\sigma_i)$ are identical and equal $\frac{1}{N}$, then

$$E_\pi\left(\pi\left(\sigma\right)\right) = \frac{1}{N} = \frac{1}{N}\sum_{i=1}^{N} \pi_i \quad .$$ It is analogous to

$$E_P\left(p_s\left(\sigma\right)\right) = \sum_{i=1}^{N} p_s^2\left(\sigma_i\right) \quad . \tag{2.111}$$

2.10. Subjective probability and expected utility

2.10.1. Relative probability

In this paragraph, we will continue discussion of the questions of those begun in chapter 1 in the paragraph, dedicated to the brief survey of the results, which are contained in the book Fishbourn [149]. Here we rest on the monograph M. de Grooth [48], which treats the concepts, mentioned in the title under somewhat different visual angle, which must contribute to forming more complete idea about the object. In a number of cases, the account is supplied with additional interpretations and examples.

Let selective space is S and A is σ- algebra on S. The basic non-deductive concept is relative likelihood. If A and B are the subsets of S: A, $B \subset S$ and respectively A, $B \in$

A, the relation $A \langle B$ means that „*B* is more plausible than *A*", and relation $A \sim B$ - „*A and B* are equally plausible". $A \underset{\sim}{\langle} B$ - „*A is* not more plausible than *B*".

$$A \underset{\sim}{\langle} B \Leftrightarrow A \cup D \underset{\sim}{\langle} B \cup D \quad .$$

The probabilities of events are determined so that for any probability distribution on σ- algebra the double-sided sequence occurs:

$$A \underset{\sim}{\langle} B \Leftrightarrow P(A) \le P(B) \quad . \tag{2.112}$$

Such distribution of likelihood $P(A)$ is called coordinated with the relation $\underset{\sim}{\langle}$.

The assumptions are accepted:

1. for $\forall A, B \in$ A either $A \langle B$, or $A \rangle B$, or $A \sim B$.
2. if $A_1, A_2, B_1, B_2 \in$ A and $A_1 \cap A_2 = B_1 \cap B_2 = \varnothing$

   and $A_i \underset{\sim}{\langle} B_i$ $(i \in \overline{1,n})$, to $A_1 \cup A_2 \underset{\sim}{\langle} B_1 \cup B_2$,

   if $A_1 \langle B_1$, and $A_2 \langle B_2$, then $A_1 \cup A_2 \langle B_1 \cup B_2$.

The following assertions occur:

Lemma. Let $\forall A, B, D \in$ A and $A \cap B = B \cap D = \varnothing$, then

$$A \underset{\sim}{\langle} B \Leftrightarrow A \cup D \underset{\sim}{\langle} B \cup D.$$

Theorem 1

If for $\forall A, B, D \in$ A the relations $A \underset{\sim}{\langle} B$ $B \underset{\sim}{\langle} D$, are valid, then $A \underset{\sim}{\langle} D$ (condition of transitivity).

The following theorem enlarges the assertion of lemma on *n* events.

Theorem 2

From $$A_i \langle B_i \; (\forall i \in \overline{1,n}) \Rightarrow \bigcup_{i=1}^{n} A_i \underset{\sim}{\langle} \bigcup_{i=1}^{n} B_i \; ,$$

from the condition $$A_i \langle B_i \;\text{ for }\; \forall i \in \overline{1,n} \Rightarrow \bigcup_{i=1}^{n} A_i \langle \bigcup_{i=1}^{n} B_i \quad .$$

Theorem 3

For $\forall A, B \in$ A the relation $A \underset{\sim}{\langle} B \Leftrightarrow \bar{A} \underset{\sim}{\rangle} \bar{B}$ is true.

Let $\forall A, B \in$ A, $\varnothing \underset{\sim}{\langle} A$ and $\varnothing \underset{\sim}{\langle} B$. Then the following theorem is valid.

Theorem 4

If $A \subset B \Rightarrow A \underset{\sim}{\langle} B$, including $\varnothing \underset{\sim}{\langle} A \underset{\sim}{\langle} B$

Theorem 5

If $A_1 \subset A_2 \subset \ldots$ is the infinite ascending sequence, and B are such subsets, that for $\forall i \in \overline{1,\infty}$ the relation is carried out

$$A_i \precsim B, \text{ then } \bigcup_{i=1}^{\infty} A_i \precsim B.$$

Theorem 6

Let A_i are incompatible and B_i are incompatible and for $\forall i \in \overline{1,\infty}$ $A_i \precsim B_i$, then

$$\bigcup_{i=1}^{\infty} A_i \precsim \bigcup_{i=1}^{\infty} B_i.$$

If for $\forall i \in \overline{1,\infty}$ $A_i \prec B_i$, then

$$\bigcup_{i=1}^{\infty} A_i \prec \bigcup_{i=1}^{\infty} B_i.$$

Let $X(s)$, where $s \in S$ is random variable, which takes values in the interval:

$$0 \le X(s) \le 1$$

for $\forall s \in S$. The value $X(s)$ is uniform in distributed on [0, 1], if for any intervals I_1, $I_2 \subset [0, 1]$ occurs the relation:

$$I_1, I_2 \subset [0, 1], \qquad (X \in I_1) \precsim (X \in I_2),$$

when $\lambda(I_1) \le \lambda(I_2)$, where $\lambda(I) = b - a$ is the length of interval $[a, b]$.

It is assumed that there is a random variable with uniform distribution in interval [0, 1]. It establishes a correspondence (biection) between the points of selective space S and the points of the unit interval [0, 1].

Assume that the triplet (S, A, $\rho : \precsim$), is given i.e., selective space S, σ- algebra and the binary relation $\rho : \precsim$. are determined.

There is a distribution of probabilities $P(A)$, coordinated with the relation $\rho : \precsim$. which is introduced by the following theorem.

Let A and B — two events: A, $B \in$ A. Relation $A \precsim B$ □ is valid if and only if $P(A) \le P(B)$. Along with the relation *of unconditional likelihood there* is a relation *of conditional likelihood*.

Let's examine three events A, B, $D \in$ A and also events $A| D$ and $B| D$, (appearance A or B when D already took place or for sure it takes place). The relation between them

$$A|D \precsim B|D$$

is established.

This means that when event D occurred (or will occur) event B is at least preferable as much as event A. The conditional distribution of probabilities, coordinated with the given above relation could be introduce. It is necessary to accept that

$$P(D) > 0$$

and the assumption that for $\forall A, B, D \in$ A the relation occurs:

$$A| D \precsim B| D \Leftrightarrow A \cap D \precsim B \cap D$$

have to be carried out.

The relationship between conditional relation of preferences assigned on σ-algebra and condition a probability distributions on the interval [0, 1], is established by the following theorem.

Theorem 7

If the relation $\precsim$ satisfies all assumptions made above, the distribution $P(A)$, assigned by the relation:

$$A \sim G\left[0, P\left(A\right)\right] \quad ,$$

where

$G(a, b)$ - events $X \in [a, b]$, is the only probability distribution with the following properties:
for $\forall A, B, D \in$ A with $P(D) > 0$ the relation $A| D \precsim B| D$ and $P(A| D) \le P(B| D)$ are equivalent, i.e.

$$A| D \precsim B| D \Leftrightarrow P(A| D) \le P(B| D).$$

Let's continue below the analysis of the expected utility following [149].

2.10.2. Objective utilities and subjective preferences

For simplicity the case, when alternative state σ_i is characterized by the quantitative parameter, which takes only real values, is examined; this real parameter we will designate by the same symbol σ. Let $P(\sigma)$ is the probability distribution of value σ.

It is assumed that the subject can put in order distributions P from a certain set of distributions, assigned on the set of alternatives $S_a|_{\sigma_0} - \Re_a\left(S_a|_{\sigma_0}\right)$.

Let $U(\sigma_i)$ is the real function of the utility of alternative σ_i for the subject. In contrast to the function of preferences the function of utility is being treated as objective utility (σ_i - quantity of calories of food product, the utility of vitamins, the utility of medicine, profit, income, flying range, the specific consumption of fuel of engines and so forth)

Dividing the concepts „objective utility — $U(\sigma_i)$ and subjective preference” — $\pi(\sigma_i)$ we obtain the unquestionable advantage from the point of view of possibility of more detailed preparation and analysis of problem - resource situations, and which is especially important, the prediction of preferences.

The ordering of probabilistic distributions is certainly much more complex intellectual problem, than, let’s say, the comparisons of the utility of two alternatives with the paired comparison. In the second case, „error” of expert is less probable than with the comparison of probabilistic distributions.

Let $P_a\left(S_a|_{\sigma_0}\right) \in \Re\left(S_a|_{\sigma_0}\right)$ is a certain probability distribution on the set of alternatives, and $U(\sigma)$ is the utility function of the alternative $\sigma \in S_a|_{\sigma_0}$. The function of utility appears a monotonic in accordance with definition, while the function of the preferences cannot be.

In accordance with the distribution P $\left(S_a|_{\sigma_0}\right)$ the probability $P\left(\sigma_i \in S'_a|_{\sigma_0}\right)$, can be calculated, where $S'_a|_{\sigma_0} \subset S_a|_{\sigma_0}$ is the subset of $S_a|_{\sigma_0}$. If $S_a|_{\sigma_0}$ is finite, then this probability is equal

$$P\left(\sigma_i \in S'_a|_{\sigma_0}\right) = \sum_{\left(S'_a|_{\sigma_0}\right)} P\left(\sigma_k\right) \quad .$$

where $k \in \{k\}'$ passes entire set of the numbers of the alternatives, which belong to the subset $S'_a|_{\sigma_0}$. Since $U(\sigma_i)$ is the utility of alternative σ_I , $P\left(\sigma_i \in S'_a|_{\sigma_0}\right)$ is probability that the utility possesses value in the limits

$$\min_{\{k\}} U(\sigma) \le U(\sigma_i) \le \max_{\{k\}} U(\sigma) \quad \text{or} \quad \min_{\{S'_a\}} U(\sigma) \le U(\sigma_i) \le \max_{\{S'_a\}} U(\sigma).$$

Let $P_1 \in \Re\left(\sigma_i \in S_a|_{\sigma_0}\right)$ and P $_2 \in \Re\left(S_a|_{\sigma_0}\right)$ are two non identical probability distributions on $S_a|_{\sigma_0}$. If the subset $S'_a|_{\sigma_0}$ is assigned and there is a strict partial order on it in accordance with the relation $\precsim$, than the correspondence of bijection between utilities and probability distributions is established by correspondence

$$\mathcal{P}_1 \in \Re\left(S_a|_{\sigma_0}\right) \precsim \mathcal{P}_2 \in \Re\left(S_a|_{\sigma_0}\right) \Leftrightarrow P\left(\sigma_i \in S'_a|_{\sigma_0} \middle| \mathcal{P}_1\right) \le P\left(\sigma_i \in S'_a|_{\sigma_0} \middle| \mathcal{P}_2\right). \quad (2.113)$$

This means that the distribution $\mathcal{P}_2$ is not less preferable than $\mathcal{P}_1$ if and only if the probability of the event $\sigma_i \in S'_a|_{\sigma_0}$ or, that is the same, the probability of the event

$U(\sigma_i) \in \left[U(\sigma)_{\min}, U(\sigma)_{\max}\right]$ on $S'_a|_{\sigma_0}$ for distribution $\mathcal{P}_2$ is not less one than for distribution $\mathcal{P}_1$. In this case it is certainly assumed that there is an ordering of utilities $U(\sigma)$ on $S_a|_{\sigma_0}$. If $S'_a|_{\sigma_0}$ is the class of the equivalence of the set $S_a|_{\sigma_0}$, i.e., $\sigma_i \sim \sigma_j$, when $\sigma_i, \sigma_j \in S'_a|_{\sigma_0}$ (equivalence on the utilities $U(\sigma_i) = U(\sigma_j)$), then instead of relationship (2.113) we should write:

$$\mathcal{P}_1\left(S_a|_{\sigma_0}\right) \precsim \mathcal{P}_2\left(S_a|_{\sigma_0}\right) \Leftrightarrow P\left(U(\sigma_\sim)\middle|\mathcal{P}_1\right) \le P\left(U(\sigma_\sim)\middle|\mathcal{P}_2\right), \qquad (2.114)$$

where $\sigma_\sim$ is one of the alternatives $S'_a|_{\sigma_0}$.

The aforesaid can be illustrated by examples. On Fig. 2.10 two uniform distributions $\mathcal{P}_1(\sigma)$ and $\mathcal{P}_2(\sigma)$ are depicted.

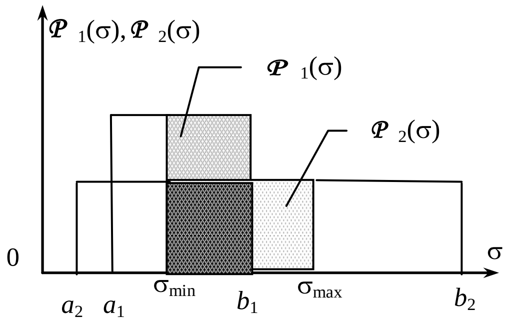


Fig. 2.10

On this figure the areas of those shaded in different ways are equal, i.e., the probabilities of hit σ within the limits of section $[\sigma_{\min}, \sigma_{\max}] = S'_a$ are equal as well.

$$P\left(\sigma_i \in S'_a|_{\sigma_0} \middle|\mathcal{P}_1\right) = P\left(\sigma_i \in S'_a|_{\sigma_0} \middle|\mathcal{P}_2\right) \quad .$$

It is possible to say that the distribution $\mathcal{P}_1(\sigma)$ and $\mathcal{P}_2(\sigma)$ relative to the subset $S'_a([\sigma_{\min}, \sigma_{\max}])$ are equivalent.

Fig. 2.11*a*, shows also two equivalent distributions relative to the section $S'_a = [\sigma_1, \sigma_2]$. Here also areas (probabilities) shaded in different ways are equal with ears others and consequently the distributions $P_1(\sigma)$ and $P_2(\sigma)$, assigned on the real axis (σ) are equivalent relative to the intercept of this axis $S'_a = [\sigma_1, \sigma_2]$:

$$\mathcal{P}_1(\sigma) \sim \mathcal{P}_2(\sigma)\middle|S'_a|_{\sigma_0} \quad .$$

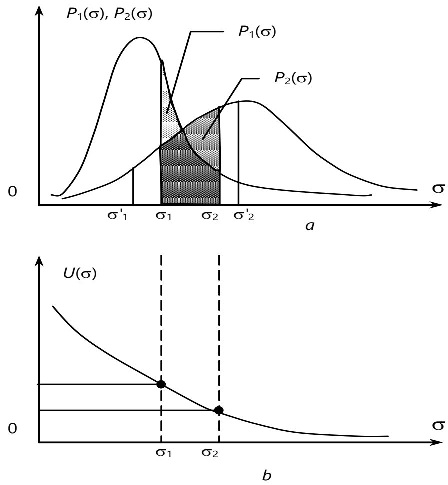


Fig. 2.11

On the lower graph the monotonic utility function $U(\sigma)$ is shown. We see that the correspondence occurs

$$P\left(\sigma \in \left[\sigma_1, \sigma_2\right] \middle| \mathcal{P}_1\right) = P\left(\sigma \in \left[\sigma_1, \sigma_2\right] \middle| \mathcal{P}_2\right) \Rightarrow P\left(U \in \left(U_1, U_2\right) \middle| \mathcal{P}_1\right) \le P\left(U \in \left(U_1, U_2\right) \middle| \mathcal{P}_2\right) \quad .$$

The distributions $\mathcal{P}_1(\sigma)$ and $\mathcal{P}_2(\sigma)$ are not equivalent relative to section [σ'$_1$, σ'$_2$], moreover from Fig. 2.11, *b* it is evident that $\mathcal{P}_1(\sigma) \underset{\sim}{<} \mathcal{P}_2(\sigma)$.

Let's examine the first example more detailed. For the distributions $\mathcal{P}_i(\sigma)$

$$P\left(\sigma \in \left[\sigma_1, \sigma_2\right] \middle| \mathcal{P}_1\right) = \int_{\sigma_1}^{\sigma_2} P_1(\sigma) d\sigma = \begin{cases} \dfrac{\sigma_2 - a_i}{b_i - a_i};\ a_i > \sigma_1, \\ \dfrac{\sigma_2 - \sigma_1}{b_i - a_i};\ a_i < \sigma_1;\ b_i > \sigma_2, \\ \dfrac{b_i - \sigma_1}{b_i - a_i};\ a_i < \sigma_1;\ b_i < \sigma_2, \end{cases} \quad \left(i \in \overline{1,2}\right).$$

In the first case the condition of the equivalence $P_1(\sigma)$ and $P_2(\sigma)$ is reduced to the form

$$\frac{d_1}{d_2} = \frac{b_1 - d_1}{b_2 - a_2} = \frac{a_1 - \sigma_2}{a_2 - \sigma_2} \quad .$$

In the second case ($[\sigma_1, \sigma_2] \in [b_1 - a_1] \cap [b_2 - a_2]$) — to the form

$$\frac{\sigma_2 - \sigma_1}{b_1 - a_1} = \frac{\sigma_2 - \sigma_1}{b_2 - a_2} \Rightarrow d_1 = d_2 \quad .$$

In the third case

$$\frac{d_1}{d_2} = \frac{b_1 - a_1}{b_2 - a_2} = \frac{a_1 - \sigma_1}{a_2 - \sigma_2} \quad .$$

It following from these examples that the ordering of probabilistic distributions from $\Re\left(S_a \left|_{\sigma_0}\right.\right)$ can be achieved with respect to a certain event *A*: $\sigma \in S'_a \left|_{\sigma_0}\right.$.

One more method of determining the expected utility is based on the use of a mathematical expectation of utility on entire set of alternatives: $E(U(\sigma)|P_i;\ \sigma \in S_a|_{\sigma 0})$, where $P_i \in \Re\left(S_a \left|_{\sigma_0}\right.\right)$. We will briefly designate this mathematical expectation $E(U|\ P_i)$. The expected utility exists, when there is bijection

$$\mathcal{P}_1 \precsim \mathcal{P}_2 \Leftrightarrow E\left(U \middle| \mathcal{P}_1\right) \le E\left(U \middle| \mathcal{P}_2\right) \quad . \tag{2.115}$$

If all distributions from $\Re\left(S_a \left|_{\sigma_0}\right.\right)$ appear δ-shaped:

$$\mathcal{P}_j\left(\sigma\right) = \sum_{i=1}^{N} P_j\left(\sigma_j\right)\delta\left(\sigma - \sigma_i\right) \quad ,$$

where $\delta(\sigma - \sigma_i)$ — Dirac's δ-function, and $P_{ij} = P_i(\sigma_j)$ such, that $\sum_{i=1}^{N} P_{ij} = 1$ for all *j*, the correspondence (2.115) takes the form

$$\mathcal{P}_1 \precsim \mathcal{P}_2 \Leftrightarrow \sum_{i=1}^{N} U\left(\sigma_i\right) P_1\left(\sigma_i\right) \le \sum_{i=1}^{N} U\left(\sigma_i\right) P_2\left(\sigma_i\right) \quad ,$$

which is carried out on *„the subset of comparison"* $S'_a \left|_{\sigma_0}\right.$. Finally, if all distributions P_i are singular

$$\mathcal{P}_j\left(\sigma_i\right) : P_j\left(\sigma_i\right) = 1 \ ; \qquad P_j\left(\sigma_k\right)\Big|_{k \ne i} = 0 \, .$$

for $\forall k : i \in \overline{1,N}$, then the determined utilities occur and order is assigned by the correspondence:

$$\sigma_1 \precsim \sigma_2 \Leftrightarrow U\left(\sigma_1\right) \le U\left(\sigma_2\right).$$

In view of the monotony of the utility function $U(\sigma)$ it follows that the function

$$V(\sigma) = aU + b \, ,$$

where $a > 0$, is also the utility function.

3. SUBJECTIVE ENTROPY OF INDIVIDUAL PREFERENCES VARIATION PRINCIPLE, SUBJECTIVE INFORMATION

3.1. Subjective entropy

Similarly, as it is done in information theory [137], there is a possibility to introduce an entropy of preferences on set of alternatives S_a in the subjective analysis. Let's name this entropy *subjective entropy*. It is tightly connected with a concept *of value* of information, which is discussed, for example, in [138]. In works on the theory of information, for example, in [13, 21, 49, 58, 110] entropy and information are expressed with respect to the probability distributions. In the present work entropy and information are expressed trough subjective preferences, distributed on a set of alternatives, essence and number of which are results of subjective ideas, preliminary analysis of different quantitative and qualitative characteristics of a virtual object, which we named above a „*problem-resource situation*". In this sense it is possible to talk about „*the entropy of problem-resource situation*". Distributions of preferences have formal similarity to probabilistic distributions, which leads to the far going analogies. Similarity is, however, incomplete and it is manifested in smaller enumeration of preferences properties, superimposed on an axiomatic level. As we will see later, a subjective - probabilistic model can be introduced, in which *the function of preference* depends on a certain *probabilistic distribution*. In this case the latter is received as an objective characteristic of state or problem-resource situation.

Let $S_a|_{\sigma_0}$ is set of alternatives of dimension N, the initial state $\sigma_0 \,\overline{\in}\, S_a|_{\sigma_0}$, and function $\pi(\sigma_i)$ assigns the distribution of preferences on $S_a|_{\sigma_0}$. We will use Boltzmann's entropy in the following form:

$$H_\pi = -\sum_{i=1}^{N} \pi(\sigma_i) \ln \pi(\sigma_i); \quad \sigma_i \in S_a|_{\sigma_0} \quad . \tag{3.1}$$

In the Shannon's information theory this form is determined as average information for one message, expressed through particular probabilities p_i [161, a].

Entropy in the form (3.1) has following properties:

1. When all values of function $\pi(\sigma_i)$ are identical, alternatives are equally preferable (in this case from normalization condition it follows, that $\pi_i = \frac{1}{N}$) value of an entropy $H_{\max} = \ln N$ is the maximum entropy value with a normalizing condition

$$\sum_{i=1}^{N} \pi\left(\sigma_i\right) = 1 \quad . \tag{3.2}$$

2. With a singular distribution, when preferences of all alternatives are equal to zero, except the preference of one alternative, value of preference of which is one:

$$\pi\left(\sigma_i\right) = \begin{cases} 0;\, i \neq k \\ 1;\, i = k \end{cases}, \quad i \in \overline{1,N} \quad ,$$

entropy has its minimum value and equal zero.

These conditions mean that subjective entropy has its maximum value, when the set $S_a|_{\sigma_0}$ constitutes *one class of preferences equivalence* and, in the case of singular distribution of preferences, there is complete certainty in an alternative choice (two classes of equivalence).

3. Subjective entropy is always positive.

Let the set $S_a|_{\sigma_0}$ contain k classes of equivalence, L_s is a quantity of alternatives in *the s-th* class, and π_{Ls} is value of preference function of elements (alternatives), which belong to this class. Then the entropy takes the form:

$$H_\pi^N = -\sum_{s=1}^{k} L_s \pi_{Ls} \ln \pi_{Ls}\,; \quad \left(k \in \overline{1,N}\right) \quad . \tag{3.3}$$

or, designating $\pi_s = L_s\pi_{Ls}$ – „*preference of class*",

$$H_\pi^k = -\sum_{s=1}^{k} \pi_s \ln \pi_s + \sum_{s=1}^{k} \pi_s \ln L_s\,; \quad \sigma_i \in S_a|_{\sigma_0} \quad . \tag{3.4}$$

First term is always positive and presents entropy of preferences on classes, second term it also always positive - this entropy of sizes (power) of classes weighed on preferences.

An entropy H^k_π with assigned $k<N$ reaches maximum, if all π_s are identical and equal $\pi_s = \frac{1}{k}$. Actually it follows from the general condition of normalization

$$\sum_{i=1}^{N} \pi\left(\sigma_i\right) = \sum_{s=1}^{k} \pi_{Ls} L_s = 1 \Rightarrow \sum_{s=1}^{k} \pi_s = 1$$

and with the condition $\pi_s = \frac{1}{k}$ for $\forall s \in \overline{1,k}$ we find from (3.4):

$$H_\pi^k = \frac{1}{k}\left(k \ln k - \ln\left(L_1 L_2 \cdot \ldots \cdot L_k\right)\right) = \ln k - \ln \sqrt[k]{L_1 L_2 \cdot \ldots \cdot L_k}\,. \tag{3.5}$$

It is evident from this formula, that H^k_π reaches maximum when $k = N$. In this case $L_s = 1$ for $\forall s \in \overline{1,N}$ and the second term becomes zero. Consequently

$$H^k_\pi\left(k<N\right)<H^N_\pi \quad \text{and} \quad H^k_\pi\left(k=N\right)=H^N_\pi=\ln N \quad .$$

As we see a presence of classes of an equivalence such that at least one of $L_s > 1$ leads to entropy decrease. An entropy is maximum, when the number of classes of equivalence $k = N$ for $\forall s \in \overline{1,N}$ $L_s = 1$.
In the information theory, the Hartley's information is examined

$$H_\pi = k \ln N,$$

where N is number of equally probable results of experiments. If $k = 1$, then Hartley's information is measured in natural units, if $k = (\ln 2)^{-1}$, then H_n is expressed in binary units – bits. If states are non-equiprobable, then the information corresponding to each state is:

$$H_\pi(\sigma_i) = -\ln p(\sigma_i),$$

where $p(\sigma_i)$ is probability of the „appearance" of a state σ_i. In our case, instead of the probability $p(\sigma_i)$ the function of preferences $\pi(\sigma_i)$ is used and an entropy $H_\pi(\sigma_i)$ reflects uncertainty connected with an alternative σ_i and can be treated as a „frozen" subjective information, which is loosen if σ_i is selected as a goal. An entropy $H_\pi(\sigma_i)$ after selection σ_i as the goal is turned to zero and, correspondingly, an information $I(\sigma_i) = H_\pi(\sigma_i)$ is released.

An entropy (3.1) is a result of averaging of Hartley's entropies „on preferences".
If is possible to attempt to use subjective entropy as a quantitative characteristic of mental condition of subject, which is located in a problem- resource situation.

Natural is the assumption that a level of the entropy of preferences, being the level of uncertainty of desires, characterizes the degree of mental tension. *In this case, the higher the subjective entropy, the higher this tension is*. The maximum entropy ln N can serve as one of the intellect criteria, since it is determined through quantity of simultaneously considered alternatives.

Most likely, for each individual j there exists a maximum mentally „bearable" by him entropy $H^{**}_{\pi j}$, such, that an individual entropy cannot exceed this limit. The result of an achievement of limit $H^{**}_{\pi j}$ must be the modification of set of alternatives S_a or a redistribution of preferences on S_a.

3.2. About non-unit normalizations.

In the probability theory, from a mathematical point of view, the selection of a certain real number as a normalizing constant is arbitrary. Usually the one is taken as this number.

Arbitrariness disappears, if we put an additional condition to the probability distribution and in our case - to the distribution of preferences.

A subjective entropy H_π in its form coincides with Shannon's entropy. This value reaches its maximum, when all particular values of a preference function are equal with each other, i.e., the distribution of preferences is uniform:

$$\pi(\sigma_i) = \frac{1}{N}; \quad \forall \sigma_i \in S_a|_{\sigma_0}$$

and this maximum value exists

$$H_{\max} = \ln N,$$

this corresponds to the normalizing condition

$$\sum_{i=1}^{N} \pi(\sigma_i) = 1 .$$

Let's now take normalizing condition in the form

$$\sum_{i=1}^{N} \pi(\sigma_i) = \varphi, \tag{3.6}$$

where φ is arbitrary positive number.

Let's examine the following task: to find this value of normalizing constant φ so that the subjective maximum entropy $H_{\max}$ would take the greatest value of all possible:

$$H_{\max,\max} = \arg\max H_{\max}(\varphi) \quad .$$

We find from a condition of normalization (3.6) in the case of uniform distribution:

$$\pi(\sigma_i) = \frac{\varphi}{N} \quad ; \quad \forall i \in \overline{1,N} \quad .$$

Let's substitute these values in an expression for an entropy. We will get

$$H_{\max}(\varphi) = -\varphi \ln \frac{\varphi}{N} \quad . \tag{3.7}$$

Let's define φ^* as a solution of an equation:

$$\frac{dH_{\max}(\varphi)}{d\varphi} = 0 \quad ,$$

from where we find that $\varphi^* = \frac{N}{e}$. Then

$$H_{\max}(\varphi^*) = \frac{N}{e} \ln e = \frac{N}{e} = \varphi^* \quad . \tag{3.8}$$

The second derivative $\left.\frac{d^2 H_{\max}(\varphi)}{d^2 \varphi}\right|_{\varphi^*} = -\frac{1}{\varphi^*} < 0$, i.e., with $\varphi = \varphi^* = \frac{N}{e}$ $H_{\max}(\varphi)$ reaches absolute maximum.

The values $H_{\max,\max}$ for some values N are given in Table 3.1.

Table 3.1

№	$\varphi^* = \frac{N}{e} = H_{\max,\max}$	$H^{\circ}{}_{\max}(\varphi = 1)$
1	0,367879…	0
2	0,7357588…	0,6931471…
3	1,1036832…	1,0986122…
4	1,4716776…	1,3862943…
5	1,8393972…	1,60943791…
…	…	…

Figure 3.1 shows the dependence $H_{\max}(N)$ and $H_{\max,\max}(N)$.

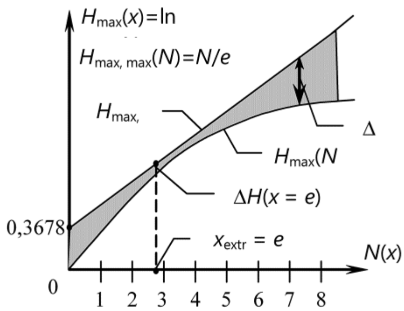


Fig. 3.1

In order to determine at what „point" a difference $|H_{max}(N) - H_{max\,max}(N)| = |\Delta H|$ reaches its minimum, replace N by variable x, which assumes any values on a semi axis in the interval [1, +∞). Let

$$f(x) = \frac{x}{e} - \ln x.$$

From the condition $f'(x) = 0$, we find $x_{extr} = e$, $f''(x) = x^{-2} > 0$, therefore, at „point" $x = e$

$$|\Delta H| = \left|\frac{x}{e} - \ln x\right| \Bigg|_{x=0} = 1 - 1 = 0 \quad .$$

An obtained result can be written as follows: an entropy reaches value $H_{max,\,max}$, if

$$\sum_{i=1}^{N} \pi_i(\sigma_i) = H_{max,max}(N) \quad , \tag{3.9}$$

i.e., if the distribution of preferences is normalized to maximally possible entropy, then

$$\varphi^* = \frac{N}{e} = H_{max,max}(N) \quad . \tag{3.10}$$

We can see that *the number of alternatives is equal to maximally possible entropy multiplied by the number e*. If in the case of this extreme normalization all particular values of the preference function are identical, then they are equal

$$\pi(\sigma_i) = \frac{1}{e} \quad . \tag{3.11}$$

Let's examine the same task, if σ is real variable that is determined on the half-interval [0, +∞) or in the interval [0, a]. In the first case the condition of a uniformity of distribution π (σ) is reduced to an equality $\pi(\sigma)|_{H_{max}} \equiv 0$, in the second case the uniformity of distribution π(σ) in the section [0, a] and for normalization by one gives $\pi(\sigma) = \frac{1}{a}$ (the finite distribution). Let now the normalization take the form:

$$\int_0^a \pi(\sigma) d\sigma = \varphi \quad , \tag{3.12}$$

then $\pi(\sigma)|_{H_{max}} = \frac{\varphi}{a} = \text{const}$ and the entropy

$$-\int_0^a \pi(\sigma)\ln\pi(\sigma)d\sigma$$

with $\pi(\sigma)=\frac{\varphi}{a}$ is equal

$$H_{\max}(\varphi)=-\int_0^a \frac{\varphi}{a}\ln\frac{\varphi}{a}d\sigma=\varphi\ln\frac{\varphi}{a} \quad .$$

Hence from an equation $\frac{dH_{\max}(\varphi)}{d\varphi}=0$ we find $\varphi^*=\frac{a}{e}$.

Consequently

$$H_{\max\max}=-\frac{a}{e}\ln\frac{a}{e}=\frac{a}{e} \quad . \tag{3.13}$$

Let us name value $a = e$. $H_{\max,\max}$ the *„maximum region of discussion"*. *The region of the discussion is equal to maximum possible entropy multiplied by the base of natural logarithm*. Maximally possible information, which appears with a change *in the region of the discussion* Δa or ΔN is, correspondingly

$$\left.\begin{aligned} I(\Delta a)&=\pm\frac{\Delta a}{e}; \\ I(\Delta N)&=\pm\frac{\Delta N}{e}. \end{aligned}\right\} \tag{3.14}$$

With a change of the number of alternatives in S_a to one $I(\Delta N=1)=\pm\frac{1}{e}$.

The result obtained above can be found by another method. We have:

$$\frac{\partial H}{\partial \pi_i}=-\ln\pi_i-1=0.$$

Hence $\pi_{i\text{opt}} = e^{-1}$.

Since $\frac{\partial^2 H}{\partial \pi_i^2}=-\frac{1}{\pi_i}<0$, that $\pi_{i\text{opt}}$ reaches the maximum of function H.

The interpretation of the non-unit normalization of preferences can be nontrivial from the point of view of the manifestations of the properties of psyche.

3.3. Analogies to Shannon entropy

The entropy of the preferences distribution in the form of Boltzmann or Shannon is not the only function, which satisfies requirements, claimed to the entropy. Some functions can be named pseudo-entropy. Such functions include, for example, following

$$H_A = \sum_{i=1}^{N} \left(1 - \pi_i\right) \pi_i \quad ; \tag{3.15}$$

$$H_B = -\sum_{i=1}^{N} \left(1 - e^{1-\pi_i}\right) \pi_i \quad ; \tag{3.16}$$

Both of these functions in the case of the degenerate (singular) distribution become zero, and in the case of uniform distribution $\pi_i = N^{-1}$ they reach their maximum value.

We investigate function H_A (3.15). Let the normalization condition be

$$\sum_{i=1}^{N} \pi_i = 1 \quad . \tag{3.17}$$

Taking this into account, it is possible to represent H_A in the following form

$$H_A = 1 - \sum_{i=1}^{N} \pi^2{}_i \quad . \tag{3.18}$$

When all alternatives have the same preferences (set S_a is the class of equivalence), from normalization condition $\pi_i = \frac{1}{N}$, and it follows from (3.18) that:

$$H_A = 1 - \frac{1}{N} = \frac{N-1}{N} \quad .$$

With $N \to \infty$ $H_A \to 1$, with $N = 1$ $H_A = 0$, with $N = 2$, $H_A = \frac{1}{2}$.

From (3.15) we can find

$$\frac{\partial H_A}{\partial \pi_i} = 1 - 2\pi_i = 0 \quad , \quad \left(\forall i \in \overline{1,N}\right) \quad .$$

from which $\pi_{i\text{opt}} = \frac{1}{2}$ and, since $\frac{\partial^2 H_A}{\partial \pi^2_i} = -2 < 0$, this value $\pi_{i\text{opt}}$ does not correspond to a normalization condition with $N > 2$. Let's assume $\sum_{i=1}^{N} \pi_i = \xi$ and $H_A = \sum_{i=1}^{N} (\varphi - \pi_i)\pi_i$. If distribution π_i is singular, then $\pi_i = 0$ for $\forall i \in \overline{1, N}$, with one exception $\pi_k = \xi$. Since measure of uncertainty H_A in this case must be equal 0, $\varphi = \xi$ and then

$$H_A = \sum_{i=1}^{N} (\xi - \pi_i)\pi_i \quad .$$

Hence it follows that $H_A = H_{A\max}$, if $\pi_{i\text{opt}} = \frac{\xi}{N}$ for $\forall i \in \overline{1, N}$, and

$$H_{A\max} = \sum_{i=1}^{N} \left(\xi - \frac{\xi}{N} \right) \frac{\xi}{N} = \xi^2 \frac{N-1}{N}.$$

In this case, the normalization condition is

$$\sum_{i=1}^{N} \pi_i = \frac{\sqrt{2}\sqrt{H_{A\max}}}{\sqrt{N-1}}.$$

Let's examine quasi-entropy H_B (3.16). At first, let's accept standard normalization (3.17) and entropy in the form (3.16). Derivative

$$\frac{\partial H_B}{\partial \pi_i} = -1 + (1 - \pi_i) e^{1-\pi_i} \text{ for } \forall i \in \overline{1, N} \quad ,$$

We see that a solution of equation $\frac{\partial H_B}{\partial \pi_i} = 0$ is $\pi_{i\text{opt}} \cong 0{,}443$, moreover all values are identical. This value $\pi_{i\text{opt}}$ does not correspond to the normalization condition, since for any N, $\sum_{i=1}^{N} \pi_{i\text{opt}} \neq 1$. Let's examine the case of the arbitrary normalization $\sum_{i=1}^{N} \pi_i = \varphi$ and the function H_B of the form

$$H_{B\max} = -\sum_{i=1}^{N} \left(1 - e^{\pi_{\text{opt}}(N-1)} \right) \pi_{\text{opt}} = -\left(1 - e^{\pi_{\text{opt}}(N-1)} \right) \pi_{\text{opt}} N$$

Substituting discrete variable N to continuous variable x, it is possible to find that

$$\frac{\partial H_{B\max}(x)}{\partial x} \geq 0\,;\ \ \forall x \geq 1\ \ .$$

Consequently, with any π_{opt} $H_{B\max}$ is monotonically increasing function N. For each N the normalizing constant

$$\xi_N = \pi_{opt}(N)N\ \ .$$

For example, if $\pi_{iopt} = 0{,}443$, then the dependence $H_{B\max}(N)$ over N is assigned in Table 3.2.

Table 3.2

N	1	2	3	4	5	6	…
$H_{B\max}(N)$	0	0,49383…	1,89437…	4,9213…	10,81497…	18,35102…	…

Let's examine the dependence $H_{B\max}$ over N with $\pi_i = \pi = \frac{1}{N}$, then

$$H_{B\max} = -\sum_{i=1}^{N}\left(1-e^{1-N^{-1}}\right)\frac{1}{N} = -\left(1-e^{1-\frac{1}{N}}\right)\ \ .$$

Calculations give following results (tab. 3.3.):

Table 3.3

N	1	2	3	4	5	10	100	1000
$H_{B\max}$	0	0,64872…	0,94773…	1,1170…	1,22934…	1,45060…	1,69123…	1,71801

$$\lim_{N\to\infty} H_{B\max} = -(1-e) = 1{,}718281\ \ .$$

Let's examine one additional function, which satisfies requirements claimed to the entropy; however, maximum entropy nonmonotonically depends on the number of alternatives N.

Let

$$H_C = \sum_{i=1}^{N}\left(1-\pi_i^{\pi_i}\right)\pi_i$$

with the normalization $\sum_{i=1}^{N}\pi_i = 1$. This function becomes zero in the case of a singular distribution

$$\pi(\sigma_i) = \begin{cases} 0; \forall i \neq k\ , \\ 1; i = k\ \ . \end{cases}\quad i,k \in \overline{1,N}\ \ .$$

Note that the function

$$H'_C = \sum_{i=1}^{N}\left(1-\pi_i^{\pi_i}\right)$$

also satisfies necessary conditions: it becomes zero during in the case singular distribution, since $\lim\limits_{\pi_i \to 0}\left(\pi_i^{\pi_i}\right) = 1$.

$$\frac{\partial H'_C}{\partial \pi_i} = -\pi_i^{\pi_i}\left(\ln \pi_i - 1\right) \quad .$$

All $\pi_i \in [0, 1]$ and therefore the second derivative

$$\frac{\partial^2 H'_C}{\partial \pi^2_i} = -\pi_i^{\pi_i}\left(\ln \pi_i + 1\right)^2 - \frac{1}{\pi_i}\pi_i^{\pi_i} < 0, \quad \text{for } \forall \pi_i \in [0, 1].$$

If we assume that $S_a|_{\sigma_0}$ - the class of the equivalence of power N and everything $\pi_i = \pi = \dfrac{1}{N}$, then

$$H'_{C\max} = \sum_{i=1}^{N}\left(1-\left(\frac{1}{N}\right)^{\frac{1}{N}}\right) = \left(1-\left(\frac{1}{N}\right)^{\frac{1}{N}}\right)N \quad .$$

It is possible to show that with $N \to \infty$ the value $H'_{C\max} \to \infty$, while $H_{C\max} \to 0$. $H_{C\max}(N)$ has maximum value with $N = 3$. Fig. 3.2 shows the dependence of the maximum values of the functions from the number of alternatives in the case when $S_a|_{\sigma_0}$ is a class of equivalence.

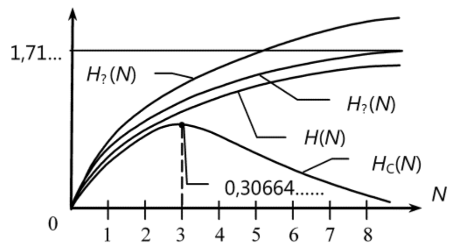


Fig. 3.2

In a certain cases it is convenient to use the normalized entropy, which is obtained by a division on the maximum value:

$$\bar{H}_\pi = -\frac{1}{\ln N}\sum_{i=1}^{N}\pi_i \ln \pi_i \quad ; \tag{3.19}$$

$$\bar{H}_A = \frac{N}{N-1}\sum_{i=1}^{N}(1-\pi_i)\pi_i \quad ; \tag{3.20}$$

$$\bar{H}_B = -\frac{1}{1-e^{1-N^{-1}}}\sum_{i=1}^{N}\left(1-e^{1-\pi_i}\right)\pi_i \quad ; \tag{3.21}$$

$$\bar{H}_C = \frac{1}{1-(N-1)^{N-1}}\sum_{i=1}^{N}\left(1-\pi_i^{\pi_i}\right)\pi_i \quad . \tag{3.22}$$

The Important property of entropy is its sensitivity with respect to changes in the value of preferences. On Fig. 3.3 the functions of sensitivity of three different entropies relative to the preferences are shown:

$$S_{\pi_i} = \frac{\partial H_\pi}{\partial \pi_i} \quad ; \quad S_{\pi A_i} = \frac{\partial H_A}{\partial \pi_i} \quad ; \quad S_{\pi B_i} = \frac{\partial H_B}{\partial \pi_i} \quad ;$$

in the range $\pi_i \in [0, 1]$. All functions have regions of positive and negative values.

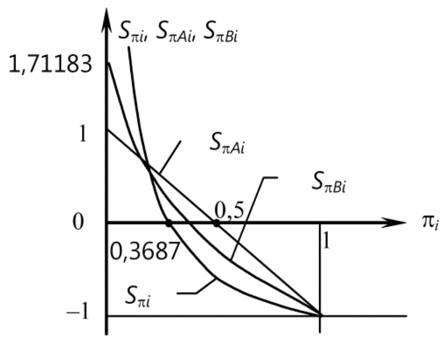


Fig. 3.3

From the number of pseudo-entropy functions, the entropy H_A appears the most convenient from a purely „technical" point of view, since it leads to easily resolvable linear relationships during construction of preference functions models, on basis of variation principles. Shannon's entropy has essential advantage over functions H_A, H_B and similar: it has a property of hierarchical additivity, which makes it especially convenient during studying hierarchical systems.

3.4. Subjective information. Entropy of ways.

We will use Shannon's entropy (3.1). Let A be a certain message, or an event, as a result of which changes the subject distribution of preferences, and also, possibly, the composition of alternatives set $S_a|_{\sigma_0}$.

As a message (an event) A, a change in a resource state can come out, and also change in the utilities - distribution of utilities, any qualitative change in an subject environment and in the subject itself, for example, a change in needs and tastes with age, political changes, change in the situation, competition and many others, that affect in one way or another interests of the subject.

An introduction of subjective entropies and subjective information allows any events, including those, which are not bagger to quantitative description, to reflect by a certain quantitative measure through their influence over preferences of subject.

As we introduced above concepts of the set of alternatives and preference functions, determined (by subject) on this set, so tracing changes, taking place as a result of a message *A*, we obtain a possibility to place in the correspondence to each such event, a certain number, which we will call subjective information. With regard to the distribution of preferences, it is determined exclusively on the subjective level. For explaining the value of preference there is a large number of methods of expert estimations, rank criteria. Statistics gives methods of processing and determining the estimations.

The second positive moment, which appears in connection with an introduction of formalized concepts in the subjective analysis, is the possibility of constructing models of functions of the preferences of subjects and groups of subjects by using variation principles, at basis of which lie the results and axioms of the theory of categories.

In turn, the presence of the models of the functions of preference offers the possibilities of further study within the framework of subjective analysis and which is very substantial, the possibility to accomplish a prognostication of the behavior of subjects, and, therefore, substantiated *control of the active systems through control of preferences*.

Let the new function of the preference of subject after message *A* be $\pi(\sigma_i|_A)$ and the corresponding entropy

$$H_\pi(|A) = H(\pi|A) = -\sum_{i=1}^{K} \pi(\sigma_i|A) \ln \pi(\sigma_i|A) \quad . \tag{3.23}$$

Suppose initial entropy is

$$H_\pi = H(\pi) = -\sum_{i=1}^{K} \pi(\sigma_i) \ln \pi(\sigma_i) \quad ,$$

then let's determine the information, which is contained in the message (event) *A* for this subject, by the formula

$$I_{\text{subj}}(A) = H(\pi) - H(\pi \mid A) \quad . \tag{3.24}$$

The determination of information from formula (3.24) is insufficiently universal. In certain cases it does not recover proceeding changes. Such cases are illustrated in Fig. 3.4.

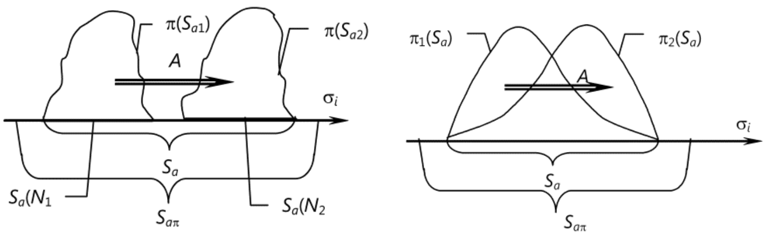


Fig. 3.4

In the scheme (a) this conversion of preferences is shown, when shapes of the distribution of preferences are similar, but the regions of their location change:

$$\pi_1(S_{a1}) \sim \pi_2(S_{a2}), \text{ but } \; S_a(N_1) \not\sim S_a(N_2).$$

In the scheme (*b*) the domain of distributions definition $\pi_1(S_a)$ and $\pi_2(S_a)$ does not change; however, the shapes of distributions change as a result of *A*:

$$\pi_1(S_a) \not\sim \pi_2(S_a),$$

but in such way that the entropy, calculated by the same formula for $\pi_2(S_a)$ is equal to the entropy, calculated for $\pi_1(S_a)$.

In these cases the entropy determined by formula (3.23) is equal to entropy $H(\pi)$ and in spite of occurred changes, an information is absent. We will obtain another formula for the subjective information, which will recover any changes in a problem-resource situation, including „pure shear", which corresponds to the case (a) in Fig. 2.3.

Let as a result on event *A* shape of the form of the preferences distribution changes, and also the set of alternatives changes:

$$\pi(\sigma_i) \to \pi(\sigma_i \mid A) ;$$
$$S_{a1}(N_1) \to S_{a2}(N_2 \mid A) .$$

If $\pi(\sigma_i, \sigma_j)$ is the distribution of preferences of „one-step" paths on the Cartesian product $S_{a1}(N_1) \times S_{a2}(N_2 \mid A)$, then

$$\pi(\sigma_i, \sigma_j) = \pi(\sigma_i)\pi(\sigma_j \mid \sigma_i),$$

where $\pi(\sigma_j \mid \sigma_i)$ – the distribution, normalized by the condition:

$$\sum_{j=1}^{N_2} \pi\left(\sigma_j \mid \sigma_i\right) = 1\left(\forall j \in \overline{1, N_2}\right) \quad .$$

Let's determine entropy $\pi(\sigma_j, \sigma_i)$ by the relationship:

$$H\left(\pi \ , S_{a1} \to S_{a2} \mid A\right) = -\sum_{i=1}^{N_1}\sum_{j=1}^{N_2} \pi\left(\sigma_i, \sigma_j\right) \ln \pi\left(\sigma_i, \sigma_j\right).$$

It is easy to show that

$$H\left(\pi, S_{a1} \to S_{a2} \mid A\right) = H\left(\pi, S_{a1} \mid A\right) + \bar{H}\left(\pi, S_{a1} \to S_{a2} \mid A\right) \quad , \tag{3.25}$$

Where

$$\bar{H}\left(\pi, S_{a1} \to S_{a2} \mid A\right) = \sum_{i=1}^{N_1} \pi\left(\sigma_i\right) \tilde{H}\left(\pi \mid A\right) \text{ and}$$

$$\tilde{H}\left(\pi \mid A\right) = -\sum_{j=1}^{N_2} \pi\left(\sigma_j \mid \sigma_i\right) \ln\pi\left(\sigma_j \mid \sigma_i\right) \text{ for } \forall i \in \overline{1, N}.$$

The formula (3.25) attests to the fact that the subjective entropy of new problem-resource situation with the assigned initial $S_{a1}(N_1)$ and final $S_{a2}(N_2| A)$ set of alternatives is equal to the sum of the entropy of initial unconditional distribution $\pi_1(\sigma_i)$ on $S_{a1}(N_1)$ and averaged entropy of conditional preferences $\pi(\sigma_j| \sigma_i)$ on $S_{a2}(N_2| A)$. The subjective information, caused by an event *A*, determine by the formula:

$$I_{subj} = H\left(\pi\right) - H\left(\pi, S_{a1} \to S_{a2} \mid A\right) \quad . \tag{3.26}$$

Even if $H(\pi) = H(\pi| A)$, the information, calculated by the formula (3.26) proves to be different from zero, in that number in cases of (a) and (*b)* of those shown in Fig. 3.3. In accordance with (3.26) the information „appears", which signals about the occurred changes, reason of which was an event *A*.

The formula (3.26) can be used as diagnostic means when subjective analysis of active systems has to be made.

A special case of entropy is the case when message *A* lies in the fact that system is in the state $\sigma_i \in S_a |_{\sigma_0}$. Let's designate $H(\pi| A) = H(\pi| i)$

$$H\left(\pi \mid i\right) = -\sum_{j=1}^{K} \pi\left(\sigma_j \mid \sigma_i\right) \ln \pi\left(\sigma_j \mid \sigma_i\right) \quad .$$

This expression coincides with $\tilde{H}(\pi \mid A)$. Quantity of information, which contains message, that the system is in state σ_i is determined by the formula

$$I_{subj}\,|_i = -\sum_{j=1}^{N} \pi(\sigma_j)\ln\pi(\sigma_j) + \sum_{j=1}^{K} \pi(\sigma_j \mid \sigma_i)\ln\pi(\sigma_j \mid \sigma_i) \quad . \tag{3.27}$$

Generalizing somewhat this formula we can write:

$$\begin{aligned} I_{subj}\,|_{s,r} = &-\sum_{i=1}^{N} \pi_1(\sigma_i \mid \sigma_s)\ln\pi_1(\sigma_i \mid \sigma_s) + \\ &+\sum_{i=1}^{K} \pi_2(\sigma_i \mid \sigma_r)\ln\pi_2(\sigma_i \mid \sigma_r) \quad . \end{aligned} \tag{3.28}$$

It is assumed here that π_1 and π_2 are assigned above different set of alternatives $S_{a2}(N)$ and $S_{a2}(K)$. In the particular case, if π_1 and π_2 coincide and are assigned on the same set of alternatives

$$I_{subj}\,|_{s,r} = -\sum_{i=1}^{N} \left(\pi(\sigma_i \mid \sigma_s)\ln\pi(\sigma_i \mid \sigma_s) - \pi(\sigma_i \mid \sigma_r)\ln\pi(\sigma_i \mid \sigma_r)\right) \quad . \tag{3.29}$$

We will consider this value as subjective information, connected with a passage (transfer) of a system from the state σ_s in the state σ_r – *„information of the states connection"*. Now we can talk about *„an entropy equivalence"* of two states σ_s and σ_r, if $I_{subj}\,|_{s,r} = 0$, or, in other words, the transfer from the state σ_s in the state σ_r: $\sigma_s \to \sigma_r$ does not change the degree of uncertainty of subject preferences.

Examine some examples illustrating the above statement.

Let S_a: (σ_1, σ_2, σ_3, σ_4) and the distribution of preferences is uniform. Since N = 4, then $\pi(\sigma_i) = N^{-1} = 0{,}25$; $\forall i \in \overline{1,4}$ (Fig. 3.5).

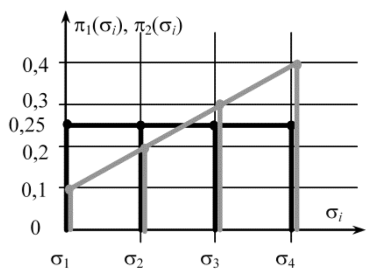


Fig. 3.5

The entropy of absolute preferences is maximum and is equal ln$N$ = ln4 = 1,38629… Assuming that as a result of certain A „message", preferences were redistributed $\pi(\sigma_i \mid A)$ and became linear, namely: $\pi(\sigma_1) = 0{,}1$; $\pi(\sigma_2) = 0{,}2$; $\pi(\sigma_3) = 0{,}3$;

$\pi(\sigma_4)$ = 0,4 ($\sum_{i=1}^{N} \pi_i = 1$), but a set of alternatives S_a did not change. It is easy to calculate entropy $H(\pi| A)$: $H(\pi| A)$ = 0,546814... Thus, it is possible to say that the subjective information, which corresponds to an event (message A) is equal

$$I_{subj} = H(\pi_1) - H(\pi_2| A) \cong 1{,}38629... - 0{,}546814... = 0{,}839480...$$

Let's examine another example: the initial distribution is uniform and S_a: (σ_1, σ_2, σ_3, σ_4) also contains four alternatives (N = 4), $\pi(\sigma_i)$ = 0,25, $\forall i \in \overline{1,4}$.

As a result of an event A a new set of alternatives also contains four alternatives, but the shift of all S_a to the right to „two steps", i.e., S_a: (σ_3, σ_4, σ_5, σ_6) occurred. All alternatives are possible and accessible. Concerning the distribution of preferences, let's suppose that it stayed uniform, i.e., $\pi_2(\sigma_i)$ = 0,25 $\forall i \in \overline{3,6}$ (Fig. 3.6).

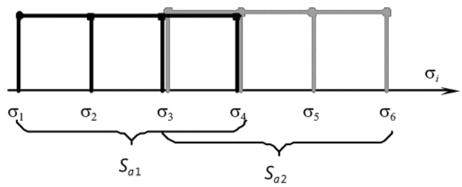


Fig. 3.6

In this task calculations on formulas (3.24) give zero information. In other words, an event A is unimportant for the subject: both problem- resource situations are entropy equivalent.

An application of formula (3.26) gives, in this case, nontrivial subjective information. This information depends on the matrix of conditional preferences $||\pi(\sigma_j| \sigma_i)||$. Let's examine two versions. In the first case let conditional preferences of all particular passages are identical, i.e., the matrix $||\pi(\sigma_j| \sigma_i)||$ has the form:

j/i	1	2	3	4
3	0,25	0,25	0,25	0,25
4	0,25	0,25	0,25	0,25
5	0,25	0,25	0,25	0,25
6	0,25	0,25	0,25	0,25
Σ	1,0	1,0	1,0	1,0

Let's find for each i

$$\tilde{H}_i(\pi \mid A) = 4\pi(j \mid i)\ln\pi(j \mid i) = 4 \cdot 0{,}25\ln 0{,}25 = 1{,}386294...$$

Further

$$\bar{H}\left(\pi, S_1 \to S_2 \mid A\right) = \sum_{i=1}^{4} \pi_1\left(\sigma_1\right)\tilde{H}_i\left(\pi \mid A\right) = 4 \cdot 0{,}25 \cdot 1{,}386294\ldots = 1{,}386294\ldots$$

Let now it be significant for the subject, from what state („the point of departure") and in what state („the point of arrival") the passage is accomplished, in other words, „the way" they are evaluated differently. Let's examine the following matrix of conditional preferences

j/i	1	2	3	4
3	0,1	0,1	0	0
4	0,2	0,2	0,2	0
5	0,3	0,3	0,3	0,4
6	0,4	0,4	0,5	0,6
Σ	1,0	1,0	1,0	1,0

.

Calculations give a following value $\tilde{H}_i\left(\pi \mid A\right)$

i	1	2	3	4
$\tilde{H}_i\left(\pi \mid A\right)$	0,546514…	0,546817…	1,04454…	0,673016…

Then

$$\bar{H}\left(\pi, S_{a1} - S_{a2} \mid A\right) = 0{,}25 \cdot 0{,}546514\ldots + 0{,}25 \cdot 0{,}546814\ldots + \\ + 0{,}25 \cdot 1{,}04454\ldots + 0{,}25 \cdot 0{,}673016\ldots \cong 0{,}702796\ldots$$

The entropy of way has features of a hierarchical additivity:

$$H\left(\pi\left(\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{iN}\right)\right) = H\left(\pi\left(\sigma_{i1}\right)\right) + H\left(\pi\left(\sigma_{i2} \mid \sigma_{i1}\right)\right) + \\ + H\left(\pi\left(\sigma_{i3} \mid \sigma_{i1}, \sigma_{i2}\right)\right) + \ldots + H\left(\pi\left(\sigma_{iN} \mid \sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{iN-1}\right)\right) \quad . \tag{3.31}$$

This function corresponds to the case, when as a result each particular passage set of alternatives S_a doesn't change. If factorization is *Markovian*, particular entropy is represented by the formula:

$$H\left(\pi\left(\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{iN}\right)\right) = H\left(\pi\left(\sigma_{i1}\right)\right) + H\left(\pi\left(\sigma_{i2} \mid \sigma_{i1}\right)\right) + \\ + H\left(\pi\left(\sigma_{i3} \mid \sigma_{i1}\right)\right) + \ldots + H\left(\pi\left(\sigma_{iN} \mid \sigma_{iN-1}\right)\right) \quad . \tag{3.32}$$

With the analogy of information theory, determine conditional subjective entropy in the case „of the way" of arbitrary length. Let ways of length N are examined:

$$Tr(N) = \sigma_{i1} \to \sigma_{i2} \to \sigma_{i3} \to \ldots \to \sigma_{iN-1} \to \sigma_{iN} = (\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{iN}).$$

This way is the element of the Cartesian product of set S_a to itself

$$Tr(N) \in \underbrace{S_a^N \times S_a^N \times \ldots \times S_a^N}_{N}$$

when, after each „step" the set of alternatives remain in the initial form.

If, for example, after each step already passed state (or alternative), that corresponds to certain problem already solved is excluded, the dimensionality of set S_a^M is decreased by one. So after k steps $M = N - k$ alternatives remains in the disposal of subject. (If, of course, S_a is not supplemented by other alternatives, in accordance with some rule). For the way of length N in the last step set of alternatives will be reduced to one element (N- (N- 1) = 1).

For the way of length K a set S_a before the last step will have, with the indicated above assumption $N - K$ alternatives. The function of the preference of route segment of length $K - 1 < M$ designate through $\pi(\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{i, K-1})$. Conditional function of the subjective preference of the remaining part of the way

$$Tr(N - (K - 1)) = (\sigma_{iK}, \sigma_{iK+1}, \ldots, \sigma_{iN-1}, \sigma_{iN})$$

let us determine by the relationship:

$$\pi(\sigma_K, \sigma_{K+1}, \sigma_N \mid \sigma_1, \sigma_2, \ldots, \sigma_{K-1}) = \frac{\pi(\sigma_1, \sigma_2, \ldots, \sigma_N)}{\pi(\sigma_1, \sigma_2, \ldots, \sigma_{K-1})} \quad . \tag{3.33}$$

Indices, which designate number of alternatives, are omitted here and indices of number of step are left. The particular entropy for the way Tr (N-(K-1)) let us define as

$$H\left(\pi(\sigma_K, \ldots, \sigma_N \mid \sigma_1, \ldots, \sigma_{K-1})\right) = -\ln \pi(\sigma_K, \ldots, \sigma_N \mid \sigma_1, \ldots, \sigma_{K-1}).$$

For an averaged entropy on the remaining part of the way (σ_K, ..., σ_N) (on entire set of route segments ($K < N$)) let us accept the formula

$$\begin{aligned} &H_{\sigma_K, \ldots, \sigma_N}\left(\pi(\sigma_K, \ldots, \sigma_N \mid \sigma_1, \ldots, \sigma_{K-1})\right) = \\ &= -\sum_{\sigma_K \in S_a} \ldots \sum_{\sigma_N \in S_a} \pi(\sigma_K, \ldots, \sigma_N \mid \sigma_1, \ldots, \sigma_{K-1}) \ln \pi(\sigma_K, \ldots, \sigma_N \mid \sigma_1, \ldots, \sigma_{K-1}) \,. \end{aligned}$$

Finally, let's determine the result of an averaging of particular entropy over the set of all complete ways (σ_1, ..., σ_N) by the relationship:

$$\begin{aligned} &H\left(\pi(\sigma_K, \ldots, \sigma_N \mid \sigma_1, \ldots, \sigma_{K-1})\right) = \\ &= -\sum_{\sigma_1 \in S_a} \ldots \sum_{\sigma_N \in S_a} \pi(\sigma_1, \ldots, \sigma_N) \ln \pi(\sigma_K, \ldots, \sigma_N \mid \sigma_1, \ldots, \sigma_{K-1}) \quad . \end{aligned}$$

If we use a formula (2.35), then it will be found that

$$H_{\sigma_K,\ldots,\sigma_N}\left(\pi\left(\sigma_1,\ldots,\sigma_{K-1}\right)\right)=H\left(\pi\left(\sigma_1,\ldots\sigma_K\right)\right)+$$
$$+\sum_{\sigma_1\in S_a}\ldots\sum_{\sigma_N\in S_a}\pi\left(\sigma_1,\ldots,\sigma_N\right)\ln\pi\left(\sigma_1,\ldots,\sigma_{K-1}\right) \quad .$$

From above it is clear, that conditional mean entropy, determined on the set of ways, is smaller than the complete average entropy.

$$H\left(Tr\left(K,N\right)\mid Tr\left(1,K-1\right)\right)\leq H\left(Tr\left(K,N\right)\right) \quad . \tag{3.34}$$

In the particular case of two-step „ways"

$$\pi\left(\sigma_2\mid\sigma_1\right)=\frac{\pi\left(\sigma_1,\sigma_2\right)}{\pi\left(\sigma_1\right)} \quad ,$$

where $\pi(\sigma_2| \sigma_1)$ is conditional preferences of passage in σ_2, if previously the passage in σ_1, was realized $\pi(\sigma_1, \sigma_2)$ is the distribution of preferences of ways $\rightarrow \sigma_1 \rightarrow \sigma_2$, $\pi(\sigma_1)$ is distribution of preferences of one-step ways $\rightarrow \sigma_1$, from where it is possible to find that

$$H\left(Tr\left(1\rightarrow 2\right)\mid Tr\left(-1\right)\right)\leq H\left(Tr\left(-1\rightarrow 2\right)\right) \quad . \tag{3.35}$$

A following regularity is here confirmed, that conditional entropy does not increase in the case of the addition of conditions.

The following important property of averaged entropies occurs: let $\pi_i \in [0, 1]$ and $\eta_i \in [0, 1]$; $\forall i\in\overline{1,N}$ is two normalized distributions of preferences [137].

Then the inequalities take place:

$$-\sum_{i=1}^{N}\pi_i\ln\pi_i\leq-\sum_{i=1}^{N}\pi_i\ln\eta_i \quad ; \tag{3.36}$$

$$-\sum_{i=1}^{N}\eta_i\ln\eta_i\leq-\sum_{i=1}^{N}\eta_i\ln\pi_i \quad . \tag{3.37}$$

In connection with distributions of preferences, in contrast to the probability distribution, „limit theorems" are not proved, since $\pi(\sigma_i)$, and ξ_j (see Chapter 4) are not considered as random variables. In any case, for this formalism – they are an optional conditions. Something similar can only be postulated, but with a required semantic treatment. In connection with this the concept of „entropy stability" is absent here. A question about the entropy stability of active systems, can be possibly solved with an examination of the dynamics of preferences, in particular taking into account Prigogine's theorem about the minimum of speed of an entropy production.

3.5. Variational principle. Canonical distributions of preferences for a discrete set of alternatives.

There is a formal analogy between the distribution of preferences and the probability distributions. We can use a set of results of the probability theory, mathematical statistics, and also information theory, giving them, however, each time an interpretation in the terms of subjective analysis.

In an information theory, for example, for obtaining the canonical distributions of probabilities the variation principle of the maximum of entropy [137] is used. Standard variational problems, solutions of which are „canonical" probability distributions are examined.

The use of an analogous approach leads to „canonical" functions of the preferences distribution. In this case we postulate the principle of optimality. It is assumed that preferences of subject are distributed on the set S_a in such a way that a certain criterion, which contains preference functions, acquires extremal value. We will consider that the mentioned criterion is a weighted sum of three components. The principal part of the criterion is the entropy of the distribution of preference H_π. As it will be seen further on, the discussion deals with „the maximum principle" entropy.

This principle, as it was already said, is postulated in particular in same applications of the theory of categories (see Chapter 2).

Second additive component of criterion is the function of effectivenessε, which depends both on subjective preferences and on the certain objective characteristic of alternatives.

We will assume that in the sufficiently general case the function of utility $U(\sigma_i)$; $\sigma_i \in S_a$ can be used as such characteristic. In special cases probabilities $p(\sigma_i)$ can come out as the function of utility , either some of resources, described in chapter 1 or even some other functions, which reflect distributions of preferences in the past and making it possible to consider in the current stage a priori tastes, „point of view" and persuasions which were established.

Third term is caused by the presence of a normalization condition. Both, a function of effectiveness and a normalization condition in each certain case take a certain form, which corresponds to conditions of task.

It is necessary to make some remarks of general kind, if not basing, then, at east, justifying of extreme principle application to the task of the preferences forming. The discussion actually deals with assigning a certain method of optimum functioning to human psyche that, of course, is not obvious.

There are two methods of an introduction of extreme principles. The first one can be named „phenomenological", when at the beginning for a studied phenomenon, a mathematical model is constructed on the basis of experimental data and then variational problem of such kind is selected, that equations of the model are a necessary conditions for where a steady-state solution.

Then, the corresponding variation problem acquires an independent importance as an extreme principle. Gnosiology of variational principles of mechanics is such one.

In the functional analysis it is known, that to each self -adjoined operator it is possible to place in correspondence such a functional, that the equations, which correspond to the operator, are an Euler-Lagrange equations.

The work of Sedov, where he attempts to obtain equations of fluid and gas mechanics, from the variational principle, analogous to Hamilton's principle are known. Complexity consisted in the fact that the corresponding processes are dissipative.

For some classes of dissipation systems with concentrated parameters variational principle and, correspondingly, quasi-conservative mechanics have been proposed by the author [64].

A different way consists in the postulation of certain extreme principle, from a priori persuasion, that „the sense of any maximum or minimum is visible in everything". Instead of words „on a priori persuasion" it would be better to use an expression „on a suspicion". Exactly such way is used in this work, in particular still because we have not available mathematical models of functioning of human psyche, for which it would be possible to select an appropriate variational problem.

In this case the following plan of theory construction is used:

- the postulation of a priori variational principle;
- obtaining a mathematical (quantitative) model, basing on this principle;
- an experimental study for the purpose to confirm the acceptability (likelihood) of an obtained model and, therefore, a variation principle.

Confidence, that this approach has right to exist, results from the following circumstances.

1. Collection, processing and production of information (in the broad sense) is one of functions of the man's psyche, at the same time in an information theory a similar variational principle is used.
2. In the theory of categories, the brief account of basic concepts of which is given above, a variational principle and corresponding generalized concept of an information, in the form of functions of cardinal number of sets *of elements* and sets of *morphism* is formulated and proposed.
3. An „experimental" fact, for some applications at every moment of time man forms a set of alternatives S_a, distributes his preferences on this set, and solves „the problem of selection", and this process has certain general properties for different situations.

Variational principles similar to that which is used in the present work have been examined previously by a number of authors. In particular in the Haken's book [152] a criterion was formulated, whose fundamental component is an entropy of the probability distribution. There is known works in which an entropy approach use in the economy, the theoretical geography, demographics, in analysis of social and economic systems, executed in the middle and the second-half of past century [30, 33, 126, 127, 134, 136, 173, 187, 194].

The concept of an entropy or „an expected information", connected with losses during aggregation in economical tasks was studied in the work [200].

A subjective nature of average values and corresponding probability distributions, a question how to handle with the entropy in the course of quantitative analysis in the social and economical systems has being discussed in the work [188].

An entropy was used for analysis of systems with the maximum utility [198].

In the book of Wilson [33] the variational principle, very similar to one discussed here in basis of which lies an entropy approach, is applied to the simulation of complex transport systems.

We will briefly present a content of this investigation in the part, where it agrees with those realized in our work. The work of Wilson, as a whole, is dedicated to the problem of the migration of population in the cities.

If T_{ij} is number of individuals, who live in the locality i and working in the zone j, Q_i is total number of working and living in the zone i, D_j is total number of work sites in the zone j, c_{ij} is expenditure for movement from the zone i in the zone j, total expenditures and T the total number of inhabitants in the city, then following conditions must be carried out:

$$\left.\begin{aligned} Q_i &= \sum_j T_{ij}; \\ D_j &= \sum_i T_{ij}; \\ C &= \sum_i \sum_j c_{ij} T_{ij}; \\ T &= \sum_i \sum_j T_{ij}. \end{aligned}\right\}$$

Let's designate through W the greatest number of possible versions with limiting conditions noted above.

$$W = \frac{T!}{\prod_i \prod_j T_{ij}!}$$

Then

$$\ln W = \ln T! - \sum_i \sum_j \left(T_{ij} \ln T_{ij} - T_{ij} \right) = \ln T! - T \ln T + T - T \sum_i \sum_j p_{ij} \ln p_{ij} \quad .$$

Here $p_{ij} = T^{-1}T_{ij}$. It corresponds to the classical definition of probability, but it can, apparently, treat as the characteristic of averaged preferences.

Last term

$$S = -\sum_i \sum_j p_{ij} \ln p_{ij}$$

represents the entropy. The following variational problem is proposed: to determine probabilities p_{ij} , so that the entropy would reach the maximum value

$$S \to \max$$

with limitations [33]:

$$\sum_i p_j f(x_i) = \varepsilon \quad ;$$

$$\sum_i p_j = 1 \quad ,$$

when $p_i = \sum_j p_{ij}$.

The optimum distribution, which is obtained as a result of an application of variation principle, takes the form:

$$p_{i\text{opt}} = \frac{e^{-\mu f(x_i)}}{\sum_j e^{-\mu f(x_j)}} \quad .$$

If x_1, x_2, ..., x_n – collection „of goods", and $I = \sum_i x_i p_i$ - budget, then individual utility $U = U(x_1, x_2, ..., x_n, I)$. The task of the utility maximization is solved via the extremalization of the functional

$$L = U(x, I) + \lambda\left(I - \sum_i p_i x_i\right) \quad ,$$

from where

$$\frac{\partial U}{\partial x_i} = \lambda p_i \quad .$$

Here p_i plays the role of „goods" prices.

These equations, together with a budget equation make it possible to determine an optimum collection of „goods"

$$x_{i\text{opt}} = \varphi_i(p_1, p_2, ..., p_N, I).$$

Since „goods" have different dimensionality, appropriate dimensionless „goods" are introduced

$$y_i = \frac{x_i p_i}{I} \quad .$$

They present the entropy in the form

$$S = -\sum_i y_i \ln y_i$$

with the normalization condition:

$$\sum_i y_i = 1 \quad .$$

A system with the maximum utility is determined by an extremalization of criterion

$$L = U + \lambda\left(1 - \sum_i y_i\right) \quad , \quad \text{where} \quad U = U\left(\frac{y_1 I}{p_1}, \frac{y_2 I}{p_2}, \dots, \frac{y_n I}{p_n}, I\right) \quad .$$

From the system of equations

$$\frac{\partial U}{\partial y_i} = \lambda \quad ; \quad \sum_i y_i = 1 \quad .$$

we find

$$y_{i\mathrm{opt}} = \psi(p_1, p_2, \dots, p_N, I).$$

Let the entropy approach be now used for the search of an optimal solution and, besides the normalizing condition, there are a number of limitations:

$$f_k(y_1, y_2, \dots, y_N) = g_k\left(k \in \overline{1, L}\right) \quad .$$

The criterion of optimality is selected in the form:

$$L^* = S + \lambda\left(1 - \sum_i y_i\right) + \sum_{k=1}^{L} \mu_k\left(g_k - f_k(y)\right) \quad .$$

Equations

$$\frac{\partial L^*}{\partial y_i} = 0 \quad \left(i \in \overline{1, n}\right) \quad ;$$

have to be added to normalizing condition and equations, which express limitations $g_k = f_k(y)$.

A wide variety of optimization tasks, which are characterized by the specific form of additional constraints, appears.

The author of work [33] also examines some questions of dynamics, uses thermodynamic analogs and introduces a concept of „the amount of work", accomplished over the system. If

$$\bar{A}_j = E\left(\frac{\partial S}{\partial \alpha_i}\right)$$

- „external forces", α_i – some parameters of „position", then work is $\bar{A}_j\, d\alpha_j$. In the last expression, S is analog of an entropy. In the work [33], the so-called gravitational model is developed. As it is evident, in spite of a certain formal similarity of the variational principle, which uses the entropy as an important component of functional, there are essential differences in the treatment between the work [33] and the variational principle, developed in the present monograph, in particular, in the treatment of functions $\pi(\sigma_i)$ as *subjective preferences*, their connection with utilities, ethical imperatives, the structure of functions of effectiveness and with number of other elements of theory.

Let's continue the analysis of the variational principle, basing on the use of subjective entropy.

Conclusions and methods, that follow as a result of a postulation of variational principle , can be useful from our point of view in order to have a capability of more definite forecast of subject's behavior and to obtain certain, more concrete, certain scheme (task), conducting psychological and sociological experimental studies. I have in mind, that presence of a theoretical model, in which „variable" and „constant" parameters, in principle statistically measured are determined, and also relationships, that connect them (for example, the function of preference), gives the possibility to plan the corresponding experiment, namely: to select measured values, to select recipients, to determine metrological characteristics of measurements, to plan the process of experiment and finally to use results of theory to control active systems.

In the sufficiently common form the functional can be undertaken in the form

$$\Phi_\pi = \alpha H_\pi + \beta\varepsilon + \gamma\mathcal{N}, \tag{3.38}$$

*W*here: H_π is subjective entropy; $\varepsilon = \varepsilon(\pi, U, \ldots)$ – the function of subjective effectiveness; $\mathcal{N}$ – the normalizing condition.

Let's name the function of effectiveness ε linear, if it is linear with respect to the function of preference π (.). Otherwise ε is nonlinear. Function $\mathcal{N}$ forms a normalizing condition: $\mathcal{N} = A_0$, where A_0 – a normalizing constant (in the majority of cases $A_0 = 1$). Function $\mathcal{N}$ depends only on the function of preferences π(.). Structural parameters α, β, γ can be considered in different situations as Lagrange's coefficients, or as weight coefficients. Subsequently, structural parameters α, β, γ… will be defined as endogenous parameters, which reflect, certain properties of psyche. The endogenous dynamics of preferences will be connected in particular with them.

The variational problem, connected with a criterion, (3.38) is a task on the conditional extremum.
We will examine two combined tasks.
1. To find distribution $\pi(.)$, delivering outer limit of the subjective entropy H_π with the presence of „isoperimetric conditions":

$$\varepsilon(\pi, U, \ldots) = \varepsilon_0 \ ; \tag{3.39}$$

$$N(\pi) = A_0 \ ; \tag{3.40}$$

$$\pi_{extr}(.) = \arg \underset{\pi(.)\in\Pi}{\text{extr}} H_\pi \quad ,$$

Where Π is class of the preference functions, from which an extreme distribution $\pi_{extr}(.)$ is selected.
2. To find distribution $\pi(.)$ delivering outer limit of the function of effectiveness $\varepsilon(\pi, U, \ldots)$ with the presence of „isoperimetric" conditions

$$H_\pi = H_0 \ ; \tag{3.41}$$

$$N(\pi) = A_0 \ ; \tag{3.42}$$

$$\pi_{extr}(.) = \arg \underset{\pi(.)\in\Pi}{\text{extr}} \varepsilon(\pi, U\ldots) \quad .$$

In the first task the extreme function $\pi_{extr}(.)$ is being searched for on the linear manifold, given by isoperimetric conditions (3.39), (3.40). In the second case the manifold is nonlinear, since the entropy H_π is nonlinear function of $\pi(.)$, and it assigned by equations (3.41) and (3.42).

In the first case it is possible to consider, that the parameter $\alpha = \pm 1$, parameters β and γ are Lagrange's coefficients, which in the final analysis can be expressed through ε_0 and A_0, in the second case $\beta = \pm 1$, parameters α and γ are Lagrange's coefficients, which are the functions of H_0 and A_0.

As it has already been mentioned above, normalizing condition is considered as model presentation of one of the subject psyche properties: with the shift of „desires" (preferences) on the set of alternatives an increase in preference of one of alternatives unavoidably leads to the decrease of the preference of some other. In other words the presence „of the balance" of preferences - certain weights on the set S_a is assumed.

Let's go back to the problem of „the principle of the entropy maximum". „The principle of entropy maximum" was initially formulated in the thermodynamics, then it was proposed in information theory, and later it was postponed by the broader class of objects.

The thermodynamic entropy H has following properties [42]:

Increase in the entropy

$$dH = d_eH + d_iH \quad , \tag{3.43}$$

where d_eH is the entropy, which enters the system from outside; d_iH is the entropy appearing inside the system. According to the second law of thermodynamics $d_iH = 0$ for reversible processes in the system, and

$$d_iH \geq 0 \tag{3.44}$$

for irreversible processes.

Increase d_eH can be positive, negative, or equal zero. For the open systems according to Carnot- Clausius theorem $d_eH = \frac{1}{T}dQ$, where dQ is transfer of heat to the system from outside. Consequently

$$dH \geq \frac{dQ}{T} \quad , \quad \text{if} \quad dQ \geq 0 \ .$$

If system is continuous medium with the density ρ and entropy s of the unit of mass, then a quantity of entropy in the volume V

$$\left.\begin{aligned} H &= \iiint_V \rho s \, dV \ ; \\ \frac{d_eH}{dt} &= \oiint_\Omega \vec{q}_s \, d\Omega \ ; \\ \frac{d_iH}{dt} &= \iiint_V \sigma \, dV \ . \end{aligned}\right\} \tag{3.45}$$

where $\vec{q}_s$ - certain entropy flow per unit of time through the Ω surface area unit of the limiting volume *V*; σ - the intensity of local entropy sources, distributed in the volume *V*.

The integral relationship occurs

$$\iiint_{(V)} \left(\frac{d(\rho s)}{dt} + \operatorname{div} \vec{q}_s - \sigma \right) dV = 0 \quad . \tag{3.46}$$

Hence the equation follows:

$$\frac{d(\rho s)}{dt} = -\operatorname{div} \vec{q}_s + \sigma \quad , \tag{3.47}$$

Introducing the flow $\vec{I}_s = \vec{q}_s - \rho s\vec{v}$, where $\vec{v}$ is the speed of element of the medium, equation (3.47) can be transformed to the following:

$$\rho\frac{ds}{dt} = -\text{div}\,\vec{I}_s + \sigma \quad , \tag{3.48}$$

if the thermodynamic equilibrium occurs in the system.

We already noted above (in chapter 2) the principle of the entropy maximum, postulated in the categories theory [96] and the information principle of the entropy maximum [137] being a special case.

Generally, the entropy as an attribute of different theories from physics to economy, sociology, psychology and even history, attracts ever closer attention recently. In some works the role of entropy is absolutized and raised to the rank of essence and attribute, which lies on the basis of the processes of proceeding in the universe as a whole.

There are opinions that *„material becomes „active": it generates an irreversible processes, and an irreversible processes organize material", and more of that „the material is not only a passive substance, described within the framework of the mechanical picture of nature, but it is also characteristic with a spontaneous activity" [133]*.

In the latest works of Panchenkow [129, 130, 131] the principle of the maximum entropy is introduced and discussed in the very common form. The entropy is represented in the form of the sum of „the entropy of structure" and „the entropy of the pulse":

$$H = H_f + H_p \ ,$$

where H_f is „the entropy of structure", connected with a configurative space of system, H_p is „entropy of pulses", connected with a momentum space.

The author of mental works proves for conservative systems, that the entropy H is invariant $H=$ const and changes in the system are reduced to „the overflow" H_f in H_p and vice versa. The principle of maximum is established for this entropy and different generalized versions.

Returning to the discussion about this book subject, we note that the author does not claim to be giving all the material world (Universum) the „active system" status, the class of the „active systems", which we are considering, is much narrower, both conceptually and spatially, and in temporal sense.

We postulate the following important provision, which may be named *„entropy principle of formation of subjective preferences"*.

1. Distribution of subjective preferences is formed on the basis *of the variational principle*. Functional of an extremed problem depends both on subjective

preference functions and on values and functions, which are objective characteristics of problem-resource situation.

This functional appeared as a connecting link between mental processes of subject and the objective circumstances, which in „total" compose problem- resource situation.

2. The basic additive component of the functional is subjective entropy.
3. Depending on circumstances, a subject forms preferences on the set S_a in such way that the entropy of preferences takes *the maximum* value (task (3.30), (2.40)) or *the fixed* value (task (3.41), (3.42)).

Each of enumerated three points is hypothesis, more or less plausible, which does not fall out, however, from the general „flow" of entropy studies.

The formulated principle is consonant with the statement of Leonard Euler about that „… nothing in all of the world will occur in which no maximum or minimum rule is somehow shining forth", and also observations, that „… our world is the best of all possible worlds, and therefore the laws, which control them, can be described by extreme principles".

There is a large suspicion, that it is so in reality. Newton's law made it possible to obtain equations of motion of material systems but then it turned out that at least in one special case, namely, in the case of conservative systems, these equations can be obtained basing on the variational principle of Hamilton. For a long time it was considered that the variational principle is absent for dissipation systems. It was recently shown that, at least formally, it is possible to construct variational principle for some types of dissipation systems [66, 68].

An introduction in the subjective analysis of variational principle does not indicate attachment to the subject of the certain method of analysis and decision making. It is an advanced assumption, hypothesis, that the certain variational principle is organically inscribed in the human psyche and functions, so to say, „independent from his will". A question consists of that, „which is this principle?"

We assume this subject forms and solves in differential stages (appearance, prognostic analysis and the use of results), that the subject one of two enumerated variational problems.

Examine special cases of variational problems.

Let it is necessary to determine the canonical distribution of absolute preferences, which corresponds to the maximum of subjective entropy with the given value of the function of effectiveness ε_0.

We define a sense of the effectiveness function ε (π, …) slightly more concretely. Let us make an additional assumption about the fact, that together with the utility function $U(\sigma)$, which has positive sense and which establishes the preference relation on the set S_a, „growing" together with the numerical value of $U(\sigma)$:

$$\sigma \langle \eta \Leftrightarrow U(\sigma) < U(\eta) \quad , \tag{3.49}$$

subject can use loss function („harmfulness”) (Losses function or harm function or ingery function) $L(\sigma)$, having negative sense and establishing „diminishing” preference relation on the set S_a with increase of $L(\sigma)$

$$\sigma \langle \eta \Leftrightarrow L(\sigma) > L(\eta) \quad . \tag{3.50}$$

Introduction of this function enlarges and facilitates analysis of alternatives.

Let’s imagine to ourself the student of arts, who is proposed to examine N objects of art and to formulate, not only advantages, but also disadvantages of each of them. *Such* analysis will be more complete and, it is necessary to assume, it facilitates the task of the ranking of objects on the preference in comparison with the case, when only advantages or only disadvantages would be considered.

Another example of positive and negative preferences can be meditations of epic hero in front of stone on the crossroads. It is written on the stone: if „go right - you will find wealth, left - glory, directly - authority”, then the hero forms positive preference and makes a positive selection (selection of the greatest benefit), but if an inscription says: „turn right – lose honor, left - freedom, directly - head”, this is a negative selection (selection of the smallest evil).

A set of alternatives S_a in this case contains three alternatives, and the subjective entropy of hero in the case of maximum „uncertainty” is equal In3.

Buyer’s behavior in the drugstore with the purchase of medicine from a certain collection is a good example. Two tactics are possible here. In the first case the buyer begins studying a „facial page” of the description of the medicine, where „indications” are presented, i.e., the benefit, which is expected as a result of its application, makes preliminary selection on the basis of this information and then he is turned to „opposite page”, where it is possible to read about the contra-evidence, i.e., about the dangers of treatment with medicine.

The second stage of selection is achieved while taking under consideration this „negative” information. In the second case tactics of the buyer (more careful) - reading, he starts, basing on the „opposite page”, a negative analysis and only then he is turned to the positive analysis.

Situation with the selection is consonant which in psychology is called „cognitive discord (dissonance)”.

Let the ordering be produced separately: first only on L(σ), then on U(σ). In the first case we will obtain the function of negative preferences, which we designate $\pi^-(\sigma)$, in the second case - the function of positive preferences we designate $\pi^+(\sigma)$. Accordingly, two functions of effectiveness [29,40] are used:

$$\varepsilon^-\left(\pi^-, L(\sigma), \ldots\right) = \sum_{i=1}^{N} \pi^-\left(\sigma_i\right) L\left(\sigma_i\right) \quad ; \tag{3.51}$$

$$\varepsilon^{+}\left(\pi^{+},U(\sigma),\ldots\right)=\sum_{i=1}^{N}\pi^{+}(\sigma_i)U(\sigma_i) \quad . \tag{3.52}$$

Normalizing conditions are identical:

$$\sum_{i=1}^{N}\pi^{-}(\sigma_i)=1 \quad ; \qquad \sum_{i=1}^{N}\pi^{+}(\sigma_i)=1 \quad ; \tag{3.53}$$

$$\left.\begin{array}{l} 1\geq\pi^{-}(\sigma_i)\geq 0 \\ 1\geq\pi^{+}(\sigma_i)\geq 0 \end{array}\right\} \quad \forall i\in\overline{1,N} \quad . \tag{3.54}$$

Let's select a functional in the first case in following form:

$$\Phi_{\pi}^{-}=-\sum_{i=1}^{N}\pi^{-}(\sigma_i)\ln\pi^{-}(\sigma_i)-\beta\sum_{i=1}^{N}\pi^{-}(\sigma_i)L(\sigma_i)+\gamma\sum_{i=1}^{N}\pi^{-}(\sigma_i) \quad . \tag{3.55}$$

From the necessary condition of the extremum

$$\frac{\partial\Phi_{\pi}^{-}}{\partial\pi^{-}(\sigma_i)}=0\left(\forall i\in\overline{1,N}\right)$$

we find

$$-\ln\pi^{-}(\sigma_i)-1-\beta L(\sigma_i)+\gamma=0 \quad .$$

Hence

$$\pi^{-}(\sigma_i)=e^{-1+\gamma}e^{-\beta L(\sigma_i)} \quad \text{or} \quad \pi^{-}(\sigma_i)=Ce^{-\beta L(\sigma_i)} \quad .$$

Using normalizing condition, will find constant C:

$$C=\left(\sum_{j=1}^{N}e^{-\beta L(\sigma_i)}\right)^{-1} \quad .$$

Then

$$\pi^{-}(\sigma_i)=\frac{e^{-\beta L(\sigma_i)}}{\sum_{j=1}^{N}e^{-\beta L(\sigma_j)}}. \tag{3.56}$$

is the monotonically decreasing function $L(\sigma_i)$.

Since $\dfrac{\partial^2\Phi_\pi^-}{\partial\pi^{-2}(\sigma_i)} = -\dfrac{1}{\pi^-(\sigma_i)} < 0,\quad \forall i \in \overline{1,N}$, then in this case a functional attains its maximum value on the manifold, given by the relations:

$$\sum_{i=1}^{N}\pi^-(\sigma_i)L(\sigma_i) = \varepsilon_0 \quad ; \quad \sum_{i=1}^{N}\pi^-(\sigma_i) = 1 \quad .$$

In view of the linearity of functions ε^- and N relative to π_i^- the equality occurs

$$\frac{\partial^2\Phi_\pi^-}{\partial\pi_i^{-2}} = \frac{\partial^2 H_\pi}{\partial\pi_i^{-2}} \quad .$$

It means that the subject selects the distribution of negative preferences in such a way that the entropy of distribution $\pi^-(\sigma i)$ reaches its maximum with satisfaction of isoperimetric conditions, that is, tends to equalize the negative preferences - to maximize the integration of all the negative factors.

Let now the criterion Φ_π^+ takes the form:

$$\Phi_\pi^+ = -\sum_{i=1}^{N}\pi^+(\sigma_i)\ln\pi^+(\sigma_i) + \beta\sum_{i=1}^{N}\pi^+(\sigma_i)U(\sigma_i) + \gamma\sum_{i=1}^{N}\pi^+(\sigma_i) \quad . \tag{3.57}$$

Analogically with the previous case we find:

$$\pi^+(\sigma_i) = \frac{e^{\beta U(\sigma_i)}}{\sum_{j=1}^{N} e^{\beta U(\sigma_j)}} \quad . \tag{3.58}$$

Function $\pi^+(\sigma_i)$ the monotonically increasing function $U(\sigma_i)$. As in the case of functional (3.55), the condition

$$\frac{\partial^2\Phi}{\partial\pi_i^{+2}} = -\frac{1}{\pi_i^+} < 0\,, \quad \forall i \in \overline{1,N} \quad .$$

is satisfied.

A distribution (3.58) again delivers to the entropy H_π maximum value on the appropriate manifold, given by isoperimetric conditions.

If the function of effectiveness ε is linear on $\pi(\sigma_i)$, then Hesse matrix

$$G = \left\| \frac{\partial^2\Phi_\pi^\pm}{\partial\pi_i^\pm(\sigma)\pi_j^\pm} \right\| = \mathrm{diag}\left\| \frac{\partial^2\Phi_\pi^\pm}{\partial\pi_\pi^{\pm 2}} \right\|$$

is diagonal one. Depending on the sign of the parameter α in (3.38) all diagonal elements are positive if $\alpha = +1$ and negative if $\alpha = -1$.

All principal minors of Hessen matrix are is positive, if $\alpha = +1$:

$$M_k(G) = \prod_{s=1}^{k} \frac{\partial^2 \Phi_\pi^\pm}{\partial \pi_i^{\pm 2}} > 0$$

and the sign reverses, if $\alpha = -1$:

$$M_k(G) = (-1)^k \left| \prod_{s=1}^{k} \frac{\partial^2 \Phi_\pi^\pm}{\partial \pi_i^{\pm 2}} \right|.$$

Independently on sign α and on the value of the parameter β canonical preferences deliver the maximum to the entropy H_π. The connection between preferences $\pi_i^\pm$ is achieved as a result the satisfaction of isoperimetric conditions.

Subsequently the parameter β is used as the endogenous parameter, and the parameter γ is determined by normalization condition.

Functions of preference π_i^+ and π_i^- are preferences „of the adoption" of the corresponding alternative. We can interpret π_i^+ and π_i^- as follows:

$\pi^+(\sigma)$ –preference to accept alternative σ, being oriented in the value of utility $U(\sigma)$;

$\pi^-(\sigma)$ – preference to accept alternative σ, being oriented in the value of harmfulness $L(\sigma)$.

The previous considerations lead to the idea to introduce in the examination the new preference function. However, this time it will be preference to „reject" certain alternative, which as introduction along with the utility function of the loss function or „harm", extends the methodological framework of the study. The introduction of a new preference function makes it possible to look at each alternative from two points of view and to form a kind of balance: „usefull-harmfull", „accepted-rejected".

Let's designate this new preference function υ_i^+ and υ_i^-, where υ_i^+ is positive preference „to reject", υ_i^- – negative preference „to reject". In more detail:

$\upsilon_i^+(\sigma)$ – preference to reject alternative σ being oriented on the utility $U(\sigma)$;

$\upsilon_i^-(\sigma)$ – preference to reject alternative σ being oriented on the harmfulness $L(\sigma)$.

Schematically this can be reflected by the following scheme of relations:

$$U(\sigma) < U(\eta) \begin{cases} \pi^+(\sigma) < \pi^+(\eta) \Rightarrow H_\pi \to \max \quad ; \\ \upsilon^+(\sigma) > \upsilon^+(\eta) \Rightarrow H_\upsilon \to \max \quad . \end{cases} \tag{3.59}$$

$$L(\sigma) < L(\eta) \begin{cases} \pi^-(\sigma) > \pi^-(\eta) \Rightarrow H_\pi \to \max \quad ; \\ \upsilon^-(\sigma) < \upsilon^-(\eta) \Rightarrow H_\upsilon \to \max \quad . \end{cases} \tag{3.60}$$

It is simultaneously shown on this scheme that in all cases corresponding distribution delivers the maximum of entropy.

Entropy H^+_υ and H^-_υ are given by formulas

$$H_\upsilon^+ = -\sum_{i=1}^{N} \upsilon^+(\sigma_i) \ln \upsilon^+(\sigma_i) \quad ; \tag{3.61}$$

$$H_{\upsilon}^{-} = -\sum_{i=1}^{N} \upsilon^{-}\left(\sigma_i\right) \ln \upsilon^{-}\left(\sigma_i\right) \quad . \tag{3.62}$$

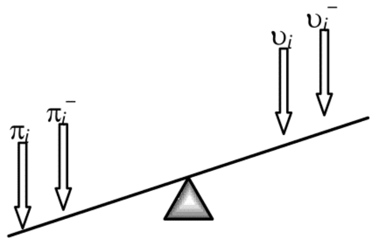


Fig. 3.7

Relative to distributions υ_i^+ and υ_i^- it is possible to repeat what was already said relative to distributions π_i^+ and π_i^-. In particular, absolute and conditional distribution, preferences of compositions of alternatives can be introduced. Evidently, it is possible to talk about preferences „to reject” trajectories. We will not repeat appropriate formulas, since they are similar to analogous formulas for distributions π_i^+ and π_i^-.

This question about how subject makes selection on the set S_a, having at its disposal the distributions of preferences π_i^+, π_i^-, υ_i^+, υ_i^- and using weights mentioned above, it can be examined and solved in the dynamics. May be the Paretto principle would be appropriate here.

It is promising to use described above four distributions of preferences for setting ordering, design and processing of the psychological experiments, considering them as toolkit, which defines concretely the task: what to investigate, what to compare, how to process and how to obtain quantitative characteristics.

It is now possible to say that the utility theory, utility function $U(\sigma)$ ($U(c^k)$, $U(T^{(k)})$...), functions of preference of the type $\pi^{\pm}(\sigma)$ are the particular case of the wider theory, which operates on the wider spectrum of the studied characteristics. We can say, that a set of alternatives S_a is, as the minimum, supplied with four distributions.

The canonical distributions of preferences depending on the resources

Above we introduced three basic kinds of resources. Required resources for each alternative – $R^{req}(\sigma_i)$, ($\sigma_i \in S_a$). In each problem-resource situation, there is some distribution of required resources. Along with the use of absolute required resources, it is possible to use normalized resources. If

$$R^{req} = \sum_{i=1}^{N} R^{req}\left(\sigma_i\right) \quad ,$$

then normalized resources exist

$$\bar{R}^{req}(\sigma_i) = \left(\sum_{i=1}^{N} R^{req}(\sigma_i)\right)^{-1} R^{req}(\sigma_i) \quad .$$

It is possible to assume that $L(\sigma_i) = R^{req}(\sigma_i)$ (or $L(\sigma_i) = \bar{R}^{req}(\sigma_i)$).

Then the negative function of the preference

$$\pi^-(\sigma_i) = \frac{e^{-\beta R^{req}(\sigma_i)}}{\sum_{j=1}^{N} e^{-\beta R^{req}(\sigma_j)}} \quad . \tag{3.64}$$

The meaning of this distribution - the more the required resources for the alternative σ_I , so it is less preferred.

Let's choose as the utility function excess of available resources above the required resources. There are two possibilities. Available resources R^{disp}, if being universal (for example, money), do not depend on what alternative subject selects. Then the excess

$$R^{d+}(\sigma_i) = R^{disp} - R^{req}(\sigma_i) \quad .$$

If the available resources are specialized, the

$$R^{d+}(\sigma_i) = R^{disp}(\sigma_i) - R^{req}(\sigma_i) \quad . \tag{3.65}$$

In the first case, with the same amount of available resources, expressed in equivalent units, the subject has more freedom in the choice of alternatives. Let

$$U(\sigma_i) = R^{d+}(\sigma_i) \quad .$$

Then the model of the preference function, which corresponds to the function of the effectiveness

$$\varepsilon = \sum_{i=1}^{N} \pi^+(\sigma_i) R^{d+}(\sigma_i) \quad , \tag{3.66}$$

has the form

$$\pi^+(\sigma_i) = \frac{e^{\beta R^{d+}(\sigma_i)}}{\sum_{j=1}^{N} e^{\beta R^{d+}(\sigma_j)}} \quad . \tag{3.67}$$

and the subjective entropy also reaches maximum.

The following possibility consists of the use in the capacity of utility function new expected resources obtained as a result of the alternative realization: $R^{exp}(\sigma_i)$, or exceeding expected resources above spent $R^{req}(\sigma_i)$:

$$R^{e+}(\sigma_i) = R^{exp}(\sigma_i) - R^{req}(\sigma_i) \quad ,$$

which can be conditionally named the expected profit. Then the positive preference function can be recorded in the form:

$$\pi^{+}\left(\sigma_i\right) = \frac{e^{\beta R^{e+}(\sigma_i)}}{\sum_{j=1}^{N} e^{\beta R^{e+}(\sigma_j)}} \quad , \quad \pi^{-}\left(\sigma_i\right) = \frac{e^{-\beta R^{e-}(\sigma_i)}}{\sum_{j=1}^{N} e^{-\beta R^{e-}(\sigma_j)}} \quad .$$

If value $R^{e-}(\sigma_i) = -R^{e+}(\sigma_i)$ can be named the expected loss. The function of the preference that corresponds to it is negative.

If subjective entropy does not change, there is no information exchange of the subject of active system with the environment and with other active systems. This situation can be considered the situation of information closure. In the particular case entropy does not change, if preferences remain constant:

$$\dot{\pi}\left(\sigma_i\right) = \dot{\pi}_i = 0 \quad , \quad \forall i \in \overline{1,N} \quad .$$

This condition is sufficient for the constancy of entropy. For absolute preferences $\pi(\sigma_i) = \pi_i$ = const, but if H_π = const, we can find:

$$\frac{dH_\pi}{dt} = -\sum_{i=1}^{N} \frac{\partial H_\pi}{\partial \pi_i} \dot{\pi}_i + \frac{\partial H_\pi}{\partial t} = 0 \quad ,$$

Since $H_\pi = -\sum_{i=1}^{N} \pi_i \ln \pi_i$, then $\frac{\partial H_\pi}{\partial t} = 0$, . Because of this

$$\frac{dH_\pi}{dt} = -\sum_{i=1}^{N} \left(\ln \pi_i + 1\right) \dot{\pi}_i = -\sum_{i=1}^{N} \dot{\pi}_i \ln \pi_i \quad .$$

It is taken under consideration, that $\sum_{i=1}^{N} \dot{\pi}_i = 0$. Thus, if $\frac{dH_\pi}{dt} = 0$, then

$$\sum_{i=1}^{N} \dot{\pi}_i \ln \pi_i = 0 \quad . \tag{3.68}$$

So, the condition of invariability of entropy is the orthogonality of vector of velocity of preferences alternation $\dot{\pi}^{\mathrm{T}} = \left(\dot{\pi}_1, \dot{\pi}_2, \ldots, \dot{\pi}_N\right)$ and vector $(\ln \pi_1, \ln \pi_2, \ldots, \ln \pi_N)$.

Let's assume that π_i is canonical distribution, i.e., it is the solution of variational problem with the functional

$$\Phi_\pi = H_\pi \pm \beta\varepsilon + \gamma N \to \text{extr} \quad ,$$

where the function of effectiveness takes the following form

$$\varepsilon = \sum_{i=1}^{N} \pi_i F_i \quad .$$

Values F_i are depending on the meaning of the problem – either utility U_i or harm L_i.

We define the canonical distribution as the solution of the equations

$$\frac{\partial \Phi_\pi}{\partial \pi_i} = 0 \quad . \tag{3.69}$$

Let's note that

$$\frac{d\Phi_\pi}{dt} = \sum_{i=1}^{N} \frac{\partial \Phi_\pi}{\partial \pi_i}\dot{\pi}_i + \frac{\partial \Phi_\pi}{\partial \pi_i} \quad .$$

Taking (3.69) under consideration, we will find that for the canonical distributions

$$\frac{d\Phi_\pi}{dt} = \frac{\partial \Phi_\pi}{\partial t} \quad .$$

For the selected function of effectiveness and taking under consideration that $\sum_{i=1}^{N} \dot{\pi}_i = 0,$ we will find:

$$\frac{\partial \Phi_\pi}{\partial t} = \pm\beta \sum_{i=1}^{N} \pi_i \dot{F}_i \quad .$$

Equation (3.69) in the expanded form gives

$$\ln\pi_i = -1 \pm \beta F_i + \gamma, \quad \left(\forall i \in \overline{1, N}\right) \quad .$$

Let's substitute the value of ln π_I , accordingly with relationship in (3.68). We will have

$$\sum_{i=1}^{N} \left(-1 \pm \beta F_i + \gamma\right) \dot{\pi}_i = 0 \quad ,$$

but $\sum_{i=1}^{N}(-1+\gamma)\dot{\pi}_i = 0$, therefore, the condition of entropy stationarity reduces to the equation:

$$\sum_{i=1}^{N} F_i\,\dot{\pi}_i = 0 \quad .$$

We can say that entropy of the canonical distribution H_π is constant, if the velocity vector of change of preferences is orthogonal to the vector $F^{\text{T}} = (F_1, F_2, \ldots, F_N)$.

In the special case, when $F_i = U_i$, where U_i is utility of alternative σ_i, subjective entropy is constant, if the vector $\dot{\pi}$ is orthogonal to the vector of utilities.

Similarly to the above, one can use the relative values $\bar{R}_i^{d\pm}$, $\bar{R}_i^{e\pm}$.

Let's show that every time the entropy H_π reaches maximum on a manifold, given by the isoperimetric conditions. Let π_i is the canonical distribution obtained by solving the variational problem. We give arbitrary small increment Δ_i to the canonical preferences. Thus the varied preference $\tilde{\pi}_i = \pi_i + \Delta_i$. The variation of entropy δH_π, due to variations in preferences, is

$$\delta H_\pi = H_\pi(\pi_i + \Delta_i) - H_\pi(\pi_i) = -\sum_{i=1}^{N}\Big(\pi_i \ln(\pi_i + \Delta_i) + \Delta_i \ln(\pi_i + \Delta_i)\Big) - \sum_{i=1}^{N}\pi_i \ln \pi_i \quad .$$

Assuming smallness of Δ_i, that

$$\ln(\pi_i + \Delta_i) \approx \ln \pi_i + \frac{1}{\pi_i}\Delta_i \quad ,$$

we will find

$$\delta H_\pi \,|_{\Delta_i \to 0} \cong -\sum_{i=1}^{N}\Delta_i = \sum_{i=1}^{N}\Delta_i \ln \pi_i - \sum_{i=1}^{N}\frac{\Delta_i^2}{\pi_i} \quad .$$

Let's introduce the general designation for $U(\sigma_i)$ and $L(\sigma_i)$: F_i. Necessary optimality conditions for the solution of variational problems take the form:

$$-\ln\pi_i - 1 \pm \beta F_i + \gamma = 0\,; \quad \forall i \in \overline{1, N} \quad .$$

Hence

$$\ln\pi_i = -1 + \gamma \pm \beta F_i \;.$$

Isoperimetric conditions must be met for any distributions π_i; so we have the relations:

$$-\sum_{i=1}^{N}(\pi_i + \Delta_i)F_i = \varepsilon_0\,; \quad \sum_{i=1}^{N}(\pi_i + \Delta_i) = 1 \quad .$$

But the canonical distribution satisfies the isoperimetric conditions, hence

$$\sum_{i=1}^{N}\Delta_i F_i = 0\,; \quad \sum_{i=1}^{N}\Delta_i = 0 \quad .$$

Given the necessary optimality condition and the last two relations, we find that

$$\delta H_\pi = -\sum_{i=1}^{N}\frac{\Delta_i^2}{\pi_i} \quad .$$

Values $\pi_i \geq 0$ for $\forall i$, hence

$$\delta H_\pi \leq 0 \quad .$$

This confirms the fact that the canonical distribution π_i provide the maximum entropy on isoperimetric manifold.

We consider an example to illustrate the considerations given above. Suppose that set S_a has three alternatives (two alternatives are excluded, as in this case, the distribution of preferences could be determined from the equations of two-dimensional isoparametric manifold) and corresponding resources are r_1, r_2, r_3.
In one case, they correspond to a positive effect, in the other – to a negative.
Let's assume for simplicity $\beta = 1$; we accept $r_1 = 0{,}7$; $r_2 = 1{,}0$; $r_3 = 3{,}5$. Then

$$\left.\begin{aligned}
\pi_1^+ &= \frac{e^{0,7}}{e^{0,7}+e^{1,0}+e^{3,5}} = 0{,}153211\ldots \\
\pi_2^+ &= \frac{e^{1,0}}{e^{0,7}+e^{1,0}+e^{3,5}} = 0{,}17183\ldots \\
\pi_3^+ &= \frac{e^{3,5}}{e^{0,7}+e^{1,0}+e^{3,5}} = 0{,}67503\ldots
\end{aligned}\right\} \sum_{i=1}^{3}\pi_i^+ = 1 \quad .$$

Negative distributions of preferences:

$$\left.\begin{aligned}
\pi_1^- &= \frac{e^{-0,7}}{e^{-0,7}+e^{-1,0}+e^{-3,5}} = 0{,}55505\ldots \\
\pi_2^- &= \frac{e^{-1,0}}{e^{-0,7}+e^{-1,0}+e^{-3,5}} = 0{,}4112\ldots \\
\pi_3^- &= \frac{e^{-3,5}}{e^{-0,7}+e^{-1,0}+e^{-3,5}} = 0{,}03375\ldots
\end{aligned}\right\} \sum_{i=1}^{3}\pi_i^- = 1 \quad .$$

Let's calculate appropriate subjective entropies

$$H_\pi^+ = -\left[\pi_1^+\ln\pi_1^+ + \pi_2^+\ln\pi_2^+ + \pi_3^+\ln\pi_3^+\right] = 0{,}8553\ldots \quad ;$$

$$H_\pi^- = -\left[\pi_1^-\ln\pi_1^- + \pi_2^-\ln\pi_2^- + \pi_3^-\ln\pi_3^-\right] = 0{,}8065\ldots \quad .$$

We vary value π_i^+ and π_i^- on the manifold M:

$$M:\sum_{i=1}^{3}\pi_i^{\pm}r_i=\varepsilon_0\ ;\quad \sum_{i=1}^{B}\pi_i^{\pm}=1\quad .$$

Let $\Delta^{\pm}{}_1$, $\Delta^{\pm}{}_2$, $\Delta^{\pm}{}_3$ variations, such that varied values of the preferences

$$\tilde{\pi}_i^{\pm}=\pi_i^{\pm}+\Delta_i^{\pm}\quad .$$

Variations $\Delta_i^{\pm}$ satisfy system of equations

$$\begin{cases}\Delta_1^{\pm}+\Delta_2^{\pm}+\Delta_3^{\pm}=0\quad ;\\ r_1\Delta_1^{\pm}+r_2\Delta_2^{\pm}+r_3\Delta_3^{\pm}=0\quad .\end{cases}$$

Hence, we find:

$$\Delta_2^{\pm}=-\frac{r_1-r_3}{r_2-r_3}\Delta_1^{\pm}\quad ;\quad \Delta_3^{\pm}=-\left(\Delta_1^{\pm}+\Delta_2^{\pm}\right)\quad .$$

Let's assume $\Delta_1^{\pm}=0{,}1$, then $\Delta_2^{\pm}=-0{,}112$ and $\Delta_3^{\pm}=-0{,}012$.

The varied values of preferences are:

$$\tilde{\pi}_1^{+}=0{,}253211...\qquad \tilde{\pi}_1^{-}=0{,}6557...$$

$$\tilde{\pi}_2^{+}=0{,}05983...\qquad \tilde{\pi}_2^{-}=0{,}2992...$$

$$\tilde{\pi}_3^{+}=0{,}6873...\qquad \tilde{\pi}_3^{-}=0{,}0457...$$

We get new varied values of entropies (positive and negative):

$$\tilde{H}_{\pi}^{+}=-\left[\tilde{\pi}_1^{+}\ln\tilde{\pi}_1^{+}+\tilde{\pi}_2^{+}\ln\tilde{\pi}_2^{+}+\tilde{\pi}_3^{+}\ln\tilde{\pi}_3^{+}\right]=0{,}77418\ldots\quad ;$$
$$\tilde{H}_{\pi}^{-}=-\left[\tilde{\pi}_1^{-}\ln\tilde{\pi}_1^{-}+\tilde{\pi}_2^{-}\ln\tilde{\pi}_2^{-}+\tilde{\pi}_3^{-}\ln\tilde{\pi}_3^{-}\right]=0{,}77934\ldots$$

As we can see

$$\tilde{H}_{\pi}^{+}=0{,}774184\ldots<H_{\pi}^{+}=0{,}8553\ldots\quad ;$$
$$\tilde{H}^{-}=0{,}779340\ldots<H_{\pi}^{-}=0{,}806548\ldots$$

Qualitatively the same result is obtained, if $r_1=-0{,}1$.

The given example illustrates the obtained conclusion, that the canonical distributions $\pi^{+}{}_i$ and $\pi^{-}{}_i$ deliver the maximum of entropy on the isoperimetric manifold.

In thermodynamics, the value:

$$Z = \sum_{i=1}^{N} e^{-\beta L(\sigma_i)}$$

is called the „*statistical sum*". A canonical negative distribution can be written in the form

$$\pi^{-}(\sigma_i) = \frac{1}{Z} e^{-\beta L(\sigma_i)} \quad ,$$

then the analog of free energy F is

$$F = -T \ln Z,$$

where $T = \beta^{-1}$ is „temperature". It can be determined from the isoperimetric condition:

$$\varepsilon_0 = \sum_{i=1}^{N} \pi^{-}(\sigma_i) L(\sigma_i) = \varepsilon^{-} \quad ,$$

where ε^{-} is „weighted mean" on the preferences of „harmfulness" (loss, losses).

The temperature can be found as following:

$$T = \frac{d\varepsilon^{-}}{dH^{-}_{\pi \max}} \quad .$$

It is evident from this formula, that, if an increase $H^{-}_{\pi \max}$ - degrees of the uncertainty of negative preferences draws an increase in the weighted losses, then $T > 0$, if inverse dependence occurs, then $T < 0$. An analog of free energy takes the form:

$$F = \varepsilon^{-} - T H^{-}_{\pi \max} \quad .$$

In information theory $H^{-}_{\pi \max}$ is called „the bandwidth" of communication channel. Given the foregoing, the „free energy" in this case can be called „free resources". The canonical distribution of preferences can be expressed as

$$\pi^{-}(\sigma_i) = e^{\frac{F - L(\sigma_i)}{T}} \quad ,$$

which is an analogue of the Gibbs distribution. In this case, the parameter T can be interpreted as „the emotional temperature of the heating". The entropy of a particular state (Hartley)

$$H^{-}(\sigma_i) = -\ln \pi^{-}(\sigma_i) \quad .$$

Models of preference functions obtained under the assumption that the function is linear in efficiency $L(\sigma)$ (or $U(\sigma)$), represent a simplified, primitive logic: „the more - the better", or „the less - the worse". These distributions present extreme hypotheses, we can say „extremist" psychological types. In fact, the distribution of preferences does not always respond to this „straight-line" principle.

More natural situation is when a very „expensive" and a very „cheap" alternatives, are less preferred than alternatives, which have a „value" corresponding to the prestige, status of the subject and at the same time commensurate with its capabilities. In order to build a canonical model of preference functions that reflect these circumstances, we will use nonlinear functions of effectiveness with respect to functions of utility $U(\sigma_i)$ or harm $L(\sigma_i)$. In order to simplify the following we will, where it is not difficult to understand, write:

$$\pi(\sigma_i) = \pi_i;\ L(\sigma_i) = L_i;\ U(\sigma_i) = U_i$$

and so on. So, let

$$\varepsilon^- = \sum_{i=1}^{N} \pi_i^- G(L_i)\ ; \quad \varepsilon^+ = \sum_{i=1}^{N} \pi_i^+ G(U_i)$$

and let, for example

$$G(L_i) = \beta L_i - \delta \ln L_i \quad . \tag{3.70}$$

The first term provides linear relation, and second term - logarithmic, „softer" relation. Functional Φ^-_π is chosen in the form:

$$\Phi_\pi^- = -\sum_{i=1}^{N} \pi_i^- \ln \pi_i^- + \sum_{i=1}^{N} \left[-\beta L_i + \delta \ln L_i + \gamma\right] \pi_i^- \quad . \tag{3.71}$$

The necessary condition of extremum gives the following model for π_i^-:

$$\pi^-(\sigma_i) = C L_i^{\delta} e^{-\beta L_i} \quad .$$

The normalizing coefficient C equals:

$$C = \frac{1}{\sum_{j=1}^{N} L_j^{\delta} e^{-\beta L_j}} \quad .$$

Thus, the model of the canonical preference function is assigned by the formula

$$\pi^-\left(\sigma_i\right)=\frac{L_i^\delta e^{-\beta L_i}}{\sum_{j=1}^{N} L_j^\delta e^{-\beta L_j}} \quad . \tag{3.72}$$

The shape of this function is shows on Fig. 3.8.

Fig. 3.8 represents the function $\pi^-_1 = \pi^-(\sigma_1)$, when S_a contains two alternatives σ_1, σ_2 and $\delta = 1$. If $\delta \neq 1$, then $L_1^* = \frac{\delta}{\beta}$.

In this case

$$\pi_1^- = \frac{L_1 e^{-\beta L_1}}{L_1 e^{-\beta L_1} + L_2 e^{-\beta L_2}} ; \quad \pi_2^- = \frac{L_2 e^{-\beta L_2}}{L_2 e^{-\beta L_1} + L_2 e^{-\beta L_2}} \quad .$$

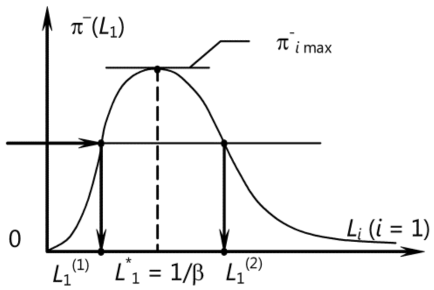


Fig. 3.8

It could be find that

$$\frac{\partial \pi_1^-}{\partial L_1} = L_2 e^{-\beta(L_1+L_2)} - \beta L_1 L_2 e^{-\beta(L_1+L_2)} = 0 \quad ,$$

hence $L^*_1 = \beta^{-1}$; in addition, we see that

$$\pi_1^- \,|_{L_1^*} = \frac{1}{1+\beta L_2 e^{-\beta L_2+1}} = \pi^-_{1\max}$$

and following conditions are satisfied:

for $L_2 = 0$ $\quad \pi^-_{1\max} = 1$;

for $L_1 = 0$ $\quad \pi^-_1 = 0$;

for $L_1 \to \infty$ $\quad \pi^-_1 \to 0$.

If $L_2 > 0$, then $\pi^{-}_{1max} < 1$. For each value $\pi^{-}_1 \in (0, 1)$ there are two equally preferable states $L_1^{(1)} \sim L_1^{(2)}$. This means that on the ascending branch $L_1 \in [0, L^*_1)$ the function π^{-}_1 establishes a correspondence:

$$L_{1a} > L_{1b} \Rightarrow \pi_1^-\left(L_{1a}\right) > \pi_1^-\left(L_{1b}\right); \quad \left(L_{1a}, L_{1b}\right) \in \left[0, L_1^*\right) \quad .$$

On the descending branch: $L_1 \in (L^*_1, +\infty)$ – the correspondence:

$$L_{1a} > L_{1b} \Rightarrow \pi_1^-\left(L_{1a}\right) < \pi_1^-\left(L_{1b}\right); \quad \left(L_{1a}, L_{1b}\right) \in \left(L_1^*, +\infty\right) \quad .$$

In the „point" $L^*_1 = \beta^{-1}$ there is a single (preferred) value π^{-}_1, for two-dimensional manifold:

$$M: \begin{cases} \sum_{i=1}^{N} \pi_i^-\left(\delta \ln L_i - \beta L_i\right) = \varepsilon_0 \quad ; \\ \sum_{i=1}^{N} \pi_i^- = 1 \quad . \end{cases}$$

The function (3.72) fulfills the maximum of subjective entropy.

The distribution (3.72) can correspond to the following logic of decisions of the subject. Let's assume that L_i are required resources $R^{req}(\sigma_i)$, then

a) alternatives, which require low expenditures - „cheap" and therefore are easily accessible, can be „non-prestige" for the subject, that has considerably bigger opportunities. Therefore „the intensity" of corresponding desires, expressed by the value of the function will be low;

b) alternatives too „expensive" for the subject, also have small preferability in view of presumably high expenditures, comparable to the subject possibilities;

c) there is a range of L_i values, commensurate with the opportunities and appropriate for a given subject, and therefore with the greatest appeal (preferred).

Subsequently certain amplification in formulation of functions $L(\sigma_i)$ and $U(\sigma_i)$ through the resources will be introduced. This is connected with the fact that we would like to examine only finite value of all types of resources, whereas utility functions can take infinite values.

3.6. Mixed type variational problems

We examined several variational problems above. In each of them the extremized functional was selected in a certain way, being determined different from others, and although the functional forming principle is general and necessarily involve the inclusion of entropy, the specific content of functional components may be different.

One concern arising in connection with this approach, lies in the fact, that it is very doubtful that the subject, which even has highly developed intelligence would be able „to solve" at each moment more than one task of the best version selection. This doubt is completely justified. Most likely, the more complex forms of variational problems of subjective analysis have smaller capability to claim the right to be model of the functioning subject psyche.

Nevertheless, we will continue analysis in this direction and examine also whole series of alternative variational problems, sometimes more complex in comparison to those described above.

We will proceed from following prerequisites:

- the assumption that the subject always solves one and the same problem to the optimum is equivalent to assumption about „the unidimensionality", unifiednesses of human psyche. Most likely the exception take place;
- for sure it is possible to assume that the subject properly solves problems to the optimum, using each time the information, obtained as a result of the previous solutions. The development of problem-resource situation bears iterative nature.

Let's recall a legend about St Vladimir: in what way he choose the faith for the Russian people. From the point of view of the subjective analysis (but entire process and the solution bore the clearly expressed subjective nature) several sequential steps were made, so to say - the certain „algorithmic" procedure was realized.

First step - several versions have been selected, and set of alternatives S_a have been formed.

The second step - preferabilities „to accept" one or another faith are studied.

The third step - preferabilities „to reject" one or another faith are studied.

The fourth step - the comparative analysis of different preferences was produced and „*the problem of selection*" was solved.

The dependence of one distribution of preferences on another does not compulsorily realize as time-sequential routine.

It could be happen so that different distributions realize, only once, in other words relate to one and the same problem-resource situation, although the analysis is achieved in the form of a now of stages.

Examination of positive and negative distributions of preferences π^+, π^-, υ^+, υ^- separately made above assumes that the subject accomplishes studying of situation step by step, passing from one type of preferences to another. Thus, the process of analysis is developed in the time. In this case in the subsequent stage it is possible to consider results of the previous stage. This assumption and this scheme do not exclude the possibility, when preferences are formed is such away that in each stage both positive and negative factors are taken into account. The criterion of the form corresponds to this scheme has form:

$$\Phi_\pi = -\sum_{i=1}^{N}\pi(\sigma_i)\ln\pi(\sigma_i) + \sum_{i=1}^{N}\pi(\sigma_i)F\big(U(\sigma_i)L(\sigma_i)\big) + \gamma\sum_{i=1}^{N}\pi(\sigma_i).$$

Here function $F\big(U(\sigma_i)L(\sigma_i)\big)$ depends both on the utility $U(\sigma_i)$ and on the harmfulness $L(\sigma_i)$. Distribution $\pi(\sigma_i)$ depends on the structure of F (.).

In order to get a meaningful specification of the function F(.), we need to accept certain realistic assumptions, namely, let $\pi(\sigma_i)$ has the following properties:

1. The bigger $U(\sigma_i)$, the bigger $\pi(\sigma_i)$,
2. If $U(\sigma_i) = 0$, then $\pi(\sigma_i) = 0$,
3. If harmfulness $L(\sigma_i) \to \infty$, then $\pi(\sigma_i) \to 0$,
4. If $L(\sigma_i) = 0$, then $\pi(\sigma_i)$ does not depend on $L(\sigma_i)$, but can depend on other harmfulness $L(\sigma_j)$ ($j \neq i$).

The following function fulfills above conditions:

$$\pi(\sigma_i) = \frac{U(\sigma_i)^\alpha e^{-\beta L(\sigma_i)}}{\sum_{j=1}^{N} U(\sigma_j)^\alpha e^{-\beta L(\sigma_j)}} \quad .$$

For example, if the set S_a, contains two alternatives: σ_1 and σ_2 with condition $L(\sigma_1) = 0$, and $L(\sigma_2) \neq 0$, then

$$\pi(\sigma_1) = \frac{U(\sigma_1)^\alpha}{U(\sigma_1)^\alpha + U(\sigma_2)^\alpha e^{-\beta L(\sigma_2)}} \quad .$$

If also $L(\sigma_2) = 0$, then

$$\pi'(\sigma_1) = \frac{U(\sigma_1)^\alpha}{U(\sigma_1)^\alpha + U(\sigma_2)^\alpha} \quad .$$

Since $e^{-\beta L(\sigma_2)} \geq 1$, we note that $\pi(\sigma_1) \leq \pi'(\sigma_2)$. It answers to „the common sense". We can also see that if all utilities are identical

$$U(\sigma_i) = \text{idem } (\sigma_i \in S_a(N)),$$

than $\pi(\sigma_i)$ do not depend on utilities and is determined only by harmfulness. Thus, if „indications" are identical for N different medicines, and contraindications are different, then precisely they will determine the selection of a subject. Distribution takes the form:

$$\pi^{-}\left(\sigma_i\right)=\frac{e^{-\beta L\left(\sigma_i\right)}}{\sum_{j=1}^{N} e^{-\beta L\left(\sigma_j\right)}} \quad .$$

Let's examine other possibilities of the idea of utility and harmfulness in the form of functions from different resources: available R^{disp}, required R^{req}, expected R^{exp}. Thus, for instance, if we assume that

$$U\left(\sigma_i\right)=\frac{R_i^{exp}}{R_i^{req}}=\overline{r}_i^{\,e} \quad ;$$

$$L\left(\sigma_i\right)=\frac{R_i^{req}}{R_i^{\exp}}=\left(\overline{r}_i^{\,e}\right)^{-1} \quad ,$$

the distribution takes the form:

$$\pi\left(\sigma_i\right)=\frac{\left(\overline{r}_i^{\,e}\right)^{\alpha} e^{-\beta\frac{1}{\overline{r}_i^{\,e}}}}{\sum_{j=1}^{N}\left(\overline{r}_j^{\,e}\right)^{\alpha} e^{-\beta\frac{1}{\overline{r}_j^{\,e}}}} \quad .$$

In the general case the isoquant of distribution $\pi(\sigma_i) = \pi(F(U_i, L_i))$ is realized in the curve $F(U_i, L_i)$ = const.

Marginal preferences $\frac{\partial\pi\left(\sigma_i\right)}{\partial U_i}$ and $\frac{\partial\pi\left(\sigma_i\right)}{\partial L_i}$ are determined by the relation

$$S_{\pi_i}^{U_i}=\frac{\partial\pi\left(\sigma_i\right)}{\partial U_i}=\frac{\partial F}{\partial U_i}\pi\left(\sigma_i\right)\left(1-\pi\left(\sigma_i\right)\right) \quad ;$$

$$S_{\pi_i}^{L_i}=\frac{\partial\pi\left(\sigma_i\right)}{\partial L_i}=\frac{\partial F}{\partial L_i}\pi\left(\sigma_i\right)\left(1-\pi\left(\sigma_i\right)\right) \quad .$$

As we can see, they are equal zero in the „points" $\pi(\sigma_i) = 0$ and $\pi(\sigma_i) = 1$. The case of $\pi(\sigma_i) = 1$ corresponds to zero entropy.

Calculating subjective sensitivities on utility and the harm, we find their relationship:

$$\frac{S_{\pi_i}^{U_i}}{S_{\pi_i}^{L_i}}=\frac{\frac{\partial F}{\partial U_i}}{\frac{\partial F}{\partial L_i}} \quad .$$

For the particular distribution this relationship takes the form:

$$\frac{S_{\pi_i}^{U_i}}{S_{\pi_i}^{L_i}} = -\frac{\alpha}{\beta}\frac{1}{L_i} \quad .$$

This means that $S_{\pi}^{U_i}$ and $S_{\pi}^{L_i}$ has opposite signs. If $S_{\pi}^{L_i} = \text{const} > 0$, then $S_{\pi}^{U_i}$ is negative and inversely proportional to harmfulness.

The following task, connected with distributions of „mixed" type is a study of the behavior of the entropy H_π on the manifolds, which define constraints.

Hereinafter, we will examine a question about the elasticity of preferences.

Interdependence of the different distributions can be expressed in terms of correlation characteristics, structurally similar to coefficients of correlation and correlation functions in the probability theory and mathematical statistics.

Interdependent sequences of distributions will be called „chains".

Many problems are not formed as a purely economic or non-economic at all (technical, scientific, social, demographic, ...). Let us consider in this regard, some productions of variational problems, where distribution of preferences depend not only on the distribution of resources (for example $R^{req}(\sigma_i)$).

Let the preference function path $Tr^{(0)}$: $\sigma_i \to \sigma_j$ factorable:

$$\pi(\sigma_i, \sigma_j) = \pi(\sigma_i)\pi(\sigma_j | \sigma_i).$$

Required resources depend on the „point of departure" σ_i and on the „point of arrival" σ_j $R^{req} = r(\sigma_i, \sigma_j)$. Partial Hartley's information can be represented in the form:

$$\begin{aligned} I(\sigma_i,\sigma_j) &= H_\pi(\sigma_i) + H_\pi(\sigma_j) - H_\pi(\sigma_i,\sigma_j) = \\ &= -\ln\pi(\sigma_i) - \ln\pi(\sigma_j) + \ln\pi(\sigma_i,\sigma_j) \quad . \end{aligned} \tag{3.73}$$

This formula follows from the property of the additivity of entropy and hypothesis of the factorization of distribution $\pi(\sigma_i, \sigma_j)$.

Let's average particular entropy over the conditional distribution $\pi(\sigma_j | \sigma_i)$ and select the functional of optimality for this distribution in the form:

$$\begin{aligned} \Phi_\pi(\sigma_i) &= -\sum_{j=1}^{N}\pi(\sigma_j | \sigma_i)\ln\pi(\sigma_j) - \sum_{j=1}^{N}\pi(\sigma_j | \sigma_i)\ln\pi(\sigma_i) + \\ &+ \sum_{j=1}^{N}\pi(\sigma_j | \sigma_i)\ln\pi(\sigma_i,\sigma_j) + \beta\sum_{j=1}^{N}\pi(\sigma_j | \sigma_i)r(\sigma_i,\sigma_j) + \gamma\sum_{j=1}^{N}\pi(\sigma_j | \sigma_i) \quad . \end{aligned} \tag{3.74}$$

Since third term can be changed:

$$\sum_{j=1}^{N}\pi(\sigma_j | \sigma_i)\ln\pi(\sigma_i,\sigma_j) = \sum_{j=1}^{N}\pi(\sigma_j | \sigma_i)\left[\ln\pi(\sigma_i | \sigma_j) + \ln\pi(\sigma_i)\right] \quad ,$$

We find

$$\frac{\partial \Phi_\pi(\sigma_i)}{\partial \pi(\sigma_i \mid \sigma_j)} = -\ln \pi(\sigma_j) + \ln \pi(\sigma_i \mid \sigma_j) + 1 + \beta r(\sigma_i, \sigma_j) - \gamma = 0 \quad .$$

Hence we obtain the canonical distribution of the conditional preferences:

$$\pi(\sigma_j \mid \sigma_i) = C_i \pi(\sigma_j) e^{-\beta r(\sigma_i, \sigma_j)} \quad .$$

This distribution depends not only on the resource component, but also on the absolute preference of state σ_j, which can be determined not only by economic considerations. Normalizing coefficient

$$C_i = \left(\sum_{k=1}^{N} \pi(\sigma_k) e^{-\beta r(\sigma_i, \sigma_k)} \right)^{-1} \quad .$$

Now let the subjective criterion (functional) is:

$$\Phi_\pi = -\sum_i \sum_j \pi(\sigma_i, \sigma_j) \Big[\ln \pi(\sigma_j) + \ln \pi(\sigma_i) - \\ - \ln \pi(\sigma_i, \sigma_j) - \beta r(\sigma_i, \sigma_j) - \gamma \Big] \quad . \tag{3.75}$$

Here a distribution $r(\sigma_i, \sigma_j)$ and absolute distributions $\pi(\sigma_i)$, $\pi(\sigma_j)$ are considered known. The distribution of the preferences of ways $T_r^{(2)}$: $\sigma_i \to \sigma_j$ have to be formed from the condition.

$$\frac{\partial \Phi_\pi}{\partial \pi(\sigma_i, \sigma_j)} = 0 \quad .$$

Canonical distribution takes the form:

$$\pi(\sigma_i, \sigma_j) = \frac{\pi(\sigma_i) \pi(\sigma_j) e^{-\beta r(\sigma_i, \sigma_j)}}{\sum_{p=1}^{N} \sum_{q=1}^{N} \pi(\sigma_p) \pi(\sigma_q) e^{-\beta r(\sigma_p, \sigma_q)}} \quad .$$

The parameter β is the function of the information I, which is equal to the sum of three terms in (3.75) (if we open brackets): $\beta = \beta(I)$, when this value is set as a constraint. In [137] (p. 313) is presented the theorem that, under a certain conditions, which can be made in this case, the obtained distribution either corresponds to the maximum information of Shannon with the assigned weighted mean resources or to weighted average of minimum resources for the fixed information, if $\beta > 0$, and minimal resources, if $\beta < 0$.

A function $\pi(\sigma_i, \sigma_j)$ is the distribution of the preferences of way $\sigma_i \to \sigma_j$. It is determined on the Cartesian product $S_a \times S_a$, and it depends not only on the distribution of resources, which correspond to alternative „ways", but also on the absolute preferences of state σ_i and σ_j (initial and final). The higher is the preference of way $\sigma_i \to \sigma_j$, the fewer resources it requires for provision of transfer and when the „points" σ_i and σ_j that constitute the way have higher preferences, the more resources it requires.

If $\beta < 0$, then is possible to examine, for example, the expected gain $r(\sigma_i, \sigma_j)$ of transition $\sigma_i \to \sigma_j$. Analogous results occur if, instead of resources $r(\sigma_i)$, $r(\sigma_j)$, $r(\sigma_i, \sigma_j)$ we will use objective probabilities $p(\sigma_i)$, $p(\sigma_j)$, $p(\sigma_i, \sigma_j)$. Let's select criterion in the form:

$$\begin{aligned}\Phi_\pi = \sum_i \sum_j \Big[-\ln\pi(\sigma_i) - \ln\pi(\sigma_j) + \ln\pi(\sigma_i,\sigma_j) - \ln p(\sigma_i) - \\ -\ln p(\sigma_j) + \ln p(\sigma_i,\sigma_j) + \beta p(\sigma_i,\sigma_j) + r(\sigma_i) + \delta(\sigma_j)\Big]\pi(\sigma_i,\sigma_j) \quad .\end{aligned} \tag{3.76}$$

Value $\sum_i \sum_j p(\sigma_i,\sigma_j)\pi(\sigma_i,\sigma_j)$ - is weighted average on preferences of ways $\sigma_l \to \sigma_j$ is the probability of the realization of a certain event, associated with this transition, such as the likelihood of the transition itself. To determine β, $\delta(\sigma_i)$, $\gamma(\sigma_j)$, the following conditions can be used:

$$\begin{aligned} I &= \sum_i \sum_j \Big[\ln\pi(\sigma_i,\sigma_j) - \ln\pi(\sigma_i) - \ln p(\sigma_j)\Big]\pi(\sigma_i,\sigma_j) = \\ &= \sum_i \sum_j \pi(\sigma_i,\sigma_j) \ln\frac{\pi(\sigma_i,\sigma_j)}{\pi(\sigma_i)\pi(\sigma_j)} \quad ; \end{aligned} \tag{3.77}$$

$$\sum_i \pi(\sigma_i,\sigma_j) = \pi(\sigma_j); \quad \sum_j \pi(\sigma_i,\sigma_j) = \pi(\sigma_i) \quad .$$

From the last two relations it follows:

$$\delta(\sigma_i) = \ln\left[\sum_j \pi(\sigma_j) e^{-\gamma(\sigma_j)-\beta p(\sigma_i,\sigma_j)}\right] \quad ; \tag{3.78}$$

$$\gamma(\sigma_j) = \ln\left[\sum_i \pi(\sigma_i) e^{-\delta(\sigma_i)-\beta p(\sigma_i,\sigma_j)}\right] \quad ; \tag{3.79}$$

which gives 2N transcendental equations relative to $2N$ values $\delta(\sigma_i)$ and $\gamma(\sigma_j)$.

If, following [137] we introduce a „subjective potentials"

$$\Gamma_1 = \sum_i \sum_j \delta(\sigma_j)\pi(\sigma_i,\sigma_j); \quad \Gamma_2 = \sum_i \sum_j \gamma(\sigma_i)\pi(\sigma_i,\sigma_j) \quad .$$

and identify $\Gamma = \Gamma_1 + \Gamma_2$, and also define the weighted mean probabilities (or in the particular case - resources) $\mathfrak{R}$ - analogous to „*internal energy*" in the thermodynamics, then using the conditions

$$\frac{\partial \Phi}{\partial \pi(\sigma_i,\sigma_j)} = 0$$

you can come to the relation

$$\beta\mathfrak{R} + \Gamma = -I.$$

Denoting $\Gamma = -\beta F$, where F is interpreted in the thermodynamics as free energy, we come to the relation

$$F = \mathfrak{R} + \beta^{-1} I,$$

where I is the given amount of subjective Shenon's information. Thermodynamic entropy is analogous to it, β^{-1} is interpreted as temperature.

The following analogues of thermodynamic relations occur:

$$\frac{d\Gamma}{d\beta} = -\mathfrak{R} \quad ; \quad \beta\frac{d\Gamma}{d\beta} - \Gamma = I \quad ; \quad \frac{d\mathfrak{R}}{dI} = -\beta^{-1} \quad .$$

If we take „temperature" T instead of β^{-1}, then these relations take the form:

$$T^2 \frac{dI}{dT}\frac{d\Gamma}{dI} = \mathfrak{R}; \quad \frac{dI}{dT}\frac{d\Gamma}{dI} + \frac{1}{T}\Gamma = -\frac{I}{T}; \quad \frac{d\mathfrak{R}}{dI} = -T.$$

Since a change I leads to the change $\mathfrak{R}$ (generalized weighted mean „resources", which takes into account the possibility of probabilistic representation), then the value of T can be interpreted also as a differential „*value of information*".

Actually, from the last relation we see that T signals, how much resources change (probability of certain objective event) due to the „increment" of information.

3.7. On the threshold values of entropy, the moments of distributions of preferences „switching" and the „moments of selection"

We must consider two types of threshold characteristics, „thresholds" participating in decision making or „selection" from the S_a.

We shall distinguish between the „objective" threshold (altitude of missed approach, the critical speed of takeoff run, the moment of the train departure... they can also agree to call them „physical" thresholds) and „subjective" thresholds - related to the choice of an alternative „before" the onset of a physical threshold (the decision on a go-around, the decision to terminate or continue takeoff, a decision on whether to go or not to go by train, ...).

We are naturally interested in the subjective thresholds.

We associate a *subjective threshold* with the value of the entropy $H\pi$ of preferences distribution. We deliberately assume that for each subject of active system there is an individual threshold of the entropy H^*_π such, that, if

We associate a subjective threshold with the value of entropy of preferences distribution $H\ \pi$. We obviously assume that for each system there is a subject of active individual threshold of entropy $H^*\pi$ such that if

$$H_\pi(k) \leq H^*_\pi \quad , \tag{3.80}$$

subject makes or can make a decision. This statement is taken as another *postulate*. Then we name H^*_π the lower threshold of the 1 st kind.

Functioning of an active system we can represent as the flow of two variable processes:

1) objective process (physical conversion of resources with the help of existing technologies;
2) the subjective process of the preferences modification of the subject who basing on folding ideas accomplishes control of physical process.

Both processes are tightly interconnected, since the subject is „the part" of active system, and to the decisive degree determine its functioning.

The system at each moment of time is in the certain (current) state $\sigma_i(x)$, where x is vector of the quantitative and qualitative system characteristic, changing in accordance with the equation

$$\mathcal{A}(x, u) = 0 \quad , \tag{3.81}$$

where $\mathcal{A}$ – operator; u - the control action, formed by the subject $u = u(\sigma_i)$.

The second parallel process - process of the preferences modification, which conditionally could be written in the form

$$\mathcal{B}(\pi, x) = 0 \quad , \tag{3.82}$$

Both operators $\mathcal{A}$ and $\mathcal{B}$ depend on alternative σ_j and corresponding problem P: $\sigma_l \rightarrow \sigma_j$ which is solving at the given moment.

Both processes bear continuous - explosive nature.

If time is considered discrete: $t = k$, then $x = x(k)$, $\pi = \pi(k)$. Equations (3.81), (3.82) together with condition (3.80) compose one of versions of the model of system functioning.

$$\mathcal{A}\left(x,u\left(\sigma_i\left(k\right),\sigma_j\right),k\right)=0 \quad ; \tag{3.83}$$

$$\mathcal{B}\left(\pi,x,k\right)=0 \quad . \tag{3.84}$$

In the operator $\mathcal{B}$ x (k) is passive control

$$H_\pi = H_\pi(k) = H_\pi\,(\pi(k)) \; .$$

Both operators $\mathcal{A}$, $\mathcal{B}$ in the general case are nonlinear and lead to bifurcational processes. It is here appropriate to talk about possible „objective” and „subjective” bifurcations. Moment of an approach of subjective threshold (in accordance with an inequality (3.80)) it can anticipate the moment of objective bifurcation, or vice versa - be late.

The construction of dynamic models of processes, which include dynamics of preferences, and some examples of simulation are given in chapter 5.

A change in the set of alternatives S_a can be a result of attainment of some subjective threshold or objective threshold (for example, in the case „of the withdrawal of train” alternative „go”disappears).

Threshold values H^*_π could be established for each type of the distribution:

$$\pi_i^+,\ \pi_i^-,\ \upsilon_i^+,\ \upsilon_i^- \ldots$$

Fig.3.9 shows the position of the threshold value H^*_π.

In proportion to the modification of the preferences distribution $\pi(k)$ the entropy decreases with the increase of k. In certain moment it reaches the threshold value H^*_π and subject after this achieves „a selection” of alternative σ_i and it takes up a solution of problem P: ($\sigma_i(k) \rightarrow \sigma_j$). Decrease of entropy occurs as a result

a) of obtaining by the subject of additional information $I(k)$ (although it is possible to imagine that the certain information will hamper decision making, i.e., lead to an increase in the entropy). Fig. 3.10 shows the case when entropy does not reach threshold value since incoming information beginning from the moment k^* produces an increase in the uncertainty of preferences and, respectively an increase in the entropy.

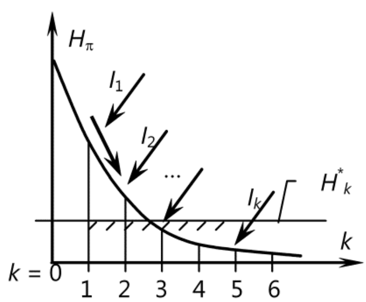


Fig. 3.9

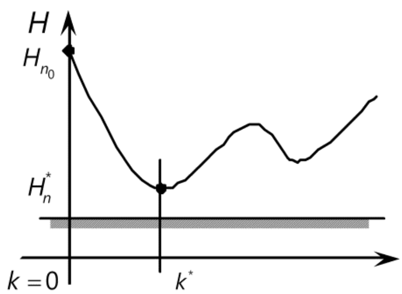


Fig. 3.10

b) of a change in the number of alternatives N, i.e., the modification of set S_a, for example,

$$S_a(N) \to S_a(N-1),\ldots$$

It is possible to propose the large number of scheme s of sequential analysis and forming of preferences. We will examine two, as it seems to us, typical patterns.

In the first scheme A, at the initial step a positive preference $\pi_1^+(\sigma_i)$ are formed, further analysis occurs in accordance with the scheme, given below:

$$S_a \to \pi_1^+(\sigma_i) \to \pi_1^-(\sigma_i) \to \upsilon_1^+(\sigma_i) \to \upsilon_1^-(\sigma_i) \to \pi_2^+(\sigma_i) \to \ldots$$

Evidently, the passage from one stage of analysis to another is each time connected with overcoming of the corresponding entropy barrier. In this case for the scheme A the sequence of barriers is

$$H^*_{\pi^+}, H^*_{\pi^-}, H^*_{\upsilon^+}, H^*_{\upsilon^-} \quad ,\ldots$$

Barriers are defined as endogenous factors of individual psyche, as well, and exogenous factors, for example, the growing shortage of time resources. The subject, who realizes the first scheme *A*, can be named „decisive" (resolute person, courageous, daring,...).

The second scheme *B* is more characteristic for the careful subject:

$$S_a \to \upsilon_1^-(\sigma_i) \to \upsilon_1^+(\sigma_i) \to \pi_1^-(\sigma_i) \to \pi_1^+(\sigma_i) \to \upsilon_2^-(\sigma_i) \to \ldots$$

In this case the study of situation begins from the analysis of possible negative consequences for the purpose to reject alternative σ_i. Here also the sequence of passages from one stage to another is connected with the entropy barriers:

$$H^*_{\upsilon^-}, H^*_{\upsilon^+}, H^*_{\pi^-}, H^*_{\pi^+} \quad ,\ldots$$

The current passages from one stage of analysis to another are also related, as we see, with overcoming of entropy thresholds, which we name lower thresholds of the 2nd kind. Each time the passage is achieved, if the corresponding inequality is fulfilled, for example:

$$H_{\pi^-} \le H^*_{\pi^-} \quad \text{or} \quad H_{\upsilon^-} \le H^*_{\upsilon^+} \quad .$$

The final solution (selection on S_a) is not yet achieved, but only the passage from one stage of analysis to another occurs.

Thus, the threshold of the 1st kind – this is the entropy threshold, which determines the readiness of subject to carry out an alternative choice $\sigma_j \in S_a$ or purpose.

The threshold of the 2nd kind determines moment t^*, when in the process of the analysis of problem- resource situation, the subject spasmodically changes the direction of analysis.

It is clear that entire process of analysis, a decision making, solution of the selected problem (achievement of the objective) is the object of subjective dynamics.

From our point of view apart from the lower entropy thresholds there are upper entropy thresholds, which as lower thresholds are individualized and are important characteristics of the subject psyche.

Almost for sure for each subject there is an upper maximum entropy of the preferences H_π^{**} such, that such high magnitude of entropy (uncertainty) the subject cannot „stand". It simply can't be realized in this subject psyche. When approaching this limit, a „defense mechanism" of consciousness appears, which is reduced either to the rejection of some alternatives in order to decrease of entropy or a change in the distribution of preferences with „strong-will method" for the same purpose.

So until you select the target (that is, during the analysis of the situation) the subject is located (in the psychological sense) in the layer

$$\delta H_\pi = \left[H_\pi^{**}, H_\pi^{*} \right] \quad .$$

Fig. 3.11 shows schematically the entropy space „double-layer" structure. In time, the process is developing in such a way, that every time the subject is making decisions - choosing a goal, that channels part (may be - significant) of the available resources in a particular direction, thus narrowing the set of „left" alternatives S_a and decreasing their subjective entropy.

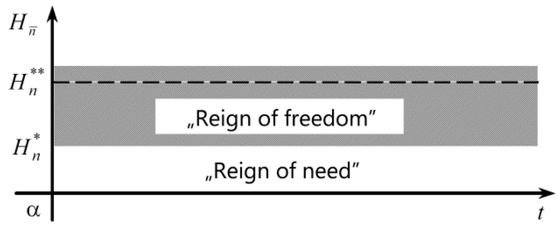


Fig. 3.11

In this case he temporarily passes into the „Reign of need", in order, to obtain new resources and, in the case of objective achievement to return into the „Reign of freedom" and again to prove to be before the complex selection in the „wide" field of alternatives S_a.

For some the return to the „Reign of freedom" proves to be inaccessible, and they perhaps forever remain in the „Reign of need".

The more detailed analysis of category „freedom", the varieties of treatments of this category and their connection with the subjective entropy will be discussed later in chapter 7.

The more detailed analysis of the „freedom" category, the varieties of the treatments of this category and their connection with the subjective entropy will be discussed later in chapter 7.

3.8. Entropy of ordinal distributions of preferences

Above we determined the entropy of the cardinal distribution of preferences $\pi(\sigma_i)$ on S_a, and also different modifications: $\pi(\sigma_i | \sigma_j)$..., which under certain conditions generate orderings of alternatives on S_a. In the present paragraph, the entropy of ordinal distributions is introduced. Let ρ: $\langle$ – relation of a strict preference, and ρ: $\underset{\sim}{\langle}$ – the relation of lax preference. In the first case we have the following ordering:

$$\sigma_1 \langle \sigma_2 \langle \sigma_3 \langle \ldots \langle \sigma_{N-1} \langle \sigma_N.$$

However, information about preferences bears a sequence of positive integers

$$i: 1\ 2\ 3\ \ldots\ N-1\ N,$$

which are the numbers of alternatives (ranks). Sum of the ranks

$$S_N = \frac{N(N+1)}{2} \quad .$$

Let's determine the normalized number of the alternative

$$\bar{i} = \frac{i}{S_N} = \frac{2i}{N(N+1)} \quad .$$

Then $\sum_{i=1}^{N} \bar{i} = 1$. Base entropy of a number of normalized ranks

$$\bar{H}_{\langle}^{*} = -\sum_{i=1}^{N} \frac{2i}{N(N+1)} \ln \frac{2i}{N(N+1)} = \frac{H^*}{S_N} + \ln S_N \quad , \qquad (3.85)$$

where $H^* = -\sum_{i=1}^{n} i \ln i$.

Another limiting case is a complete indifference or a complete equivalence:

$$\sigma_1 \sim \sigma_2 \sim \sigma_3 \sim \ldots \sim \sigma_{N-1} \sim \sigma_N \ .$$

In this case all alternatives have the identical rank N^{-1}:

$$\frac{1}{S_N};\frac{1}{S_N};\frac{1}{S_N}\dots\frac{1}{S_N};\frac{1}{S_N},$$

and the corresponding entropy is equal:

$$\bar{H}^*_{\sim}=-\sum_{i=1}^{N}\frac{1}{N}\ln\frac{1}{N}=-\ln\frac{1}{N}=\ln N \quad . \tag{3.86}$$

We see that since $H^* < 0$, then

$$\bar{H}^*_{\langle}\le\bar{H}^*_{\sim}=H_{\max} \quad .$$

It is clear that the order of complete indifference has maximum entropy.

Here we assign to all alternatives the identical non-normalized rank equal 1.

Let's examine order with the presence of one class of the equivalence of that containing of more than one alternative:

$$\sigma_1 \langle \sigma_2 \langle \sigma_3 \langle \dots \langle \sigma_k \sim \sigma_{k+1} \sim \dots \sim \sigma_{p+k} \langle \sigma_{p+k+1} \langle \dots \sigma_{N-1} \langle \sigma_N.$$

Let this class of equivalence stretch from *k-th* to *(p + k)-th* of alternative inclusively. Let's number of elements a sequence is such that all alternatives from σ_k to σ_{p+k} have identical rank k. Then the sum of ranks r_i

$$S_N(k,p)=\frac{1}{2}\left[k(k-1)+2k(p+1)+(N-k-p)(N+k-p+1)\right] \quad .$$

For example,

for the order $\sigma_1 \langle \sigma_2 \sim \sigma_3 \sim \sigma_4 \langle \sigma_5$ $S_N(k, p) = S(2, 3) = 10$,
for the order $\sigma_1 \sim \sigma_2 \sim \sigma_3 \sim \sigma_4 \langle \sigma_5$ $S_N(k, p) = 6$,
for the order $\sigma_1 \langle \sigma_2 \langle \sigma_3 \sim \sigma_4 \langle \sigma_5$ $S_N(k, p) = 13$.

It is convenient to use „corrected" ranks

$$r_i^*=r_i+\frac{S_N-S_N(k,p)}{N} \quad . \tag{3.87}$$

Sum of corrected ranks:

$$\sum_{i=1}^{N}r_i^*=\sum_{i=1}^{N}\left(r_i+\frac{S_N-S_N(k,p)}{N}\right)=S_N(k,p)+S_N-S_N(k,p)=S_N \quad .$$

Let's further introduce normalized corrected ranks

$$\bar{r}_i^*=\frac{1}{S_N}r_i^*=\frac{r_i}{S_N}+\frac{1}{N}\left(1-\frac{S_N(k,p)}{S_N}\right) \quad . \tag{3.88}$$

We see that

$$\sum_{i=1}^{N} \overline{r}_i^* = 1 \quad .$$

Thus, the values $\overline{r}_i^*$ play the role of the quantitative measure of the preference of alternative σ_i. Analogy is, however, incomplete.

A strict order corresponds to the case, when the number of classes of equivalence is equal to the number of alternatives N in S_a. If the number of classes of equivalence $m < N$, then at least in one of them more than one element is contained. Let's assume that the normalized entropy on S_a in the case of strict order must be equal zero, since „complete certainty" occurs. In this case $m = N$. The entropy of order on S_a must grow as the number of classes of equivalence will decrease takes maximum value since $m = 1$, i.e., in the case, when set S_a is one class of equivalence. The entropy, which satisfies the indicated conditions (for $1 \le m \le$ N), is expressed by the formula

A strict order corresponds to the case, when the number of equivalence classes is equal to the number of alternatives N in S_a. If the number of equivalence classes of $m < N$, then at least in one of them more than one element is contained. Let us assume that the standardized entropy on S_a in the case of strict order must be equal to zero, since the „complete certainty" occurs. In this case $m = N$. Entropy of order on S_a should increase as the number of equivalence classes will be reduced and takes the maximum value at $m = 1$, that is, if the set S_a is one equivalence class. The entropy, which satisfies the indicated conditions (for $1 \le m \le N$) is expressed by the formula

$$\overline{H}_{\underset{\sim}{\langle}} = \overline{H}^* - \overline{H}_{\langle}^*,$$

where H^* - is defined as the value

$$H^* = -\sum_{i=1}^{N} \overline{r}_i^* \ln \overline{r}_i^* \quad , \tag{3.89}$$

and $\overline{H}_{\langle}^*$ is a basic entropy, introduced by relation (3.85). Thus, if there is one equivalence class containing more than one element (alternative), then

$$\overline{H}_{\underset{\sim}{\langle}} = -\sum_{i=1}^{N} \overline{r}_i^* \ln \overline{r}_i^* - \frac{1}{S_N} H^* - \ln S_N \quad . \tag{3.90}$$

Let now $S_{a\sim}$ is factor- set of alternatives, which contains m equivalence classes S_{jk} $(k \in \overline{1,m})$, while the *k-th* class contains n_k elements. If a lexicographic order is established in $S_{a\sim}$, then to each class (each element of the class S_{ak}) rank k is assigned, and the sum of ranks is equal

$$\sum_{k=1}^{N} kn_k \quad .$$

The biggest possible sum of ranks corresponding to a strict order when $m = N$ and weight $n_k = 1$, is equal $\frac{N(N+1)}{2}$.

In general case the scarcity of sum of ranks $S_{j\sim}$ equals

$$\frac{N(N+1)}{2} - \sum_{k=1}^{m} kn_k = S_N - \sum_{k=1}^{m} km_k \quad ,$$

and correcting rank supplement

$$\delta_N = \frac{S_N - \sum_{k=1}^{m} km_k}{N} \quad .$$

Then the corrected rank

$$r_k = r_k + \delta_N = r_k + \frac{S_N - \sum_{q=1}^{m} qn_q}{N} \quad , \tag{3.91}$$

and the relative corrected rank

$$\bar{r}_k^* = \frac{k}{S_N} + \frac{1}{N}\left(1 - \frac{S_N - \sum_{q=1}^{m} qn_q}{S_N}\right) \quad . \tag{3.92}$$

Given this relationship, we obtain

$$\bar{H}_{\sim} = -\sum_{k=1}^{m} n_k \left[\frac{k}{S_N} + \frac{1}{N}\left(1 - \frac{\sum_{q=1}^{m} qn_q}{S_N}\right)\right] \ln\left[\frac{k}{S_N} + \frac{1}{N}\left(1 - \frac{\sum_{q=1}^{m} qn_q}{S_N}\right)\right] - \frac{H^*}{S_N} - \ln S_N \quad . \tag{3.93}$$

In the particular case, when m = N and $\forall n_q = 1$, from (3.96) we find

$$\bar{H}_{\sim} = -\sum_{i=1}^{N} \frac{i}{S_N} \ln\frac{i}{S_N} - \left(\frac{1}{S_N} H^* + \ln S_N\right) = 0 \quad . \tag{3.94}$$

In the case, when S_a is the unitary equivalence class (m = 1)

$$\bar{H}_{\succsim} = \bar{H}^{*}_{\succsim} = \frac{2}{N(N+1)}\sum_{i=1}^{N} i\ln i - \ln\frac{N+1}{2} \quad . \tag{3.95}$$

We can see that $\bar{H}_{\succsim} = 0$ only, when N = 1.

We have the following table

N	$\bar{H}^{*}_{\succsim}$	$\bar{H}^{*}_{\succsim\max} = \ln N$	
1	0	0	~1
2	0,0567…	0,69315…	12,2249…
3	0,08721…	1,09861…	12,5239…
4	0,10644…	1,38629…	13,02414…
5	0,11969…	1,60944…	13,4467…
6	0,12838…	1,79176…	13,84881…

As we see, the entropy $\bar{H}^{*}_{\succsim}$ is equal to zero, only when $S_{a\sim}$ consists of only one element - there is only one alternative. When N> 1, entropy $\bar{H}^{*}_{\succsim} > 0$ and it grows with N increase.

The entropy $\bar{H}_{\succsim}$ is high and much greater than $\bar{H}^{*}_{\succsim}$ entropy. If we consider the entropy as a subjective measure of decision-making „complexity", we can conclude that strict rules are more „simple" objects to analyze than the factor rules, i.e. orders with no singular equivalence classes (m < N).

Introduced in this paragraph entropy of the lexicographic orderings has somewhat outstanding properties from the entropy defined by the normalized function preferences.

It is possible to determine the lexicographic entropy differently, namely so, that in the case of „complete indifference" (S_a is the equivalence class: m = 1) this entropy coincides with H_{max} = ln N.

Let's assume

$$\bar{H}_{\succsim} = -\sum_{k=1}^{m} n_k \bar{r}_k \ln \bar{r}_k \quad , \tag{3.96}$$

where $\bar{r}_k = r_k\left(\sum_{s=1}^{m} s n_s\right)^{-1}$, $\sum_{s=1}^{m} n_s = N$. Then

$$\sum_{k=1}^{m} \bar{r}_k n_k = 1 \quad .$$

In the case of the maximum uncertainty (S_j is a wholly equivalence class) m = 1, n_1 = N, formula (3.96) gives

$$\bar{H}_{\precsim} = \bar{H}_{\sim} = \ln N \quad .$$

In the case of a strict order (m = N, n_k = 1, $\forall_k$):

$$\bar{H}_{\precsim} = \bar{H}_{\prec} = -\sum_{k=1}^{N} \frac{k}{S_N} \ln \frac{k}{S_N} > 0 \tag{3.97}$$

and $\bar{H}_{\prec} = 0$ only for N = 1. In the general case from (3.97)

$$S_N \ln S_N \geq \sum_{k=1}^{N} k \ln k \quad .$$

Finally, the lexicographic entropy can be determined in such a way that by both of limiting conditions will be carried out:

for m = 1, n = N $\quad \bar{H}_{\precsim} = \bar{H}_{\sim} = \ln N \quad ,$

for m = N, n_k = 1,$\forall k$ $\quad \bar{H}_{\precsim} = \bar{H}_{\sim} = \dfrac{1}{S_N} H^* + \ln S_N \quad ,$

where $\quad H^* = -\sum_{k=1}^{N} k \ln k \quad .$

In addition, we require that for m = 2, n_1 = N – 1, n_2 = 1 the ranks of all alternatives in the class n_1 are equal zero. Then the sum of ranks SN(m) = 1, also, for $\forall k \in$ [1, N – 1], $\bar{r}_k = 0$, but for $k = n$ $\bar{r}_N = 1$ and the entropy $\bar{H}_{\precsim} = \bar{H}$ (m = 2, n_1 = N – 1, n_2 = 1) = 0. In order to satisfy all described conditions, it is enough to assume that the rank of least preferred item (σ_1) would be equal zero.

Let's rewrite a formula (3.96) in the form:

$$\bar{H}_{\precsim} = -\sum_{k=1}^{m} n_k \frac{r_k}{S_N(m)} \ln \frac{r_k}{S_N(m)}, \tag{3.98}$$

where r_1 = 0, and $S_N(m)$ is a sum of ranks:

$$S_N(m) = \sum_{k=1}^{m} (k-1) n_k \quad .$$

The equality of certain rank r_k to zero does not lead to the divergence in formula (3.98), since

$$\lim_{x\to 0}\left(x\ln x\right)=0 \quad .$$

The lexicographic entropy in this form can prove to be most convenient, if it will be possible to formulate variation principle for the lexicographic orders. Apparently, the function of effectiveness (generalization „of risk") can be recorded in the following form

$$\varepsilon_\pi = \sum_{k=1}^{m} n_k \bar{r}_k U_k \quad ,$$

where U_k is utility of the alternative σ_k, which belongs to the class n_k.

3.9. Two optimization tasks: optimization of utilities and optimization of preferences

In this chapter we examine the sequence of two interconnected optimization tasks:
1. Optimization of selection on the basis of the utility function, and
2. „Natural" optimization of preferences due to conditions, when solution of the first problem is already known.

These inserted optimization tasks have different purpose. The selection of an optimal utilitarian solution is a solution of the first problem. The second task lead to the optimum distribution of preferences on the set of alternatives S_a, which is determined by conditions of the first task.

In the trivial version, when preferences represent only utilitarian component, the sense of solution of the second problem is to ensure that it is in contrast to the first task, not only indicates the best version of the solution, but also gives the condition of decision making:

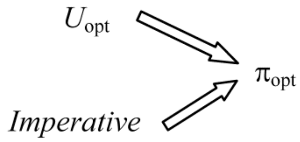
$$H_\pi \le H^*_\pi.$$

Actually, the obtained optimal solution can differ very little from an „adjacent" non-optimal solution; the entropy of the preferences distribution will be big $H_\pi \approx H_{\max} = \ln N$ and the solution will not be accepted. Besides the entropy H_π there are some other functions, which integrally present distribution of preferences and possessing a sufficient elasticity in a vicinity of distribution extreme, for example, analogies of entropy, examined earlier (Chapter 3).

In the trivial version, a logical connection between two tasks has form of sequence:

$$U_{\text{opt}} \Rightarrow \pi_{\text{opt}}$$

Nontrivial formulation lies in the fact that π_{opt} are formed not only on the basis of utilitarian component, i.e., the utility function, but also on certain non-utilitarian, for example, ethical components, which is shown on the following scheme:



Thus, π_{opt} is determined not only by rational utility, but also by the certain collection of imperatives, which in each certain case are not subject to optimization, but they are a priority. We can see here a fundamental difference between utilities and preferences.

The connection of two optimization tasks resembles connection on two task with the solution of the problem of the dynamic object identification , when the problem of the optimum test program selection (in this case usually a mathematical model of an object and previously obtained experimental data are used a priori) is solved at first.

Then founded optimal plan of an experiment is being realized, and resulting data is used for the solution of the second optimization problem - the determination of such estimations of the object model, which would minimize the error of simulation with respect to real dynamics modeling.

There are several versions of the connection between two optimization problems described above. Let's examine a trivial version using an example of the utility optimization, undertaken from the book [199]. It is necessary to find such vector $x = (x_1, x_2, \ldots, x_n)$, which delivers the maximum of the utility function $U(x_1, x_2, \ldots, x_n)$ on the manifold $M(x_i, p_i, M)$:

$$x_{opt} = \arg\max_{x \in M \subset X} U\left(x_1, x_2, \ldots, x_n\right) \quad . \tag{3.99}$$

The manifold reflects the equality of budget M and expenditures for the acquisition of the collection of goods $p_i x_i$, where p_i is price of the goods unit, x_i is quantity of goods units of i-th type.

Let at first the preferences are determined by quantities of acquired goods x_i:

$$\pi^+\left(\sigma_i\right) = \frac{e^{\beta x_i}}{\sum_{k=1}^{N} e^{\beta x_k}} \quad . \tag{3.100}$$

Define how preferences are distributed, if in the capacity of x_i the optimum quantities x_{iopt} , found as a result of optimization problem solution (3.99) will be undertaken. In the form (3.100) preferences depend on only utilitarian component. Assuming that the manifold M is assigned by relation:

$$M = p_1x_1 + p_2x_2 \ , \qquad (3.101)$$

and the function of utility

$$U = x_1 x_2 \ . \qquad (3.102)$$

Both goods are needed and more of each is, the better. Let's form the criterion of optimality in the conditional extremum task:

$$\Phi^* = U(x_1x_2) + \lambda(M - x_1p_1 - x_2p_2) \ , \qquad (3.103)$$

where λ – Lagrange's coefficient. Variational problem takes the form:

$$(x_ix_j)_{opt} = \arg\max \Phi^* \ . \qquad (3.104)$$

Find the solution from necessary conditions:

$$L_1 = \frac{\partial\Phi^*}{\partial x_1} = \frac{\partial U(x_1x_2)}{\partial x_1} - \lambda p_1 = 0\,; \quad L_2 = \frac{\partial\Phi^*}{\partial x_2} = \frac{\partial U(x_1x_2)}{\partial x_2} - \lambda p_2 = 0 \quad ; \qquad (3.105)$$

$$M - p_1x_1 - p_2x_2 = 0 \ .$$

Sufficient condition for the existence of solution is the requirement, that:

$$G = \begin{vmatrix} \dfrac{\partial^2\Phi^*}{\partial x_1^2} & \dfrac{\partial^2\Phi^*}{\partial x_1\partial x_2} & \dfrac{\partial^2\Phi^*}{\partial x_1\partial\lambda} \\ \dfrac{\partial^2\Phi^*}{\partial x_2\partial x_1} & \dfrac{\partial^2\Phi^*}{\partial x_2^2} & \dfrac{\partial^2\Phi^*}{\partial x_2\partial\lambda} \\ \dfrac{\partial^2\Phi^*}{\partial\lambda\partial x_1} & \dfrac{\partial^2\Phi^*}{\partial\lambda\partial x_2} & \dfrac{\partial^2\Phi^*}{\partial\lambda^2} \end{vmatrix} > 0 \quad . \qquad (3.106)$$

We have

$$G = \begin{vmatrix} 0 & 1 & -p_1 \\ 1 & 0 & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix} = 2p_1p_2 > 0 \quad .$$

Equations (3.105) take the form:

$$L_1 = x_2 - \lambda p_1 = 0; \quad L_2 = x_1 - \lambda p_2 = 0 \ .$$

Substituting x_1 and x_2 in (3.101), we'll find: $\lambda = \dfrac{M}{2p_1p_2}$.

Then

$$x_{i\,opt} = \frac{M}{2p_i}; \quad x_{j\,opt} = \frac{M}{2p_j} \quad .$$

Here M plays the role of available resources, and prices – of required resources. Preferences are determined by formula:

$$\pi^+(\sigma_i) = \frac{e^{\beta\frac{M}{2p_i}}}{e^{\beta\frac{M}{2p_1}} + e^{\beta\frac{M}{2p_2}}} \quad .$$

Let $M = 10$; $p_1 = 2$; $p_2 = 5$; $\beta = 1$, then $\pi^+(\sigma_1) = 0{,}9526$; $\pi^+(\sigma_2) = 0{,}0474$. We can see that $\pi^+(\sigma_1) > \pi^+(\sigma_2)$. The entropy $H^{(1)}_{\pi+} = 0{,}1907$ is small in comparison to $H_{\pi\max} \cong 0{,}693$. Let's decrease the budget in half with the same prices: $M = 5$. Preferences take values: $\pi^+(\sigma_1) = 0{,}81757$; $\pi^+(\sigma_2) = 0{,}018240$, and the entropy $H_{\pi+}^{(2)} = 0{,}475 > H^{(1)}_{\pi+}$, therefore, the decrease of budget in this case leads to an increase in the entropy.

More adequate formulation of the second optimization problem lies in the fact, that as arguments of functions of preferences distributions the utility $U(x')$ appears, where x' is vector of dimensionality $m < n$ being selected from the vector x of dimensionality n. Number of versions $C_n^m = \frac{n!}{m!(n-m)!}$. If non-utilitarian component of preferences is not included (or it is absent), we accept the distribution $\pi^+(\sigma_i)$ in the form:

$$\pi^+(\sigma_k) = \frac{e^{\beta U(x'_k)}}{\sum_{j=1}^{N} e^{\beta U(x'_j)}} \quad . \tag{3.107}$$

Here $N = C_n^m$. If with the course of formation of preferences the subject is oriented not only to the utilitarian, but also to non-utilitarian, for example, the ethical component, we will use, for example, as the preference model the distribution in the form

$$\pi^+(\sigma_k) = \frac{\pi(I_k) e^{\beta U(x'_k)}}{\sum_{j=1}^{N} \pi(I_j) e^{\beta U(x'_j)}} \quad . \tag{3.108}$$

where $\pi(I_j)$ is the function, that reflects the influence of a certain a priori imperative, or irrational considerations.

Let's examine a quantitative example. Suppose that $m = 2$, and $n = 3$, hence $N = C_3^2 = 3$, 3 alternative collections of „goods" from $x = (x_1, x_2, x_3)$: $x'_1 = (x_1, x_2)$;

$x'_2 = (x_1, x_3)$; $x'_3 = (x_2, x_3)$. Assume that „prices" of goods are distributed as shown in the table below

$$\begin{array}{c|ccc} x_k & x_1 & x_2 & x_3 \\ \hline p_k & 1 & 2 & 3 \end{array} \quad , \quad \beta = 1 \quad ,$$

and „budget" M = 10. Each time, with the determination of optimum utilities, as x_k their optimum values are taken, obtained as a result of problem solution (3.104):

$$U(\sigma_1) = U(x'_1) = x_{1opt} x_{2opt} = \frac{M}{2p_1}\frac{M}{2p_2} = \frac{M^2}{4p_1p_2} \quad ;$$

$$U(\sigma_2) = U(x'_2) = x_{1opt} x_{3opt} = \frac{M}{2p_1}\frac{M}{2p_3} = \frac{M^2}{4p_1p_3} \quad ;$$

We find $U(\sigma_1)$ = 12,5; $U(\sigma_2)$ = 8,333; $U(\sigma_3)$ = 4,1667. Accordingly for utilitarian preferences we obtain (formula 3.107):

$$\pi_0^+(\sigma_1) = 0{,}9845 \quad ; \quad \pi_0^+(\sigma_2) = 0{,}0153 \quad ; \quad \pi_0^+(\sigma_3) = 0{,}00024 \quad ,$$

and the entropy

$$H_{\pi+}^{(0)} = 0{,}08128 \quad .$$

Let's suppose further, that the function $\pi(I_k)$ reflects a tendency to decrease a total quantity of „goods" in the „basket" (unique asceticism) and takes the form:

$$\pi(I_k) = \frac{1}{x_r^2 + x_s^2} \quad , \text{ where } r,s \in \overline{1,2,3} \quad .$$

Results of calculations are shows in the table.

σ_k	σ_1	σ_2	σ_3
$U(\sigma_k)$	12,5	8,333...	4,1667...
$\pi(I_k)$	0,032...	0,036...	0,111...
$\pi^+(\sigma_k)$	0,9820	0,01712	0,00082

Entropy

$$H^{(1)}_{\pi+} = 0{,}09327 > H^{(0)}_{\pi+} = 0{,}08128 \quad ,$$

which corresponds „to the export" of information

$$I_{\pi+} = H^{(0)}_{\pi+} - H^{(1)}_{\pi+} = 0{,}08128 - 0{,}09327 = -0{,}01199 \quad .$$

In both cases, the entropy is small, the transition from the utilities to the utilitarian preferences „reinforce" the differentiation options and facilitates decision-making.

Also see that the inclusion of non-utilitarian component, leads in this case to an increase in entropy and therefore complicates the decision. The value $I_{\pi+}$ is a „ price information" taking into account a priori imperatives.

Decrease the budget in half and place M = 5. Let's calculate the distribution of preferences, excluding non-utilitarian component, and we find:

$$U(\sigma_1) = 3{,}125; \qquad \pi^+_0(\sigma_1) = 0{,}6770;$$
$$U(\sigma_2) = 2{,}0833; \qquad \pi^+_0(\sigma_2) = 0{,}2389;$$
$$U(\sigma_3) = 1{,}0417; \qquad \pi^+_0(\sigma_3) = 0{,}0842;$$
$$H^{(0)}_{\pi+} = 0{,}8144.$$

Let calculate $\pi(I_k)$:

$$\pi(I_1) = \frac{1}{x^2_{1opt} + x^2_{2opt}} = 0{,}0624\,; \quad \pi(I_2) = \frac{1}{x^2_{1opt} + x^2_{3opt}} = 0{,}0709 \quad ;$$

$$\pi(I_3) = \frac{1}{x^2_{2opt} + x^2_{3opt}} = 0{,}0944 \quad .$$

Taking under consideration non-utilitarian component we will obtain new preferences:

$$\pi^+_1(\sigma_1) = 0{,}7039 \quad ; \quad \pi^+_1(\sigma_2) = 0{,}2172 \quad ; \quad \pi^+_1(\sigma_3) = 0{,}0789 \quad ,$$

the entropy

$$H^{(0)}_{\pi+} = 0{,}7792 \ .$$

As we see, the account of non-utilitarian components has led to a decrease in entropy

$$I_\pi = H^{(0)}_{\pi+} - H^{(1)}_{\pi+} = 0{,}8144 - 0{,}7792 = 0{,}0352 \ .$$

Let's perform calculations for different distribution of „goods" prices with smaller differentiation, as shown in the following table:

	x_i	x_1	x_2	x_3
M = 5	p_i	0,8	1,0	1,2
	$x_{i\ opt}$	3,125	2,500	2,083

Utilities and utilitarian preferences are given in the table:

	σ_k	σ_1	σ_2	σ_3
M = 5	$U(\sigma_k)$	7,8125	6,509	5,2075

$\pi^{+}{}_{0}(\sigma_k)$	0,7432	0,20183	0,0549

$$H^{(0)}{}_{\pi+} = 0{,}7029.$$

Following table presents preferences, calculated with the utilitarian component, which takes into account the introduction of coefficients

$$\pi\left(I_k\right) = \frac{1}{x_{ki}^2 + x_{kj}^2}$$

	σ_k	σ_1	σ_2	σ_3
M = 5	$\pi(\sigma_k)$	0,0624	0,0709	0,0944
	$\pi^{+}{}_{1}(\sigma_i)$	0,7039	0,2172	0,0789

$$H^{(1)}{}_{\pi+} = 0{,}7791 \quad .$$

The „information price" of the imperative I_k calculation is

$$I_{\pi+} = H^{(0)}{}_{\pi+} - H^{(1)}{}_{\pi+} = -0{,}0762.$$

Consequently, the calculation of an imperative I_k leads in this case to an increase in the „information export". The sense of the imperative action lies in the fact that it stimulates smaller weight of the „consumer's basket", abstention in consumption. In this case it was not possible to take $\pi(I_k)$ in the form of the product $x_i x_j$ for the utility $U(\sigma_i)$, since with this structure one of quantities x_i (or x_j) can take infinitely big significance, if another is equal zero.

4. GROUP OF SUBJECTS. FUNCTION OF RATING PREFERENCES. AGGREGATION OF PREFERENCES

4.1. Problems, related to the subjective analysis of a group of interacting subjects.

An active system with one subject was predominantly studied in the work [64] and also in chapters of 1-3 of this book. The functions of individual preferences (first kind) were introduced, as well as individual subjective entropy on a set S_{aj} of alternatives of subject j ($j \in \overline{1,M}$), where M is number of subject in the group. Variational problems were formulated, solution of which were *canonical functions of individual preferences*. Different models of preferences functions of the first kind are examined, when either functions of utility (individual utility) or resources (required available, expected as a result of problem solution) come out as their arguments.

In this chapter the group of individuals (subjects), who in some way, interact between each other [27, 28, 59, 79, 81, 110, 111, 113, 117, 118, 141, 142, 144], is the object of analysis. A lot of new tasks and concepts appears in connection with this problem. The task of aggregation of preferences is one of basic ones. When we talk about "the group", we mean that there is a certain link objective and subjective between the members of the group. Basis of this link are common alternatives and problems, common resources (consolidated resources), coinciding or close "mechanisms" of the forming of individual preferences distributions, "flows" of resources of all forms inside the group. The group has a structure and professes the certain principle "of prosperity" (for example - utilitarism or egalitarism).

Furthermore, it is completely obvious that in "the group" there are reciprocal effects, a mutual dependence of individual preferences of different subjects. Mechanisms of interaction, being realized in the time as dynamic process can be postulated. Completion corresponding models is the task of an experimental study.

At last, group can be connected by the common ethnic, political, cultural imperatives. In this paragraph we will briefly comment only some features and problems of the analysis of group subject's preferences.

Some of these problems are worth to be an independent subject of a study and the reference of them is here caused by the desire to show that common and sufficiently effective basis can be the subjective analysis, in particular, canonical preferences distributions.

The most important question is forming of the alternatives set of a group on the basis of comparison and combining of individual sets S_{aj}. In this case it is possible that the composition of individual collections of alternatives inside the group is strongly correlated. The presence of identical problems and the appearance of corporative problems brings in the common case to a change in distributions of available and required resources, the creation of corporative (consolidated) resources, the appearance of the replacement of resources between members of group and as consequence to a change in the group and individual subjective entropies and "flows" of subjective information.

During the study of available resources replacement and their corporatization, there are several versions:

a. Available resources R_j^{disp} ($j \in \overline{1,M}$) are all universal (for example - money).

b. Available resources R_j^{disp} (σ_i) are specialized for particular alternatives.

c. There is a resource, both of the first and the second type.

In more common case of distribution they can be expressed through utility, including "corporate" utility.

We already know that the form and the content of the individual function of preferences model on S_{aj} depend on the form the extreme functional was selected in. In particular, functions of preferences can be monotonic and non-monotonic; they can be positive or negative. Therefore, characteristics of group and processes, that take place, depend; on the basis of identical or different functional structures models of functions of individual preferences of group members are built. It can be interpreted as a congruence or a difference in psychological types of subjects of the group or their natural needs.

The function of the preference distribution of the II kind or rating function introduced in this chapter is the essential feature of a conceptual and analytical mechanism of the subjective analysis of a group. We will study versions of this function representation, the canonical models of rating distributions, corresponding normalization, and formulas for the rating entropy calculation. Study of ratings is basis of the subjective group structurization, study "of the motion" of resources inside the group and other questions.

Dynamic processes which take place in the group present a significant interest. In this direction we will base on a number of postulated settings of variational problems, from which it is possible to obtain recurrent scheme of a change of the preferences distribution, and some additional results. With this problem a question about the chronological variation of an individual set of alternatives and, correspondingly, the collection of corporative alternatives, and also calculation of previous preferences arise is connected.

4.2. About ordinal theory of corporate decisions. Theory of aggregated utility.

There is an extensive literature in the field of the theory of corporate decisions making, which is mainly based on the use of ordinal preferences distributions, aggregate utility, expected utility, expected subjective utility .

Usually in the basis of theory certain system of axioms lies, which reflects a priori ideas about properties of the subjects psyche, the adopted concept of justice - ethical concept.

The attempt to give the complete review of all directions and even basic results, is hopeless goal which is impossible to undertaking. Therefore, in our brief review we will refer to one of the fundamental monographs about the theory of cooperative solutions - work E. Mullen [113], and also simultaneously to some other works. Since this monograph can perform the role of the textbook, entering in the problem a brief survey is an appropriate one. Reference to the work of E. Mullen relates to entire chapter, which is in the substantial part the quick summary of this book, augmented by judgments of the author about the connection between the theory of cooperative solutions and by the theory, in the basis of which canonical preferences distributions lies.

The theory of cooperative solutions is the mechanism of "the theory of welfare" (term by A. Sen [142]). This is one of the theories directed to solution of long age problem - "the tendency of people to the equality is passionate, insatiable, eternal and invincible" (Tocqueville 1860), from itself, let`s add - and to those absolutely unreal, and, in the individual sense to superiority over others.

"A tendency to the equality" can be taken under consideration while postulating functional for the task of obtaining canonical distributions of preferences of the II kind (ratings). In the form of an ethical imperative "tendency to the equality" is close to the standard; do not „create an idol" for itself. However, we know, that together with this imperative tendency to search "for the leader", to search for that "idol" both in the religious and in the rationalistic sense is "assembled" in the consciousness of people.

The capacious communizing concept is a collective justice which does not require the equality of association members at all, but, one way or another, in one or other form it exists in people`s consciousness, in the collective practice, it reflects in the economic and political structure of associations.

Moreover, in the same association different groups ("coalition") can profess different concepts of justice.

In hierarchical organizational systems "collective justice" undergoes "*to the vertical line*" and "*along horizontal*" decomposition. It is necessary to note that if we talk about the role of state, *then it hasn`t another task besides realization in the society of certain* *concept of collective justice. If "state" does not have a concept of collective justice and does not realize it, then it is not a state.*

Without going in the retrospective analysis of category "collective justice" let`s pause at two extreme concepts: the egalitarism and the utilitarianism, to which the certain collections of the ethical postulates, expressed in theoretical analysis in the form of axioms correspond.

Between the egalitarism and the utilitarianism there are many series of intermediate compromise concepts.

The egalitarism - is the principle of justice, which is briefly reduced to the fact that for equal subjects there must be an equal relation. From a formally mathematical point of view it lies in the fact that, if $U = (u_1, u_2, \ldots, u_m)$ — the vector of individual utilities, then principle requires the maximization of minimum utility ("the dictates" of the poorest). As the criterion of optimality the function of collective utility $W(U) = W(u_1, u_2, \ldots, u_m)$ is used, which aggregates the individual utilities (in this case on one and the same set of alternatives S_a). Egalitarian criterion takes the form:

$$W_{ed}(U) = \min_{j \in \overline{1,M}} u_j \quad . \tag{4.1}$$

Egalitarian strategy is a solution of optimization problem.

$$\sigma_{\text{opt}} = \underset{\sigma \in S_a}{Sup} W_{\text{ed}}(U) = \underset{\sigma \in S_a}{Sup}\left(\min_{j \in \overline{1,M}} u_j\right). \tag{4.2}$$

Egalitarism leads to the leveling off individual utilities, but it does not deny inequality. Considerably more complex situation appears, if an individual set of alternatives S_{aj} doesn`t coincide $S_{aj} \neq S_a$ ($\forall j \in \overline{1,M}$), and also when we have as a goal to construct dynamic models.

In "the economy of welfare" the principle of unanimity is the leading one, according to which poor Paretto-solution have to be rejected unanimously.

Optimality on Paretto (1907) consists of the following: selection (solution) $\sigma \in S_a$ is Paretto - optimum if for $\forall \xi \in S_a$, when someone counts that $\xi \rangle \sigma$, then someone other counts that $\sigma \rangle \xi$. The Paretto - optimum solutions are called also effective.

The Paretto principle can come into conflict with the leveling of utility, and dilemma "equality- effectiveness" appears as its result.

It is shown (John Rolls), that the maximin procedure leads to the Paretto - optimum (In the weak sense) solution. Egalitarism does not worry originally about an increase in the welfare of the society (group) as a whole; this can be reached by mediations, through mechanisms of the redistribution of utilities.

Another principle - classical utilitarianism maximizes the sum of utilities, i.e., total benefit. A utilitarian function of the collective utility

$$W_* = \sum_{j=1}^{M} u_j, \tag{4.3}$$

and the selection of optimum version is achieved by a solution of the optimization task

$$\sigma_{\text{opt}} = \underset{\sigma \in S_a}{Sup} W_*\left(U\right) = \underset{\sigma \in S_a}{Sup}\left(\sum_{j=1}^{M} u_j\right). \tag{4.4}$$

In this form the principle will be coordinated with the principle of the unanimity: any vector $U = (u_1, u_2, \ldots, u_M)$ on S_a is optimal on Paretto.

Described optimization tasks are formed with the set of additional constraints and conditions, so that "naked" functional, expressed through the function of collective utility, is rarely used. In particular, members of the group can have a distinguished set of alternatives S_{aj}.

Vector U is *Paretto* - optimal, if from $V > U \Rightarrow V \,\bar{\in}\, S_a^M$, where the inequality $V > U$ means that $v_j \geq u_j$ and $V \neq U$. The vector U is weakly Paretto - optimal, if from $V >> U \Rightarrow V \,\bar{\in}\, S_a^M$, where the inequality $V >> U$ means that $v_j \geq u_j$ for $\forall j \in \overline{1,M}$, S_a^M —is Cartesian product

$$\underbrace{S_a \times S_a \times \ldots \times S_a}_{M}.$$

Classical utilitarianism does not limit the inequality of subjects in the group.

From our point of view *egalitarism* and *utilitarianism* are the economic projection of joined ethical principles, genetically joined in depths of human psyche: collectivism and individualism. From the point of view of our theory it would be desirable to convert this dichotomy at the working hypothesis (postulate).

It is not excluded, that inside the group the coalitions confessing different ethical principles can exist, or all subjects of group confess the certain compromise principle of justice. In the second case it would be conditionally possible to express this with of formula

$$W = \alpha W_{\text{ed}} + (1 - \alpha) W_* \ (0 \leq \alpha \leq 1)\ . \tag{4.5}$$

In the first case, assuming that in the set of subjects there are coalitions T_1 and T_2, such that $T_1 \cup T_2 = M$, it is possible to assume that the coalition T_1 solves egalitarian problem, while the coalition T_2 solves utilitarian one using assigned resources and technologies.

There are different schemes (algorithms) of resolution of the dispute between coalitions.

The function of collective utility CUF induces "*the order of collective welfare*" (SWO). Let`s note that utilities are expressed everywhere as objective characteristics (distance, cost, incomes,...). The presence of corporate purposes is considered (erection of bridge, hospital, the creation of army,...) in the process of aggregation. The requirement of congruency and transitivity are presented to individual utility. CUF is means for evaluation of the orders of a collective welfare SWO answers the certain ethical postulate.

Pigu - Dalton principle is one of the most frequently used ethical postulates, which consists of the following: *"the transfer" of utility from one subject i to another j increases (does not decreases) collective welfare, if*

$$u_i > u_j$$

before and after "transfer".

In other words transfer is accomplished from the subject with the greater utility to the subject with the smaller utility so that after the transfer the utility of someone giving would remain more than the utility of someone obtaining.

Let`s note that this principle can be refined due to conditions of using the entropy- information approach, where the "collective welfare" concept assumes somewhat different and more meaningful sense.

We see that during "revolutionary" expropriation the Pigou - Dalton principle as a rule is violated. If it was not disrupted, then the corresponding "revolution" would be, conditionally talking, *"valid" - by egalitarian revolution*. The concept of "egalitarian revolution": is such revolution, with which the Pigu - Dalton principle is not violated.

The work [64] gives several versions of optimization tasks, connected with indices of collective welfare. In particular, as most simple egalitarian circuit is "the dictatorship" of the poorest, and also utilitarian circuit is described. However, it is simultaneously noted that other less radical settings are possible.

Utilitarianism with the limited break of the utility levels of richest and poorest is an example. The corresponding task is a task on the conditional extremum:

$$\sigma_{\text{opt}} = \underset{\sigma \in S_a}{Sup} W_* \left(U \right) = \underset{\sigma \in S_a}{Sup} \left(\sum_{j=1}^{M} u_j \right). \tag{4.6}$$

with additional condition

$$\Delta_{\text{cryt}} \geq u_{\max} - u_{\min}, \tag{4.7}$$

where Δ_{cryt} is assigned break of extreme values of individual utility.

If $u_{\min} < \Delta_{\text{cryt}}$, it is possible to talk about the practical equality. Inequality (4.7) expresses ethical postulate and, of course, it is reflected in the effectiveness of active system.

Let`s assume that the functioning of an active system is combined with the possibility of catastrophic events, i.e., events, which put a question about existence of entire system or very serious worsen its state. Due to these conditions the system is forced to produce expenditures to assure the certain level of safety, which, in return, decreases individual utility. Taking as a basis, the previous optimization task, let`s add one more limitation:

$$P_{em} < 1 - P_s = \delta_s << 1,$$

where P_{em} is probability of extraordinary unfavorable event, P_s is probability of the safe functioning of system.

Another class of strategies is position strategies, oriented for the maximization of the rate of the growth of utilities. "Egalitarian" position CUF $V_e = \frac{dW_e}{dt}$:

$$\sigma_{\text{opt}} = \underset{\sigma \in S_a}{Sup} \frac{dW_e}{dt} = \underset{\sigma \in S_a}{Sup} \left(\frac{du_{\min}}{dt} \right) \tag{4.8}$$

when all remaining utility do not diminish:

$$\frac{du_k}{dt} \geq 0 \quad (k \in \overline{2, M}).$$

There are a lot of possible settings of variational a problems of the conditional extremum with the utility's CUF as the basic component of functional. For example, in the case of limited resources it is necessary to ensure a maximal exuberance in the average welfare (utilitarian position criterion) when the rate of growth in the welfare of the unhappiest subject will be not less than the given one: $\frac{du_{\min}}{dt} \geq v_{\min}$, and for all the rest $\frac{du_k}{dt} \geq 0$ ($k \in \overline{2, M}$). Here, as additional conditions, the necessary level of safety can be taken under consideration, some criteria having demographic nature and so forth.

A number of indices "of justice" can be formulated in the sense of the games theory, including antagonistic or non-antagonistic game with nature the probability of natural catastrophes calculation: earthquake, flood, tsunami, drought and so forth.

From these and similar sufficiently common formulations, the ethical requirements of second level follow to the point of status and behavior of individual (coalition) and their interrelation with other individuals (coalitions), association as a whole.

What is primary: individual or group ethical principles? Answer to this question lies in the concept of "*collective justice*", which is compatible with the formalization in the form of compromise criterion.

"*The rational selection of association must be as close as possible to the methods of selection of its separate members*". (Kondorse 1785). This ancient principle formulates a set of treatments and formalizations and, naturally, their leading role in this case (right after a basic criterion CUF), different additional conditions and limitations, in particular those, which are determined "by an openness" of an active system - by action of external factors.

Let`s continue the examination of different facts of the theory of collective decisions making. To functions of collective utility and the order of collective prosperity are subjected usually with certain requirements. Most frequently these requirements (axiom) are reduced to a separate, an anonymity, an unanimity, an independence from the common scale of utilities, an independence from the common utilities zero and others.

Let`s assume that for the group of M subjects the set of utility profiles $U(\sigma_i)$ on the set of alternatives S_a , and the order of collective welfare SWO that is relation $\rho \succsim$ on the set ε_u^M complete, reflexive and transitive are determined and let ($\rangle$) is a strict component while (~) is the relation of indifference:

$$U \rangle V \Leftrightarrow \left\{U\rho V; V\overline{\rho}U\right\}; \quad U \sim V \Leftrightarrow \left\{U\rho V; V\rho U\right\}.$$

Anonymity means that, if U and V are distinguished only by the transposition of subject's numbers, then they are identical with respect to the eligibility

$$U \sim V.$$

Thus, anonymity requires symmetry CUF with respect to the transposition of components $u_1, u_2, ..., u_m$.

Unanimity means that if U and $V \in \varepsilon_u^M$ are such, that $U \geq V$, ($U >> V$), then $W(U) \geq W(V)$, ($W(U) >> W(V)$), where W (.) - the monotonically increasing function.

Separability is determined as follows. Let there be the group, which consists of M participants (subjects). The order of collective welfare ρ, assigned on ε_u^M is separable, if for any subgroup T of group M and of any vectors of utilities U, V, U', V' the condition is fulfilled:

$$\{(u_i = u'_i, v_i = v'_i \text{ for } \forall i \in T) \text{ and } (u_j = v_j, u'_j = v'_j \text{ for } \forall j \in M\backslash T)\} \Rightarrow$$

$$\Rightarrow \{U\rho V \Leftrightarrow U'\rho V'\} \ (\rho \succsim).$$

Let`s explain this definition.

$$U = (u_1, u_2, ..., u_\tau, u_{\tau+1}, ..., u_M) = (U_T, U_{M\backslash T});$$
$$U' = (u'_1, u'_2, ..., u'_\tau, u'_{\tau+1}, ..., u'_M) = (U'_T, U'_{M\backslash T});$$
$$V = (v_1, v_2, ..., v_\tau, v_{\tau+1}, ..., v_M) = (V_T, V_{M\backslash T});$$
$$V' = (v'_1, v'_2, ..., v'_\tau, v'_{\tau+1}, ..., v'_M) = (V'_T, V'_{M\backslash T}).$$

Separability occurs, if

$$\left(U_T, U_{M\backslash T}\right) \succsim \left(V_T, U_{M\backslash T}\right) \Leftrightarrow \left(U'_T, V'_{M\backslash T}\right) \succsim \left(V'_T, V'_{M\backslash T}\right).$$

This means that the relation ρ is carried out independently on the distribution of utilities on the subgroup $M \setminus T$, i.e., the comparison of welfare does not depend on "not examined agents".

The order of collective welfare (SWO) ρ, assigned on ε_u^M, does not depend on the common zero of utilities, if for $\forall U$, $V \in \varepsilon_u^M$ and any $\rho \in R$ the condition is satisfied:

$$U\rho V \Leftrightarrow (U + \beta I)\rho(V + \beta I),$$

where $I = \underbrace{(1,1,\ldots,1)}_{M}$ - vector with the unit elements.

SWO ρ, assigned on the positive orthant ε_u^M does not depend on common scale, if for $\forall U$, $V \in \varepsilon_u^M$ and $\forall \alpha > 0$ the condition is satisfied:

$$U\rho V \Leftrightarrow (\alpha U)\rho(\alpha V).$$

SWO ρ does not depend on the common scale of utility, if the condition is satisfied for any monotonically increasing function $f(\cdot)$:

$$U\rho V \Leftrightarrow f(U)\rho f(V).$$

It is possible to show that egalitarian and utilitarian functions of collective utility do not depend on common zero and common scale. The egalitarian CUF does not depend on the common scale of utility, but the utilitarian CUF - on zero of each individual utility. Such CUF is the only function represented SWO, which satisfies the property indicated above. It is usually assumed that CUF satisfies properties of anonymity and unanimity. Egalitarian W_e and utilitarian W_* CUF are such CUF.

However, in the case of weighed utility aggregation, for example, for the function

$$W_{**} = \sum_{j=1}^{M} \xi_j u_j, \tag{4.9}$$

where ξ_j is weight coefficients, which assign "weight" to each subject, the axiom of anonymity is not carried out. This function is also inseparable.

Since in further theory all analogous functions will take the form of weighed "risks", a set of axioms of the usual theory of collective welfare will not be carried out. The summary given here presenting the basic axioms is necessary in order to have an idea about those victims, which we have to bring within the framework of the group preferences theory developed in this chapter.

It is proven that if ρ is continuous and separable order of collective welfare on ε_u^M, then it does not depend on the common zero utilities when and only when it is presented with one of following CUF:

$$\sum_{j=1}^{M} e^{\beta u_j} \ (\beta > 0); \quad -\sum_{j=1}^{M} e^{\beta u_j} \ (\beta < 0); \quad \sum_{j=1}^{M} u_j \tag{4.10a}$$

and it does not depend on the common scale of utility, when and only when it is presented with one of CUF

$$\sum_{j=1}^{M} u_j^{\gamma} \ (\gamma > 0); \quad -\sum_{j=1}^{M} u_j^{\gamma} \ (\gamma < 0); \quad \sum_{j=1}^{M} \log u_j. \tag{4.10b}$$

It is indicated that the function of collective utility *generates* the order of collective welfare (SWO) ρ, if the condition is satisfied:

$$U\rho V \Leftrightarrow W(U) \geq W(V)$$

and weakly generates the order of collective welfare (SWO) ρ, if the condition is satisfied

$$W(U) > W(V) \Rightarrow U\rho V \ \text{for} \ \forall U, V \in \varepsilon_u^M.$$

Theorem (Roberts). Let SWO ρ is such one, that its strict component P satisfies the condition: for any U, $V \in \varepsilon_u^M$, for which $U\rho V$, exist U' and V' any amount close to U and V, moreover $U' >> U$; $V' >> V$ and $U'\rho V'$, exist other vectors U'' and V'', for which $U'' << U$; $V'' << V$ and $V''\rho U''$, then ρ is weakly presented. with continuous function of collective welfare CUF .

It is that SWO ρ does not depend on common zero, if it satisfies one of the equivalent properties

a) for $\forall U, V, W \in \varepsilon_u^M$: $U\rho V \Leftrightarrow (U + W)\rho(V + W)$;

b) for $\forall U, V \in \varepsilon_u^M$: $U\rho V \Leftrightarrow (U - V)\rho$.

The following theorem bases the weak representation of SWO with utilitarian W^*.

Theorem (Atkinson, Gevers). Utilitarian CUF W^* does not depend on utilities zero. Conversely, independent on zero SWO. Let`s weakly present utilitarian CUF .

In particular, there is only independent variable from zero and continuous of SWO. It is represented by a utilitarian CUF W^*.

It is indicating that SWO ρ, determined on the positive orthant ε_+^M ($u_j > 0$ for $\forall j \in \overline{1, M}$), does not depend on scale, if one of the conditions is carried out:

a) for $\forall U, V, W \in \varepsilon^M$: $U\rho V \Leftrightarrow (U \bullet W)\rho(V \bullet W)$;

b) for $\forall U, V \in \varepsilon^M$: $U\rho V \Leftrightarrow (U:V)\rho e$;

where

$$U \cdot V \equiv (u_1 \cdot v_1, \ldots, u_M \cdot v_M); \quad U : V \equiv \left(\frac{u_1}{v_1}, \ldots, \frac{u_M}{v_M} \right);$$

$$e = (1, \ldots, 1).$$

Together with functions W_e and W_* the function of the collective welfare of Nesh is also examined:

$$W_N = u_1 u_2 \ldots u_M = \prod_{j=1}^{M} u_j. \tag{4.11}$$

The Nesh function satisfy condition that if at least one of u_j ($j \in \overline{1,M}$), tends to zero, then entire function W_N becomes equal zero.

The Nesh theorem asserts that the Nesh function W_N (radical) does not depend on scale. Conversely, if SWO is determined on ε_+^M and does not depend on scale, then it is weakly presented by Nesh CUF. In particular, there is only SWO on ε_+^M independent on the scale and continuous, represented by the Nesh function.

From the further analysis point of view an important problem of distributions aggregation of individual preferences at collective (group) preferences appears. Within the framework to the theory of ordinal distributions this task has considerable difficulties, and in certain, cases, has no solutions at all. There is a whole series of "*negative*" results. "*The theorem of Arrow about the impossibility*" is most known. The sense of this theorem lies in the fact that in the common case and under certain conditions it is not possible to coordinate reasonably the individual preferences and to obtain orderings of collective welfare.

The simplest version of this theorem is an assertion, that "it is not possible to obtain the transitive order of collective welfare on the basis of paired comparisons with the rule of majority".

In the approach of Arrow an insoluble contradiction between "*resoluteness*" and *anonymity appears*.

The ordering of collective welfare (collective preferences) is being achieved on the basis of the paired comparison of alternatives from *the S_a - binary* selection.

Previously requirements at the point of the procedure of voting inside the group are superimposed, which reflect certain ethical principles, namely: "voting" is accomplished *about the rule of majority*, and in this case must be *anonymous*, *neutral* and *monotonic*.

Let`s say immediately that these requirements are not carried out from our point of view of the overwhelming majority of active systems with the group subject. Nevertheless they can be the starting point during the study of other scheme and other principles, particularly those which are developed in this work, on the basis of entropy approach.

Let`s note that functions of collective utility examined above are, commonly speaking, a particular case of weighed function (4.9)

From this function we will obtain egalitarian CUF, if we will place entire $\xi_K = 0$ with exception of one ξ_K — coefficient with the smallest u_K and utilitarian CUF, if we assume $\xi_K = 1$ for $\forall k \in \overline{1,M}$. If we somewhat communize Nesh CUF (radical) and to examine functions

$$W'_N = \prod_{j=1}^{M} u_j^{\xi_j}, \tag{4.12}$$

that, after taking the logarithm, we`ll find CUF of the form

$$\ln W'_N = W'_{**} = \sum_{j=1}^{W} \xi_j \ln u_j. \tag{4.13}$$

Now it suffices to accept $u'_j = \ln u_j$ (since ln (.) is monotonic function) as a new utility. We will obtain

$$W'_{**} = \sum_{j=1}^{W} \xi_j u'_j. \tag{4.14}$$

Let`s examine, following to mentioned monograph Mullen [114], the procedure of binary selection on the set S_a.

Let σ and $\eta \in S_a$. It is assumed that each subject realizes strict preferences. Indifferences in the stage of individual selection are not admitted. Let $(\sigma, \eta)^M$ are the Cartesian products of two-element sets, and the profile of the group preferences be vector $U = (u_1, u_2, \ldots, u_M)$ with values from $(\sigma, \eta)^M$, i.e., $u_j = \sigma$, or $u_j = \eta$. The rule of voting G places in the correspondence to each profile U the non-empty subset from $(\sigma, \eta)^M$. $G(U) = (\eta)$ indicates selection η, $G(U) = (\sigma)$ - selection σ, $G(U) = \{\sigma, \eta\}$ - equivalence (indifference) or draw as a result "of voting".

The rule of selection G (.) *is anonymous*, if it "*realizes the symmetrical mapping, which depends on M variables*".

The rule of selection G (.) *is neutral*, if the transposition of preferences on $(\sigma, \eta)^M$ of each subject leads to the transposition of collective preference. If A is vector of the transposition of preferences on $(\sigma, \eta)^M$, then

$$G\left(A(u_1), A(u_2), \ldots, A(u_M)\right) = AG\left(u_1, u_2, \ldots, u_M\right).$$

Rule of selection G (.) is *monotonic*, if the new supporter of the certain selection does not bring harm. If U and V are two profile such, that $u_j = v_j$ ($j \neq i$), $u_i = a$, $v_i = b$, then the following relations is valid:

$$\{\eta \in G(U) \Rightarrow \eta \in G(V)\};$$

$$\{\sigma \in G(V) \Rightarrow \sigma \in G(U)\}.$$

The consequence *of monotonic* is *strategy proof rotting-role*: the subject has no motives to report false opinion.

Arrow`s theorem about independence from irrelevant alternatives is expressed as follows.

Let S_a a set of alternatives consist, at least, of three alternatives and ρ - ordering of collective welfare, which satisfies the condition of unanimity: for all profiles U and alternatives σ, η

$$\{M(U, \sigma, \eta) = M\} \Rightarrow \sigma\rho\eta.$$

Then ρ satisfies the axiom of independence from irrelevant alternatives (NIIA (э)) when and only when it is dictatorial. The latter means that there is such $j \in \overline{1,M}$ - "dictator" (subject, who has number j) for whom $\rho(U) = u_j$ for $\forall U$.

Here $M(U,\sigma,\eta) = \left\{ j \in \overline{1,M} \mid u_j(\sigma) > u_j(\eta) \right\}$.

Preordering ρ for set S_a and M satisfies the Arrow axiom of independence from irrelevant alternatives, if for $\forall\sigma, \eta \in S_a$ and $U, V \in L(S_a)$

$$M(U,\sigma,\eta) = M(V,\sigma,\eta) \Rightarrow \left\{ \sigma\rho(U)\eta \Leftrightarrow \sigma\rho(V)\eta \right\}.$$

The problem of non-aggregating, connected with of assumptions described above (axioms), in particular anonymity, can be admitted, if certain ratings are assigned to subjects, for example, as a result of different scope of subjects. Nitzan and Paroush (1982), Sherm and Grofman (1984) the estimation (preference) of subjects considered with a weight log $[p_j(1 - p_j)^{-1}]$, where p_j is probability of correct judgment of an expert with the number j. According to Kondorse there is an objective ranking of alternatives in accordance with "the most probable combination of opinions". The corresponding criterion takes the form of the maximum likelihood criterion.

4.3. Group of interacting subjects. Preference functions of the 2[nd] kind.

The object of study is the group of M subjects, between whom there is a definite dependence (link) and interaction. Each subject at the given moment has an individual set of alternatives $S_{aj}(j \in \overline{1,M})$, that contains N_j alternative $\sigma_k(k \in \overline{1,N_j})$. Commonly speaking, the alternatives of subject should be noted with two indices σ_k^j. We will consider explicitly this special feature, where this will be necessary. As already mentioned σ_k are either desirable terminal states or strategies of the problem- resource situation solution.

Among the group consist factors it would be possible to name the following:

— common (corporate) problems;

— influence of the ratings (preferences of the II kind) on the distribution of resources;

— the interdependence of utility;

— the presence of common ethical imperatives;

— the influence of events "of the past" (In the retrospection) - its kind "the load of the past" - past habits, connections of incomplete problematic situations and so forth;

— the straight reciprocal effect of preferences.

If an individual set of alternatives doesn`t intersect: $S_{aj} \cap S_{ai} = \varnothing$ for $\forall i, j \in \overline{1,M}$, those subjects are not problematically connected and the dependence (a group creating factor) is available in the region of distributing resources, interdependence of utility. If there is a non-empty intersection of individual set of alternatives, then subjects in the group are problematically connected. Forms of problematic link can be different. Let their only twin "linear" link (Fig. 4.1) occur.

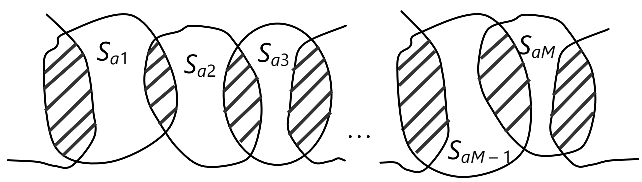


Fig. 4.1

Then each set has non-empty intersection not more than other two sets. In this case triple intersections and intersections of the greater number of set don`t occur. However, the whole set S_{aj} participates in the connectedness, if from

$$\left.\begin{aligned} S_{ai} \cap S_{aj} \neq \varnothing \\ S_{ai} \cap S_{ak} \neq \varnothing \end{aligned}\right\} \Rightarrow S_{aj} \cap S_{ak} = \varnothing,$$

which means that there are no cycles of the link: situation shown in Fig.4.2.

If there are sets of the type S_{ai}, then we have a situation, when each set of alternatives can have common paired intersection with other three sets.

We will indicate that in the group G there are corporate problems, if they are dual, triple, fourfold,..., *m-fold* intersections (Fig4.3) of an individual set of alternatives.

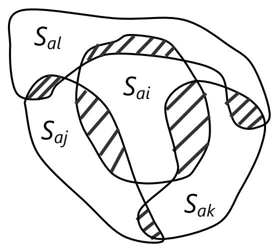


Fig.4.2

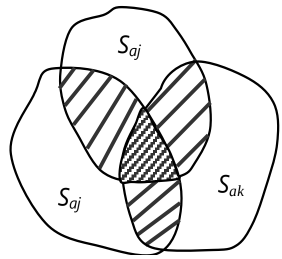


Fig.4.3

Commonly speaking, versions of possible mutual arrangement of individual sets there are set. Moreover, if set of alternatives is countable, then set of versions have power of continuum.

Each of subjects in the group at each moment (or "section") of time "has" the individual preferences distribution to S_{aj}: $\pi_j(\sigma_i)$ ($j \in \overline{1,M}$). Thus, for each group of subjects (if any link between subjects inside the group occurs) the presence of the collection of individual distributions of preference $\pi_j(\sigma_i)$ is postulated. These distributions are formed under the action of "external" circumstances and "internal" reciprocal effects between individual subjects of the group. Kinds and forms of these factors action will be examined subsequently.

Analogically, with *internal* and *external* forces, which act on the elements of material system would not be complete and adequate, since, the total result of internal interactions does not bring in the active system to the zero result (clear example - football team); there is also a certain reciprocal effect *of external* and internal factors.

Using a concept of an individual set S_{aj} it is possible to isolate certain types of groups from the point of view of link subjects nature.

The negative result of Kenneth Arrow about the impossibility of aggregation of individual ordinal preferences is intended.

One of possibilities of the link determination consists of the following: let there are two subjects and their sets of alternatives S_{a1}, S_{a2}. Let $S_a = S_{a1} \cap S_{a2}$. Then, if in S_a there is at least one alternative $\sigma_k \in S_a$ such, that preferences of both subjects i

and j – $\pi_i(\sigma_k)$ and $\pi_j(\sigma_k)$ are not equal zero, we will consider that the subjective link occurs. Pearson correlation coefficient, could serve the measure of link.

An individual function of preferences of the first kind $\pi_j(\sigma_i)$: $\sigma_i \in S_{aj}$ as this was described above, is determined by the individual preferences distribution on individual set of alternatives S_{aj}. It could be examine as measure to S_{aj}, if following conditions are satisfied:

1. S_{aj} is a semicircle of subsets $\Omega \subset S_{aj}$.
2. $\pi_j(\Omega)$ is real and

$$\pi_j(\Omega) \geq 0 \text{ for } \forall\Omega \subset S_{aj}.$$

3. $\pi_j(\Omega)$ is additive - for any finite expansion $\Omega=\Omega_1\cup\Omega_2\cup...\cup \Omega_n$ as disjoint sets ($\Omega_r \cap \Omega_s = \varnothing;\ \forall r,s \in \overline{1,n}$) the equality occurs

$$\pi_j(\Omega) = \sum_{i=1}^{n} \pi_j(\Omega_i).$$

Hence it follows in particular, that $\pi_j(\varnothing) = 0$. The normalization condition is satisfied:

$$\pi_j(S_a) = 1.$$

Fulfilling the same requirements is necessary during the construction of the aggregated functions of preference.

The need and the expedience of introduction of the new function of preference, which reflects the rating of each individual subject "at the background" of the entire group, is an essential difference of the group of subjects analysis:

$$\xi(j) = \xi_j \ (j \in \overline{1,M}). \tag{4.15}$$

We will count this function of that normalized:

$$\sum_{j=1}^{M} \xi_j = 1 \tag{4.16}$$

and let`s name the function of positive preferences of the second kind (II kind) or absolute integral positive ratings.

A rating function is an integral characteristic of subject, as a participant in the group and it „absorbs" as itself set sides of his manifestations - its activity. This is actually a certain integral measure, which evaluates all "graphs" of individual characteristic in total. In connection with this a natural desire to introduce more detailed "measure" appears. For example, vector functions of the preference $\vec{\xi}_j$ (vector rating). This desire is completely natural, but in this stage one must be dismantled with an integral rating ξ_j.

Together with the absolute positive ratings determine the function of conditional rating $\xi(j|\,i) = \xi_{j|\,i}$. *This rating assigned for all participants group, including* i-th *member of the group itself. A normalization condition for this distribution exists has form:*

$$\sum_{j=1}^{M} \xi_{j|i} = 1 \ \ (\forall i \in \overline{1,M}). \tag{4.17}$$

Even if a subject "does not follow" the fulfillment of normalization and estimations of ratings are accomplished in an arbitrary scale, and then there is always a possibility of normalized assigned to the given number (for example, to 1).

The rating of subject can be differentiated with respect to alternatives $\sigma_k \in S_{aj}$ (from its individual set of alternatives). In this case functions $\xi_j(\sigma_k)$, $\xi_{j|\,i}(\sigma_k)$, ... are examined. This means that the subject in a group is evaluated differently, depending on examined problem P: $(\sigma_0 \to \sigma_k)$. Strictly speaking, in this case one should consider its initial "state" σ_0. The case, when alternative set of all subjects coincide, is the simplest:

$$\bigcap_j S_{aj} = \bigcup S_{aj} = S_a,$$

or starts the circuit of analysis, when an united set of alternatives is introduced $S_a = \bigcup_{j=1}^{M} S_{aj}$, and individual distributions of preferences of the I kind are defined on everything S_a by zero values.

With respect to the distribution it is rating so, as this was done for a set of alternatives S_a, it is possible to introduce ordinal relations of rating preference ρ_ξ:$\langle$ or $\rho_\xi : \underset{\sim}{\langle}$. The relation of strictly rating preference ρ_ξ *is transitive*, *asymmetric* and *continuous*. The relation of rating equivalence $\rho_\sim$: ~ *is transitive*, *reflexive* and *symmetrical*. If we use for the subject in the group a designation Σ_j, then the relation ρ_ξ:$\langle$ generates the order:

$$\Sigma_1 \langle \Sigma_2 \langle \Sigma_3 \langle \ldots \langle \Sigma_{M-1} \langle \Sigma_M,$$

and the relation $\rho_\xi : \lesssim$ - the order:

$$\Sigma_1 \langle \Sigma_2 \langle \ldots \langle \Sigma_k \sim \Sigma_{k+1} \sim \ldots \sim \Sigma_{k+l} \langle \ldots \langle \Sigma_{M-1} \langle \Sigma_M.$$

The zone of equivalence from k to $k + l$ assumes identical ratings of subjects equal to rating Σ_k. Zones of equivalence there can be somewhat finite groups, whose subjects having identical ratings belong to one group of equivalence or class of equivalence (in accordance with the terminology of chapter 1).

The group of subjects can be *weakly-ordered* (see chapt.1.3), if relation ρ_ξ: $\Sigma_i \rho_\xi \Sigma_j$ $(\Sigma_i, \Sigma_j \in M)$ is asymmetric and negatively transitive.

The group of subjects *is strictly-ordered*, if additionally to asymmetry and negative transitivity for the relation ρ_ξ a weak link occurs.

A group *is strictly partially ordered* if the relation ρ_ξ is non-reflective and transitive. As we can see, there is a certain typology of groups of subjects depending on what relation ρ_ξ is fulfilled. It would be possible to talk about the function of utility of subject in the group by analogy to the utility of alternatives. However, we prefer to talk about the function of preferences of the II kind or about the rating function, which, as let`s see further it is possible to determine mutual utility's dependence.

As an example let`s examine two special cases: ρ_ξ — the relation of competition and ρ_ξ — the relation of antagonism.

The relation "salesman- buyer" - this relation is antagonistic: the first one wants to sell expensive, the second - to buy chipper. At the same time the relation between two salesmen, who want to sell goods to one and the same buyer there is a relation of competition between sellers.

Let $\rho_\xi = \rho_c$ — the relation of competition and *A*, *B*, *C*,... - competitors. Relation ρ_c

— is no reflexive $A\bar{\rho}_c A$ ("I not competitor to itself");

— is symmetrical $A\rho_c B \Leftrightarrow B\rho_c A$ (if *A* —competitor *B* , then *B* — competitor *A*);

— is transitive $\left(A\rho_c B; B\rho_c C\right) \Rightarrow A\rho_c C$ (if *A* competitor *B* , and *B* — competitor *C*, then *A* - competitor *C*).

The relation of *antagonism* $\rho_\xi = \rho_a$ can be counted

— nonreflexive $A\bar{\rho}_a A$;

— asymmetric $A\rho_a B \Rightarrow B\bar{\rho}_a A$;

— negative - transitive $\left(A\bar{\rho}_a B; A\rho_a C\right) \Rightarrow D\rho_a C$.

One of possible determinations of an antagonism: "The enemy of our enemy is our enemy" or "who`s not with us, is against us".

Antagonisms are sometimes useful from the points of view of both subjects; competitors are always harmful to each other. The solution of contradiction between competitors it is not possible to name "revolution", the solution of contradiction between the antagonists sometimes takes the form "of revolution". Relation between employer and worker is antagonism.

There can be a basic problem in the context of the question. It is a question about by whom - "carrier" (and user) of ratings, who operates with ratings of individual subjects and decisions are made. In other words, who has certain imperious authorities, sufficient in order to make decisions on the basis of information about the integral ratings?

As we can see here the allocation problem *of imperious authorities* naturally appears inside the group. What does it follow to understand under the imperious authorities, and also turn what does the task about aggregation of preferences of the second kind consist? In the case of ordinal rating this task encounters, as it was already said with the difficulties of the Arrow theorem type "about the impossibility". The use of continuous rating distributions, in the first, removes the substantial part of similar difficulties, and secondly gives the possibility to postulate of certain variation principles, assigned to human psyche.

Just as in the case of object preferences (preferences of the first kind $\pi(\sigma_i)$) together with "positive" ratings, which can be designated ξ_j^+ it is possible to assume forming "of negative" ratings ξ_j^-, when based on two (or several) "ashes" the smallest one is selected. There are such democracies worldwide, when each citizen during elections the absolute democratic right from two-three bandits to select best. It is possible to visualize the sequential procedures of the evaluation of members of the group, when distributions ξ_j^+ and ξ_j^- are obtained as a result of following stages of evaluation. If t - moments of time, then the corresponding chain can appear as follows

$$\xi_j^+(t) \to \xi_j^-(t + 1) \to \xi_j^+(t + 2) \to \ldots$$

In this case results of evaluation on the previous stage are considered during the production of ratings on the following stage.

4.4. Individual ratings. Functions of group effectiveness

A study of functions of the preference of the second kind or rating preferences assumes several initial conditions. It is necessary to establish the connection between individual "ratings" ξ (j| i), carriers of which are subjects i, which assign ratings to remaining members of group ($j \in \overline{1,M}$), and also to indicato methods of aggregation of individual distributions (conditional ratings ξ (j| i)), for the purpose of the development of integral ratings ξ (j) and their "carriers" ratings as well. It is completely obvious that there is a connection between preferences of the first kind ($\pi(\sigma_k)$, ...) and preferences of the second kind (ξ (j| i),...). What is connection of preferences of the second kind with utilities, functions of effectiveness, with resources? This one and following paragraphs are dedicated to the study of these problems. It is not possible to propose universal rating distributions for "all life cases". The form of distributions ratings in the group. It depends on what "is a set a question" for thous, who assigns ratings. This assertion is important, since it offers the possibility to examine the wide variety of rating preferences, substantially enlarges the field of studies and the field of application. "It is half of knowledge to ask right question". The form of canonical distribution in essence is determined by the form of the effectiveness function, which in this case we will call the group rating function of effectiveness and designate by a symbol ε_ξ, or simpler, by "a group rating effectiveness". Talking about the group effectiveness, we, following the concept of the required "carrier accepted", must indicate, who certain ally interests "a group effectiveness" - only " leader" or all participating in the solution of a corporate problem.

Let`s recall that with object preferences forming functions "utility" U and function "of harmfulness" L were used. They, by the agreement, are expressed as objective characteristics: all forms of resources, "speed" of conversion or translation of resources… We will talk here about utility only, taking under consideration that the form of canonical distributions, which depend on harmfulness differs little from the form of distributions, which depend on utility. Other factors, which are reflected in canonical distributions, include a priori imperatives, mutual influence of subjects, influence of the previous distributions.

Regarding functions of effectiveness that reflects an influence of objective factors on subjective preferences.

If with forming of preferences of the first kind $\pi(\sigma_i)$ we talk about *utility* $U(\sigma_i)$ of one or other alternative or another, to be more precise, about the solution of the problem, connected with realization of an alternative σ_i , then in the case the forming of rating preference ξ (j), we speak about the utility of this subject (for another subject, for the group solwing the corporate problem and forth).

Let`s examine diverse variants of the function of group effectiveness.

1. $$\varepsilon_\xi\left(S_a\right)=\sum_{j=1}^{M}\xi^+\left(j\right)U_j\left(S_a\right) \tag{4.18}$$

- integral, utility weighed on absolute ratings ξ^+ (j) referred to entire set S_a .Here $U_j(S_a)$ the utility of subject "j" with respect to entire set of alternatives S_a. Since as "the purpose" will be selected a certain alternative σ_k or the subset $S'_a \subset S_a$, characteristic $U_j(S_a)$ is too rough.

2. $$\varepsilon_\xi\left(S'_a\right)=\sum_{j=1}^{M}\xi^+\left(j\right)U_j\left(S'_a\subset S_a\right) \tag{4.19}$$

- the integral utility weighed on the absolute ratings of subject "j", referred to the subset $S'_a \subset S_a$.

3. $$\varepsilon_\xi\left(\sigma_k\right)=\sum_{j=1}^{M}\xi^+\left(j\right)U_j\left(\sigma_k\right) \tag{4.20}$$

- the weighed on the absolute ratings differential utility of subject "j", referred to the alternative σ_k.

When functions of effectiveness are expressed as integral absolute ratings ξ (j), it is assumed that as a "carrier" this rating appears either certain hierarch (manager), which stands above the group, or virtual subject which could be called "collective reason", or that recently was implied by "information essence".

From the point of view of subjective analysis "collective reason" can be described as this psychological state of the group, when there are effective mechanisms leveling both the objective ranks and rating preferences of members of the group.

In connection with an object of preferences the additional condition of existence "collective reason" is the presence of the subset $S'_a \subset S_a$, on which preferences of different members of the group practically coincide and, furthermore, S'_a contains the most preferable alternatives. If we allow the presence of "collective reason", then the measure "ideas identity" is possible to consider the entropy of the form

$$H_{\pi}^{\Sigma}(\sigma_k) = -\sum_{j=1}^{M} \overline{\pi}_j(\sigma_k) \ln \overline{\pi}_j(\sigma_k), \tag{4.21}$$

which characterizes the degree "ideas identity" or also the degree "divergence in views".

Here

$$\overline{\pi}_j(\sigma_k) = \left(\overline{\pi}(\sigma_k)\right)^{-1} \pi_j(\sigma_k),$$

where $\overline{\pi}(\sigma_k) = \sum_{j=1}^{M} \pi_j(\sigma_k)$. The normalized on the group preferences of the I kind can be introduced for the subset $S'_a \subset S_a$ of alternatives:

$$\overline{\pi}_j(S'_a) = \left(\sum_{j=1}^{M} \pi_j(S'_a)\right)^{-1} \pi_j(S'_a),$$

where $\pi_j(S'_a) = \sum_{\sigma_k \in S_a} \pi_j(\sigma_k)$ and the corresponding entropy:

$$H_{\pi}^{\Sigma}(S'_a) = -\sum_{j=1}^{M} \overline{\pi}_j(S'_a) \ln \overline{\pi}_j(S'_a). \tag{4.22}$$

Supposing "by the carrier" of these entropies ($H_{\pi}^{\Sigma}(\sigma_k)$ and $H_{\pi}^{\Sigma}(S'_a)$) appears a virtual subject – a carrier of "collective reason". It is possible to talk about a sufficient unanimity, if $\pi_j(\sigma_k) \to \frac{1}{M}$, (or $\pi_j(S'_a) \to \frac{1}{M}$).

It is possible to visualize that the contribution of each member of the group to the forming "means" of collective reason is determined not only by the value of his preference, but also by "his own certain weight" in the group, i.e., by his rating. Let`s designate the given preference

$$\pi_j^{\xi}\left(\sigma_k\right) = \xi\left(j\right)\pi_j\left(\sigma_k\right).$$

These values are not normalized, moreover since for $\forall j \in \overline{1,M}$; ξ (j) $\leq$ 1, then $\pi_j^{\xi}\left(\sigma_k\right) \leq \pi_j\left(\sigma_k\right)$.

Let`s introduce values normalized on the set S_{ξ}

$$\overline{\pi}_j^{\xi}\left(\sigma_k\right) = \frac{\pi_j^{\xi}\left(\sigma_k\right)}{\overline{\pi}^{\Sigma}\left(\sigma_k\right)},$$

where $\overline{\pi}^{\Sigma}\left(\sigma_k\right) = \sum_{j=1}^{M}\pi_j^{\xi}\left(\sigma_k\right) = \sum_{j=1}^{M}\xi\left(j\right)\pi_j\left(\sigma_k\right)$, then the entropy “collective reason”

$$H_{\pi}^{\xi}\left(\sigma_k\right) = -\sum_{j=1}^{M}\overline{\pi}_j^{\xi}\left(\sigma_k\right)\ln\overline{\pi}_j^{\xi}\left(\sigma_k\right).$$

The proximity is also the sign of identity of ideas in this case $H_{\pi}^{\xi}\left(\sigma_k\right)$ to 1. analogously it is possible to examine the entropy, in reference to the fragment S'_a of set S_a.

Functions of the group effectiveness take more particular form, if they are expressed, directly through resources, for example available

4. $$\varepsilon_{\xi}\left(S_a\right) = \sum_{j=1}^{M}\xi^{+}\left(j\right)R_j^{disp}. \qquad (4.23)$$

This function is the sum of individual available resources, weighed on the absolute preferences. In this case it is counted, that all R_j^{disp} are universal.

If available resources by any means are specialized, we will use functions of effectiveness:

5. $$\varepsilon_a\left(S'_a\right) = \sum_{S'_a}\sum_{j=1}^{M}\xi^{+}\left(j\right)R_j^{disp}\left(S'_a\right);\;\; R_j^{disp} = \sum_{S'_a \subset S_a}R_j^{disp}\left(S'_a\right) \qquad (4.24)$$

It is function of effectiveness of the sum of individual available resources on the set $S'_a \subset S_a$, weighed on absolute ratings. Summing is accomplished on all those being not intersecting $S'_a \subset S_a$.

6. $$\varepsilon_{\xi}\left(\sigma_k\right) = \sum_{k=1}^{N}\sum_{j=1}^{M}\xi^{+}\left(j\right)R_j^{disp}\left(\sigma_k\right);\;\; R_j^{disp} = \sum_{S'_a \subset S_a}^{N}R_j^{disp}\left(\sigma_k\right) \qquad (4.25)$$

It is function of effectiveness of the sum of individual available resources, intended for the alternative σ_k (specialized for the separate alternative), and weighed on the absolute ratings.

We already noted that integral absolute ratings ξ (*j*), carrier of which is virtual either real hierarchy , are assigned these by hierarch on the basis of common and it is not compulsory differentiated on the elements S_a and according to the subjects from S_ξ the considerations or they are the result of aggregation of differential it is rating.

If "the carrier" of function of effectiveness is subject $\Sigma_i \in S_\xi$, then we need to use conditional ratings ξ^+ (*j*| *i*), which are the rating of the subject "*j*" "In the eyes" of subject "*i*" ("Σ_j by eyes Σ_i").

Functions of effectiveness (chapt.1-6) given above can be "given" to the subject "*i*".

7. $$\varepsilon_{\xi_i} = \sum_{\substack{k=1 \\ S'_a}}^{N} \sum_{j=1}^{M} \xi^+ \left(j \mid i \right) R_j^{disp} \begin{pmatrix} S_a \\ S'_a \subset S_a \\ \sigma_k \end{pmatrix} \tag{4.26}$$

This function is the sum of individual available resources of those specialized, weighed on conditional ratings ξ (*j*| *i*).

8. $$\varepsilon_{\xi_i} = \sum_{\substack{k=1 \\ S'_a}}^{N} \sum_{j=1}^{M} \xi^+ \left(j \mid i \right) U_j \begin{pmatrix} S_a \\ S'_a \subset S_a \\ \sigma_k \end{pmatrix}. \tag{4.27}$$

As in the case 7 this is function of individual of subjects from S_ξ, weighed on conditional ratings ξ^+ (*j*| *i*) ("from the point of view of *i*").

In cases 5, 7, 8 the first sum $\sum\limits_{\substack{k=1 \\ S'_a}}^{N}$ indicates summation over $S'_a \subset S_a$, or on $k \in \overline{1,N}$, when available resources are not universal.

In expressions given above, as already mentioned, the utility *U* makes another sense in comparison with utility, used in object preferences (preferences of the I kind).

If in preferences of the I kind the utility of resources directed is intended for the solution of one problem or another, then in rating preferences (preferences of the II kind) the utility of the member of group is intended (subject). As it seems to us, one of the vital differences in the concepts of "utility " in the first and second cases consists of it.

Further detailing is connected with deeper differentiation of rating preferences. It is easy to see that one and the same subject (member of the group) can be preferable to different degree from the point of view of possibility and ability to solve different problems (being a good specialist in physics, it can prove to be a poor manager, a good athlete in one kind of sport, will not reach positive results In other area and so forth). Therefore examination together with functions ξ (*j*) and ξ (*j*| *i*), which we call integral, preferences $\xi^+(j|\ i,\ S'_a \subset S_a)$ specialized on subsets of alternatives is completely natural. In connection with these reasoning we have foundations for introducing ratings, differentiated on alternatives $\sigma_k \subset S_a$ or on groups of alternatives $S'_a \subset S_a$

$$9. \qquad \varepsilon_\xi\left(\sigma_k\right) = \sum_{j=1}^{M} \xi^+\left(j \mid \begin{Bmatrix} S'_a \subset S_a \\ \sigma_k \end{Bmatrix}\right) R_j^{disp}\begin{pmatrix} S'_a \subset S_a \\ \sigma_k \end{pmatrix}. \tag{4.28}$$

$$10.\ \varepsilon_{\xi_i}\left(S'_a\right) = \sum_{j=1}^{M} \xi^+\left(j \mid i, \begin{Bmatrix} S'_a \subset S_a \\ \sigma_k \end{Bmatrix}\right) R_j^{disp}\left(\begin{Bmatrix} S'_a \subset S_a \\ \sigma_k \end{Bmatrix}\right). \tag{4.29}$$

Curly braces indicate two possibilities: $S'_a \subset S_a$ or $\sigma_k \in S_a$.

If an allocation problem appears, it is rating in the group with respect to expenditures of resources by different subjects for the realization of one and the same alternative σ_k or the subgroup of alternatives, then the required resources $R_j^{\text{req}}(\sigma_k)$ come out as the objective characteristic. The function of effectiveness in this case reflects negative properties of subjects. (for example, if service lives - this is required time, then that subject who on the realization of alternative will spend shorter time, must be of higher rank).

$$11.\ \varepsilon_\xi = -\sum_{k=1}^{N}\sum_{j=1}^{M} \xi^-\left(j\right) R_j^{req}\left(\sigma_k\right). \tag{4.30}$$

$$12.\ \varepsilon_\xi = -\sum_{k=1}^{N}\sum_{j=1}^{M} \xi^-\left(j \mid i\right) R_j^{req}\left(\sigma_k\right). \tag{4.31}$$

$$13.\ \varepsilon_{\xi_i}\left(\sigma_k\right) = -\sum_{j=1}^{M} \xi^-\left(j \mid i, \sigma_k\right) R_j^{req}\left(\sigma_k\right). \tag{4.32}$$

$$14.\ \varepsilon_{\xi_i}\left(\sigma_k\right) = \sum_{j=1}^{M} \xi^-\left(j \mid i, \sigma_k\right)\left[\alpha \ln R_j^{req}\left(\sigma_k\right) - \beta R_j^{req}\left(\sigma_k\right)\right]. \tag{4.33}$$

In the last formula parameters α and β are structural parameters of psyche, stable, or changing with the time (with the age, under the effect of external circumstances,...).

In a number of cases instead of absolute resources it is possible to use relative resources, for example

$$\overline{r}_j^{\,r}\left(\sigma_k\right) = \frac{R_j^{req}\left(\sigma_k\right)}{R_j^{disp}\left(\sigma_k\right)};$$

$$\bar{r}_j^e\left(\sigma_k\right)=\frac{R_j^{\exp}\left(\sigma_k\right)}{R_j^{req}\left(\sigma_k\right)};$$

Relative given resources

$$\bar{r}_j^d\left(\sigma_k\right)=\frac{R_j^{req}\left(\sigma_k\right)}{R_j^{disp}\left(\sigma_k\right)-R_j^{req}\left(\sigma_k\right)};\;\; R_j^{req}\left(\sigma_k\right)<R_j^{disp}\left(\sigma_k\right);$$

$$\bar{r}_j^e\left(\sigma_k\right)=\frac{R_j^{\exp}\left(\sigma_k\right)-R_j^{req}\left(\sigma_k\right)}{R_j^{req}\left(\sigma_k\right)}.$$

In formulas recorded above $R_j^{\exp}\left(\sigma_k\right)$ is the value of expected, newly formed resources as a result of the solution of problem P: $\sigma_0 \to \sigma_k$. In the latter case $\bar{r}_j^e\left(\sigma_k\right)$ is "exceeding of incomes above expenditures" referred to expenditures. Following functions are an example of functions of effectiveness, expressed through relative expenditures:

$$15.\;\; \varepsilon_\xi\left(\sigma_k\right)=\sum_{j=1}^{M}\xi\left(j\mid\sigma_k\right)\bar{r}_j^r\left(\sigma_k\right). \tag{4.34}$$

$$16.\;\; \varepsilon_\xi\left(\sigma_k\right)=\sum_{j=1}^{M}\xi\left(j\mid\sigma_k\right)\bar{r}_j^e\left(\sigma_k\right). \tag{4.35}$$

As it has already been said, in the certain situation the role "of resources" can play probability $p_j(\sigma_k)$ of a certain event, connected with an alternative σ_k, for example, the probability of the solution of problem P by subject "j": $\sigma_0 \to \sigma_k$ with assigned available resources and perhaps the random required resources, in reference to the subject "j".

Let`s write the function of effectiveness in this case in the form:

$$17.\;\; \varepsilon_\xi\left(\sigma_k\right)=\sum_{j=1}^{M}\xi\left(j\mid\sigma_k\right)p_j\left(\sigma_k\right). \tag{4.36}$$

Here we deal with respect to the mixed problem of subjective- probabilistic analysis. Since $0 \le p_i \le 1$, it is expedient to introduce as the examination the given probability

$$\bar{p}_j=\frac{p_j}{1-p_j};\;\; 0\le\bar{p}_j<+\infty,$$

(point $p_j = 1$ "is cut"). Let`s represent the function of effectiveness in the form:

$$18.\;\; \varepsilon_\xi\left(\sigma_k\right)=\sum_{j=1}^{M}\xi^+\left(j\mid\sigma_k\right)\bar{p}_j\left(\sigma_k\right)=\sum_{j=1}^{M}\xi^+\left(j\mid\sigma_k\right)\frac{p_j\left(\sigma_k\right)}{1-p_j\left(\sigma_k\right)}. \tag{4.37}$$

The different way of the introduction of chance on scheme of subjective analysis lies in the fact that endogenous and exogenous characteristics are considered random variables or random functions. Then connected preferences prove to be random. The study of stochastic properties of preferences and statistical methods of their estimations obtaining by conducting psychological tests is separate problem and is not the object of this work. We will only concise touch some questions from this region.

If instead of function (4.36) we use the function of effectiveness

$$19.\ \varepsilon_{\xi}(\sigma_k)=\sum_{j=1}^{M}\xi^{+}(j\,|\,\sigma_k)\ln p_j(\sigma_k), \tag{4.38}$$

more tolerant with respect to $p_j(\sigma_k)$, the corresponding canonical distribution will be presented by the formula

$$\xi^{+}(j\,|\,\sigma_k)=C_j p_j(\sigma_k), \tag{4.39}$$

where C_j is the normalizing constant

$$C_j=\frac{1}{\sum_{q=1}^{N}p_j(\sigma_k)}=1,$$

since $\sum_{q=1}^{N}p_j(\sigma_k)=1$ for $\forall j\in\overline{1,M}$. In such a manner, as we can see in this special case rating $\xi(j|\ \sigma_k)$ simply coincides with probability $p_j(\sigma_k)$. But if this is so, than the function of effectiveness (4.38) is equal to the entropy H_p and is equal to the subjective entropy $H_{\xi,\,j}$. The same occurs, if we determining canonical distribution of the I kind $\pi_j(\sigma_k)$ the quality "of resources" select the probability $p_j(\sigma_k)$ of the problem solution, and function of individual effectiveness take in the form

$$\varepsilon_{\pi_j}=\sum_{j=1}^{N}\pi_j(\sigma_k)\ln p_j(\sigma_k), \tag{4.40}$$

Then

$$\pi_j(\sigma_k) = p_j(\sigma_k),$$

and

$$\varepsilon_{\pi_j}=-H_{p_j}=\sum_{k=1}^{N}p_j(\sigma_k)\ln p_j(\sigma_k)=-H_{\pi_j}=\sum_{k=1}^{N}\pi_j(\sigma_k)\ln\pi_j(\sigma_k). \tag{4.41}$$

We differentiate preferences of the I kind $\pi(\sigma_k)$ and objective utility $U(\sigma_k)$ (or harmfulness $L(\sigma_k)$). With respect to subjective rating the role *an objective character-*

istic it the rank of subject in the group can play. The rank of subject or "the measure of Diocletianus" is determined by the formal position of subject and by connected with of these authorities. "The measure of Diocletianus" we call rank, since for the first time "table about the ranks" was introduced by Diocletianus, and was legalized by Peter the I in Russia. Further information about the ranks is led in p. 4.4.2 of present division.

It is obvious that the ratings ξ (*j*) depend on the ranks $\overline{\eta}(j)$ - this is observed everywhere. Function of the effectiveness

$$20.\ \varepsilon_{\xi} = \sum_{j=1}^{M} \xi(j)\overline{\eta}_j \ \text{ or } \ \varepsilon_{\xi} = \sum_{j=1}^{M} \xi(j)\ln\overline{\eta}_j, \tag{4.42}$$

where $\overline{\eta}_j$ is the rank of subject "*j*", and the function of effectiveness is the sum of ranks $\overline{\eta}_j$ weighed on preferences of the II kind.

Dispersion of ranks

$$D_M(\overline{\eta}_i) = \frac{1}{M}\sum_{i=1}^{m}\left(\overline{\eta}_i - \eta(s'_{\xi})\right)^2\eta_i = \frac{1}{M}\sum_{i=1}^{m}\overline{\eta}_i^2\eta_i - \left(\eta(s'_{\xi})\right)^2. \tag{4.43}$$

Finally, conditional ratings can be connected with the rank of the subject, who assigns ratings (Σi: $i \in \overline{1,M}$), if we introduce effectiveness in the form:

$$21.\ \varepsilon_{\xi,i} = \sum_{j=1}^{M} \xi(j \mid i)\left(\overline{\eta}_j - \overline{\eta}_i\right)^{\alpha},\ (\alpha = 1 \text{ or } \alpha = 2). \tag{4.44}$$

All given functions of effectiveness do not give the direct coupling between ratings and object preferences. At the same time, it is obvious, that they are interdepended. One of the ways of this dependence organizing consists of postulation of effectiveness functions, in which both kinds of preferences "are mixed". Functions of effectiveness of such type are for example the following:

$$22.\ \varepsilon_{\pi}^{\xi}(\sigma_k) = \sum_{j=1}^{M} \xi(j)\pi_j^{\pm}(\sigma_k); \tag{4.45}$$

$$\varepsilon_{\pi}^{\xi}(\sigma_k) = \sum_{j=1}^{M} \xi(j \mid \sigma_k)\pi_j^{\pm}(\sigma_k); \tag{4.46}$$

$$\varepsilon_{\pi}^{\xi}(\sigma_k) = \sum_{j=1}^{M} \xi(j \mid i, \sigma_k)\pi_j^{\pm}(\sigma_k). \tag{4.47}$$

These functions are the object preferences averaged on ratings.

Procedures of the canonical distributions ξ and π determination sequential in time are described in chapter 5.

Completing the survey of different effectiveness functions, let`s note that the set of functions given above do not countable all possible versions for groups and correspondingly - possible types of canonical distributions. In particular, it is possible to examine additive combinations, for example:

$$\varepsilon_{\xi,\pi,t} = \alpha\sum_{j=1}^{M}\xi_t\left(j \mid \sigma_k\right)\pi^{\pm}_{jt-1}\left(\sigma_k\right) + \beta\sum_{j=1}^{M}\xi_t\left(j \mid \sigma_k\right)\xi_{t-1}\left(j \mid \sigma_k\right), \tag{4.48}$$

where t is moment of time.

The second term in the sum represents one more type of effectiveness functions:

$$\varepsilon_{\xi t,t-1}\left(\sigma_k\right) = \sum_{j=1}^{M}\xi_t\left(j,\sigma_k\right)\xi_{t-1}\left(j \mid \sigma_k\right),$$

which are used in recurrent scheme , fitted out to account the influence of the rating preferences distribution at the previous moments of time. Examples of such scheme are examined in chapter 5.

Let`s give several examples of obtaining of canonical rating distributions with the use of functions of effectiveness introduced above.

For function (4.18) let`s form the criterion

$$\Phi_{\xi} = -\sum_{j=1}^{M}\xi^{+}\left(j\right)\ln\xi^{+}\left(j\right) + \beta\sum_{j=1}^{M}\xi^{+}\left(j\right)U_j\left(S_a\right) + \gamma\sum_{j=1}^{M}\xi^{+}\left(j\right). \tag{4.49}$$

Canonical distribution in this case takes the form:

$$\xi^{+}\left(j\right) = \frac{e^{\beta U_j\left(S_a\right)}}{\sum_{k=1}^{M} e^{\beta U_k\left(S_a\right)}}; \tag{4.50}$$

the distribution corresponding to the function (4.23) is

$$\xi^{+}\left(j\right) = \frac{e^{\beta R_j^{disp}}}{\sum_{k=1}^{M} e^{\beta R_k^{disp}}}; \tag{4.51}$$

To the function (4.28) the distribution corresponds

$$\xi^+\left(j \mid S'_a\right) = \frac{e^{\beta R_j^{disp}\left(S'_a\right)}}{\sum\limits_{S'_a \subset S} e^{\beta R_j^{disp}\left(S'_a\right)}}. \tag{4.52}$$

A summation over entire set of subsets $S'_a \subset S_a$ is here implied. As far as S_a contains finite number of elements σ_k, a set of subsets S'_a is finite. If each subset S'_a contains only one element σ_k, then

$$\xi^+\left(j \mid \sigma_k\right) = \frac{e^{\beta R_j^{disp}\left(\sigma_k\right)}}{\sum\limits_{q=1}^{M} e^{\beta R_q^{disp}\left(\sigma_k\right)}}. \tag{4.53}$$

The function of effectiveness (4.32) leads to the distribution

$$\xi^-\left(j \mid i, \sigma_k\right) = \frac{e^{-\beta_i R_j^{req}\left(\sigma_k\right)}}{\sum\limits_{q=1} e^{-\beta_i R_q^{req}\left(\sigma_k\right)}}, \tag{4.54}$$

and function (4.33) - to the distribution

$$\xi^-\left(j \mid i, \sigma_k\right) = \frac{\left(R_j^{req}\left(\sigma_k\right)\right)^{\alpha_i} e^{-\beta_i R_j^{req}\left(\sigma_k\right)}}{\sum\limits_{q=1}^{M}\left(R_q^{req}\left(\sigma_k\right)\right)^{\alpha_i} e^{-\beta_i R_q^{req}\left(\sigma_k\right)}}. \tag{4.55}$$

A function (4.37) generates the following functional:

$$\Phi_\xi\left(\sigma_k\right) = -\sum_{j=1}^{M} \xi^+\left(j \mid \sigma_k\right) \ln \xi^+\left(j \mid \sigma_k\right) + \beta \sum_{j=1}^{M} \xi^+\left(j \mid \sigma_k\right) \bar{p}_j\left(\sigma_k\right) + \gamma \sum_{j=1}^{M} \xi^+\left(j \mid \sigma_k\right), \tag{4.56}$$

and the corresponding canonical distribution takes the form:

$$\xi^+\left(j \mid \sigma_k\right) = \frac{e^{\beta \bar{p}_j\left(\sigma_k\right)}}{\sum\limits_{q=1} e^{\beta \bar{p}_q\left(\sigma_k\right)}}. \tag{4.57}$$

With the determination of an integral ratings in the group it can prove to be essential the value of the ratio of available resources of this subject to the sum of available resources of the group

$$\bar{R}_j^{disp} = \frac{R_j^{disp}}{\sum\limits_{q=1}^{M} R_j^{disp}}, \tag{4.58}$$

or the ratio R_j^{disp} to the maximum value $R_{\max}^{disp} = \max\limits_{j \in \overline{1,M}} R_j^{disp}$:

$$\overline{R}_{j,\max}^{disp} = \frac{R_j^{disp}}{R_{\max}^{disp}}. \tag{4.59}$$

In other case it will be of interest with the ratings designation the ratio R_j^{disp} to individual available resources minimum on the group:

$$\overline{R}_{j,\min}^{disp} = \frac{R_j^{disp}}{R_{\min}^{disp}}, \tag{4.60}$$

where $R_{\min}^{disp} = \min\limits_{j \in \overline{1,M}} R_j^{disp}$.

It is most natural compare resources of this subject R_j^{disp} with resources of subject "*i*", who assigns conditional ratings $\xi\,(j|\,i)$ or $\xi(j|\,i,\,\sigma_k)$:

$$R_{j|i}^{disp} = \frac{R_j^{disp}}{R_i^{disp}}. \tag{4.61}$$

The use in the effectiveness function of each of these indices introduces the appropriate type of ratings distribution. Study of correlation between of conditional rating distributions, carriers of which are subjects of one and the same group is of interest.

Let`s examine the following situation: hierarch, being out of the group wants to obtain the estimation of averaged object preferences (in other words, - "consolidated desire of group"). Let`s designate this consolidated estimation through $\pi^{\Sigma}(\sigma_k)$. As the weight coefficients it is possible to take integral ratings $\xi\,(j)$, or differential ratings $\xi(j|\,\sigma_k)$. Let`s assume

$$\pi^{\Sigma}\left(\sigma_k\right) = \sum_{j=1}^{M} \pi_j\left(\sigma_k\right)\xi\left(j\right) \tag{4.62}$$

or

$$\pi^{\Sigma}\left(\sigma_k\right) = \sum_{j=1}^{M} \pi_j\left(\sigma_k\right)\xi\left(j \mid \sigma_k\right). \tag{4.63}$$

We see that these functions coincide with functions of effectiveness defined with formulas (4.45), (4.46).

The second version is preferable, since rating estimations are used, which consider the qualification of subjects, compared with an alternative σ_k.

Let there be two alternatives σ_1 - to be participant of in the football match, σ_2 - to give lecture on higher mathematics in the group of 2 subjects: Lobachevski and Lobanovski. It is clear that use for the construction $\pi^{\Sigma}(\sigma_k)$ of integral ratings ξ (j) in this case will lead to errors. The use of function (4.63) as the distribution is impossible, since it is not normalized. Here from equalities

$$\sum_{k=1}^{N} \pi_j\left(\sigma_k\right) = 1, \quad \forall j \in \overline{1,M} \tag{4.64}$$

and

$$\sum_{j=1}^{M} \xi\left(j \mid \sigma_k\right) = 1, \quad \forall k \in \overline{1,N}$$

it does not follow that $\sum_{k=1}^{N} \pi^{\Sigma}\left(\sigma_k\right) = 1.$ If is possible to normalize function (4.63), i.e., to introduce the new function $\overline{\pi}^{\Sigma}\left(\sigma_k\right)$ by the relationship:

$$\overline{\pi}^{\Sigma}\left(\sigma_k\right) = \frac{\pi^{\Sigma}\left(\sigma_k\right)}{\sum_{q=1}^{N} \pi^{\Sigma}\left(\sigma_k\right)} = \frac{\sum_{j=1}^{M} \pi^{\Sigma}\left(\sigma_k\right)\xi\left(j \mid \sigma_k\right)}{\sum_{q=1}^{N}\sum_{j=1}^{M} \pi_j\left(\sigma_q\right)\xi\left(j \mid \sigma_q\right)}. \tag{4.65}$$

Distributions (4.65) and (4.64) determine weighed preferences of alternative σ_k for whole group. If hierarch determines his preferences oriented to the total available resources of the group $R^{disp}\left(\sigma_k\right) = \sum_{j=1}^{M} R_j^{disp}\left(\sigma_k\right)$, where $R^{disp}\left(\sigma_k\right)$ are not universal resources, then we have to assume that the weight of alternative are considered by him as corporative, and he has necessary authorities in order to carry out consolidation of resources. Preferences in this case we will be noted by sign "+": $\pi^{\Sigma+}\left(\sigma_k\right)$ or $\overline{\pi}^{\Sigma+}\left(\sigma_k\right)$. Note that, if hierarch distributes his preferences and the result of "natural" extremalization of the functional

$$\Phi_{\pi^{\Sigma}} = -\sum_{k=1}^{N} \overline{\pi}^{\Sigma+}\left(\sigma_k\right)\ln \overline{\pi}^{\Sigma+}\left(\sigma_k\right) + \beta\sum_{k=1}^{N} \overline{\pi}^{\Sigma+}\left(\sigma_k\right)R^{disp}\left(\sigma_k\right) + \gamma\sum_{k=1}^{N} \overline{\pi}^{\Sigma+}\left(\sigma_k\right), \tag{4.66}$$

Then the obtained canonical distribution

$$\bar{\pi}^{\Sigma+}\left(\sigma_k\right)=\frac{e^{\beta R^{disp}\left(\sigma_k\right)}}{\sum_{q=1}^{N} e^{\beta R^{disp}\left(\sigma_q\right)}} \tag{4.67}$$

does not coincide in the common case with distribution, calculated by formula (4.62) or (4.63), where $\pi^{\Sigma+}\left(\sigma_k\right)$ are defined as canonical individual preferences, and $\xi(j|\ \sigma_k)$ are defined as the solution of the corresponding variational problem for the differential rating.

The question arises: is it possible to select structures of functional so that both distributions of type (4.65) and (4.67) coincide. Analogous task occurs for preferences $\pi^{\Sigma-}\left(\sigma_k\right)$ (or $\bar{\pi}^{\Sigma-}\left(\sigma_k\right)$), when as the objective factor required resources $R^{req}(\sigma_k) \le R^{disp}(\sigma_k)$ are taken and all σ_k are corporative.

Then

$$\bar{\pi}^{\Sigma-}\left(\sigma_k\right)=\frac{e^{-\beta R^{req}\left(\sigma_k\right)}}{\sum_{q=1}^{N} e^{-\beta R^{req}\left(\sigma_q\right)}}. \tag{4.68}$$

In this case any member of the group cannot solve the problem $P = (\sigma_0 \to \sigma_k)$ in view of the insufficiency of individual available resources. It`s possible to assume that individual preferences $\pi_j(\sigma_k)$ and differential ratings $\xi(j|\ \sigma_k)$ will be determined through the value partial resources $R_j^{disp}(\sigma_k)$, of isolated by subject (“j”).

It is analogous to that as this was done for individual preferences. In this case “collective reason” or hierarch can distribute preferences on the set of corporative alternatives taking into account of the prestige considerations:

$$\bar{\pi}^{\Sigma}\left(\sigma_k\right)=\frac{\left(R^{req}\left(\sigma_k\right)\right)^2 e^{-\beta R^{req}\left(\sigma_k\right)}}{\sum_{q=1}^{N}\left(R^{req}\left(\sigma_q\right)\right)^2 e^{-\beta R^{req}\left(\sigma_q\right)}}. \tag{4.69}$$

The relative or relative reduced resources, as earlier, can come out instead of absolute resources.

The problem noted above, about possibility of concordance of two methods group preferences obtaining is reduced to a question about solution existence of the system of equations (4.62), which consists of N linear equations. In this case left side is considered known and expressed, for example, by formulas

$$\pi^{\Sigma}\left(\sigma_k\right)=\frac{e^{-\beta R^{req}\left(\sigma_k\right)}}{\sum_{q=1}^{N} e^{-\beta R^{req}\left(\sigma_k\right)}}, \tag{4.70}$$

$$\pi^{\Sigma}\left(\sigma_k\right)=\frac{e^{\beta R^{disp}\left(\sigma_k\right)}}{\sum_{q=1}^{N} e^{\beta R^{disp}\left(\sigma_k\right)}}, \tag{4.71}$$

$$\pi^{\Sigma}\left(\sigma_k\right)=\frac{e^{-\beta \bar{r}^{r}\left(\sigma_k\right)}}{\sum_{q=1}^{N} e^{-\beta \bar{r}^{r}\left(\sigma_k\right)}}, \tag{4.72}$$

where $\bar{r}\left(\sigma_k\right)=\frac{R^{req}\left(\sigma_k\right)}{R^{disp}\left(\sigma_k\right)}.$

The system of N equations (4.62) is supplemented with normalizing conditions for $\pi_j(\sigma_k)$ and $\xi(j)$. This system of equations can be considered as a system for determination of rating ξ (j). Taking under consideration normalizing condition for ξ (j) we have N + 1 connections superimposed on M variables ξ (j). Three cases are possible

$$M = N + 1;\ M > N + 1;\ M < N + 1.$$

It is assumed that $\pi_j(\sigma_k)$ is determined from the solution of individual variational problems. In the case $M = N + 1$ is a unique solution, if the rank of the matrix

$$\prod=\begin{bmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1M} \\ \pi_{21} & \pi_{22} & \dots & \pi_{2M} \\ \dots & \dots & \dots & \dots \\ \pi_{N1} & \pi_{N2} & \dots & \pi_{NM} \\ 1 & 1 & \dots & 1 \end{bmatrix}$$

is M. If $M > N + 1$, task is indetermined and additional conditions for the isolation of unique solution are necessary. Finally, if $M < N + 1$, task is over determinated.

From a practical point of view, a problem lies in connection of individual interests and group ideas. Their correlation is not obvious in advance. Clearing out of this question composes the fundamental element of investigation in each special case. The determination of covariances between individual and group preferences is one of tools of such investigation.

Any time it should be explained up in which “natural” way variationed problems for individual and for group are formed, how they are connected with each other, what variation principle either for a group, or for an individual has a priority and in what logical and time-sequential routine group and individual variational problems are being solved.

The removal of uncertainty described above probably, consists of an acknowledgment of the fact that the rating and object preferences, determined according to different scheme are different preferences. So to say, ratings found from linear system of (4.62) described above, together with normalizing condition, can be treated as such, that their "carrier" and "user" is hierarch, and preferences, obtained directly from the variational problem with functional (4.49) - as the reflection "of public opinion". In all cases, it is assumed that somebody who forms the group preferences or ratings of members of the group is informed about the value of arguments of canonical distributions (resources, utility, individual object preferences…), or possesses with their approximate estimations.

Due to conditions of inaccurate information about these factors the task *about the degree of the adequacy of preferences distributions* immediately appears: to what degree the distortion of initial information distorts preferences. Partially, for small distortions of initial information this question could be solved through an analysis "of the elasticity of psyche".

One of possible scheme, which make it possible to resolve the discussed uncertainty, consists of an attraction of temporary factor, i.e., In the assumption that different from of preferences are determined not simultaneously, but In the certain sequence.

A simplification occurs if we assume, for example, the following:

1. Individual preferences of the I kind $\pi_j(\sigma_k)$ and a group preferences $\pi^{\Sigma}(\sigma_k)$ are determined on the whole by required resources for σ_k and their derivatives.

2. Rating preferences $\xi\,(j|\,i)$, or ratings, established by hierarch, are determined in on the whole by available resources.

A rating "from below -upward", shown in the following circuit , is a logic circuit of an introduction:

$$
\begin{array}{lcl}
extr\Phi_{\pi_j} \to \pi_j(\sigma_k) & \Rightarrow & \\
 & & extr\Phi_{\pi}^{\Sigma} \to \pi^{\Sigma}(\sigma_k) \\
\left.\begin{array}{l} extr\Phi_{\xi(j)} \to \xi(j) \\ extr\Phi_{\xi(j|i)} \to \xi(j\,|\,i) \end{array}\right\} & \Rightarrow &
\end{array}
$$

It is completely obvious that all versions proposed are altogether only models, which give certain basis for psychological experiments organizing, and also, subsequently of the simulation of the dynamics of the problem- resource situations development.

4.5. Ratings and ranks. "Well" organized groups.

Rating preferences like object preferences are subjective characteristics in the first case - subjects in the group, in the second case - alternatives.

"The problem" is not yet defined on "the set" of subjects, similarly, as it was done on the set of object alternatives S_a. In order to do it, suppose that the group is structured - the system of ranks is assigned. A rank is considered as objective characteristic of subject, whereas a rating is subjective characteristic. We will assume that, finding a rank, a subject occupies the specific position in the group and imperious authorities connected with this are obtained. Who, when, how and on the basis of what, appropriates to this subject rank - this is a separate question, which we will touch later.

Let`s at first examine the relation between ranks and ratings. The measure of rating is the function of preference of the II kind $\xi(j)$, $\xi(j|\ i)$, $\xi(j|\ i, \sigma_k)$… (to simplify we will name it "*ratings*"). We will further use a quantitative measure of the significance of a rank or "a measure of Diocletian". Naturally this measure would be connected with "*the volume of imperious authorities*". There are several possibilities of a quantitative assessment of this "*volume*". In this work we will correlate this measure with a relative quantity of available resources, and also with the right of awarding ranks to others members of the group.

The number of rank, determined by natural number in the ascending order in the hierarchy of ranks, is the simplest measure.

Let M_η is a set of all possible systems of ranks in the group, which consists of M subjects. The system of ranks we will characterize by parameters $\{m, q_1, q_2, ..., q_m\}$, where m is the number of different ranks (classes of rank equivalence); $q_1, q_2, ..., q_m$ are quantities of subjects, which obtain respectively ranks $A_1, A_2, ..., A_m$.

The condition is satisfied: $\sum_{s=1}^{m} q_s = M$. Let's, for example, everything $q_s = 1$, $\forall s \in \overline{1,m}$, then the non-normalized Diocletian measure can be values

$$\eta_s = s;\ (s \in \overline{1,m}). \tag{4.73}$$

It`s possible to consider the value the normalized measure

$$\eta_s = \frac{2s}{m(m+1)}. \tag{4.74}$$

The weight of rank can be defined as the quantitative measure of imperious authorities, which display the subject of the given rank. One of the methods of such measure lies in the fact that a use a relative percentage of available resources, which subject has right to manage. The discussion can deal with the portion of consolidated resources. Simplifying ideas about dynamic processes, let`s isolate two stages: before making a decision about the purpose selection, i.e., a period of an problem - resource situation analysis, and after the purpose selection – a period, when available resources are partially or completely directed to the solution of the selected problem.

Let $R^{disp}\left(\bigcap_{j=1}^{M} S_{aj}\right)$ are all available resources of a group , consolidated at an intersection of individual problem sets S_{aj}. Suppose that all alternatives, which are contained in this intersection $\sigma_k \in \left(\bigcap_{j=1}^{M} S_{aj}\right)$ - are corporative for all members of the group (although this, of course, is significant simplification, since corporative problems can exist only for the part of the members of the group). If $R_s^{disp}\left(\bigcap_{j=1}^{M} S_{aj}\right)$ is a part of consolidated resources, which the subject of rank A_s manages, then the weight of rank η_s can be determined by the relation

$$\eta_s = \frac{R_s^{disp}\left(\bigcap_{j=1}^{M} S_{aj}\right)}{R^{disp}\left(\bigcap_{j=1}^{M} S_{aj}\right)} = \frac{R_{sc}^{disp}}{R_c^{disp}}, \tag{4.75}$$

where R_{sc}^{disp} and R_c^{disp} - available consolidated resources (*sc*), which the subject of rank *s* manages with, and complete consolidated resources.

If an integral corporative problem is decomposed in the form of hierarchic structure, which corresponds to a rank hierarchy, then analogously available resources are decomposed.

When an alternative σ_k choice from the set S_a and the problem P: $\sigma_0 \to \sigma_k$ occurs and a purpose is designated, then a change of the set $S_a = \bigcap_{j=1}^{M} S_{aj}$, and also available resources occurs, from which a part is moved away, directed for of the solution of the selected problem. Consequently, each time with decision making a change in the weights of ranks η_s is connected.

The possible diagram of the distribution of consolidated available resources, oriented to corporative problems, is represented in Fig. 4.4.

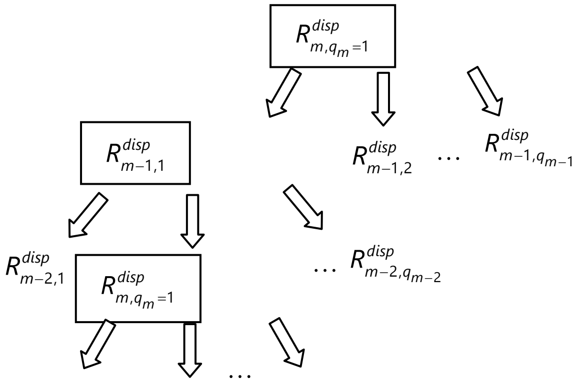


Fig.4.4

On each level of hierarchy each subject, who has rank A_s and "place" on this level q_k has at his disposal available consolidated resources R^{disp}_{s,q_s}, which, naturally, in the general case, are revealed from his personal available resources R^{disp}_{j}, part from which can be included in the composition of consolidated resources.

The weight of rank A_s in connection with the diagram, depicted on Fig. 4.4 can be, in this case, determined by the formula

$$\eta_s = \frac{\min\limits_{q_s} R^{disp}_{s,q_s}}{R^{disp}\left(\bigcap\limits_{j=1}^{m} S_{aj}\right)}, \tag{4.76}$$

i.e., as the ratio of the minimum (at the level s) volume of available consolidated resources, which are at the disposal of all subjects of this rank, to the total volume of consolidated resources. The estimation of the weight of rank look like this with the view, so to speak, "from above" i.e., from the position of subject, who has the highest rank m, since the denominator in the formula (4.75) is this consolidated resources, which are at his disposal.

It`s possible to conective that "the weight" of rank with the view "from below", i.e., from the position of subjects, who are on steps of hierarchy lower than A_s, will have another form and another numerical value. Further detailing in this direction, we do not conduct here.

Returning to ratings, let`s note that they can be determined depending on

1) the relation of total available resources of a subject, which include both not consolidated and consolidated part, to complete available resources of an whole group , either to the richest resources or to the poorest ones;

2) the ratio of only consolidated part of individual resources of subject to total consolidated resources of whole group .

In the first case (formula (4.59)):

$$\xi(j) = f_j\left(\bar{R}_j^{disp}\right); \quad \bar{R}_j^{disp} = \frac{R_j^{disp}}{\sum_{k=1}^{M} R_k^{disp}}, \tag{4.77}$$

in the second case:

$$\xi(j) = f_j\left(\bar{R}_j^{disp}\right); \quad \bar{R}_{jc}^{disp} = \frac{R_{jc}^{disp}}{\sum_{k=1}^{M} R_{kc}^{disp}}. \tag{4.78}$$

In the latter case a rating considers participation of a subject *j* through his resources in the solution of corporative problems.

Connection of ratings with available resources only is not certainly universal and obligatory for all social and for all situations. In the community Nazoreev in Jerusalem directed by apostle Peter, regulations were absolutely socialist, including no one possessed passive resources; however, there was a hierarchy of ranks.

The following point of view is important: ranks are not defined as solutions of variational problem with the entropy functional and play the role the more or less stable characteristics of an active system similar to some imperatives. The distribution of ranks and the structure named "*take of value*" are formed "historically"; on the basis of the accumulation of experience. They remain constants for certain time. Besides of experience at the basis one or an other system of "the group ethics" is put down, which regulates not only individual, but also collective decisions.

As the temporary retreat from the basic theme we want to continue the consideration of the fact that was named above "*collective reason*" or "*virtual subject*" as it seems to us, always being present in the group, in social, and corresponding at the point of the collective coordinated behavior. We often observe it in life in different forms of life: a behavior of a flock of bees, a flight of a flock of birds, a motion of school of fish, a migration of deer herds, and so on. In the highest form of life - in human associations, the collective coordinated behavior is manifested both in the most perfected and in the most primitive forms. An example of the primitive manifestation of a "*virtual subject*" is the behavior of a crowd on the square appearance of cumulative effect, manifestation of "*public opinion*".

It should be recognize that a "virtual subject" exists objectively. This means that each the group , each social is not simply the sum of individuals, but it appears simultaneously a certain mega-organism, possessing "collective reason" - specific "parts" of a consciousness of individuals in socials work coordinated and from results of this work, to a considerable extent , all attributes of subjective manifestations of individual psyches depend on.

In contrast to ranks ratings are more variable, more dynamic, they react to any changes in the problem - resource situation, and as we see, they can be defined each time as the solution of variational problem with the functional, whose main component is subjective entropy.

Inasmuch as we recognize an existence of "virtual reason", we must allow an existence of preferences, both of the I and the II kind, whose carrier is this subject and for their determining we have right to postulate the certain variation principle. This subject can be, for example, the carrier of preferences $\pi^{\Sigma}(\sigma_k)$ and integral ratings $\xi\,(j)$...

As we explained, there is an essential difference between ratings (preferences of the II kind) and ranks (Diocletian's measures). Hence it follows that the study of the relation between distribution of rating and distribution of ranks in each social must give the important information, which characterizes this group (social).

If one assumes that in the group the condition is satisfied

$$\xi_j\left(\sigma_p\right) \geq \eta_j\left(\sigma_p\right); \;\; \forall j \in \overline{1,M}; \;\; \forall p \in \overline{1,N\left(S_a\right)}, \tag{4.79}$$

where $\sigma_p \in S_a$ is a corporative alternative, this would indicate that from the point of view of subject possessing higher value in the value hierarchy ranks are distributed "correctly": no one has the undeserved high rank, in each case above the rating of subject is equal his ranks to.

However, it easy to say, if distributions ξ_j and η_j are normalized, then this "correct" structurization of an entire hierarchy is impossible. Actually, if at least for one of subjects an absolute inequality is fulfilled, then at least one subject, for whom an opposite inequality will be fulfilled, will be available. It`s evident from the following examples:

1. let the group consists of two subjects Σ_1 and Σ_2, their ranks and ratings are normalized by one $\xi_1 + \xi_2 = 1$; $\eta_1 + \eta_2 = 1$, the group is structured, i.e., $\eta_1 \neq \eta_2$. It`s evident from the table that strict inequality cannot be carried out simultaneously for both subjects

	Σ_1	Σ_2
ξ_j	0,2	0,8
η_j	0,3	0,7

$M = 2$; $m = 2$

The same occurs for any M and m. Only an equality can be fulfilled there. Let`s discuss definitions, given to p.181, more detailed.

In chapter 4.3 we determined concepts of "weakly ordered group", "the strictly ordered group" and "the strictly partially ordered group" relative to binary rating relation ρ_ξ. In this case properties of were used:

- of the asymmetry:

$$\Sigma_i \rho_\xi \Sigma_j \Rightarrow \Sigma_j \bar{\rho}_\xi \Sigma_i; \tag{4.80}$$

— of the tranistivity:

$$\left(\Sigma_i \rho_\xi \Sigma_j ; \Sigma_j \rho_\xi \Sigma_k\right) \Rightarrow \Sigma_i \rho_\xi \Sigma_k \tag{4.81}$$

— of the negative tranistivity:

$$\left(\Sigma_i \bar{\rho}_\xi \Sigma_j ; \Sigma_j \rho_\xi \Sigma_k\right) \Rightarrow \Sigma_i \rho_\xi \Sigma_k. \tag{4.82}$$

In the first case of the weak ordering the relation of indifference ~, defined as absence of a strict preference on S_ξ is relation of equivalence (reflexive, symmetrical and transitive). If the set of classes of equivalence is countable, then if ρ_ξ:〈 is a weak ordering, it`s possible to assign to subjects $\Sigma_j \in S_a$ quantitative measure $\xi_j = \xi(j)$, which we call rating such, that

$$\Sigma_i \rho \Sigma_j \Leftrightarrow \xi(i) < \xi(j) \tag{4.83}$$

and

$$\Sigma_i \sim \Sigma_j \Leftrightarrow \xi(i) = \xi(j). \tag{4.84}$$

In the case of strictly partial ordering the relation of indifference cannot be transitive, but relation≈, determined by the condition

$$\Sigma_i \approx \Sigma_j \Leftrightarrow (\Sigma_i \sim \Sigma_k \Leftrightarrow \Sigma_j \sim \Sigma_k,\ \forall \Sigma_k \in S_\xi), \tag{4.85}$$

is equivalence.

Then, if a relation ρ_ξ:〈 is strict partial ordering, and $S_{\xi\approx}$ is of classes of equivalence in sense (4.85) is countable (in actuality – it is always finite), ratings ("usefulness") can be put in correspondence to the elements of a set S_ξ (the subjects of group).

$$\Sigma_i \langle \Sigma_j \Rightarrow \xi(i) < \xi(j) \tag{4.86}$$

and

$$\Sigma_i \approx \Sigma_j \Rightarrow \xi(i) = \xi(j). \tag{4.87}$$

We see that, in this case, the equivalence of subjects Σ_i and Σ_j is each time installed via comparison with the certain third subject.

In order to refer the group "well" structured, it`s as the minimum desirable that subjects, allotted by different ranks, would belong to different classes of rating equivalence.

This is possible, if the number of ranks does not exceed the number of classes of rating equivalence. It is visible in particular, the importance of the rating studies, which give important initial information for designing of the administrative arrangement in the group, if it`s assigned "from above".

Let`s examine the table, which reflects the structurization of the group of three subjects, in which three ranks A_1, A_2, A_3 are determined.

Σ_j	Σ_1	Σ_2	Σ_3
ξ_j	0,2	0,3	0,5
η_j	0,1	0,2	0,7

Here, as we see, the condition (4.79) is not fulfilled, but the condition

$$\xi_1 < \xi_2 < \xi_3 \Leftrightarrow \eta_1 < \eta_2 < \eta_3 \tag{4.88}$$

is satisfied. Following table shows the case. When the group consists of three subjects $M = 3$ and two ranks $m = 2$, $q_1 = 2$, $q_2 = 1$ are determined.

Σ_j	Σ_1	Σ_2	Σ_3
ξ_j	0,2	0,3	0,6
$\eta_j^{(1)}$	0,15	0,15	0,7
$\eta_j^{(2)}$	0,23	0,23	0,4
$\eta_j^{(3)}$	0,10	0,10	0,8
$\eta_j^{(4)}$	0	0	1,0

Assuming that there are only two ranks ($m = 2$), $M = 3$ and, furthermore, a one-man management, $q_1 = 2$; $q_2 = 1$. in each of versions subjects *of the first* (lowest) rank manage identical portions of consolidated resources, in the fourth version these portions are equal zero and the total volume of the consolidated resources manages the subject *of the second* (highest) rank. In the third case the portion of consolidated resources of those being at the disposal of each of subjects *of the first* rank, corresponds to the lowest rating in the first class of rank equivalence. In the second case this portion corresponds to the highest rating in the first class of rank equivalence. The first case in this sense takes an intermediate position. Different versions are possible. We can see that, generally speaking, in each of versions the psychological tension, connected with the disturbance of a "complete ordinal agreement" of ranks and it`s rating can appear. It`s most likely possible, to ensure a complete agreement only when the number of subjects M is equal the number of ranks m, or when the number of ranks is equal the number of classes of rating equivalence and in (4.79) is carried out rigorous equality.

This table illustrates (does not prove) the assertion, which denies the possibility of fulfilling an absolute inequality (4.88).

A group can be considered "well-structured", if for $\forall j \in \overline{1,M}$ the condition is satisfied:

$$\xi_1 < \xi_2 < \dots < \xi_M \Rightarrow \eta_1 \leq \eta_2 \leq \dots \leq \eta_M.$$

Possible versions of the structurization of groups are shows on diagrams, given below, (Fig. 4.5).

In the case (a) a quantity of ranks m equals to a quantity of classes rating of equivalence. $M^{\sim}_k$ ($k \in \overline{1,m}$) is number of classes of equivalence. "Manager" by consolidated resources of class $m = k$ is selected from the class of equivalence $m = k + 1$. Then $\xi_j < \eta_i$; $j \in M^{\sim}_k$; $i \in M^{\sim}_k + 1$. Both the "manager" by class $m - 1 = 6$, (in this case $m = 7$) and supreme hierarch , which governs all resources through the subject with the same rank m, which controls the class $m - 1 = 6$ have the highest rank. If in the highest class there is only one subject: $M^{\sim}_m = 1$, then the manager of class $m - 1$ is simultaneously supreme hierarch.

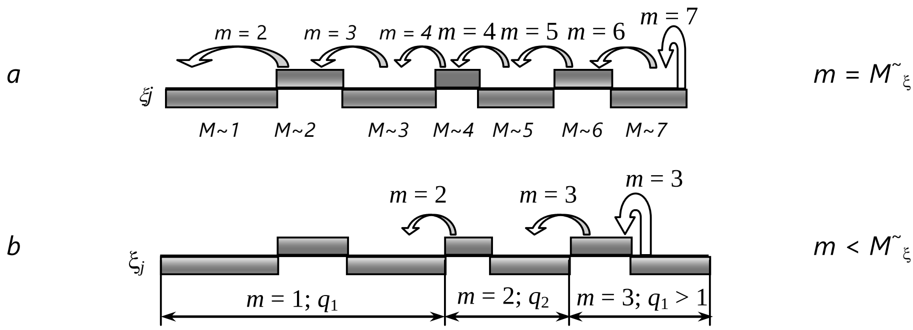


Fig. 4.5

In the case (b) $m < M^{\sim}_{\xi}$. Also here, if in the upper class $M^{\sim}_3 > 1$, an uncertainty in the selection of supreme hierarch occurs. In any case it is impossibly to solve this question by rating comparison. But if in both cases (a) and (b), the number of upper classes is equal one, a selection of hierarch can be made unambiguously, if we achieve a selection on the basis of comparison of ratings, and not due to any other considerations.

Fig. 4.6 shows that on each level of hierarchy q_k ($k \in \overline{1,m}$) subjects exists, who have rank A_k.

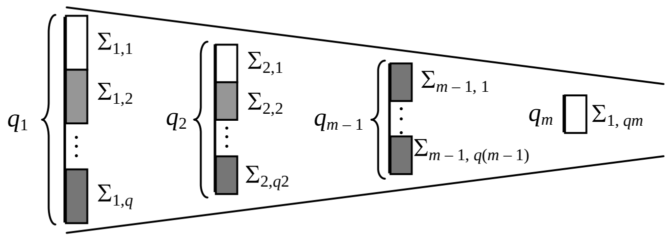


Fig. 4.6

Given diagrams by no means fulfill possible diagrams of groups structurization. In their turn rating preferences $\xi(j)$, $\xi(j|\ i)$, $\xi(j|\ i, \sigma_k)$, ... depend on conditions of the decided task and, although, supposedly, each time they can be defined as the solution of variational problem, they will be different depending on what form and sense makes the function of effectiveness, entering the functional, as it`s expressed through resources, functions of utility or function of harmfulness. Furthermore, ratings depend on that, who is their "carrier", in whose consciousness the task is formed, and finally what the diagram of aggregation of individual preferences is accepted and who carries out an aggregation.

Since we postulate the presence of a "virtual subject" (M + 1) the carrier of "collective reason" or the agent, which achieves aggregation, then together with individual preferences in each group, we must consider the presence of preferences of those generated by a "virtual subject". Latter ones, in accordance with the hypothesis realize as the component of the individual preferences; however, they must be the result of extremisation of the specific functional, which characterizes the collective component of the psyche of M + 1 subjects.

As the characteristics of the structured and group a rating entropy, a rank entropy and a correlation coefficient of its rating ξ_i also of ranks η_s ($j \in \overline{1,M}$, $s \in \overline{1,m}$) serve.

Rating entropy in different versions was examined above for example, for the integral ratings

$$H_\xi = -\sum_{j=1}^{M} \xi(j) \ln \xi(j).$$

On the analog, we introduce the rank entropy

$$H_\eta = -\sum_{s=1}^{m} q_s \eta_s \ln \eta_s \tag{4.89}$$

with the fulfillment of conditions

$$\sum_{s=1}^{m} q_s = M; \quad \sum_{s=1}^{m} q_s \eta_s = 1. \tag{4.90}$$

The rating entropy characterizes the degree of the rating heterogeneity of a group. If all ratings are equal

$$\xi(j) = \frac{1}{M}, \quad \forall j \in \overline{1,M}, \tag{4.91}$$

then, that group is uniform and rating entropy is maximum:

$$H_\xi = H_{\xi\max} = \ln M.$$

In other cases $H_\xi < \ln M$.

The rank entropy H_η is the characteristic of the degree of an objective structurization of a group. The less the entropy, the clearer the group is structured. Thus, if among the ranks is a rank, whose weight $\eta_s = 1$ (for example, when $q_s = 1$, $s = 1$), then the rank entropy becomes zero, when with $m = 2$, $M - 1$ subjects, they have the rank weight $\eta_1 = 0$ and only one subject has rank weight $\eta_2 = 1$. In this case it corresponds to the situation, when no one subject, including the manager, has any consolidated resources at his disposal, but the manager manages all resources.

The correlation coefficient $r_{\xi\eta}$ between ξ (j) and η_s makes it possible to judge a relation of distributions of subjective preferences of the II kind ξ (j) and ranks. The coefficient $r_{\xi\eta}$ can be expressed by the formula

$$r_{\xi\eta} = \frac{\sum_{s=1}^{m}\left(\bar{\eta}_s - \frac{1}{M}\right)\left(\sum_{j=1}^{q_s}\xi_j^{(s)} - \frac{s}{M}\right)}{\sqrt{\sum_{j=1}^{M}\left(\xi_j - \frac{1}{M}\right)^2 \sum_{j=1}^{m} q_s\left(\bar{\eta}_s - \frac{1}{M}\right)^2}}, \tag{4.92}$$

where $\xi_j(s)$ is normalized ratings of subjects, which belong to the class s of the rank equivalence, $\bar{\eta}_s$ - a normalized rank, q_s is number of class of rank equivalence, M the number of the group .

If $r_{\xi\eta} \to 1$, the distribution of ranks in the group reflects rating distribution, i.e., subjective ideas about “merits” of subjects is coordinated with an objectively existing “administrative” structure.

If $r_{\xi\eta} \to -1$, the rank organization of the group is in contradiction with rating distribution. This group “is badly” structured and it`s necessary to take into account the possibility of psychological and, as a result, social tension.

"*A table about ranks*" and rank weights distribution doesn`t completely reflect the structure of the group. We should also define "the zone of responsibility" for each rank. Above we connected "the zone of responsibility" with the volume of consolidated available resources, what defined-by-example characterizes "the volume" of imperious authorities, and also with specified additional conditions - number of classes of rank equivalence and, consequently - the number q_s of directly "subordinates" subjects, who have the rank A_{s-1}.

In the first chapter resources were divided as the passive R_p (including available R_p^{disp}) and *active* R_a (including available active R_a^{disp}). It`s seemed that ratings are passive determined by the presence of both the first and the second. If passive resources in the majority of cases can be easily measured and even represent their value in commensurate scales, for example, in the money equivalent, then active resources in the majority of cases, on the contrary, yield to measurement with difficulty, and the universal scale for them similar to money doesn`t exist.

Two possible ways for measuring of active resources in commensurate units, at least, for subjects, who have close specializations, consist of:

1. Testing with the help of previously developed general and professional tests and

2. Calculation of previous achievements in the form of objects of intellectual property, ranked also on those matched to coordinated procedures.

For example, if the discussion deals with the scientist, generally speaking, it is not difficult to define quantity and quality of his scientific achievements, for example as a quantity and the value of scientific publications, a quantity of references, a quantity of prepared postgraduate students and so forth, which usually is being done in Institute of Higher Education and scientific research establishments.

In connection with the task of the quantitative assessment of active resources of subjects through objects of their intellectual property of more important becomes the problem of the protection of these objects, and the protection of association from the unconscientiously use of some belonging objects of intellectual property.

Three versions occur:

1. Ratings are determined exclusively by the presence of passive available resources (in this case they say: "It the cost that ammount ").

2. Ratings are determined by exceptionally active available resources (examples: Christ, Moses , Einstein,…).

3. With the determination of rating both kinds of resources are considered.

As an example let`s examine consequences of organizational reform, which was conducted by Moses, after outcome from Egypt was completed. We will take some data from the book of Skousen [193] "The making of America".

It`s known that some of basic sources of ideas, which "fathers founders" of the USA placed as basis of constitution, was the administrative device established by Moses for the large number of fugitives, who proved to be themselves in the improbably severe, extreme conditions in the Sinai desert.

The "Book of Numbers" gives the possibility to estimate the number of entire population, which would after Moses: it`s asserted that there was 600000 men, capable to bear weapon. Adding women, old men, children and adult men, at the point of any reasons capable of being soldiers, it`s simple to estimate total number as 3000000 people. The number of families is evaluated as equal to the number of soldiers, i.e., 600000. Under severe desert conditions a set of the most varied problems appeared and, since at first mass - association of fugitives was not structured, there was no control system, Moses was formed to solve all problems on his own. All day long queue people stood to Moses with their problems and adversities. Moses had no experience in controlling such hugs number of people and did not manage different administrative tasks, which required the urgent solution.

The carried out on the council of God, speaking in modern language, administrative reform. The ordered hierarchical system for the control was created: at the lower level of 3000000 people were naturally divided as 600000 families. An the following level the groups of 10 families, only of 60000 the group s was created. In each such group the head of the group was separated, 5 groups of 10 families composed the group of 50 families, in which the elder was also selected. There were 12000 such leaders. Then groups of 100 families was created. There were 6000 leaders of these groups, and finally - the group of 1000 families. 600 leaders of such groups composed, the council of elective representatives. This structure was reflected in the American political system as "House of Representatives". In the Moses structure there was also "a council of seventies", analog of which is the senate of the USA. Finally, there existed two "deputy" of Moses - Aaron, who carried out the role "of Vice President in foreign affairs" and Joshua - "Vice President on the servicemen to the matters".

The hierarchical system for control, which contains 9 levels, was created. The disaggregation of control was achieved and 9 "ranks" was introduced. Thus, unique "table about the ranks" was invented long before Diocletian, but in contrast to "tables about Diocletian's ranks" it contains only 9 ranks. Using our designations and terminology, to the series member of association it`s possible to assign the first rank A_1, and to Moses - ninth rank A_9.

Before the administrative reform the association of fugitives had three hierarchical levels, schematically depicted on Fig. 4.7,.

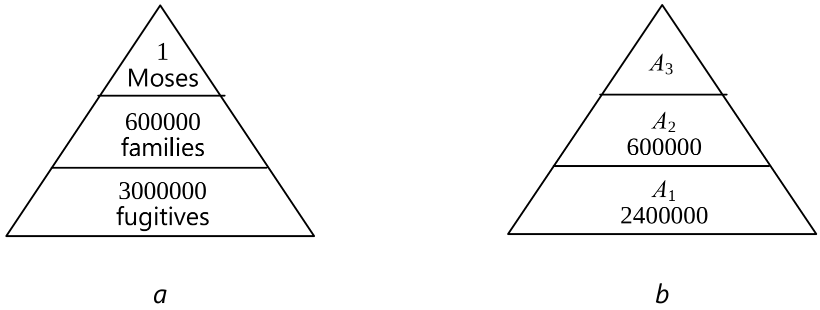


Fig. 4.7

Fig.4.7 (*b*) shows the number of subjects of each rank: 2400000 people had the lowest rank A_1, 600000 heads of families, which theoretically associated with Moses, had a rank A_2 finally Moses himself had a rank A_3.

After the reform the structure represented in Fig. 4.8 came out.

Fig. 4.8, a shows the number of team leaders of different ranks. Fig. 4.8, *b* shows the number of subjects, that have certain ranks under assumption that leaders of higher rank were selected from the number of leaders of the previous rank (this assumption, of course, is not necessary).

Let`s try to calculate how the rank entropy changed as a result given reform.

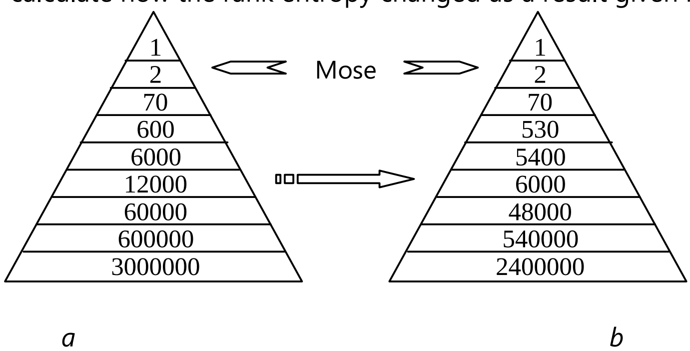


Fig. 4.8

In order to make this it`s necessary, in any manner, to estimate the weight of the subject of given rank. Since the volume of imperious authorities of each leader in the hierarchical Moses system is unknown and it`s not possible to express rank weights through consolidated resources. Let`s assume that the weight of rank can be determined as value the inversely proportional to the number of subjects of those having this rank. Assuming that the sum of weights of ranks is normalized by one, let`s find weights, represented in the tables:

Before the reform			
the rank	the role	$\bar{\eta}_s = \dfrac{1}{3q_3}$	$-q_s\left(\bar{\eta}_s \ln \bar{\eta}_s\right)$
A_3	Moses	0,333(3)	0,3162...
A_2	the head of families	0,000000556	4,80467...
A_1	members of families	0,000000138	5,23164...
			$H_\eta - \Sigma q_s\left(\bar{\eta}_s \ln \bar{\eta}_s\right) = 10{,}4025$

In the case of the unstructured association, when all ranks are identical and equal $\bar{\eta}_1 = 0{,}000000333$, entropy is maximum and for M = 3000000

$$H_\eta = 14{,}8992.$$

After the reform			
the rank	the role	$\bar{\eta}_s = \dfrac{1}{3q_3}$	$-q_s\left(\bar{\eta}_s \ln \bar{\eta}_s\right.$
A_9	Moses	0,111(1)	0,2441...
A_8	two ministers	0,055555(5)	0,32113...
A_7	member of council	0,001589...	0,7168...
A_6	representative (the group of 1000 families)	0,00021...	0,9428...
A_5	head of the group of 100 families	0,0000206...	1,2003...
A_4	head of the group of 50 families	0,0000186...	1,2156...
A_3	head of the group of 10 families	0,00000233	1,4505...
A_2	head of family	0,000000205	1,7048...
A_1	members of family	0,000000046	1,8646...
			$H_\eta = -\Sigma q_s\left(\bar{\eta}_s \ln \bar{\eta}_s\right) = 8{,}2098$

Relative entropy $H_\eta = H_\eta\left(H_{\eta\max}\right)^{-1}$ in the first case is equal

$$\bar{H}_\eta = \frac{10,4025}{14,8992} = 0,6982...,$$

in the second case

$$\bar{H}_\eta = \frac{8,2098}{14,8992} = 0,5510...,$$

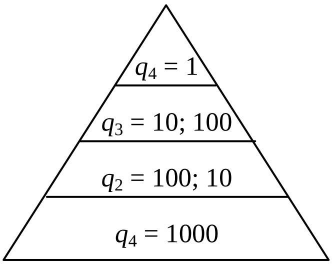


Fig. 4.9

As we see, the reform of Moses led to the decrease of rank entropy in 1,27 times. That can be considered as evidence of the deeper structurization of association and, apparently, larger certainty in decision making.

Let`s examine the following abstract example, when the hierarchic structure of control has four levels and is represented on Fig. 4.9.

In accordance with the diagram Fig. 4.9 on "lower" level there are q_1 = 1000 members of the association for whom rank A_1 is assigned, on the second from below level there are q_2 = 100 subjects of the rank A_2. To each of them the group of 10 people is subordinated, the third level there are q_3 = 10 subjects of the rank A_3. Each of governs the group of 100 people and finally them the fourth level there is a "supreme leader" - the subject of the rank A_4 and q_4 = 1.

Let`s examine two methods of determining the weight of the rank:

1. The weight of the rank of subject is directly proportional to the number of directed and it`s inversely proportional to the number of subjects of the same rank, i.e., the number of class of rank equivalence, to which this subject belongs to.

Normalized satisfy the normalizing condition

$$\sum_{s=1}^{m} q_s \bar{\eta}_s = 1. \tag{4.93}$$

Let`s calculate weights of ranks for the diagram on Fig. 4.9, we have:

$$\tilde{\eta}_1 = \frac{1}{1000};\ \tilde{\eta}_2 = \frac{10}{100};\ \tilde{\eta}_3 = \frac{100}{10};\ \tilde{\eta}_4 = \frac{1000}{1}.$$

A complete composition of the association sown on Fig. 4.9 M = 1111 subjects; therefore normalized ranks:

$$\bar{\eta}_1 = \frac{0,001}{1111} = 0,0000009;\ q_1 = 1000;$$

$$\bar{\eta}_2 = \frac{0,1}{1111} = 0,00009;\ q_2 = 100;$$

$$\bar{\eta}_3 = \frac{10}{1111} = 0{,}009;\ q_3 = 10;$$

$$\bar{\eta}_4 = \frac{1000}{1111} = 0{,}9001;\ q_4 = 1{,}0.$$

As we see, the condition (4.93) is satisfied.

2. The weight of the rank of subject is inversely proportional to the number "of associates" - the number of class of rank equivalence, which this subject belongs to, and it does not depend on the number of those controlled. In our example

$$\tilde{\eta}_1 = \frac{1}{1000};\ \tilde{\eta}_2 = \frac{1}{100};\ \tilde{\eta}_3 = \frac{1}{10};\ \tilde{\eta}_4 = 1.$$

For obtaining normalized ranks it`s necessary to divide the value $\tilde{\eta}_s$ by 4, and

$$\bar{\eta}_1 = 0{,}00025;\ q_1 = 1000;$$

$$\bar{\eta}_2 = 0{,}0025;\ q_2 = 100;$$

$$\bar{\eta}_3 = 0{,}025;\ q_3 = 10;$$

$$\bar{\eta}_4 = 0{,}25;\ q_4 = 1.$$

These weights also satisfy the normalizing condition (4.93).

3. The weight of the rank of subject is directly proportional to the number of those controlled and does not depend on q_s is number of its own class of the rank equivalence:

$$\tilde{\eta}_1 = 1;\ \tilde{\eta}_2 = 10;\ \tilde{\eta}_3 = 100;\ \tilde{\eta}_4 = 1000.$$

For obtaining normalized weights, each value $\tilde{\eta}_s$ should be divide by 4000. As a result the weight distribution analogous to previous one will come out.

Let`s calculate rank entropies for each of three cases.

In the first case we have:

$$H_{\eta_1} = -0{,}9001 \cdot \ln 0{,}9001 - 10 \cdot 0{,}009 \ln 0{,}009 - 100 \cdot 0{,}00009 \ln 0{,}00009 -$$

$$-1000 \cdot 0{,}0000009 \ln 0{,}0000009 = 0{,}61504\ .$$

In the second and third cases we will obtain the identical entropy ($H_{\eta 2} = H_{\eta 3}$).

$$H_{\eta_2} = H_{\eta_3} = -0{,}25 \ln 0{,}25 - 10 \cdot 0{,}025 \ln 0{,}025 - 100 \cdot 0{,}0025 \ln 0{,}0025 -$$

$$-1000 \cdot 0{,}00025 \ln 0{,}00025 = 4{,}84016\,.$$

We see that in the first case rank entropy is considerably less (~ 8 times) than the entropy, which corresponds to the 2 or 3 case. This circumstance can be considered indirectly as an evidence of the greater certainty of the structured system and greater certainty and, apparently, the effectiveness of leaders of each rank.

Since in all cases M = 1111 subjects,

$$H_{\eta\max} = \ln(1111) = 7{,}0130...,$$

and relative entropies is are

$$\bar{H}_{\eta_1} = \frac{H_{\eta_1}}{\bar{H}_{\eta\max}} = \frac{0{,}61504}{7{,}0130} = 0{,}0877;$$

$$\bar{H}_{\eta_2} = \frac{H_{\eta_2}}{\bar{H}_{\eta\max}} = \frac{4{,}84016}{7{,}0130} = 0{,}69026.$$

Let M_s is number "of those controlled", and q_s is number of class, to which the subject of rank A_s belongs, and the normalized weight is determined by the formula

$$\eta_{s1} = \frac{M_s}{q_s M},$$

then the entropy $H_{\eta 1}$ takes the form:

$$H_{\eta 1} = -\frac{1}{M\ln M}\left(\sum_{s=1}^{m} M_s \ln M_s + \sum_{s=1}^{m} M_s \ln q_s\right) + 1.$$

When the weight is determined by the formula

$$\eta_{s2} = \frac{1}{q_s m},$$

the entropy $H_{\eta 2}$ is assigned by the formula:

$$H_{\eta 2} = -\frac{1}{\ln M}\left(\frac{1}{m}\sum_{s=1}^{m} \ln q_s + \ln m\right).$$

Both entropies are turned into one, if in the first case $q_s = M_s = \frac{M}{m}$ and in the second case $q_s = \frac{M}{m}$:

$$H_{\eta 1} = H_{\eta 2} = 1.$$

At the beginning of this division we compare weights of ranks with the portion of consolidated resources at the disposal of given subject.

In economic theories the result frequently is characterized with the help of the production function (Kobba- Douglas, Sollou,...), where capital funds and labor come out as basic arguments. Both that and, etc is resources. In the last examples in labor resources are used, as resources.

Let the hierarchical system have only two levels: one leader, whom weight of rank of η_1 and $M - 1$ "ordinary subjects". From normalization condition:

$$\eta_1 + (M - 1)\eta_2 = 1$$

We can find that

$$\eta_2 = \frac{1 - \eta_1}{M - 1},$$

then entropy H_η takes the form:

$$H_\eta = -\eta_1 \ln \eta_1 - (M - 1)\left(\frac{1 - \eta_1}{M - 1} \ln \frac{1 - \eta_1}{M - 1}\right) =$$

$$= -\eta_1 \ln \eta_1 - (1 - \eta_1)\ln(1 - \eta_1) + (1 - \eta_1)\ln(M - 1)..$$

We see that

$$\lim_{\eta_1 \to 1} H_\eta = 0; \quad \lim_{\eta_1 \to 0} H_\eta = \ln(M - 1).$$

Value H_η reaches the maximum, when $\eta_1 = \frac{1}{M}$, in this case $H_\eta = \ln M$. Fig. 4.10 shows dependence $H_\eta(M, \eta_1)$. If we identify the value of weight $\eta_1 \in [0, 1]$ with the volume of imperious "authorities" of subject, then, as it follows from the figure, it`s possible to consider "the leader" of the group if $\eta_1 \in (\frac{1}{M}, 1]$. Moreover, when $\eta_1 = \frac{1}{M}$, then "authorities" are distributed between members of the group evenly and "leader" is absent, when $\eta_1 = 1$, the volume "of authorities" of all the rest is equal zero - they appear as "slaves".

"of authorities" of all the rest is equal zero - they appear as "slaves".

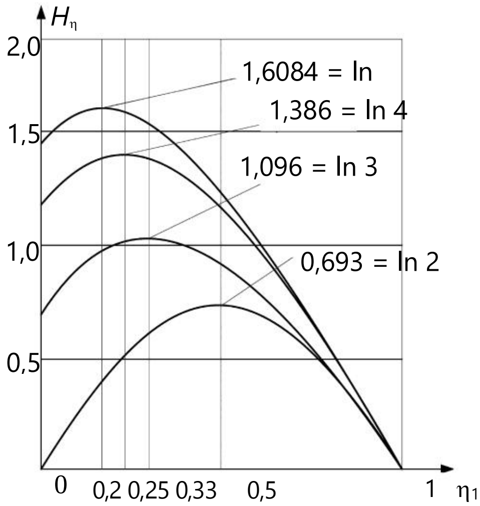


Fig. 4.10

Let`s examine the following situation: there are M of subjects in the group and three classes of equivalence

$M_1^{\sim}$ - number q_1;

$M_2^{\sim}$ - number q_2;

$M_3^{\sim}$ - number q_3 = 1 (leader).

Since $q_1 + q_2 + q_3 = M$ and the normalization condition takes form (4.93), designating $\overline{\eta}_k = x_k$, we can write, that

$$x = \frac{1 - x_3 - x_2 q_2}{M - 1 - q_2},$$

where $M - 1 - q_2 = q_1$.

The entropy of ranks takes the form:

$$H_\eta = -x_3 \ln x_3 - q_2 x_2 \ln x_2 - \left(M - 1 - q_2\right) \frac{1 - x_3 - x_2 q_2}{M - 1 - q_2} \ln \frac{1 - x_3 - x_2 q_2}{M - 1 - q_2}.$$

A necessary condition of the extremum

$$\frac{\partial H_\eta}{\partial x_3} = 0; \ \frac{\partial H_\eta}{\partial x_2} = 0$$

lead to the solution $x_2 = x_3 = \dfrac{1}{M}$, i.e., to the equality of all weights and, correspondingly, maximum uncertainty and, apparently, maximum “lack of organization". This indicates the uniform distribution of "imperious authorities". Let`s put additional constraint on the weight distribution: let $\bar{\eta}_3 = \alpha\bar{\eta}_2$ or $x_3 = \alpha x_2$. Optimum value $\bar{\eta}_2 = x_2$ is determined by the formula

$$x_{2opt} = \frac{1}{\left(M - 1 - q_2\right)\alpha^{\frac{\alpha}{q_2+\alpha}} + q_2 + \alpha}.$$

Calculations show that $x_{2opt} < x_{3opt}$, but $x_{1opt} > x_{2opt}$. This result is a consequence of the requirement of a maximality of entropy.

Dispersion and the entropy of the rank hierarchies

Let`s examine special cases of rank hierarchies and determine dispersions and entropies of ranks on these structures. These values can serve the comparative characteristics of hierarchies.

Let there be L ranks in the hierarchy and, respectively, L classes of rank equivalence. The non-normalized rank coincides with the number of the rank j, which is calculated, as above, beginning from the lowest rank $\eta_j = j = 1$. A normalized rank is determined by the formula

$$\bar{\eta}_j = \frac{\eta_j}{\sum_{k=1}^{L} \eta_k q_k} = \frac{j}{\sum_{k=1}^{L} k q_k},$$

where q_k — number of *k-th* class of rank equivalence. With this determination of the normalized rank the average normalized rank

$$\bar{\bar{\eta}} = \frac{1}{\sum_{k=1}^{L} q_k} \sum_{k=1}^{L} \bar{\eta}_j q_j = \frac{1}{\sum_{k=1}^{L} \eta_k} \frac{\sum_{k=1}^{L} j q_j}{\sum_{k=1}^{L} k q_k} = \frac{1}{\sum_{k=1}^{L} q_k} = \frac{1}{M},$$

where M is total number of the group. Dispersion of ranks is determined by the formula:

$$D_\eta = \frac{1}{M} \sum_{j=1}^{L} \left(\bar{\eta}_j - \bar{\bar{\eta}}\right)^2 q_j,$$

and entropy of ranks - by the formula:

$$H_\eta = -\sum_{j=1}^{L} q_j \bar{\eta}_j \ln q_j \bar{\eta}_j = -\sum_{j=1}^{L} q_j \bar{\eta}_j \ln \bar{\eta}_j - \sum_{j=1}^{L} q_j \bar{\eta}_j \ln q_j.$$

Let`s examine two types of rank hierarchies:

1. A "geometric" hierarchy is such hierarchy, in which numbers of classes of rank equivalence form geometric progression with the assigned denominator q.

2. A “parabolic” hierarchy, numbers of classes of equivalence q_j of which are defined as degrees of the reverse number of the rank

$$q_j = (L + 1 - j)^m,$$

where m is the index of parabola. A table gives numbers of classes of equivalence for the hierarchy, which includes 5 ranks.

j	q^{1-j}	$s_j^{m=1}$	$s_j^{m=2}$	$s_j^{m=3}$	$s_j^{m=4}$
5	1	1	1	1	1
4	q	2	4	8	16
3	q^2	3	9	27	81
2	q^3	4	16	64	244
1	q^4	5	25	125	625
$\sum_{j=1}^{L}$	$\dfrac{q^L - 1}{q - 1}$	15	55	225	967

Here $S_j = L + 1 - j$.
Normalized ranks for geometric and parabolic hierarchies are respectively equal:

$$\bar{\eta}_j = \frac{j}{\sum_{k=1}^{L} q^{L-k} k}; \quad \bar{\eta}_j = \frac{j}{\sum_{k=1}^{L} (L + 1 - k)^m k}.$$

For the „geometric” hierarchy

$$D_\eta = \frac{q-1}{q^L - 1} \sum_{j=1}^{L} \left(\frac{j}{\sum_{k=1}^{L} q^{L-k} k} - \frac{\eta - 1}{q^L - 1} \right)^2 q^{L-j};$$

$$H_\eta = -\frac{1}{\sum_{k=1}^{L} q^{L-k} k} \left(q^{L-j} j \ln j + j n^{L-j} \ln n^{L-j} \right) + \ln \sum_{k=1}^{L} q^{L-k} k.$$

For the “parabolic” hierarchy:

$$D_\eta = \frac{1}{N_m}\sum_{j=1}^{L}\left(\frac{j}{\sum_{k=1}^{L}(L+1-k)^m k} - \frac{1}{N_m}\right)^2 (L+1-k)^m .$$

$$H_\eta = -\frac{1}{\sum_{k=1}^{L}(L+1-k)^m k}\sum_{j=1}^{L}\left(\left(L+1-k\right)^m j\ln j + jn^{L-j}\ln n^{L-j}\right) +$$

$$+ j\left(L+1-k\right)^m \ln\left(L+1-k\right)^m + \ln\sum_{k=1}^{L}\left(L+1-k\right)^m k.$$

Results of calculation for the system with three classes of rank equivalence L = 3 are represented below. For the geometric hierarchy D_η and H_η they are represented as functions of the denominator q:

L = 3		
q	D_η	H_η
1	0,01801	0,114
2	0,00438	1,090
3	0,001204	1,927

In the case of the parabolic hierarchy D_η and H_η were determined depending on the index of parabola.

It`s evident from Fig. 4.11 that with an increase of the denominator q *dispersion* D_η decreases, and the entropy H_η grows. An increase in the entropy can be treated as indirect evidence of an increase of a disaggregation of imperious authorities in “the hyperbolic” hierarchy with the increase of q.

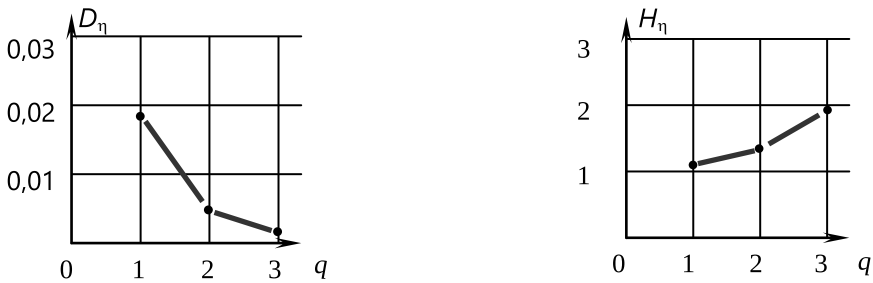


Fig. 4.11

$L = 3$		
q	D_η	H_η
1	0,004073	1,089
2	0,00097	1,0104
3	0,000121	0,8507
4	0,000013	0,7006

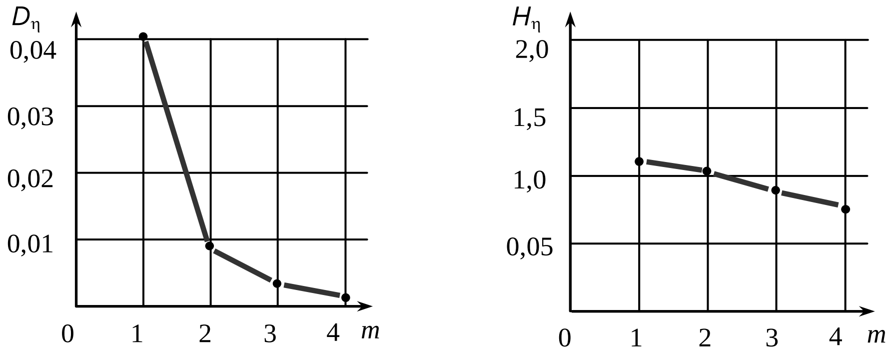


Fig. 4.12

In the case of the "parabolic" hierarchy we see that an increase in the index m, i.e., the differentiation of the number of classes of equivalence, leads to slow decrease of entropy and, apparently, to larger concentration of imperious authorities in higher ranks (Fig.4.12).

We already gave above as an example "Moses hierarchy". A calculation shows (diagram on Fig. 4.13) that this structure is close enough to the "geometric" hierarchy with nine classes (L = 9) and the denominator q = 5,25

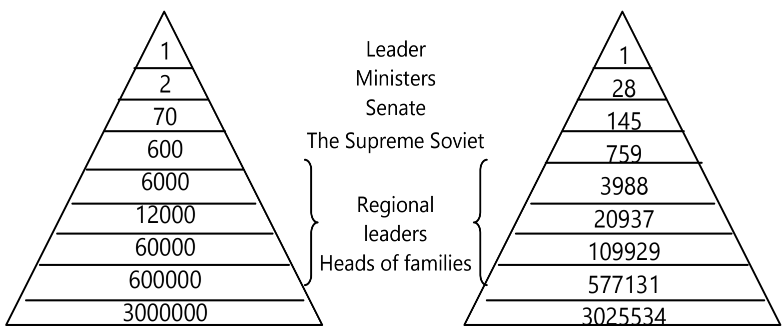


Fig. 4.13

One of the problems lies in the clearing up to what degree the formal hierarchy of the ranks $\overline{\eta}_j$ will be coordinated with the non-formal hierarchy is rating ξ_j. The index of agreement is natural to consider the appropriate correlation coefficient of Pearson $\rho_{\xi\eta}$. In this case there is a wide field for studies, since we have different models of the distribution of ranks and different models of rating indices $\xi(j)$, $\xi(j|\ i)$, $\xi(j|\ i,\ \sigma_k)$, ..., $\xi(j|\ i,\ S'_a)$ ..., available. For each type of rating indices it`s possible to calculate the appropriate correlation coefficient. It was already said, that ratings are determined not only by utilitarian factors, but also by personal factors, which we conditionally named "active" resources. Ratings, which depend on mutual utilities, will be introduced below. Finally, analogously we can investigate a rank- rating coordination from the point of view of an individual subject "*i*", or of subgroup of subjects (a coalition).

Since, in each case we have a quantitative model both rank and rating preferences this study becomes to a considerable degree an object of quantitative analysis. If we recall that models of rating preferences contain endogenous parameters, then the influence of endogenous factors (reflecting properties of psyche) on appropriate estimations happens to be an object of analysis.

It is possible to assume that the proximity of the correlation coefficients of the type $\rho_{\xi\eta}$ to +1 will be condition of subjective consensus in the group, on the contrary the basis of social tension, the index of danger of social internal conflict is proximity $\rho_{\xi\eta}$ to - 1.

Let`s touch a question about the dependence of the of rating preferences distribution on endogenous factors. In the described models of preferences, structural parameters α, β,... come out as such factors. It has already been said that the structural parameters of this type are a priori with respect to the variation principle, which forms quantitative models of preferences and, it`s most likely, individualized. However, in the following model task, for the purpose of simplification, they are considered those coinciding for all subjects of the group .

Let`s assume that the group consists of three subjects. As the model of the distribution of integral rating preferences $\xi_j$ let`s select the simplest model; assume that ξ_j depends on available resources and one endogenous parameter β:

$$\xi_j(\beta) = \frac{e^{\beta R_{dj}}}{\sum_{k=1}^{3} e^{\beta R_{dk}}},$$

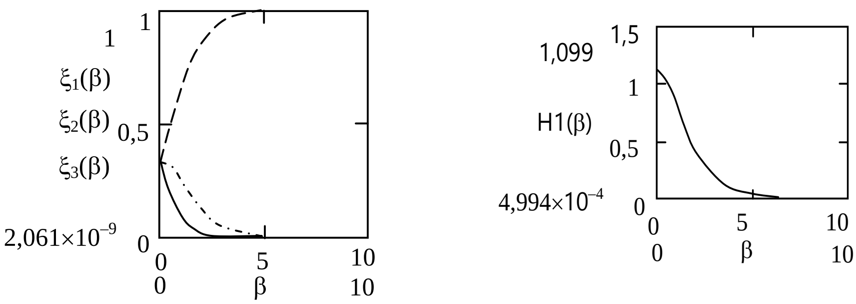
where $R_{dj} = R_j^{disp}$ is available resources of subject "*j*".

Fig. 4.14 (a) shows the dependence $\xi_j(\beta)$ ($j \in \overline{1,3}$) on the endogenous parameter $\beta \in [0, 10]$ with fixed and different values of available resources (in the arbitrary units): R_{d1} = 1; R_{d2} = 2; R_{d3} = 3. Fig. 4.14 (*b)* shows a rating entropy of the group.

It`s evident from figures that when the endogenous parameter β is low, then even larger differences in the value of available resources (in "wealth") lead to the small divergence of subjective ratings and a rating entropy remains high (a leader is noted not patently). With the increase of β even insignificant differences in available resources, is sufficient for the appearance of large differences in rating preferences and the low value of entropy testifies about the presence of the universally recognized leader. As in the case of object preferences $\pi(\sigma_k)$, the threshold of the subjective rating entropy H^*_ξ, should exist such, that the condition

$$H_\xi \le H^*_\xi$$

proves to be necessary, but not sufficient for decision making about a change in formal ranks and, correspondingly, the number of classes of rank equivalence .



a *b*

Fig. 4.14

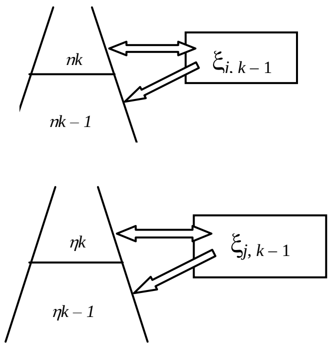
It`s necessary to note that as with respect to the object preferences $\pi(\sigma_k)$, in this case this condition "is addressed" to the subject (subjects), who is the carrier of rating preferences of this type and who is invested with by authorities about designation and past change in ranks. Without going into details, let`s say that it`s possible to examine, at least, two diagrams of the distribution of ranks:

1) "from up to bottom" by one hierarchy, or by sequential motion "down wards" on steps of hierarchy;

2) "from below to top" in the case of the electoral system of the designation of ranks. Either the system of "straight selections" or the sequential procedure of ascending the hierarchy is also fulfilled here.

In the case of sequential procedures both “from top to bottom” and “from below to top”, the carrier of distribution of the above-indicated type are subjects, who realize authorities making designation of ranks. Let`s explain the formula by diagrams, given in Fig. 4.15.

a



$$(j \in \overline{1,q_{k-1}})$$

$$H_{\xi_{k-1}}\left(k\right) \le H^*_{\xi_{k-1}}\left(k\right))$$

b

$$(j \in \overline{1,q_{k-1}})$$

$$H_{\xi_{k-1}}\left(k-1\right) \le H^*_{\xi_{k-1}}\left(k-1\right)$$

Fig. 4.15

The first case (a) realizes the scheme of the designation of ranks “from top to bottom”, i.e., subject having rank η_k accomplish selection on the lower class between subjects of ranks η_{k-1} in order to find aspirants for their transfer the level η_k. Consequently, the carrier of subjects of class q_{k-1} rating the subject of class q_k appears. For him threshold inequality is the forcing condition.

The second case (*b)* realizes the scheme of the designation of ranks “from below- top” (electoral). Selection can be accomplished according to different schemes: scheme of Condorset or Borda [113], in accordance with the model of the collective selection of Arrow. In chapter 9 of the mentioned work numerous schemes of collective selection (collective decision making) are examined. Aggregation of individual preferences is associated with essential difficulties, connected with theorem of Arrow “about the impossibility” (see also chapter 1.1).

In this case our problem does not presuppose the analysis of internally non-contradictory rules of selection. The discussion deals only with the fact that the selection is maid by taking under consideration canonical distributions of preferences of the second kind and obvious presence with each selection and for each subject of threshold values of subjective entropy.

As in the theory of utility the orderings “of candidates” based on quantitative comparisons of their ratings are possible here. In this case it`s assumed that the rating *subjective* preferences continuously depend on their arguments and in each situation, and at each moment *objectively exist* in the consciousness of subjects (members of the group). The theory of collective selection (presented in particular, in the mentioned work) can be substantially augmented, if we, with the forming of procedures of selection, consider individual rating thresholds and influence on "the elasticity" of preferences of endogenous factors.

Let`s give the calculated distribution of preferences in the same group, if they are determined not only by utilitarian factors (for example, by available resources R_j^{disp}), but also by a certain ethical principle. Let the rating depend from one side on "relative wealth", i.e., from the relation $\frac{R_j^{disp}}{R_{\max}^{disp}}$,. From the other side the tendency to "the equality" is founded in the consciousness. Let`s express this in such a way that the rating would be, with other equal conditions, the greater, the nearer "individual wealth" R_j^{disp} to the average value on the group

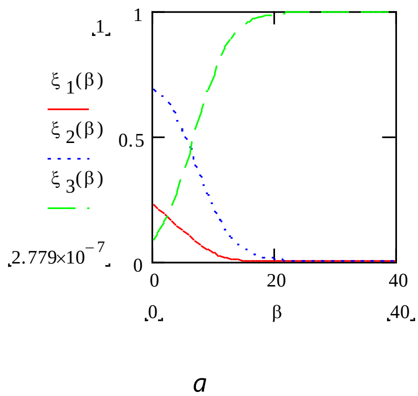
$$\bar{R}^{disp} = \frac{1}{N}\sum_{j=1}^{N} R_j^{disp}.$$

For $\xi_j(\beta)$ let`s accept the expression:

$$\xi_j(\beta) = \frac{\dfrac{\bar{R}^2}{\left(R_{dj} - \bar{R}\right)^2} e^{\beta \frac{R_{dj}}{R_{\max}}}}{\sum\limits_{k=1}^{N} \dfrac{\bar{R}^2}{\left(R_{dk} - \bar{R}\right)^2} e^{\beta \frac{R_{dk}}{R_{\max}}}},$$

where $\bar{R} = \bar{R}^{disp}$, $R_{\max} = R_{\max}^{disp}$, $R_{dj} = R_j^{disp}$.

We see, that with $R_{dj} \to \bar{R}$, $\xi_j(\beta) \to 1$ for $\forall\beta$. In the case $N = 3$, $R_{d1} = 1{,}8$; $R_{d2} = 2$; $R_{d3} = 3$; $\beta \in [0, 40]$ results of calculation are shown on Fig. 4.16.



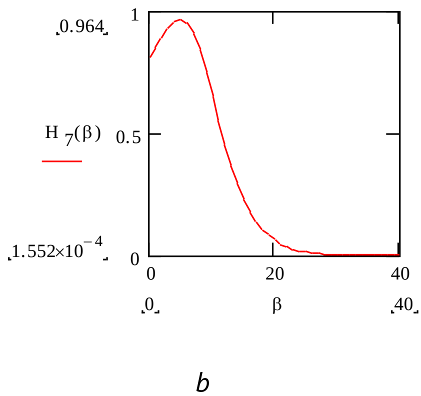


Fig. 4.16

It follows from Fig. 4.16 of (*a*) that for small β the influence of ethical factor - equalizing tendency predominates entropy grows and reaches maximum, where as with $\beta > \beta'$ distribution of ratings on the larger degree is determined by relative wealth.

Returning to the thermodynamic analogy (chapter 3.5, 3.6) and to the treatment of the parameter $T_\pi = \beta^{-1}$ as psychological "temperature" or "the temperature of emotional heating", we can make the following observation.

Let the distribution of preferences takes the form:

$$\pi_i = \frac{e^{-\beta x_i}}{\sum_{q=1}^{N} e^{-\beta x_q}}; \ \forall x_q > 0,$$

then, if $T_\pi \to \infty$ ($\beta \to 0$), then $\pi_i \to \frac{1}{N}$ — to uniform distribution. Entropy approaches the maximum value: $H_\pi \to H_{\max} = \ln N$. If $T_\pi \to 0$ ($\beta \to \infty$), distribution of π_i tends to singular distribution: $\pi(x_{\max}) \to 1$ and $\pi(x_i) \to 0$ for $\forall i \neq j$.
Entropy $H_\pi \to 0$.

Thus with an increase of "temperature" and constant exogenous situation (x_i =const) the degree of uncertainty increases, reaction on a change in ambient conditions becomes more weak.

With sufficiently high "temperature" the entropy doesn't conquer from top to "bottom" the threshold H^*, as a result of which decision making becomes impossible. On contrary, with a decrease of "temperature" the distribution of preferences tends to singular, and the degree of uncertainty tends to the minimum ($H_\pi \to 0$).

We come to analogous conclusions examining the rating preferences. Let, for example, distributions of individual rating preferences in the group of M subjects have the form:

$$\xi(j \mid i) = \frac{e^{-\beta f(j|i)}}{\sum_{q=1}^{N} e^{-\beta f(q|i)}}; \ \forall f(q|\, i) > 0.$$

Defining in this case $T_\xi = \beta^{-1}$ as "social temperature", we come to similar conclusions.

With $T_\xi \to \infty$ ($\beta \to 0$) distribution $\xi\,(j|\,i)$ approached the uniform one: $\xi\,(j|\,i) = \frac{1}{M}$, respectively the entropy $H_\xi \to H_{\max} = \ln M$.

Leveling of ratings leads to the rating "indistinguishability" of subjects in the group. In connection with this, apparently, subjective basis for self-organizing of the group disappears, simultaneously the influence of a change in exogenous factors and changing between subjects decreases.

With $T_\xi \to 0$, ($\beta \to \infty$) the rating distribution tends to singular, i.e., to the certain isolation of non-formal leader *j*. From the point of view of all remaining members of the group his ratings $\xi(j|\ i)$, $(\forall i)$ approaches the maximum value: $\xi\ (j|\ i) \to 1$, and $\xi\ (q|\ i) \to 0$ for $\forall q \neq j$. Conditionally speaking, in this case, it`s possible to indicate that in the group conditions for totalitarian organization appears.

These reasoning can be extended on other special cases of preferences distributing, including the preferences distribution of a "virtual subject" (p. 4.12).

The state of individual subject or group of subjects with high "temperature of emotional heating" cannot exist for a long time, if it will not forces from without, since in this case active resources must be expended intensively.

The consideration of some aspects of rating dynamics is given in chapter 5.

4.6. Mutual utilities.

In connection with two groups it`s possible, somewhat, to modify the concept of utility. We will distinguish the utilities of alternative $\sigma_k \in S_{aj}$ for the subject *j* ($j \in \overline{1,M}$): $U(j|\ \sigma_k)$ and the utility of subject *i* for the subject *j*: $U\ (j|\ i)$ on the set S_{aj} (the more complete designation: $U(j,\ i|\ S_{aj})$). When it is going about the certain alternative, then the utility $U(j,\ i|\ \sigma_k)$, $\sigma_k \in S_{aj}$ is examined. Subsequently we will also use the designation $\hat{U}(i \to j \mid \sigma_k)$ or $\hat{U}(i \to j \mid S_{aj})$, where the pointer indicates in what direction the utility (in this case from *i* to *j*) "is transferred" (*it returns*). The transfer of utility can look like sale, exchange, the rendering of services, help, affirmative vote the certain candidate and so forth

Thus, "the utility" does not coincide with "resources" and is more general common category. Let`s determine the aggregated utility of the subject $j \in \overline{1,M}$ *obtained* from other members of the group, relative to alternative σ_k as:

$$\breve{U}_j(\sigma_k) = \sum_{i=1}^{M} \theta\big(\xi(j,i \mid \sigma_k)\big)\breve{U}(j \leftarrow i \mid \sigma_k), \tag{4.94}$$

where θ is the function, which depends on the conditional "differential" rating $\xi(j,\ i|\ \sigma_k)$, $\breve{U}(j \leftarrow i \mid \sigma_k)$ is utility *i* for *j*, if the latter selects an alternative σ_k. In the particular case $\theta = \xi(j,\ i|\ \sigma_k)$, where $\xi(j,\ i|\ \sigma_k) = \xi(j \to i|\ \sigma_k)$ is the rating "of one giving" in the eyes "of one obtaining". In even a more special case $\theta = 1$ and the utilities are summarized without weights:

$$\tilde{U}_j(\sigma_k) = \sum_{i=1}^{M} \breve{U}(j \leftarrow i \mid \sigma_k). \tag{4.95}$$

This aggregated utility is not, however, the "normalized" utility, which was discussed earlier. Let`s note that in (4.94) to condition $i = j$ corresponds own "available" utility j relatively σ_k. The total available utility j exists on entire problematic set S_{aj}

$$\breve{U}_j\left(S_{aj}\right)=\sum_{k=1}^{N_j}\sum_{i=1}^{M}\theta\left(\xi\left(j,i\mid\sigma_k\right)\breve{U}\left(j\leftarrow i\mid\sigma_k\right)\right). \tag{4.96}$$

Let`s designate through $\hat{U}\left(j\rightarrow i\mid\sigma_k\right)$ - "returned" to $j$ utility for $i$, which is going to select the alternative $\sigma_k$, in other words, utility $j$ from the point of view of $i$ for the alternative $\sigma_k\in S_{ai}$. It`s possible to consider this as the hypothesis

$$\breve{U}\left(j\leftarrow i\mid\sigma_k\in S_{aj}\right)=\hat{U}\left(i\rightarrow j\mid\sigma_k\in S_{aj}\right). \tag{4.97}$$

From the other side, in the general case

$$\breve{U}\left(j\leftarrow i\mid\sigma_k\in S_{aj}\right)\neq\hat{U}\left(j\rightarrow i\mid\sigma_k\in S_{ai}\right) \tag{4.98}$$

on the intersection $S_{ai}\cap S_{aj}$ (if its not $\varnothing$), and only

$$\breve{U}\left(i\leftrightarrow i\right)=\hat{U}\left(i\leftrightarrow i\right).$$

Let`s examine an example. Let in the group there are 2 subjects Σ_1 and Σ_2, problematic sets coincide $S_{a1} = S_{a2}$ and contain one alternative σ_1. Let the matrix of differential ratings

$$\xi=\begin{bmatrix}\xi_{11} & \xi_{12}\\ \xi_{21} & \xi_{22}\end{bmatrix}=\begin{bmatrix}0,7 & 0,4\\ 0,3 & 0,6\end{bmatrix}.$$

Here ξ_{11} and ξ_{22} are self-appraisal, ξ_{12} - rating Σ_1 " in the eyes" of Σ_2 and vice versa, ξ_{21} - rating Σ_2 " in the eyes" of Σ_2.

Let it be further

$$\breve{U}=\begin{bmatrix}\breve{U}_{(1-1)} & \breve{U}_{(1\leftarrow 2)}\\ \breve{U}_{(2\leftarrow 1)} & \breve{U}_{(2-2)}\end{bmatrix}=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix};$$

$$\hat{U}=\begin{bmatrix}\hat{U}_{(1-1)} & \hat{U}_{(1\leftarrow 2)}\\ \hat{U}_{(2\leftarrow 1)} & \hat{U}_{(2-2)}\end{bmatrix}=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}.$$

Then we easily find that

$$\breve{U}_1=0,7\cdot 2+0,3\cdot 1=1,7;\;\;\breve{U}_2=0,6\cdot 4+0,4\cdot 3=3,6;\;\;\breve{U}_1+\breve{U}_2=5,3;$$

$$\breve{U}_1 = 0{,}7\cdot 2 + 0{,}3\cdot 3 = 2{,}3;\;\; \breve{U}_2 = 0{,}6\cdot 4 + 0{,}4\cdot 1 = 2{,}8;\;\; \breve{U}_1 + \breve{U}_2 = 5{,}1.$$

Consequently

$$\sum_{j=1}^{M}\hat{U}_j \neq \sum_{j=1}^{M}\breve{U}_j.$$

That is mutual aggregated utility in the general case are unequal. The aggregated utility j relative to the alternative $\sigma_k \in S_{aj}$ is composed of "that returned" and "that obtained" utilities:

$$U_j(\sigma_k) = \breve{U}_j(\sigma_k) - \hat{U}_j(\sigma_k).$$

Let`s assume that "the exchange" of utilities is accomplished with weights by the equal to the differential ratings.

Utility i "for others" $\hat{U}(j \leftarrow i)$ superimposes on i certain obligations in the fact of $j \in \overline{1,M}$ and after alternative choice requires the realization of expenditures (time, labor, money, material resources, information,...).

The utility of "others" for i increase for i the possibility to manage successfully the problem P: $(\sigma_0 \to \sigma_k)$ and, on the contrary, aggravating obligations on j ("returning") are superimposed. Therefore, the more is $\hat{U}(j \leftarrow i)$, the less can be "*rating j in the eyes of i*", and, on the contrary, the greater is $\breve{U}(j \to i)$, is the higher "*rating j in the view of i*".

Quantitatively this circumstance can be expressed as follows:

$$U_i(\sigma_k) = \sum_{j=1}^{M}\theta\big(\xi(j,i \mid \sigma_k)\big)\Big[\breve{U}(j \to i \mid \sigma_k) - \hat{U}(j \leftarrow i \mid \sigma_k)\Big]. \tag{4.99}$$

Here θ (...) is the monotonically- increasing function, $\xi(i, j| \sigma_k)$ is rating "*of j* in the eyes of *i*", relative to alternative σ_k. Total aggregated utility i on his set of alternatives S_{ai}

$$U_i = \sum_{k=1}^{N_i}\sum_{j=1}^{M}\theta\big(\xi(i,j \mid \sigma_k)\big)\Big[\breve{U}(j - i \mid \sigma_k) - \hat{U}(j \leftarrow i \mid \sigma_k)\Big] = \sum_{k=1}^{N_i} U_i(\sigma_k). \tag{4.100}$$

Below we will assume that

$$\theta = \xi(j, i| \sigma_k).$$

Since effectiveness introduced earlier (or, in the narrower sense - risk) is connected with the preferences distribution of the first kind $\pi_i(\sigma_k)$,it`s possible for the aggregated utility U_i to used instead of (4.100) another expression, which, in this case. We name the function of effectiveness for i:

$$\varepsilon_{\pi_i} = \sum_{k=1}^{N_i} \pi_i\left(\sigma_k\right) U_i\left(\sigma_k\right). \tag{4.101}$$

Positive $\breve{U}$ and negative $\hat{U}$ utilities used here are "utilities of subjects" in the group stimulated by relations between each other and, are consequently individualized utilities.

With the help of introduced of utility functions and effectiveness (risk) it`s possible to try  to obtain canonical rating distributions  similarly to current to that it was done for preferences distribution of the I kind on the set of alternatives $S_{ai}$. It`s necessary to make number of additional assumptions for it.

Since the preference distribution on the set of subjects S_ξ it is, as in the case of set S_a, the preferences distribution rating of alternatives, although another type, it`s completely possible to assume that the entropy principle is applicable and selection of distribution is optimal in a certain.

In connection with this, additional assumptions, mentioned above relate to the selection of the form of subjective entropy and to the structure of the optimized functional. Let`s examine two types of entropies, the carrier of which is the subject Σ_i:

$$H_{\pi_i} = -\sum_{k=1}^{N_i} \pi_i\left(\sigma_k\right) \ln \pi_i\left(\sigma_k\right); \tag{4.102}$$

$$H_{\xi_i} = -\sum_{j=1}^{M} \sum_{k=1}^{N_i} \xi\left(i, j \mid \sigma_k\right) \ln \xi\left(i, j \mid \sigma_k\right). \tag{4.103}$$

One of assumptions lies in the fact that the subject solves problems of the selection of distributions $\pi_i(\sigma_k)$ and $\xi(j, i| \sigma_k)$ separately (consecutively) using in this case one and the same function of effectiveness. It seems natural to assume that the distribution of preferences of the first kind $\pi_i(\sigma_k)$, if subject i is the member of the group, depends on what help he can obtain with the alternative choice σ_k. In its turn the distribution of differential it`s rating, i.e., ratings, that relates to this alternative σ_k: $\xi(j, i| \sigma_k)$ depends on an alternative and preference $\pi_i(\sigma_k)$ by subject, who assigns ratings. Fig. 4.17 shown in the geometric interpretation of the interdependence of preferences distributions of the first and second kind (the case of the group of 7 subjects: M = 7 and 4 alternatives: N = 4, is shown as an example).

We would want to construct such function of preferences of the II kind $\xi(i\,j| \sigma_k)$, which would satisfy some prior guesses:

1. If utility j for i is equal zero, then $\xi(i\,j|\,\sigma_k) = 0$.

2. If $\hat{U}(j \to i \mid \sigma_k) = \breve{U}(j \leftarrow i \mid \sigma_k)$, that is rating $\xi(i\,j|\,\sigma_k)$ is "neutral", but not equal zero. In favor of this speaks the simple reasoning: "j has goods, which i requires, i has money or another goods, which j requires". In this case j and i are mutually useful, but the presence of other members of the group makes it possible to accomplish the exchange with other subjects on the more advantageous or less advantageous conditions. If exchange conditions of exchange $\breve{U} \leftrightarrow \hat{U}$ are identical for all j, then their ratings "in eyes i" must be identical.

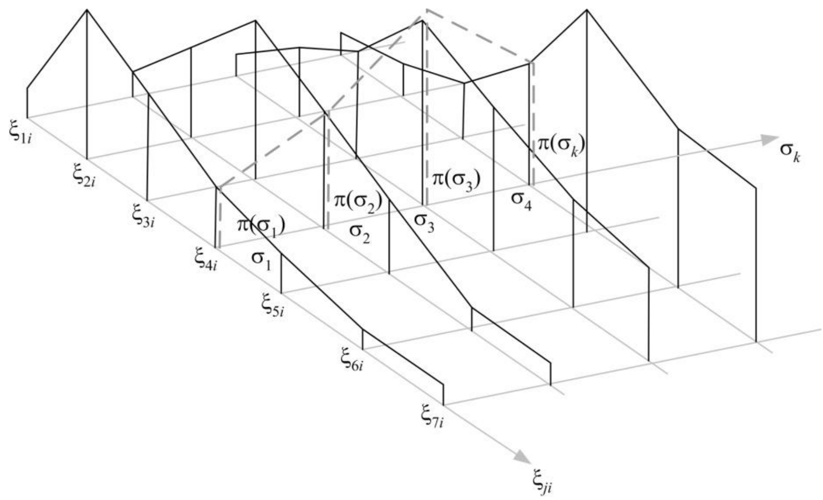


Fig. 4.17

3. Finally, it can be assumed that rating $\xi(i, j|\,\sigma_k)$ depends on a relative contribution j in comparison with the available utility i ("its own" utility): $U(i–i|\sigma_k)$, moreover $\hat{U}(i - i \mid \sigma_k) = \breve{U}(i - i \mid \sigma_k)$.

Let`s determine the relative weighed utility of subject "j in eyes i" by the formula

$$\bar{U}(i, j \mid \sigma_k) = \frac{\breve{U}(j \to i \mid \sigma_k) - \hat{U}(j \leftarrow i \mid \sigma_k) - U(i - i \mid \sigma_k)\left(M^{-1} - 2\delta_{ij}\right)}{\breve{U}(j \to i \mid \sigma_k)}, \quad (4.104)$$

where δ_{ij} is Kronecker's symbol.

This function has following properties:

$$1.\ \breve{U}(j \to i \mid \sigma_k) \to 0 \Rightarrow \bar{U}(i, j \mid \sigma_k) \to -\infty. \quad (4.105)$$

$$\breve{U}\left(j\to i\mid\sigma_k\right)\to\hat{U}\left(j\leftarrow i\mid\sigma_k\right)\Rightarrow\bar{U}\left(i,j\mid\sigma_k\right)\to-\frac{U\left(i-i\mid\sigma_k\right)}{\breve{U}\left(j\to i\mid\sigma_k\right)}\frac{1}{M},\quad(4.106)$$

The latter means that $\bar{U}$ becomes of a negative value. The greater on the module, the greater on "its own" utility $U\left(i-i\mid\sigma_k\right)$ with respect to the $\breve{U}\left(j-i\mid\sigma_k\right)$ "transferred *j* for *i*". We see that the sum of relative utilities with the fulfillment of conditions 2 is equal:

$$\sum_{j=1}^{M}\bar{U}\left(i,j\mid\sigma_k\right)=-U\left(i-i\mid\sigma_k\right)\frac{1}{M}\sum_{j=1}^{M}\frac{1}{\breve{U}\left(j\to i\mid\sigma_k\right)}.\quad(4.107)$$

If $i=j$ and $\breve{U}\left(j\to i\mid\sigma_k\right)=U\left(i-i\mid\sigma_k\right)$, that

$$\sum_{j=1}^{M}\bar{U}\left(i,j\mid\sigma_k\right)=\sum_{j=1}^{M}\left(\frac{1}{M}-2\delta_{ij}\right)=-1.\quad(4.108)$$

As we see, the similar function $\bar{U}\left(i,j\mid\sigma_k\right)$ satisfies conditions advanced above.

The next step is the construction of the effectiveness function, more precise, *subjective effectiveness*, which must connect subjective preferences of the I and II kind with utilities .

Apparently, the preferences distribution of the I kind $\pi_i(\sigma_k)$ of subject *i* is connected with utilities aggregated on group:

$$\bar{U}_i\left(\sigma_k\right)=\sum_{j=1}^{M}\bar{U}\left(i,j\mid\sigma_k\right).\quad(4.109)$$

Each subject *i* distributes preferences on two sets: the set of alternatives of the I kind σ_k: S_{ai} and the set of subject's Σ_j: $S_{\xi i}$. Both sets are subjective constructions and bear especially individual nature. As with respect to the first set, whose properties already partially were discussed, in the relation to the set S_ξ it`s necessary to say that inclusion of various subjects in it is the exceptional scope of subject *i* ("the carrier" of ratings distribution).

Formula (4.109) can be modified, if we assume that values $\bar{U}\left(i,j\mid\sigma_k\right)$ are some *specific* utility, and complete utility for *i*, in reference on *j* can be represented as the product $\bar{U}\left(i,j\mid\sigma_k\right)$ on the appropriate differential rating.

Then

$$\bar{U}_i^*(\sigma_k) = \sum_{j=1}^{M_i} \bar{U}(i,j \mid \sigma_k)\xi(i,j \mid \sigma_k). \tag{4.110}$$

If $\pi_i(\sigma_k)$ is the function of preferences of the I kind ($\sigma_k \in S_{ai}$), then the function of effectiveness on S_{ai} can be represented in the form:

$$\varepsilon_{\pi_i} = \sum_{k=1}^{N_i} \pi_i(\sigma_k)\bar{U}_i^*(\sigma_k) = \sum_{k=1}^{N_i}\sum_{j=1}^{M_i} \pi_i(\sigma_k)\xi(i,j \mid \sigma_k)\bar{U}(i,j \mid \sigma_k). \tag{4.111}$$

Product $\pi_i(\sigma_k)\xi(i, j| \sigma_k)$ resembles a relationship for the probability of the product of random events. In this case: the preference of pair $(\sigma_k, j) \in S_{ai}\times S_{\xi i}$ is equal to the product "of preference" $\sigma_k \in S_{ai}$ on "the preference" $j \in S_{\xi i}$ when σ_k is selected from S_{ai}.

In the simpler version

$$\varepsilon_{\pi_i} = \sum_{k=1}^{N_i}\sum_{j=1}^{M_i} \pi_i(\sigma_k)\bar{U}(i,j \mid \sigma_k). \tag{4.112}$$

Here with the forming of function of effectiveness differential ratings are not considered.

The function of effectiveness on $S_{\xi i}$ can be accepted in the form:

$$\varepsilon_{\xi_i} = \sum_{k=1}^{N_i}\sum_{j=1}^{M_i} \xi(i,j \mid \sigma_k)\bar{U}(i,j \mid \sigma_k). \tag{4.113}$$

It seems that the sequence of the distributions assignment is such, that at first the subject assigns rating (i.e. the preference of the II kind) depending on the expected utility $j \in S_{ai}$ (not aggregated on alternatives function of effectiveness)

$$\varepsilon_{\xi_i}(\sigma_k) = \sum_{j=1}^{M_i} \xi(i,j \mid \sigma_k)\bar{U}(i,j \mid \sigma_k). \tag{4.114}$$

Then, after ratings $\xi(j, i| \sigma_k)$ establishment subject i selects (assigns) preferences $\pi_i(\sigma_k)$, using a function $\varepsilon_{\pi i}$ (4.111) (or (4.112)). In this case the distribution $\pi_i(\sigma_k)$ and $\xi(j, i| \sigma_k)$ are not connected with each other.

Let`s examine the criterion, whose extremalization corresponds to selection of ratings

$$\Phi_{\xi_i}(\sigma_k) = -\sum_{j=1}^{M_i} \xi(i,j \mid \sigma_k)\ln\xi(i,j \mid \sigma_k) + \beta\sum_{j=1}^{M_i} \xi(i,j \mid \sigma_k)\bar{U}(i,j \mid \sigma_k) + \gamma\sum_{j=1}^{M_i} \xi(i,j \mid \sigma_k), \tag{4.115}$$

last term reflects normalizing condition

$$\sum_{j=1}^{M_i} \xi\left(i, j \mid \sigma_k\right) = 1; \ \forall\sigma_k \in S_{ai}.$$

The distribution $\xi(i, j| \sigma_k)$ we can find from the condition

$$\frac{\partial\Phi_{\xi_i}\left(\sigma_k\right)}{\partial\xi\left(i, j \mid \sigma_k\right)} = 0; \quad (j \in \overline{1, M_i}),\ \sigma_k \in S_{ai}.$$

It has from:

$$\xi\left(j \mid i, \sigma_k\right) = \frac{e^{\beta\bar{U}\left(j|i,\sigma_k\right)}}{\sum_{q=1}^{M_i} e^{\beta\bar{U}\left(q|i,\sigma_k\right)}}. \tag{4.116}$$

In formulas (4.111) and (4.112) relative utilities $\bar{U}_i^*\left(\sigma_k\right)$ must have another sense in comparison with the situation in the previous case when it was going about I forming of rating distributions. Function $\bar{U}_i^*\left(\sigma_k\right)$ must be arranged, in such a way, that under condition

$$\bar{U}_i^*\left(\sigma_k\right) \to 0, \tag{4.117}$$

the corresponding preference also became zero. If we select, for example, $\bar{U}_i^*\left(\sigma_k\right)$ in the form:

$$\bar{U}_i^*\left(\sigma_k\right) = \bar{U}_i\left(\sigma_k\right) + \frac{1}{\beta}\alpha \ln \bar{U}_i\left(\sigma_k\right),$$

then after the appropriate modification of the functional:

$$\Phi_{\pi_i} = -\sum_{k=1}^{N_i} \pi_i\left(\sigma_k\right) \ln \pi_i\left(\sigma_k\right) + \beta\sum_{k=1}^{N_i} \pi_i\left(\sigma_k\right)\bar{U}_i^*\left(\sigma_k\right) + \gamma\sum_{k=1}^{N_i} \pi_i\left(\sigma_k\right) \tag{4.118}$$

We find

$$\pi_i\left(\sigma_k\right) = \frac{\bar{U}_i\left(\sigma_k\right)^{\alpha} e^{\beta\bar{U}_i\left(\sigma_k\right)}}{\sum_{q=1}^{N_i} \bar{U}_i\left(\sigma_k\right)^{\alpha} e^{\beta\bar{U}_i\left(\sigma_q\right)}}. \tag{4.119}$$

This function of preferences ensures fulfillment of conditions (4.117): if $\bar{U}_i\left(\sigma_k\right) \to 0,$ then the preference $\pi_{iopt}(\sigma_k) \to 0$ with any $\alpha > 0$.

If, with the definition of rating preferences utility is understood on the whole as the presence of objective possibilities, and means of the solution of the corresponding problem P: $(\sigma_0 \to \sigma_k)$, then with the determination of preferences of the I kind $\pi_i(\sigma_k)$ is intended the utility for the subject, connected with solution of problem P: $(\sigma_0 \to$

σ_k). It seems that, in certain cases indicated utilities can coincide or partially coincide. It`s possible, for example, to consider that "participation" in the group of subject j increases or " decreases" the individual utility for i of alternative σ_k. However, in this case, the relative utility $\bar{U}_i(\sigma_k)$ must be "designed" in such a way that it would not become entirely zero, if contribution j $\breve{U}(j \to i \mid \sigma_k) \to 0$. If we use ourselves a model of a type (4.119), then the absolute utility can be undertaken

$$U(i, j \mid \sigma_k) = \breve{U}(j \to i \mid \sigma_k) - \hat{U}(j \leftarrow i \mid \sigma_k) - U(j - i \mid \sigma_k)\left(\frac{1}{M} - \delta_{ij}\right).$$

If an effectiveness is used in the form (4.112), the distribution $\pi_i(\sigma_k)$ is not connected straight with the distribution $\xi(i, j \mid \sigma_k)$, however ,if we accept for $\varepsilon_{\pi i}$ an expression (4.111), then, the following optimum distribution $\pi_i(\sigma_k)$ will come out:

$$\pi_i(\sigma_k) = \frac{\sum_{j=1}^{M_i} \xi(j, i \mid \sigma_k)\bar{U}(j, i \mid \sigma_k) e^{\beta \sum_{j=1}^{M_i} \xi(j, i \mid \sigma_k)\bar{U}(j, i \mid \sigma_k)}}{\sum_{q=1}^{N_i}\left(\sum_{j=1}^{M_i} \xi(j, i \mid \sigma_k)\bar{U}(j, i \mid \sigma_k) e^{\beta \sum_{j=1}^{M_i} \xi(j, i \mid \sigma_k)\bar{U}(j, i \mid \sigma_k)}\right)}. \tag{4.120}$$

It`s evident here that the distribution of preferences $\pi_i(\sigma_k)$ depends on distribution of ratings in the group $\xi(j, i \mid \sigma_k)$ and, therefore, appears after the subject distributed his rating preferences. The mentioned sequence of preferences genesis isn`t compulsorily strict most likely, the certain iterative process occurs. The complexity of given formulas must not frighten, first of all, because, the real mental processes aren`t less, but most likely are more complex, secondly, because due to actual conditions the number of alternatives $\sigma_k$ – $N_i$, studied simultaneously, and the number of subjects $M_j$, between whom ratings are distributed isn`t big. It`s considered that optimum values:

$$N_{i\text{opt}} \le 5 \ldots 6;$$

$$M_{j\text{opt}} \le 6 \ldots 7.$$

It`s obvious that both processes: the determination of distributions $\pi_i(\sigma_k)$ and the determination of distributions $\xi(i, j \mid \sigma_k)$ "are spread" in time and, that different subjects have individual habits and methods of study and solution of problem - resource situations and the special feature of subjective analysis is also manifested.

Introduction of mutual utilities $\hat{U}$ and $\breve{U}$ allows to reflect concepts of individualism and collectivism within the framework of a subjective analysis in the formalized form. If there is a subject i, for whom all $\hat{U}_i$ and $\breve{U}_i$ are equal zero, then his position answers the maximum individualism: it can manage without a group and group can manage without him. Generally, the value of mutual utilities, characterizes "the tightness" of the connection the group. *It all mutual utilities tend to zero for all members the group decomposes.*

We examined above one of the possible schemes of the generation of preferences distributions, which can be illustrated by the following logical scheme:

$$\Longrightarrow \bar{U}(i,j \mid \sigma_k) \rightarrow \xi(i,j \mid \sigma_k) \searrow \quad \swarrow \bar{U}^{*}(i,j \mid \sigma_k)$$
$$\pi_i(\sigma_k)$$

Question of utilities aggregation which must be solved in the form of functions of effectiveness all first and, secondly, the selection of the form of utilities for:

- rating estimations (preferences of the II kind);
- estimations of alternatives (preferences of the I kind)

are fundamental here.

Similarly, as it was done for preferences of the first kind $\pi(\sigma_k)$ in chapter 3, where *positive* and *negative* preferences $\pi^{-}(\sigma_k)$ and $\pi^{+}(\sigma_k)$, connected with "*utilities*" $U(\sigma_k)$ and "*the harmfulness*" $L(\sigma_k)$, were introduced we can introduce positive and negative ratings $\xi^{+}(i, j|\sigma_k)$ and $\xi^{-}(i, j|\sigma_k)$, which can be connected with expected "benefit j for i" $\bar{U}(i,j \mid \sigma_k)$ and expected "harm j for i" $\bar{L}(i,j \mid \sigma_k)$. In this case it`s assumed that the subject $i$ can carry out an appropriate analysis separately, in a certain time sequence. Criteria of selection, which organically "build" inside the consciousness of subject can also be separated, so to speak, "in time and space": $\Phi^{+}_{\xi i}$ and $\Phi^{-}_{\xi i}$. We don`t give appropriate computations, since they are practically analogous to those, which are given in chapter 3. We are ready to repeat here the *hypothesis* about the fact that with the determination of a "negative" ratings ξ^{-}_{j} carrier - the subject i is found in uncomfortable mental conditions, that correspond to maximum entropy of rating, but with the determination of a "positive" ξ^{+}_{j} is in the most comfortable state, that correspond to smaller entropy of rating.

It`s obvious that distribution ξ^{+} and ξ^{-} generate two entropies: H^{+}_{ξ} and H^{-}_{ξ}, respectively two forms of the information:

$$I^{+}_{\xi} = H^{+}_{\xi} - H^{+}_{\xi}(|A);$$

$$I^{-}_{\xi} = H^{-}_{\xi} - H^{-}_{\xi}(|A),$$

where A is certain event (or message), which leads to a change in the entropies H^{+}_{ξ} and H^{-}_{ξ}.

The problem of the agreement of the scheme "*of the exchange of utilities*" examined above with Pigu-Dalton principle, which sets substantial limitations on parameters of an exchange (see Chapter 3) is curtail.

In the conclusion of this paragraph let`s examine one special case. Let`s designate $\breve{U}\left(j \to i \mid \sigma_k\right) = U_{ji}$ and $\xi(i, j|\sigma_k) = \xi^+_{ji}$, then "a positive" criterion let`s write in the form:

$$\Phi^+_{\xi i} = -\sum_{j=1}^{M} \xi^+_{ji} \ln \xi^+_{ji} + \beta \xi_{ii} U_{ii} + \gamma \sum_{j=1}^{M} \xi^+_{ji}.$$

Assume that for all $j \neq i$ $U_{ji} = U_e$, i.e., "no one" renders help i. Criterion takes the form:

$$\Phi^+_{\xi i} = -\sum_{j=1}^{M} \xi^+_{ji} \ln \xi^+_{ji} + \gamma \sum_{M-1} \xi_{ji} U_e + \beta \sum_{j=1}^{M} \xi^+_{ji} U^+_{ji} + \gamma \sum_{j=1}^{M} \xi^+_{ji}.$$

From conditions

$$\left.\frac{\partial \Phi^+_{\xi i}}{\partial \xi_{ji}}\right|_{j \neq i} = -\ln \xi_{ji} + \beta U_e - 1 + \gamma = 0; \quad \left.\frac{\partial \Phi^+_{\xi i}}{\partial \xi_{ii}}\right|_{j = i} = -\ln \xi_{ii} + \beta U_{ii} - 1 + \gamma = 0,$$

we find

$$\xi^+_{ji} = C_1 e^{\beta U_e}; \quad \xi^+_{ii} = C_2 e^{\beta U_{ii}},$$

here $C_1 = C_2 = C = e^{-1+\gamma}$.

It can be find from normalizing condition:

$$C e^{\beta U_e}\left(M-1\right) + C e^{\beta U_{ii}} = 1,$$

from where

$$C = \frac{1}{e^{\beta U_e}\left(M-1\right) + e^{\beta U_{ii}}}.$$

Then

$$\left.\xi^+_{ji}\right|_{j \neq i} = \frac{e^{\beta U_e}}{e^{\beta U_e}\left(M-1\right) + e^{\beta U_{ii}}} = \frac{1}{M-1+e^{\beta\left(U_{ii}-U_e\right)}};$$

$$\xi^+_{ii} = \frac{e^{\beta\left(U_{ii}-U_e\right)}}{M-1+e^{\beta\left(U_{ii}-U_e\right)}}.$$

All subjects $j \neq i$ have identical ratings. If their utilities $U_e \to 0$, then all $\left.\xi^+_{ji}\right|_{j \neq i} \to \dfrac{1}{M-1+e^{\beta U_{ii}}}$ (and with $U_{ii} \to \infty$, $\xi^+_{ji} \to 0$); $\xi^+_{ii} \to \dfrac{e^{\beta U_{ii}}}{M-1+e^{\beta U_{ii}}}$ (and with $U_{ii} \to \infty$, $\xi^+_{ii} \to 1$). Thus, normalizing conditions are fulfilled. With the simultaneous tendency on zero U_e and U_{ii} $\xi^+_{ji} \to \xi^+_{ii} \to \frac{1}{M}$, i.e., all ratings are equal with the universal "utility" and *complete equality* comes.

We see that when all utilities are identical: $U_e = U_{ii}$, then all ratings are identical $\xi^+_{ji} = \xi^+_{ii} = \frac{1}{M}$. Finally if $U_{ii} = 0$ (own utility of i is equal zero), then ratings are distributed as follows

$$\xi^+_{ji} = \frac{1}{M - 1 + e^{-\beta U_e}};$$

$$\xi^+_{ii} = \frac{e^{-\beta U_e}}{M - 1 + e^{-\beta U_e}}.$$

Relations of ratings

$$\frac{\xi^+_{ji}}{\xi^+_{ii}} = e^{\beta U_e},$$

i.e., the greater absolute utility U_e, the greater this relation is. With $U_e \to \infty$, rating relation also approaches infinity.

An increase in relation of ratings bears an *exponential nature*. Analogical results are obtained, if we use a function of the utility of the form:

$$U^*_{ji} = U_{ji} + \alpha \ln U_{ji},$$

accordingly

$$U^*_{ji} = U_e + \alpha \ln U_e;$$

$$U^*_{ii} = U_{ii} + \alpha \ln U_{ii}.$$

Then ratings are determined by formulas:

$$\xi^+_{ji} = \frac{U_e e^{\beta U_e}}{(M-1) U_e e^{\beta U_e} + U_{ii} e^{\beta U_e}}; \ (j \neq i)$$

$$\xi^+_{ii} = \frac{U_{ii} e^{\beta U_{ii}}}{(M-1) U_e e^{\beta U_e} + U_{ii} e^{\beta U_{ii}}}.$$

We see here that with $U_e \to 0$; $\xi^+_{ji} \to 0$; $\xi^+_{ii} \to 1$, and with $U_e \to U_{ii}$ $\xi^+_{ji} \to \xi^+_{ii} \to \frac{1}{M}$.

Normalizing conditions in each case are satisfied.

4.7. Consolidation of resources in group.

The situation, when in a group there are corporate problems is examined. In this case members of the group can agree about the association of efforts for their solution. The discussion deals with a stage of analysis till the moment of a decision making about the purpose selection. Depending on that how individual alternatives are defined, who is the distributer of consolidated resources, how responsibility for the realization of a corporate alternative is distributed, diverse variants of the resources consolidation are possible. We will examine some special cases, but also, simple examples. Let`s define how as a result consolidations change preferences, individual entropies, connected with a change in the entropy of the portion of information.

Example 1

The group, which consists of two subjects S_ξ: (Σ_1,Σ_2) is examined. Each of them initially studies two alternatives so that

$$S_{a1}: (\sigma_1, \sigma_2); S_{a2}: (\sigma_2, \sigma_3).$$

Coinciding alternative is alternative σ_2 and let this alternative be such, one that following condition satisfies:

$$P_1: (\sigma_{01} \to \sigma_2) \Leftrightarrow P_2: (\sigma_{02} \to \sigma_2), \tag{4.121}$$

where P_1 and P_2 are corresponding problems of the first and second subject , σ_{01} and σ_{02} their initial states. With the validity of condition (4.121) the alternative σ_2 can be realized by each of subjects or by their joint efforts (with the presence of sufficient resources). If the condition occurs

$$P_1: (\sigma_{01} \to \sigma_2) \Rightarrow P_2: (\sigma_{02} \to \sigma_2), \tag{4.122}$$

the solution of corporate problem can be charged to the first subject, or it can be solved together. If, on

$$P_2: (\sigma_{02} \to \sigma_2) \Rightarrow P_1: (\sigma_{01} \to \sigma_2), \tag{4.123}$$

the solution of corporate problem charges the second subject or it is solved together.

Supplement set S_{a1} and S_{a2} so that they would coincide, i.e., contained all three alternatives

$$S'_{a1} = S'_{a2} = S_a: (\sigma_1, \sigma_2, \sigma_3).$$

This method, described before makes it possible to organize each time the overall "field" for the game for several subjects. In this case it`s necessary to assume that $\pi_1(\sigma_3) = \pi_2(\sigma_1) = 0$. In the designation $\pi_j(\sigma_i)$ j is the number of subject, i is the number of alternative. Let take the inequalities take place:

$$R_1^r\left(\sigma_2\right) > R_1^r\left(\sigma_1\right);\ R_1^r\left(\sigma_1\right) < R_1^d < R_1^r\left(\sigma_2\right);$$

$$R_2^r\left(\sigma_2\right) > R_1^r\left(\sigma_3\right);\ R_2^r\left(\sigma_3\right) < R_2^d < R_2^r\left(\sigma_2\right).$$

Thus, corporate alternative σ_2 can be realized by none of subjects separately; therefore it should be deliberately assumed that $\pi_1(\sigma_2) = 0$; $\pi_2(\sigma_2) = 0$. In the initial situation of distribution $\pi_1(\sigma_i)$ and $\pi_2(\sigma_i)$ on the extended set S_a is given in table 1:

Tabele 1

σ_i	σ_1	σ_2	σ_3
$\pi_1(\sigma_i)$	1	0	0
$\pi_2(\sigma_i)$	0	0	1

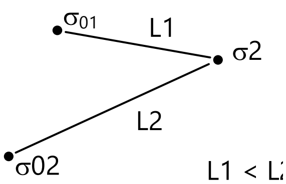
and the entropies $H_{\pi 1} = 0$; $H_{\pi 2} = 0$. We further assume that preferences are simulated by the function

$$\pi_j\left(\sigma_i\right) = \frac{e^{-\beta x_{ji}}}{\sum_{k=1}^{N} e^{-\beta x_{ji}}}, \tag{4.124}$$

where $x_{ji} = \dfrac{R_j^r\left(\sigma_i\right)}{R_j^d}$; the available resources R_j^d are universal.

We will consider available resources such that they ensure "the attainability" of alternative σ_1 for the first subject and alternatives σ_3 for the second subject. In other words problems $\sigma_{01} \to \sigma_1$ and $\sigma_{02} \to \sigma_3$ are solvable. Let, furthermore, the condition is satisfied

$$R^1{}_r(\sigma_2) < R^2{}_r(\sigma^2).$$



The latter means that it`s profitable to propose to solve the problem $\sigma_{01} \to \sigma_2$ to the first subject. The difference in required resources for achievement σ_2 is, possibly, caused by a difference in initial positions of subjects.

Then the second subject transfers resources to the first one, so that the relation $\sigma_2 \rangle \sigma_1$ would be carried out,(if is possible in the gives situation).

After the transfer of resources, available resources of the first subject $R_1^{d'} = R_1^r\left(\sigma_2\right) + \delta$; ($\delta > 0$), the transferred value, thus, is:

$$R_1^r\left(\sigma_2\right) - R_1^d + \delta.$$

As a result of consolidation

$$x_{11} = \frac{R_1^r(\sigma_1)}{R_1^d}; \; x_{12} = \frac{R_1^r(\sigma_2)}{R_1^r(\sigma_2)+\delta};$$

$$x_{21} = \frac{R_1^r(\sigma_2)}{R_1^r(\sigma_2)+\delta}; \; x_{23} = \frac{R_2^r(\sigma_3)}{R_2^d + R_1^d - R_1^r(\sigma_2) - \delta}.$$

Subject "2" transferring resources on subject "1", preserves interest to the alternative σ_2 and, in connection with this we assume that $x_{12} = x_{21}$ (subject 1 " builds a bridge", but use there will be for both. In this case "2" participates by the part of his resources). Since in the initial state S_{a1} and S_{a2} contain one alternative, each the entropies $H^0_{\pi1} = H^0_{\pi2} = 0$.

For a numerical example let`s select following initial data:

$R_1^r(\sigma_1) = 3{,}0$; $R_1^r(\sigma_2) = 3{,}5$; $R_2^r(\sigma_2) = 4{,}0$; $R_1^d = 3{,}1$; $R_2^d = 3{,}8$.

Here $R_1^r(\sigma_2) < R_2^r(\sigma_2)$. Let "2" transfer to "1" a quantity of resources 0,7 so that after the transfer $R_1^{d'} = 3{,}8$; $R_2^{d'} = 3{,}1$. In this case

$$x_{11} = \frac{3}{3{,}1} = 0{,}96774; \; x_{12} = \frac{3{,}5}{3{,}8} = 0{,}92105;$$

$$x_{22} = \frac{3{,}5}{3{,}6} = 0{,}92105; \; x_{23} = \frac{3}{3{,}1} = 0{,}96774.$$

It is clear that $x_{12} = x_{21}$, both subjects approached corporate alternative equally, as subject "1". We obtain $\pi_1(\sigma_1) = 0{,}48834$; $\pi_1(\sigma_2) = 0{,}51166$; $\pi_2(\sigma_1) = 0{,}51166$; $\pi_2(\sigma_3) = 0{,}48834$. Entropies after the transfer of resources $H_{\pi1} = H_{\pi2} = 0{,}69281$, increase in entropies $\Delta H_\pi = 0{,}00337$. This numerical example has particular nature. It not always succeeds as a result of resources transfers to ensuring the preferability of corporate problem (alternative). In order to guarantee the condition $\pi_1(\sigma_2) > \pi_1(\sigma_1)$ and $\pi_2(\sigma_2) > \pi_2(\sigma_3)$ the transfers are realized with "encouraging surplus". The scheme examined below is deprived of deficiency of scheme given above if it about problem corporatization consolidated use of resources.

Example 2

Subjects of the group consisting of two subjects ($M = 2$) have one and the same set of permissible states S_σ: (σ_1, σ_2, σ_3). States σ_1 and σ_3 are attainable by each subject individually witch available resources R_1^d and R_2^d, but state σ_2 is not attainable for each of them. This means that following conditions are satisfied:

$R_1^r(\sigma_1) = R_1^d + \varepsilon$;

$R_1^r(\sigma_2) > R_1^d$; $\qquad R_1^r(\sigma_1) > R_1^r(\sigma_3)$;

$R_1^r(\sigma_3) < R_1^d$;

$R_2^r(\sigma_1) < R_2^d$;

$R_2^r(\sigma_2) > R_2^d$; $R_2^r(\sigma_3) > R_2^r(\sigma_1)$;

$R_2^r(\sigma_3) < R_2^d$.

If $R_1^r(\sigma_2) \neq R_2^r(\sigma_2)$, then with the consolidation of resources we select greater of them: if $R_2^r(\sigma_2) > R_1^r(\sigma_2)$, then $R_2^r(\sigma_2)$ is selected. The scheme of the solution of a problem in this example assumes participation in a certain proportion in the realization σ_2: expenditures, directed for realization σ_2 are now divided between subjects so that

$$R_1^{r'}(\sigma_2) = \mu R_2^r(\sigma_2);$$

$$R_2^{r'}(\sigma_2) = (1 - \mu)R_2^r(\sigma_2);\ \mu \in [0, 1].$$

The condition must be satisfied:

$$R_2^r\left(\sigma_2\right) + \varepsilon < R_1^d + R_2^d.$$

Let $R_2^r\left(\sigma_2\right) = \eta\left(R_1^d + R_2^d\right)$ and, furthermore

$$R_2^r\left(\sigma_2\right) = k_1 R_1^d;\ \ R_2^r\left(\sigma_2\right) = k_2 R_2^d\ \ (k_i > 1).$$

We can find that $\eta = \dfrac{k_1 k_2}{k_1 + k_2}$. Results of calculations of a numerical example are given in table 2.3. Let`s accept the same, model of the preferences distribution as in the previous example, assume that $\beta = 1$. Before the solution of the resources consolidation a set of alternatives $S_{a1} = S_{a2} = S_a$ and two alternatives were included: σ_1 and σ_3. A consolidation leads on to the expansion of S_a on one alternative σ_2 ($N = 3$).

Assume that $\mu = 0,5$, i.e., subjects equally participate in reaching σ_2.

From table 2 we see that none of subjects can realize σ_2, basing only on his own resources.

$$k_1 = \frac{R_2^d\left(\sigma_2\right)}{R_1^d} = \frac{5}{3} = 1,666;\quad k_2 = \frac{R_2^r\left(\sigma_2\right)}{R_2^d} = \frac{5}{4} = 1,25;\quad \eta = \frac{1,666 \cdot 1,25}{1,666 + 1,25} = 0,7142.$$

Tabele 2

INITIAL DISTRIBUTION OF PREFERENCES

Subject 1	σ_i	σ_1	σ_2	σ_3	$H_{\pi 1}$ = 0,6795
	$R_1^{\,r}(\sigma_i)$	3	4	2	
	x_{1i}	1	∞	0,666	
	$e^{-\beta x_{1i}}$	0,3679	0	0,5134	
	$\pi_1(\sigma_i)$	0,4175	0	0,5825	
Subject 2	σ_i	σ_1	σ_2	σ_3	$H_{\pi 2}$ = ln2 = 0,6795
	$R_2^{\,r}(\sigma_i)$	3	5	3	
	X_{2i}	0,75	∞	0,75	
	$e^{-\beta x_{2i}}$	0,4724	0	0,4724	
	$\pi_2(\sigma_i)$	0,5	0	0,5	

Tabele 3

DISTRIBUTION OF PREFERENCES AFTER CONSOLIDATION

Subject 1	σ_i	σ_1	σ_2	σ_3	$H'_{\pi 1}$ = 1,07846
	$R_1^{\,r'}(\sigma_i)$	3	$\mu \cdot 5$ = 2,5	2	
	$R_1^{\,d'}(\sigma_i)$	3 + ε	5,00 + ε	3 + ε	
	x'_{1i}	1	0,5	0,666	
	$e^{-\beta x'_{1i}}$	0,3679	0,0065	0,5134	
	$\pi'_1(\sigma_i)$	0,2473	0,4076	0,3451	
Subject 2	σ_i	σ_1	σ_2	σ_3	$H'_{\pi 2}$ = ln2 = 1,09137
	$R_2^{\,r'}(\sigma_i)$	3	$\mu \cdot 5$ = 2,5	3	
	$R_2^{\,d'}(\sigma_i)$	4 + ε	5,0 + ε	4 + ε	
	x'_{2i}	0,75	0,5	0,75	
	$e^{-\beta x'_{2i}}$	0,4724	0,6065	0,4724	
	$\pi'_2(\sigma_i)$	0,3045	0,3910	0,3045	

From table 3 we see, that after consolidation available resources prove to be different for different alternatives. This explains the fact, that consolidated resources in accordance with cannot be used by each subject for other purposes, except σ_2.

We can see, that the production of entropy inside the system and the import of information occur as a result of the resources consolidation and expansion of the set S_a *entropies of both subjects increased* :

$$I_{\pi 1} = H_{\pi 1} - H'_{\pi 1} = 0{,}6795 - 1{,}07846 = -0{,}39896,$$

$$I_{\pi 2} = H_{\pi 2} - H'_{\pi 2} = 0{,}6931 - 1{,}09137 = -0{,}39827.$$

The import of information for both subjects is practically identical. Apparently, this can be explained by the fact that they participate in the realization of σ_2 on parity principles: $\mu = 0{,}5$.

Example 3

Let`s examine the previous problem; however, we assume now that the model of distribution takes the form

$$\pi_j(\sigma_i) = \frac{x_{ji}^{\alpha} e^{-\beta x_{ji}}}{\sum_{k=1}^{N} x_{ji}^{\alpha} e^{-\beta x_{ji}}} \tag{4.125}$$

with the condition that structural parameters α and β are the same for the first and second subject.

Let`s put $\alpha = \beta = 1$ for simplification of calculations. The distribution (4.125) has a maximum at the point $x^* = \frac{\alpha}{\beta}$, for $\forall j \in \overline{1,M}$. This means that if inequalities are fulfilled

$$\frac{\alpha}{\beta} < x'_{ji} < x_{ji}, \quad \text{then} \quad \pi_j\left(\frac{\alpha}{\beta}\right) > \pi'_j(\sigma_i) > \pi_j(\sigma_i),$$

and, on the contrary, if

$$x_{ji} < x'_{ji} < \frac{\alpha}{\beta}, \quad \text{then} \quad \pi_j(\sigma_i) < \pi'_j(\sigma_i) < \pi_j\left(\frac{\alpha}{\beta}\right).$$

In the first case “the direction” of inequalities for x_{ji} and $\pi_j(\sigma_i)$ is opposite, in the second case – they are equal. Results of calculations with the same initial conditions are represented in table. 4, 5.

Tabele 4

INITIAL DISTRIBUTION OF PREFERENCES

Subject 1	σ_i	σ_1	σ_2	σ_3
	$\pi_1(\sigma_i)$	0,5186	0	0,4820
	$H_{\pi 1}$ = 0,6925			
Subject 2	σ_i	σ_1	σ_2	σ_3
	$\pi_2(\sigma_i)$	0,5	0	0,5
	$H_{\pi 2}$ = 0,6931			

Tabele 5

DISTRIBUTION OF PREFERENCES AFTER THE CONSOLIDATION

Subject 1	σ_i	σ_1	σ_2	σ_3
	$\pi'_1(\sigma_i)$	0,3628	0,2996	0,3376
	$H'_{\pi 1}$ = 1,09430			
Subject 2	σ_i	σ_1	σ_2	σ_3
	$\pi'_2(\sigma_i)$	0,35015	0,2997	0,35015
	$H_{\pi 2}$ = 1,0960			

From the comparison of tables 2, 3 and 4, 5 it`s evident that in the first case (for "radical" distribution (4.123)), corporate alternative σ_2 after making a decision about the resources consolidation is more preferable than remaining alternatives (σ_1 and σ_3), in the second case the preference of σ_2 after consolidation occurs lower in comparison with preferences of σ_1 and $\sigma_{3,}$ both for the first subject and for the second subject.

In the first case, the solution about consolidation would be most likely accepted and σ_2 will be subsequently selected as purpose.

In the second case the consolidation of resources and inclusion of additional alternative σ_2 in the set S_a can be see disadvantageous for both subjects, but as the purpose it one of alternatives σ_1, σ_2 will be selected.

4.8. Aggregation of rating preferences

4.8.1. Aggregation of individual ratings

In the previous paragraph, we examined individual ratings, differentiated on alternatives $\sigma_k \in S_{ai}$. The problem, which naturally appears on the way of the preferences investigation is the problem of their aggregation, and also aggregation of preferences of I kind.

First step is aggregation of individual ratings on a set of alternatives S_{ai}, which consists of the following: subject i aggregates differential ratings of subjects j $\xi(i, j| \sigma_k)$ the united rating $\xi\ (j| i)$ ("rating j from the point of view i"), reflecting general utility j for i on an entire set of alternatives S_{ai}. Here several possibilities exist:

1. With the forming of rating $\xi\ (j| i)$ the subject i does not consider the opinion of other members of the group and does not differentiate them on the significance - the preferability of alternatives σ_k. In this case $\xi\ (j| i)$ could be determined by the formula:

$$\xi\left(j \mid i\right) = \frac{1}{N_i}\sum_{k=1}^{N_i}\xi\left(i, j \mid \sigma_k\right). \tag{4.126}$$

Values $\xi\ (j| i)$ are normalized by the condition:

$$\sum_{j=1}^{N_i}\xi\left(j \mid i\right) = 1\,.$$

Let`s note that the value $\frac{1}{N_i}$ is a preference of each of $\sigma_k \in S_{ai}$, if they all are equally preferable.

2. Natural generalization of formula (4.126) is the formula

$$\xi\left(j \mid i\right) = \sum_{k=1}^{N_i}\pi_i\left(\sigma_k\right)\xi\left(i, j \mid \sigma_k\right). \tag{4.127}$$

In this case rating $\xi\ (j| i)$ aggregates differential ratings with weights equal to the preferences of alternatives $\sigma_k \in S_{ai}$.

3. Ratings $\xi\ (j| i)$, as also $\sigma_k \in S_{ai}$ can be formed by the subject i taking under consideration opinions of other members of groups ("agents"). In this case these opinions of "others" made by functions of ratings: $\xi\ (j| q)$ or $\xi(q, j| \sigma_k)$, $q \in \overline{1, M_i}$, $j \in \overline{1, M_q}$, produced by them if, of course, this information is accessible for i.

It`s necessary to take into account that in the general case of a set of alternatives S_{aq} *they* cannot coincide from S_{ai} and between each other. Furthermore, each subject from $S_{\xi i}$ can apprehend the group "in its own way", i.e., consider itself the member of the group, different from $S_{\xi i}$. Here $S_{\xi i}$ is the group, that studies and evaluates subject *i*. Each time normalizing conditions should be specified.

Let`s examine one of scheme s, when with the determination of the rating of each subject the opinion of other members of the group is taken into account in the form "the weight" of equal rating of this subject, established by other subjects. (This aggregation can be carried out by "external" subject.) This scheme can be named "*the scheme of the mutual responsibility*".

Let rating ξ $(j|\ i)$ are determined by the rule, expressed by the formula

$$\xi(j \mid i) = \sum_{q=1}^{M} \xi(j \mid q)\xi(q \mid i). \qquad (4.128)$$

It is supposed that the sets $S_{\xi i}$ of all members of the group ($i \in$ 1, N) coincide and subject i has available precise information about ξ $(j|\ q)$, appointed to subject j by another subject of the group.

Relationships (4.128) can be considered as equations, which connect ratings ξ $(j|\ i)$, quantity of which is $M \times M$. To equations (4.128) the normalizing condition should be added:

$$\sum_{j=1}^{M} \xi(j \mid i) = 1 \text{ for } \forall i \in \overline{1,M}.$$

Thus, for $M \times M$ of variables ξ $(j|\ i)$ we have $M \times (1 + M)$ of equations. It`s possible to show that this system has the only solution

$$\xi(j \mid i) = \frac{1}{M}; \ (\forall i, j \in \overline{1,M}),$$

i.e. - all ξ $(j|\ i)$ are identical. Consequently, due to conditions "*the mutual responsibility*" the ratings of all members of the group are identical.

Let`s look, is it possible to exceed limits of this trivial case. Obviously, for this it`s necessary to modify an equation (4.128).

Let`s assume that there are a priori ratings ξ $(j|\ 0)$ either occurred in the previous stage or assigned an "external" subject - "hierarch" which can be named "*the arbitrator*". We will also consider a priori ratings also normalized by one.

Instead of system (4.128) we will obtain the non-heterogeneous system

$$\xi(j \mid i) = \alpha\xi(j \mid 0) + (1-\alpha)\sum_{q=1}^{M} \xi(j \mid q)\xi(q \mid i) \qquad (4.129)$$

with normalizing conditions of the formation

$$\sum_{j=1}^{M} \xi(i \mid 0) = 1; \quad \sum_{j=1}^{M} \xi(i \mid q) = 1 \quad (\forall j \in \overline{1,M}).$$

The parameter α reflects the distribution of the degree of confidence between "hierarchy" and members of the group. Values α and 1-α can be consider the preferences distributed between two possibilities (1) " hierarchy " believes only to itself and does not consider the opinion of other participants in the group (α = 1, 1 α = 0); (2) " hierarchy " wholly bares his position on the collective opinion of the group: (α = 0, 1 α = 1). But in the latter case, as shown above, all ratings prove to be (in this scheme of aggregation) identical and equal $\frac{1}{M}$.

The next step of aggregation is forming of "integral" ratings $\xi_j(\sigma_k)$ and ξ_j.
The question arises: to whom these ratings are necessary for who are "the carrier" of the corresponding data and their user?

Hypothesis is accepted that *the integral ratings can be aggregated from the individual*, and that different scheme s of aggregation of in the form of weighted sums are possible. From the very beginning, as it was already said, it is possible to assume the presence of "*the arbitrator*", who assigns a priori ratings to the members of the group. As this judge can come out a "*hierarchy*", i.e., the subject confronting above the group, who has authorities, relating on the activity of the group. Interesting hypothesis is an assertion, that the role of this "*arbitrator*" plays certain *ethical postulate*, or the certain "*collective reason*", which exists, as a sovereign subject and therefore, has his own preferences, being characterized at each moment of time by their entropy.

We see that in the consciousness of associations members *"coalition" component of psyche"* exists constantly being manufactured and renews. In other words, *certain component of the distribution of individual preferences, which appears common for all members of the group exists*. It`s the manifestation of the collective component of the consciousness of each person, which exists on the genetic level and is a consequence of evolution and "pressed" generalized life experience of association during its history. Using language usual in this book in proportion as "corporate" problems appeared (i.e., non-empty intersections of an individual set of alternatives: $\bigcap_{j=1}^{M} S_{aj} \neq \varnothing$), appeared and was strengthened collective consciousness as the part of the consciousness of each individual generally. The process contributed on development and strengthening, achievement of the equal right to exist both of collectivist and individual consciousness.

The invention of fire and the need of tribe to support it was for years a corporate problem and strengthened collectivism, just as joint hunting a large beast, the need for protection from an attack of an another tribe and so forth. In our times an association exist only because there are corporate problems (non-empty intersections of an individual problematic set S_{aj}), there is today a historical (it can be genetic) memory about how this collective (corporate) part of psyche helped to survive an entire tribe (people) and each individual - member of social corporation.

Ethical postulates (imperatives) influence on decision making with utilitarian factors, being most important component of safety and progress of an association. They are expressed in the form of studies, religious commandments, and philosophical concepts. The Moses commandment is an example. In the contemporary theories they are reflected in the models of humanism, socialism, communism, liberalism - models of the "social justice". In the utility theory they are represented in the form of models of the collective utility (see the beginning of this chapter), by the concept of egalitarism, utilitarianism, intermediate models CUF, principle of Pigu- Dalton, Paretto - optimum scheme s and so forth. The coalition psychology appears "on squares" as the effect of crowd, as a result of goal-directed prolonged action of the media on scales of the whole country, or the entire world scientifically substantiated technologies of a deliberate manipulation with consciousness [73].

An insufficiency in previous constructions of this work is the exceptionally rationalistic (utilitarian) nature of models of the preferences distribution, when utility or resources are considered as the argument. Certainly, the try in the mathematical form (more accurate - in the symbolic form) to consider *ethical principles* must lawfully call skeptical relation as philosophers, so also mathematicians. The most likely corresponding models will bear speculative nature. Nevertheless, absolute conviction that solutions always start not only on basis of rational factors places the author in the *dilemma*: either to completely forge the "thing" that is defined as a *subjective analysis* or try by any means to consider the essential influence *of ethical factors*. Most likely, this can be made, moving with the same way, which authors of utility theories go along. This way lies in the fact that the meaningful aspect of ethical principles is ignored (almost always), but as elements of the theory, which reflect consequences of the action of these principles on those characteristics, which allow quantitative interpretation, they are introduced.

Let`s make some general *assumptions*.

1.*Ethical postulates (imperatives) appear in associations*. It does not indicate, however, that they do not influence individual preferences of the I kind $\pi_j(\sigma_k)$... This influence can be realized in the form of limitations, superimposed on the composition of alternatives $\sigma_k \in S_{aj}$, i.e., on problematic set S_{aj} as the result of religious considerations. Most frequently this is prohibition on certain actions with respect to other members of the group (association of Fig.4.18). So if S^*_{aj} is a set of alternatives, permitted in the rationalistic sense (sufficiency of resources, for example, the $S_{aj} \subset S^*_{aj}$ is a set of alternatives obtained by the rejection of the inadmissible for ethical reasons alternatives.

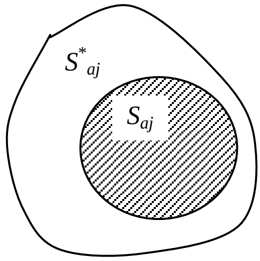


Fig.4.18

2. The *calculation of ethical postulates is possible to carry out via the calculation of the certain limitations, assigned on distributions of preferences, and a composition of problem set S_{aj}.*
3. The *calculation of ethical postulates can be carried out by the introduction of standard distributions* (imperatives).
4. For functions of the preference of the II kind (ratings) a substantial limitation can be *the permissibility of belonging of this group* (*to set $S_{\xi i}$*), *i.e., limitation superimposed on the composition of the group S_ξ or $S_\xi(\sigma_k)$*, depending, for example, on his preferences of the I kind.

Let`s examine the model of forming of integral subjects ratings in the group.

Determine integral rating by the relationship

$$\xi_j = \frac{1}{M}\sum_{i=1}^{M}\xi(j \mid i),$$

if ξ (j| i) is obtained by averaging on S_{aj}, and

$$\xi_j(\sigma_k) = \frac{1}{M}\sum_{i=1}^{M}\xi(i, j \mid \sigma_k).$$

Let now the integral rating of subject j is defined as the sum of his differential ratings, assigned by other members of the group, weighed proportionally on their integral ratings:

$$\xi_j = \sum_{i=1}^{M}\xi_i\xi(j \mid i). \tag{4.130}$$

In this case conditional ratings ξ (j| i) are normalized:

$$\sum_{i=1}^{M}\xi(j|i) = 1, \ (\forall j \in \overline{1,M}). \tag{4.131}$$

Also integral ratings are normalized:

$$\sum_{i=1}^{M} \xi_j = 1 . \quad (4.132)$$

In this relationship (4.130) normalizations are coordinated both to the left and to the right.

The assertion occurs: with the determination of an integral rating ξ_j according to scheme (4.130) with the presence of normalizations (4.131) and (4.132) there is an infinite set of decisions of uniform equations for ξ_j, with the given ξ $(j|\ i) \geq 0$, moreover all $\xi_j \geq 0$.

System (4.130) is uniform and the determinant of matrix Z - I, where I is unit matrix, when conditions for normalizing (4.131) are present, it`s equal zero. It is possible to show that due to conditions "*of the mutual responsibility*", when all $\xi(j \mid i) = \frac{1}{M}$, integral ratings ξ_j are all identical as a result of normalization condition

$$\xi_j = \frac{1}{M}, \ (\forall j \in \overline{1,M}).$$

If not all ξ $(j|\ i)$ are identical, then to each collection of conditional ratings the set solutions for the integral ratings corresponds. Thus, there is a certain freedom and for obtaining the unique solution ξ_j the knowledge of conditional ratings is insufficient. Analogical situation occurs, if ratings $\xi_j(\sigma_k)$ are examined.

Let`s examine the relationship:

$$\xi_j = \alpha \xi_j^0 + (1-\alpha) \sum_{i=1}^{M} \xi_i \xi(j \mid i), \quad (4.133)$$

which determines the system of M linear equations with respect to ξ_j, for which starts the condition of normalization (4.132) is accepted.

The system (4.133) for each $0 < \alpha < 1$ has an only solution, since the determinant of the matrix

$$I - Z = ||\delta_{ji} - \xi(j|\ i)||$$

different from zero, if not all ξ $(j|\ i)$ are identical and they are not equal $\frac{1}{M}$. In the case "of the mutual responsibility" or the absolute equality of integral ratings in the group $\xi(j \mid i) = \frac{1}{M}$, $(\forall j, i \in \overline{1,M})$, the system (4.133) does not have a solution at all. It can mean that due to conditions of the mutual responsibility

a) the interference of an "external" subject (hierarch`s) with its estimations ξ_j^0 does not lead to the disturbance "status quo", or

b) a corporate component of psyche with the rating preferences distribution different from the uniform one ($\xi_j^0 = \dfrac{1}{M}$) cannot exist.

One additional ingenious treatment of the noted fact is possible: interferences "of hierarch`s" is effective (an integral change of ratings) only then, if in the group "*the mutual responsibility*" is absent.
Let`s write the system (4.133) in the matrix form:

$$(\alpha I - (1 - \alpha)\ Z)\xi = \alpha\xi^0, \tag{4.134}$$

where Z is $M \times M$ matrix of conditional ratings $\xi\ (j|\ i)$, ξ is M-vector of integral rating and ξ^0 is M-vector of ratings, of the assigned by "hierarch". It follows from (4.134) that

$$\xi = (\alpha I - (1 - \alpha)\ Z)^{-1}\alpha\xi^0. \tag{4.135}$$

In order to remain consecutively within the framework the concept of subjective preferences we can values α and $1 - \alpha$ consider as general for all members of the group of preferences π_α and $\pi_{1-\alpha}$: $\pi_\alpha + \pi_{1-\alpha} = 1$.

Let`s try to give answers to the question presented above: who need the integral ratings $\xi\ (j)$ and who? Answer depends on that, do we count these ratings belonging to the external " hierarch" or belonging the component of the corporate psyche, which we, in this case, examine as a "virtual" subject, the place "of inhabiting" of whom is the brain of each of members of the group . Let`s designate this subject by mark (*) (or will assign to it number "0").

It`s seemed that (*) can use an information about ratings $\xi\ (j)$ in order to

a) influence the structurization of the group , for example during the selection "of the leader" of the group , the establishment of objective ranks inside the group,

b) select a partner for the solution of a cooperative problem (using ratings $\xi_j(\sigma_k)$),

c) make decision for itself about the entry as one or other coalition with other subjects,

d) inform the higher subject ("hierarch") in the hierarchy about the distribution of ratings in the subordinated group.

In connection with that we can together with the entropy of conditional of ratings $\xi\ (j|\ i)$ (or $\xi(i, j|\ \sigma_k)$) examine the "hierarch" entropy.

$$H_\xi^* = -\sum_{j=1}^{M} \xi_j \ln \xi_j \qquad \text{or} \qquad H_\xi^*(\sigma_k) = -\sum_{j=1}^{M} \xi_j(\sigma_k) \ln \xi_j(\sigma_k).$$

These entropies differ from entropy, represented by the formula (4.102).

The component ξ^0, considered as a reflection in the psyche of a "collective reason" can be exactly the carrier of ethical principles. Depending on the relationship of values π_α and $\pi_{1-\alpha}$ an "interference", of ethical principles in the decision making will be greater or smaller.

The reflection of limitations of those assigned by ethical principles can be realized through the preferences distribution of both the I kind and the II kind.

4.8.2. Ratings of subgroups of subjects

We here examine another interpretation of conditional ratings $\xi(j|\ i)$ (or $\xi(i,\ j|\ \sigma_k)$).. Let as before $\xi(j|\ i)$ is the rating *j*, established *by i*, and $\xi(j,\ i)$ is the rating of the subgroup G_2, which consists of two subjects: *i* and *j*. Let, further, someone ("hierarch", "collective reason",...) assigns integral ratings, including rating of subject *i*: $\xi(i)$, but rating of another subject in the group is determined "from the words" of this subject, i.e., hierarch judges group using the rating

$$\xi^{(1)}(j,i) = \xi(i)\xi(j\,|\,i) \tag{4.136}$$

The rating of subgroup from the point of view of the external observer, who "associates" only with one of the members of the subgroup (i)

Rating of the i-th subject, previously formed by the external observer

Rating of the j-th subject "based on words" of i, well familiar external

In scheme (4.136) it`s assumed that the external observer doesn`t associate with the entire group, but only with the part of its members (may be with half) $\frac{1}{2}M$ (if *M* is countaible).

If rating *j* does not depend on *i*, then $\xi\ (j|\ i) = \xi\ (j)$ and the rating of the group $\{j,\ i\}$ - G_2

$$\xi^{(1)}(j,\ i) = \xi(i)\xi(j). \tag{4.137}$$

How this case can realize it? Let`s say, rating *j* "from the words" *i* one and the same for $\forall i \in \overline{1,M}$.. In other words, if *i* can estimate *j* or all members of the group from the point of view "*i*" have identical ratings: $\xi\ (j|\ i) = \mathrm{Idem}\ (j)$ ($\forall j \in \overline{1,M-1}$), the "hierarch" uses his judgment - the independently manufactured rating for *j*.

The formula (4.137) is only one of the possible schemes of the formation of the group rating and it corresponds to the situation, when the subjects of the subgroup "multiplicatively" strengthen, or they weaken subgroup with the solution of a certain problem. For example, in "the machine-gun crew" departure from system of one draws the curtailment of the functioning of an entire group.

For the equivalent soldiers of those reflecting an attack from their own weapon they strengthen or weaken the subgroup G_2 "additively". Departure from system of one only decreases the force of the group, but does not reduce its effectiveness to zero . In the latter case of an additive group rating it is natural to assume

$$\xi^{(1)}(j,i) = \frac{1}{2}\left(\xi(i) + \xi(j)\right). \tag{4.138}$$

Note that in the case of model (4.136) using, generally speaking, the order of indices plays role, i.e., $\xi^{(1)}(j,i) \neq \xi^{(1)}(i,j)$, since conditional preferences in the general case do not coincide: $\xi(j\,|\,i) \neq \xi(i\,|\,j)$ (on opinion i about j does not coincide with opinion j about i). A normalizing condition for (4.138) has form:

$$\frac{1}{2M}\sum_{j=1}^{M}\sum_{i=1}^{M}\left(\xi(i) + \xi(j)\right) = 1\,. \tag{4.139}$$

For the models (4.137) and (4.136) (if $\xi(j\,|\,i) = \xi(j\,|\,i)$) normalizing conditions appear as follows

$$\sum_{i=1}^{M}\sum_{j=1}^{M-1}\xi^{(1)}(j,i) = 1\,. \tag{4.140}$$

If the order of numbers j and i is of not importance ($\xi(j\,|\,i) = \xi(j\,|\,i)$), a possible quantity of subgroups G_2, which consist of two subjects of the group S_ξ equals $\frac{1}{2}M(M-1)$, consequently, if we number subgroups G_2: G_{2q}, $q \in \left(1, \frac{1}{2}M(M-1)\right)$, the condition for normalization can be written in the form:

$$\sum_{q=1}^{\frac{(M-1)M}{2}} \xi_q(j,i) = 1. \tag{4.141}$$

If the order of numbers j, i is essential ("hierarch in the pair has place"), a quantity of different pairs in the group S_ξ equals $(M-1)\,M$. Condition of normalization has a form:

$$\sum_{q=1}^{(M-1)M} \xi_q(j,i) = 1\,. \tag{4.142}$$

If we in the number "of pairs" conditionally include value $\xi\,(i,\,i) = \xi\,(i)\,\xi\,(i|\,i)$, where $\xi(i)$ is a priori estimation of i, $\xi\,(i|\,i)$ is self-appraisal, then rating normalization signs the form

$$\sum_{i=1}^{M}\sum_{j=1}^{M}\xi^{(1)}(j,i)=1\,. \tag{4.143}$$

However, among pairs G_2 the pairs with indices {i, i} are absent. Consequently, normalizing (4.139) - (4.142) must be used.

Let`s examine all possible sets, which consist of three subjects (G_3). A quantity of such subgroups, which are characterized by the order of indices, i.e., having their internal structure ("the hierarchy of competence") is M (M – 1)(M – 2):

$$\xi(i, j, k) = \xi(i \to j \to k) = \xi(i)\xi(j|\, i)\xi(k|j).$$

Here i give an estimation j, and j in his form, gives an estimation k. Normalizing conditions appear looks like following:

$$\sum_{q=1}^{M(M-1)(M-2)}\xi_q(i,j,k)=1$$

or

$$\sum_{k=1}^{M=2}\sum_{j=1}^{M-1}\sum_{i=1}^{M}\xi(i)\xi(j\mid i)\xi(k\mid j)=1\,. \tag{4.144}$$

It`s possible in the general case, to examine subgroups, which are of $m < M$ of subjects, and also cases, when "associations" with any number of subjects are possible. Accordingly, to the introduced group ratings the entropies of distributions are determined

$$H_{\xi}^{(2)}=-\sum_{j=1}^{M-1}\sum_{i=1}^{M}\xi(i)\xi(j\mid i)\ln\xi(i)\xi(j\mid i)=H_{\xi(i)}^{M}+\sum_{i=1}^{M}\xi(i)H_{\xi(j|i)}^{M-1}; \tag{4.145}$$

where $$H_{\xi(i)}^{M}=-\sum_{i=1}^{M}\xi(i)\ln\xi(i);\quad H_{\xi(j|i)}^{M-1}=-\sum_{i=1}^{M-1}\xi(j\mid i)\ln\xi(j\mid i).$$

Analogous function occurs for trinomial groups

$$H_{\xi}^{(3)}=H_{\xi(i)}^{M}-\sum_{i=1}^{M}\xi(i)H_{\xi(j|i)}^{M-1}+\sum_{i=1}^{M}\sum_{j=1}^{M-1}\xi(i)\xi(j\mid i)H_{\xi(k|j)}^{M-2}. \tag{4.146}$$

Fig.4.19 shown different scheme s of the formation of the aggregated rating of the group s G_2 and G_3.

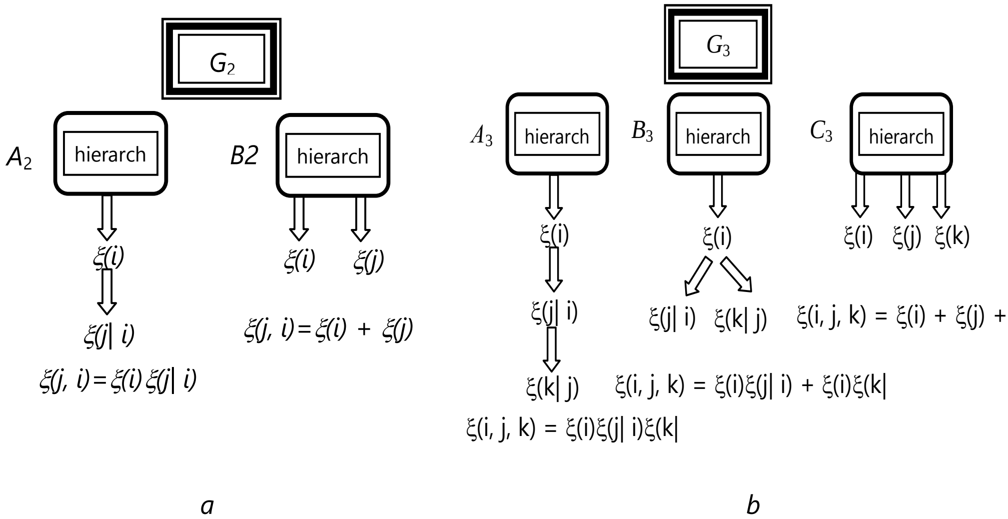


Fig. 4.19

Let`s note that, if ratings of binomial groups G_2 are equal ξ (j, i) = ξ (i, j), then condition (completely A_2) must be satisfied:

$$\frac{\xi(j\mid i)}{\xi(i\mid j)}=\frac{\xi(j)}{\xi(i)}.$$

This means that conditional ratings relate as non-conditional ratings "assigned by hierarch", members of paired group manifest a *complete conformism* with hierarch in estimations of each other. For the model A_3 the assumption about the independence of the rating ξ (i, j, k) from the order of indices leads to the conclusion about the equality of all mutual (conditional) rating type ξ (k| j), and also non-conditional ratings: ξ (i) = ξ (j) = ξ (k). Scheme s A_2, A_3 and similar once for groups with the large number of members (q > 3) are the groups with "multiplicative" hierarchy in the sense that reducing to zero of one cofactors it conducts reducing to zero ratings of the group B in the cases of schemes of type B_2 and B_3 (with" multiplicative- additive" rating) and of type C_2, C_3 (with "additive" ratings) this property does not occur.

4.9. Notion of "problem" referred to a set of ranks.

In the first chapter the concept of "problem" on a set of object alternatives was introduced. Distributions of object preferences – preferences of the I kind were subsequently connected with this concept. In this chapter we study rating preferences of subjects in the group. It would be natural to try to introduce the analogous concept, correlated with the distributions of ratings and ranks. Let`s conditionally name this problem "organizational problem" of an individual. It`s possible to propose the following scheme. A set of ranks S_η: $A_r \in S_\eta$ come out as a set of organization alternatives. Let r is number of rank ($r \in \overline{1,m}$) and simultaneously the number of the class of rank of equivalence. If the system of ranks has strictly vertical linear hierarchic a structure, then we will consider that ranks form the system, given by the relationship,

$$r < r + 1 \Rightarrow \eta(r) < \eta(r + 1), \tag{4.147}$$

in other words the higher the weight of rank (volume of imperious authorities) the higher the number of ranks and in the subjective sense rank A_{r+1} is always more preferable than rank A_r, i.e.,

$$r < r + 1 \Leftrightarrow A_r \langle A_{r+1}. \tag{4.148}$$

The set S_η, as it was already said, is considered a set of organizational alternatives and it`s "the field of battle" on which battle at the point of the advance upward on the rank hierarchy developed. "The linearity" of rank hierarchy is understood in the sense that branching about professional signs does not exist there. Actually there are set of cases and systems with the branching, when subject, finished the certain "rank step" in one professional region (for example, in the science), pass on to another "stairs" of rank hierarchy (for example, in politics).

If "branching" in the system of ranks is allowed, then a set of possibilities of rank alternatives are substantially enlarged. Upon the transfer from one branch to other both "upward" and forward, "steps" are possible.

In any event "the motion" of subject across the hierarchical system of ranks is accompanied by a change "in the volume of imperious authorities", i.e., weight $\eta_s(j)$.

If subject at the given moment has a rank $\eta_s(j)$, but he would prefer to obtain $\eta_r(j)$; $r > s$, then the "realized" desire to change rank $A_s \to A_r$ can be named "organizational problem" of subject j:

$$P_j(S_\eta): A_s \to A_r;\ \eta_s \to \eta_r. \tag{4.149}$$

The possibility of the solution of such a problem depends on objective and subjective circumstances.

Passage $A_s \to A_r$ is at the same time passage from one class of rank equivalence to another. The objective possibility of passage depends on the theoretical number of class of the rank equivalence $M^{\sim}{}_{\eta r}$ and its actual "population" q_r, and also on the presence of other aspirants, who would want to enter this class, or those already having rank A_r and not desiring from it to part. For such subject, who would want to preserve the status quo, problem could be defined as the problem of the homeostasis

$$P_j(S_\eta): A_s \to A_s. \tag{4.150}$$

Another objective factor is the presence of passive and active resources R^{disp}_{jp} and R^{disp}_{ja}, of subject j, which are necessary for him in order to realize his imperious authorities corresponding to the given rank and of those characterizing in the integral sense witch weight η_r of the desired rating A_r.

Let`s make the assumption that the passage from one class of rating equivalence to another is defined by subjective rating either integral $\xi\ (j)$, or differential $\xi(j|\ \sigma_k)$, $\xi(j|\ S'_a)$, where $S'_a \subset S_a$ is subset of corporate problems.

The set of ranks is finite, and passage $A_s \to A_r$ has spasmodic nature. The functions of rating preferences are continuous, taking values in the interval [0, 1]. Awarding rank depends on subject i or it can be the result of the collective solution.

In any case the value of a subjective criterion corresponding type of rating preference serves:

$$\xi(j|\ i),\ \xi(j|\ S_\xi) = \xi(j),\ \xi(j|\ i,\ \sigma_k)\dots$$

Thus, if subject i makes decision relative to subject j, then value $\xi\ (j|\ i)$ is used. It is obvious that one of the conditions for passage $A_s \to A_r$ from the point of view of i is the inequality

$$\xi(j|\ i) \geq \eta_r. \tag{4.151}$$

The administrative career of subject j from the point of view of the scheme proposed could be represented as "wandering" on the hierarchical "stairs" of ranks (on "the tables of the ranks").

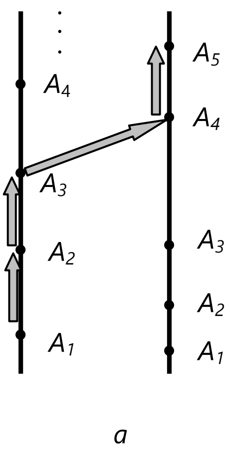
The rating of subject in the course of time changes. These changes are caused by a change in the available passive and active resources, in particular. By a change in the educational level, increase or reduction in the popularity, the accumulated personal achievements, physiological and age factors and so forth.

If "the branching" of rank stairs occurs, then this can be expressed as the dependence of ranks on the professional certain character and what is more particular, from the type of the problem, which must be slowed i.e., $\eta_s = \eta_s(\sigma_k)$, $\sigma_k \in S_a$ or $\eta_s = \eta_s(S'_a)$; $S'_a \subset S_a$.

Fig. 4.20 illustrates the versions of passages in the case "of the branching of administrative career".

In the case (*a*) subject, having a rank A_3in one professional sphere of activity passes to the higher class of rank equivalence in another professional sphere. In the case (*b)* the passage at another region of professional activity occurs with reduction in the rank. It is obvious that even in one and the same professional region the passages are being accomplished both "upward" and "downward".

It was already spoke about the entropy barriers, which determine the moments of decision making on the set of object alternatives S_a: H_π^*, H_π^{**}, In the case of rating - rank process it is possible to speak about the certain entropy barriers on the set of S_ξ: H_ξ^*, H_ξ^{**}, since speech is going about the selection of subject $\Sigma_j \in S_\xi$ as the aspirant to the rank change.



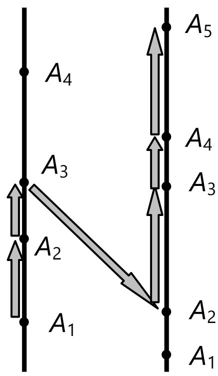


Fig. 4.20

Problem differs somewhat from the case of object preferences. Thus, with the selection of subject to the rank A_s in "the competition" either only subjects, who belong to class s - 1, whose number is q_{s-1}, or all subjects having the rank lower than A_s,i.e., A_{s-1}, A_{s-2} can participate. This requirement determines the condition of normalization of the preferences and expression for enumerating the entropy, for example:

$$H_{\xi_i} = -\sum_{j=1}^{q_1+q_2+\ldots+q_{s-1}} \xi(j \mid i)\ln\xi(j \mid i); \tag{4.152}$$

$$\sum_{j=1}^{q_1+q_2+\ldots+q_{s-1}} \xi(j \mid i) = 1 .$$

Or, if the representatives of class s - 1 are examined only, then

$$H_{\xi_i} = -\sum_{j=1}^{q_{s-1}} \xi(j \mid i)\ln\xi(j \mid i); \tag{4.153}$$

$$\sum_{j=1}^{q_{s-1}} \xi(j \mid i) = 1 .$$

Here in the sums the rating preferences only of those subjects, who enter into the appropriate classes of rank equivalence participate.

The necessary condition (4.151) should be added with the condition

$$H_\xi \le H^*_\xi. \tag{4.154}$$

Condition (4.151) bears auxiliary "consultative" nature, since if filling of the class s is necessary it can be ignored. In order to explain this term let`s recall that at the time of war in the case of failure of the commanders of more high rank, their place were assigned with people of lower rank, which frequently did not have a sufficient formation and combat experience.

The limit H_ξ^{**} is the upper limit of rating entropy, provoking the person making cadre decision in the direction of retrieval for additional information, if $H_\xi^* \ge H_\xi^{**}$, or testifying about his insufficient organizational abilities – incapability to make cadre decisions. "Cadre leap-frog" frequently speaks about uncertainty and indecisivenes of the person making decision and about the sharp fluctuations of rating entropy near the upper limit $H_\xi^{**} \lesssim H_{\xi \max}$.

Thus, a study of the process of group structuring or "cadre process" can be connected with of dynamics of the preferences of II kind, whose canonical distributions are available as the solutions of the corresponding variation problems.

4.10. Stable imperatives. Brief survey of ethics.

In p. 4.5 and 4.8, a question about the ethical component of preferences is set. Before undertaking the attempts in any manner to consider this component in the developed formalism, let`s give brief survey on the ethics as the chapter of philosophy and psychology.

"Ethics" (from the Greek ξουη - disposition, custom) - is the morals concept taken by association, including the science about the morals: principles and the standards of behavior, decision making. Ethics is the component of each philosophical system, and ethical principles also differ from each other, as differ these systems, their common base prerequisites.

It is distinguished *ethics descriptive*, which fixes the existing ideas about the morals in this association and answers a question "what exists", which in one or other association or another ethical rules and regulations are accepted, and *the ethics normative*, which answers a question "what ethical standards must be in one or other ethical system, one or other association". Hence, *ethics descriptive* is inverted in the past, sums up experience that take place, *and ethics normative* is inverted in the future and bears *the forcing* nature. The second to a considerable extent is based on the first and both of them reflect the process of development and modification of ideas about morality - passage from the retrospection to the forecast. When somebody speak about the business ethics. In essence he has in mind normative ethics.

There are two approaches to the determination of the sources of ethical principles (principles of morals) appearance:

— *Naturalistic* approach - derives moral standards from the sciences about nature and society, asserting the historical way of ethics as the result of the formation of contemporary humanity, historical experience, gradual forming and refinement. Naturalism allows different ethics of different groups, classes, ethnic, religious associations, and more of that religions as ethical studies derives from the sequence of historical epochs and sequence of the changing conditions;

— *Anti-naturalistic* approach assumes that the moral standards are given "*a priori*", they proceed "on the God" and do not refer to empirical data. Anti-naturalism allows "the dynamics" of a change in the standards, confessed by one or other association in the given historical period, but only as a process of the gradual knowledge of "highest" abstract morals, which exists out of the human psyche;

— *emotivism* - direction, which treats morals as expression and continuation of the objective properties of human psyche (like "unconditional" reflexes), in connection with which both naturalistic and anti-naturalistic approaches are being rejected.

The objectivistic theories of ethics assume that the ethical standards can be derived from the certain common, universal assumptions, independent on social formations, ethnic, historical, geographical, economic special features and can be carried to each person (individual). On the contrary, *the subjectivistic theories* of ethics assert that the standards of ethics have by their source separate "subjects" (individuals) and become conventional in this association as a result of historical procedure of "aggregations".

Following approaches' can be distinguished from the behavioral (behavioristic) point of view:

— *motivism*, which assumes that one or other solution cannot be recognized moral independent on final effect, if it was not caused by "good intentions";

— *effektsionism*, which with the evaluation of morality of the solution (action) considers exceptionally final effect, and even act, "undertaken" from "unkind motives" is recognized moral if final effect proved to be good (useful, positive,...);

— *Nominalism* is abstracted both from the motive and from the final effect. Good and evil are being acknowledged as the concepts of primary.

The construction of the ethics systems rests on *the methods of ethics*, which are the object of formal- methodological analysis and the means of the systematization of categories, concepts and connections between them.

In the 17th and 18th Century owing to the rationalism underwent development the rationalistic methods of ethics (Spinoza, Kant). Spinoza based his method on the experience, traditions, knowledge and intuition. Cant rested on the theory of clean, free from empiricism, reason. In the historical period, which can be attributed to the present, the problem arose connected with possibility in principle to construct the normative ethics on the scientific basis.

It is assumed sometimes, that *the normative ethics* is not scientific discipline: ethics is occupied by the fact "which must be, but which still is not jet", therefore, there is no real object of investigation. Neopositivist`s also not consider ethics a science, since ethical standards is not possible to base either experimentally, or by mental constructions.

Nevertheless, the possibility to use the methods of sociology within the frame of so-called "science about morality" is beginning acknowledge. *Neopositivism* and *emotivism* related to it derives morality exclusively from the individual emotions. Morality from this point of view is accessible to scientific research so, as feelings and persuasion are somewhat accessible to a study. This direction is defined as *meta ethics*.

Thus, there are two fixed points of sight. The first unites the directions, *which deny the possibility of building of the normative ethics* as science. They include *sociologism*, *neopositivism*, *and emotivism*. The second unites the directions, which allow such possibility (empirical ethics, rationalism, intuitionism, and intellectualism).

The structural classification of ethics separates

- ethics fundamental and
- ethics special.

Fundamental ethics includes:

- philosophical assumptions;
- the doctrine about the people acts;
- the doctrine about purposes and sense of existence;
- the doctrine about good and moral values (axiology);
- the doctrine about moral imperatives (deontology);
- the doctrine about moral responsibility.

Let`s examine some concepts of ethics.

Neokognetivism assumes that the rational line of reasoning of moral persuasions is impossible and, therefore, the substantiation of ethical principles is based only on group persuasions.

Kognetivism requires the rational substantiation of moral persuasions, construction of the non-contradictory system of ethical norms.

Konsekventialism asserts the need for the moral estimations of those based on the results of activity. The particular version of *konsekventionalism* is *the utilitarianism*, which we already spoke above, and also *egalitarism* and compromise systems, which find reflection in the structure of the collective utility function (CUF).

Non-konsekventialism is based on the fact that good is defined as the correspondence of the actions:

- to the hierarchy of values accepted;
- to the evaluation of positive results;
- to the requirements of haws;
- to the moral bases
- to the idea of responsibility.

Let`s examine briefly some positions *of the business ethics*.

The ethics of business as *ethics normative* is occupied by estimations and determination of the moral standards, which relate to the particular region of the activity of the subject within the frame of association.

It is possible to assign three approaches, three base positions of the business ethics.

1. *Deontological position* (deo - responsibility) - the ethics of responsibilities and virtues (Kant), which can be expressed as the system of ethical prohibitions and prescriptions, which regulate activity, the system of ethical codes (for example, the code of doctor…). Deontology is not capable to foresee a change in conditions and special features of particular situation. Its tenets are the postulates of common nature and limitedly influence on the decision making in the particular situations. The certain common imperatives of businessman follow from it: diligence, consistency, patience, truthfulness, obligation and so forth

2. *Axiological –teleontological* (axiology - is doctrine about the values, teleontology - doctrine about the purposes), is ethics of interests and purposes. The discussion does not deal in this case with of prohibitions and prescriptions. Everything that contributes to self -realization of auto-creation, leads to the success, the achievement of the objective, optimization of strategies of objective achievement is moral. One of the expressions of this type of ethical concept is *the theory of utility* intensively developing discipline of economic science. *Axiological utilitarianism* is based on the optimization of the balance of goods and losses (good and evil). Morality in this case can prove to be the inplement of the achievement of real success by the subject.

Let`s note here that the problem- resource concept, places *the problem* higher than purpose. The latter plays subordinate role. Consequently the center of gravity of ethical analysis is moved at the field of research of the problems creation, the forming of problem set S_{aj} their dynamics and connection with each other in the associations (groups).

3. *Position, based on the priority of responsibility and conscience*: the ethics personal, oriented to the individual, regulates relations with partner and relation to the labor as the important factor of the relations between the people. The purpose of ethics of this type is the humanization of economic activity.

Some essential axioms of contemporary ethics are formulated as follows:

1. Only *free act* is ethical act (the object of ethical estimation, it can be evaluated as positive or negative). For the free act subject bears responsibility.

In our understanding, as this was said earlier, *each act is free*. This means that each situation regardless of the fact is those prescription "from above" (directive) or not, in the compulsory order it is transformed as the personal *problem of subject*. Respectively his own responsibility is determined.

2. The man is, first of all, responsible witch respect to himself and this determines his merit.

3. Final instance of morality is individual conscience.

However, it is obvious that the standards of morals and, correspondingly, conscience are the product of social development, social nature of man. They become meaningless out of the association because their object, is the interaction of this given subject with others.

In the ethics of business it is possible to detect such chapter s and some negative phenomena:

1. Ethical problems of the labor:

— the phenomenon of the exploitation in the form of the wrong remuneration for labor;

— exploitation in the relations between the colleagues;

— tendency to an increase of prestige of his own and his professional group.

2. Ethics of advertisement and advance of the goods:

— the determination of the advertisement success;

— conflict between success of advertisement and the moral standards of information.

3. Ethical problems of the competition:

— the tendency to exclude competition in the given segment of market;

— buying of specialists in the competitive struggle;

— economic blackmail;

— the distribution of negative information about the competitor and his good;

— copying of goods;

— forgery and distortion of firm signs.

4. Ethical problems of interrelations between participants in the business:

— the disturbance of agreements and contracts with the partners, common, the negative aspects of relations with of partners;

— the negative aspects of relations with of collaborators, the clients, subordinated.

5. Ethical standards and the problem of the ecology:

— exceeding the standards of harmful effect on the environment for the purpose to decrease expenditures;

— the carrying out of poor quality and dangerous goods for the health;

— the use of harmful stimulators, catalysts and so forth

"Common " ethical standards are in contradiction with some, newly appearing aspects of reality; therefore a special branch of ethics - "*ethics of business*" is being developed called in any manner to adapt, to reconcile *common ethics* with the realities of contemporary business, the ideological installations of "clean" liberalism, economic rationalism, oriented to obtaining of maximum benefit. There are as minimum three approaches to the solution of contradictions.

1. Supporters of the first consider common ethics a sufficient base and "the ethics of business" by the reflection of the common crisis of morality.

2. Supporters of the second regard "*the free market*" as an absolute, and convert at *the ethical standards of business* the relations, generated by the free market.

3. Supporters of the third approach - "legal`s" - confess the slogan: "something not forbidden by law, is permissible from the morals point of view". Thus, it is assumed that the laws are also at the same time *the standards of morals*. The reduction of morals to the right level occurs.

The fact, that the ethics in the whole is in the state of insoluble contradiction with the realias "*of the free market*" (in reality free market does not exist, but using term "*free*" is fig sheet, model *of the non-ethics* hypocrisy of its authors) it is one of the main contradictions of modern world, source of social tension, conflicts, revolutions and wars. There are no single-valued answers to such questions, for example: "Is ethical commercial secret?", "is ethical, competition at all", "is ethical aggressive advertisement, as the tool of mass consciousness manipulation?", "is ethical whether usury - credits under the percentages?", "is ethical the exploitation of strange labor at all?" and so forth.

These, sufficiently common, questions have centuries-old history, and in the course of time their sharpness does not weaken, but become more strengthened.

For example, recognition of competition being permitted in the sport, the skill, in other spheres of activity and the non-recognition of the competition ethical permissible in the economy as the forms of "economic war", is the pivotal moment of a whole series of social systems and concepts. But it is difficult to present the market without the competition, when the set of mechanisms and "laws" simply cease to act. Christian ethics rejects usury.

Entire system of private banks, this important regulator of financial flows is in a sense the usury brought to the perfection. However, a whole series of the functions of banking system has nothing common with the usury. And in this sense its existence can be ethically justified.

The proposed brief and very superficial survey of ethical concepts has as a one goal here the only reason: to find the possibility to reflect in subjective analysis of active systems ethically standards.

How it can be done, having in mind the existing the set of means? Answer to this question, as it seems should be search for in the following directions:

1. In the structure of individual problem sets - selection of the ethical permissible alternatives ($\sigma'_k \in S_{aj}$).

2. In the models of the preferences aggregation both of the I and II kind.

3. in the use "of standards" or ethical imperatives and the postulation of variation principles.

4. In principles and schemes of "the utilities transfer", (such as Pigu- Dalton principle).

5. In the determination of the balance between *the individualism* and *the collectivism* with the forming *of corporate problems*. Let`s recall what the hero of E. Zola Novel "Money" K. Marx's follower speaks about individualism and collectivism: "Certainly, the existing social system is obliged by its age-long prosperity to the principle of the individualism, which because of the competition and the personal interest, causes ever greater productivity. Will collectivism be fruitful? And by what means it is possible to increase productivity of labor, if the spirit of profit does disappear? Here those are our weak place and we should for long fight so that socialism sometimes would triumph. But we will conquer, because we are justice".

This reasoning is the historical forerunner of that which was named above "the dilemma: equality - effectiveness" or in the close sense - by dilemma "egalitarism - utilitarianism".

6. On the way of more a fundamental understanding of sense and role *of subjective entropy*, for example, in the answer to a question, in what sense *subjective entropy* and such category as "the Kant's will of freedom" are available. It is possible to assume that the greater the entropy, the greater "the freedom" (sensation of freedom - more selection) and vice versa.

7. On the way of forming and interpreting the subjective sense of the functions of effectiveness it is necessary to realize what prevails in psyche and practice of decision making: "positive" analysis or "negative" analysis.

These and other possibilities of the ethical norms calculation we are to realize in further constructions to a greater or smaller extend.

From the point of view of the developed subjective analysis the compromise, which recognizes different approaches, enumerated above: naturalism, anti-naturalism, emotivism, motivism, objectivism, subjectivism, and nominalism is most acceptable. Each of these approaches reflects the properties of human psyche - that sole field of battle, on which "the conflict" between the ethics and the rationalism develops.

The most consonant and most adequate to the methodologies of subjective analysis are two of the enumerated approaches: objectivism and subjectivism. Rest of approaches from a systematic point of view supplement two mentioned above. Objectivism and subjectivism can be accepted as the ideological basis of the formalism of the preference functions.

Thus, for instance, if we stand on the soil of effektionalism`s, then, apparently, it is possible to limit our self with the criterion of utility and to count those constructions, which are given in the previous chapter, sufficient.

On the contrary, nominalism, being accepted as the sole of ideological basis, would require the construction of the preference distribution exclusively depending on abstract ideas about *the good and evil*. Here we could hope at the best case on the possibility to operate with the ordinal preferences, and quantitative estimations of the type of the utility functions would be very problem.

The assumption, taken in the present work, lies in the fact that it is possible in principle to construct some formalized models, oriented to one or other arecognized approaches and descriptive methodologies, if we use distribution of subjective preferences, technology connected with evaluations of subjective entropy and information, and, then, to estimate the plausibility of the conclusions obtained from the theoretical conceptions.

From our point of view central place in these constructions postulated *variational principle* occupies. In this case, it is assumed as it has been mentioned above, that "*the optimality*" *is originally inherent in human psyche*.

It is difficult to say, whether there is universal variational principle, which covers all forms of activity; therefore different formulations of the variational problems are examined, which are compared with either one or another type of psyche, with one or another stage of activity. As an example, "positive" $\pi^{+}(\sigma_k)$ and "negative" $\pi^{-}(\sigma_k)$ preferences, corresponding functional, entropies H^{+}_{π} and H^{-}_{π} can serve and so forth.

The possibility of the fact that formalization withhim the frame of subjective analysis will show positive action in the sense of ordering and strict logical structurization of ethics, the establishment of the connection between different approaches and concepts. The application *of synergetic* methods, the study of the preferences stability, which reflect ethical standards and imperatives, the presence of attractors on the set of ethical preferences are undoubtedly perspective, promising direction of studies. Unique view on the ethics is contained in the works [133,134], where the concept of actor (so called "geo-pathos" and their unions) is introduced as the acting subject, and reaching the reign of freedom is proposed as the global mega-historical purpose. Let`s give several quotations of the mentioned works, which determine the basic "attributive" properties of the actors:

1) "…actor must *be*, since rules of the game allow his death, for existence of actor it is necessary and sufficient that the actor would manage certain minimum quantity of resources and passionate energy". If this condition is satisfied, then

2) the second attribute of actor is his tendency to the state of maximum freedom, he makes this by forming (disintegration) unions with other actors, which also composes the essence of the informative aspect of historical process and finally

3) - the tendency of actor to control over large amount of resources. This intension of actor is being realize in the competitive struggle for resources with other actors and composes the essence of the play aspect of historical process".

In the course of implementation of these properties actor manufactures *moral code* - i.e., the certain ethics system.

The author of works [133,134] derives this system comparing it with the "categorical imperative" of E. Kant:

Actor must be, he must be free; he must be rich.

From the point of view of the author of [133, 134] the moral law of actor appears as follows:

actor - must be but not to the detriment for of other actors; he must approach the maximum of freedom but not to the detriment of others; he must approach control over a maximum amount of resources but not to the detriment of others".

Further a question about the subordination "of the attributes *of existence*, *freedom* and *wealth*" is examined.

It is completely obvious that these "maxims" cause the set of questions, since each used category (for example, "freedom", "resources"...) is received ambiguously by the supporters of different doctrine and scientific schools. So other actors can accept the given actors as "resources" - "labor resources" - labor force. In connection with the category "freedom" it is appropriate to ask the question: with what "freedom" the discussion deals: "internal" or "external" within the frame of stoics doctrine or using E. Kant's terminology in "the criticism of clean reason", to what is nearer the model of the actor: "Homo of phenomenon" or "Homo of noumenon", what is relationship between "the freedom and the independence". It general: "freedom" for whom? "freedom" from what? In the name of what? By what means?

The small excursus at the region of category, which is determined by term "freedom" we well make in chapter 7, in connection with attempt to connect certain concept of freedom with the categories of subjective analysis and, in particular, with the subjective entropies H_π, H_ξ.

In this paragraph we mention about the works [133,134] since they relate to the recent time and they are connected with attempts to reflect such non-formal concepts as freedom and ethics in the mathematical constructions.

We will return again to this topic in chapter 7 in connection with the application of the developed approach to the conflicts theory.

4.11. Stable imperatives in schemes of subjective analysis.

We conducted above brief excursus at the region of ethics. One of the important conclusions, of this survey, lies in the fact that different systems of ethics are realized in the form of system of the imperatives, which in the historical perspective can be taken as a basis of the systems of laws. Imperatives are stable in the sufficiently large intervals of time. In the psyche of each subject, they are reflected by individual means, but in this case they preserve certain community and serve "cementing grout" of associations. These are national customs, political concepts and theory, religious views and belief. Besides the ethical imperatives there are other powerful regulators, which were genetically taken root in the consciousness: tendency to the enjoyment, fear, tendency to leadership, superiority over the similar to itself, and, simultaneously, tendency to the subordination, the search of leader, patron, who could take upon himself the part of the difficult problems. The corresponding properties of psyche, if they are expressed in the explicit form, can be named the imperatives (may be - "genetic imperatives"). These imperatives are most stable, although in the common case, "they are distributed" between the individuals unevenly and yield to correction in the process of training, acquisition of experience. They change with the age.

Ethical imperatives are acquired, considerably less stable, they change more slowly than rational preferences. We know that people change religion, habits, political views, aesthetical tastes.

There are imperatives, expressed in the form of rules, orders, instructions, and laws. Such imperatives are most rigid and categorical, but also they do not determine decisions unambiguously. Let`s pass to formalization of concept "imperative", which would make it possible to enter it in the theoretical scheme , based on the canonical distributions of preferences.

We will indicate that imperative I_k ($k \in \overline{1,L}$) is quantitative (liberal), if its presence changes the numerical value of preference ($\pi(\sigma_i)$ or ξ_j...) and qualitative (radical or categorical), if its presence turn correspondent preference into zero. The influence of imperatives will be determined by measure $\pi(I_k)$. For the quantitative imperative (cardinal scheme)

$$\pi(I_k) \in [0, 1],$$

for the qualitative (ordinal scheme) -

$$\pi\left(I_k\right) = \begin{cases} 0, I_k \text{ not considered} \\ 1, I_k \text{ considered} \end{cases}$$

In the latter case the system of imperatives is singular.

Imperative is *absolute*, if it "acts" always independent on problem- resource situation (on initial σ_0 and alternative state $\sigma_i \in S_a$). Imperative is relative, if its action depends on the type of situation. For example, imperative "do not deceive" acts in peacetime, but during the war the fraud of enemy is not only permissible but even is considered heroism.

Imperative I_k is relative invariant on the subset $S'_a \subset S_a$, if "the force" of its action (measure $\pi(I_k)$) is identical for $\forall \sigma_i \in S'_a$.

Imperative is "familiar" to the alternative $\sigma_i \in S_a$ (or to the problem P: (σ_0, σ_a)), if it is connected with σ_a (or P) in the meaningful sense and "non-familiar", if there is no such connection.

Let in S_a N alternatives are contained, the dimensionality of imperatives set is L. Let\`s designate through $\pi(\sigma_i| S^L_I)$ the preference, of alternative $\sigma_i \in S_a$, when all imperatives are taken into consideration. Here S^L_I is the set of imperatives. Set S^L_I is complete, and the corresponding ethical system is balanced or complete, if

$$\sum_{k=1}^{L} \pi\left(I_k\right) = 1\,. \tag{4.155}$$

Let\`s assume that

$$\pi\left(\sigma_i \mid S_I^L\right) = \sum_{k=1}^{L} \pi\left(I_k\right)\pi\left(\sigma_i \mid I_k\right). \tag{4.156}$$

Set S_a is complete relative to imperative I_k, if normalizingn condition is satisfied

$$\sum_{i=1}^{N} \pi\left(\sigma_i \mid I_k\right) = 1,\;\; (\forall k \in \overline{1,L}). \tag{4.157}$$

It is assumed, that the system of alternatives S_a always can be supplemented in such a way that it will contain alternatives "permitted" by ethical imperative I_k. With satisfaction of this condition

$$\sum_{i=1}^{N} \pi\left(\sigma_i \mid S_j^L\right) = 1\,. \tag{4.158}$$

This means that the ethical system, accepted by subject, never creates *hopeless situations*. In this case certainly, it can occur that the added alternatives are for the subject catastrophic. The set of alternatives complete relative to imperative I_k are designated through $S_a(I_k)$, set complete relative to the collection of alternatives $(I_i, I_j, ..., I_q) \in S^L_I$ is $S_a(I_i, I_j, ..., I_q)$. It is possible to accept that the system of the inserted sets occurs

$$S_a\left(S_I^L\right) \subseteq S_a\left(S_I^{L-1}\right) \subseteq \ldots \subseteq S_a\left(S_I^1\right).$$

Adding of imperative to the initial system narrows (not expands) the set of ethically permitted alternatives.

Depending on the type of the relations between the elements of the sets S_a and S_I it is possible, relying on chapter 2.5, to speak about different types of ethical systems, for example:

- "ewrywere defined ethics";
- "surjective ethics";
- "functional ethics";
- "injective ethics".

The ewrywere defined ethics corresponds to such situation, when each alternative

$\sigma_i \in S_a$ has non-empty "image" (at least one element $I_k \in S^L{}_I$) in S_j, but can have several "images". I.e., for each alternative σ_i at least one "intimate" imperative can be found in S_j. This means that the ethics „has answers" to any questions, including new, which can arise in the future.

Surjective ethics is such ethics, when each ethical postulate (imperative) $I_k \in S^L{}_I$ finds at least one "point" of application - one alternative $\sigma_i \in S_a$, which will find ethical estimation. However, in S_a *there* can be such alternatives, relative to which the ethics (set S_I) "keeps silent". In other words in the surjective ethics there are no such ethical standards, which would not have rationalistic applications.

By "*functional ethics*" such ethics is understood, when each alternative $\sigma_i \in S_a$ either does not have ethical estimation within the frame of S_I^L, or can be evaluated from the point of view only of one ethical postulate $I_s \in S_I^L$. In this case each postulate (imperative) can be applied to several (subset $S_a(I_s)$) of alternatives: $S_a(I_a) \subset S_a$.

Projection S_I on S_a separates in S_a set of subsets $M_a(I)$, including subset $\bar{S}_a$ "non-served" by the functional ethics of alternatives.

Injective ethics is such ethics, when any ethical imperative $I_s \in S^L{}_I$ either "does attend" not one alternative $\sigma_i \in S_a$, or „serve" only one alternative. It is possible to imagine the combined relations between S_a and $S^L{}_I$. For example, if each alternative is " evaluated" by only one ethical postulate and, on the contrary, each ethical postulate is intended for "the estimation" of only one rational alternative, then here simultaneously all four formulated above assumptions are carried out, and the corresponding ethics is the so-called *biektive ethics*.

We will examine some models of the quantitative calculation of ethical imperatives.

Let the first model be determined by relationships (4.155) - (4.157). In this model of preferences the indices of the significance of ethical postulates (imperatives I_k) are considered the given ones ("a priori"). Their value can be expressed in the cardinal sense; however, more natural would be the assumption that the consciousness of subject can perceive (or to carry out) with respect to ethical postulates only *ordinal* scheme based on the relation ρ: ⟨, ⟩, ~. Inside the set $S^L{}_I$ it is possible the presence of the classes of equivalence, in that number entire set $S^L{}_I$ can be one class of equivalence. Conditionally it is possible to consider pair ($S^L{}_I$, ρ) the ethics system, where ρ is binary relation, assigned on $S^L{}_I$. It is possible to assume that ρ is weak ordering, strict ordering, and strict partial ordering (see Section 1.3). In these case we deal with different systems of ethics.

The presence of binary relation in the set of imperatives S_I tells, about the certain internal coordination of the ethical system, for example, of the possibility to arrange imperatives in the determined order with an increase of their significance. Such ethical systems have an internal link and conditionality. However, not all systems have internal logical conditionality. Imperatives "not kill" and "no steal" are not logically interdependent. It is completely obvious, nevertheless that the first of them has larger weight.

Generally speaking, the ethics system acts when certain set of alternatives is presented. Classification of ethics systems proposed above also, obviously, makes sense only in confrontation with the given set S_a. It is possible to speak about the projection of the set of imperatives S_j on the set of alternatives S_a

$$S^L{}_I \to S_a.$$

The latter is "the field of battle", on which "ethics" battles with "utilitarianism".

Competition between the ethical imperatives appears when two or more imperatives "encounter" during the estimation of one, and the same alternative as shown in Fig. 4.21.

Comparing this assertion with systems of ethics described above, we see that this competition, and, therefore, also the possibility to rank imperatives are possible only in the cases of the everywhere- determined ethics, surjective and injective ethics (more precise it goes about pains (S_I, S_a)).

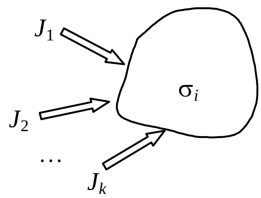


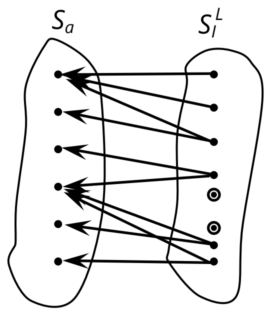
Fig. 4.21

In the cases *of functional* and *biektiv* ethics the competition of imperatives is absent. Repeating the scheme s from paragraph 2.1.3 in connection with the given case, let`s illustrate four basic types of pairs (S_j, S_a) in Fig. 4.22.

In the case (*a*) of *everywhere – defined pair there* are unequipped "latent" imperatives: S^L_I is excessive with respect to S_a. In the case (*b) of surjective pair* there are alternatives, relative to which given set of imperatives S^L_I do not give ethical estimations.

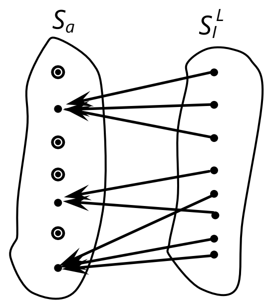
In the case (c) *of functional pair* (S_I, S_a) there are both *the unevaluated* alternatives and "*non working*" (latent) imperatives, but there is no competition of imperatives on the sets S_a. Finally, in the fourth case (*d)* as in the third case there are *unevaluated* (not serviced) alternatives and "*non working*" imperatives, but can exist the competition of imperatives on S_a. In each particular case it is possible to give the quantitative characteristics of the situations on the basis of combinatorial analysis.

The everywere defined pair (S^L_I, S_a)



a

Surjective pair (S^L_I, S_a)



b

Functional pair (S^L_I, S_a)

Injective pair (S^L_I, S_a)

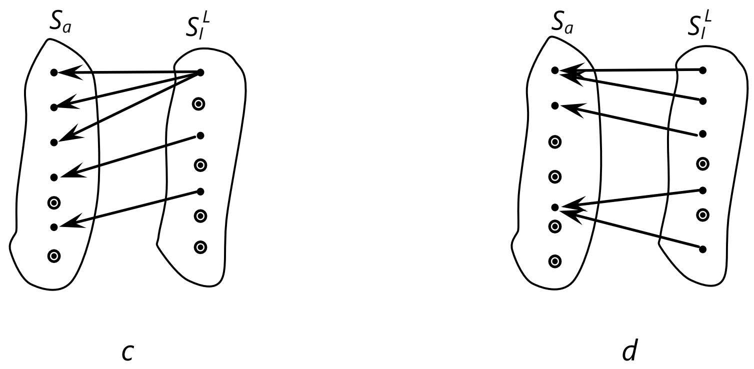


Fig. 4.22

Since the systems of ethics appeared in the retrospective during the prolonged intervals of time, it is possible to assert that each of the systems has certain common canon set of alternatives S^*_a, which serves for given the set of imperatives, as “a touchstone”, on which the set S^L_l is sharpened.

Let the ethics system consist of L postulates and ρ assigns on S^L_l preference relation non-reflexive and transitive, i.e., a strict partial order is introduced. If a quantity of classes of equivalence m does not coincide with a quantity of postulates (imperatives): $m < L$ then in accordance with point 3.10, “the entropy” of the system of the ethics

$$\bar{H}\left(S^L{}_j\right) = -\sum_{k=1}^{m} n_k \frac{r_k}{S_L\left(m\right)} \ln \frac{r_k}{S_L\left(m\right)}, \tag{4.159}$$

where r_k — rank of imperative, $S_L(m)$ — the sum of the ranks

$$S_L\left(m\right) = \sum_{k=1}^{m} k n_k$$

when $\sum_{k=1}^{m} n_k = L$. If $n_k = 1$, $\forall k$, $m = L$), then

$$\bar{H}\left(S_l^L\right) = -\sum_{k=1}^{L} \frac{k}{S_L\left(L\right)} \ln \frac{k}{S_L\left(L\right)}. \tag{4.160}$$

It is evident that $S_L\left(L\right) = \dfrac{L\left(L+1\right)}{2}$ and, consequently

$$\bar{H}\left(S_l^L\right) = -\sum_{k=1}^{L} \frac{2k}{L\left(L+1\right)} \ln \frac{2k}{L\left(L+1\right)} > 0. \qquad (4.161)$$

Entropy can be less than $\bar{H}\left(S_j\right)$ in (4.161), if the ranks of some imperatives will be equal to zero. Entropy will be equal to zero, if the ethics system is "singular": it consists of only one imperative.

Entropy $\bar{H}\left(S_l^L\right)$ is the integral characteristic of the ethics system. If $m < L$, i.e., there are classes of equivalences of imperatives, which contain more than one element, the entropy $\bar{H}\left(S_l^L\right)\Big|_{m<L} > \bar{H}\left(S_l^L\right)\Big|_{m=L}$.

However, the equivalence of imperatives is understood only from the point of view of their estimations (their weight) in the system of the ethics of subject - "the carrier" of this ethics system. It is possible to speak about the limited (weighted) equivalence. If in the condition of equivalence we include the sets $S_a(I_k)$, being "served" by the given imperatives in the system, for example, *biektiv ethics*, then equivalence acquires more wide sense and can be named functional. In any case, measure $\pi(I_k)$ of the significance of imperative I_k from the point of view of the subject " the carrier" of the ethics system is considered a priori and it does not depend on alternative $\sigma_i \in S_a$ and from S_a as a whole.

It is possible to speak about "ethical" information $J^L(S^L_l, \rho)$, which is connected with account or disregard of ethical imperatives. So, if $\pi^0(\sigma_i)$ is initial utilitarian distribution on S_a, and $\pi(\sigma_i| S^L_l)$ is the preference distribution on S_a, that is "squeezed" by the ethics system S^L_j, then

$$J^L\left(S_j^L, \rho\right) = H_{\pi^0(\sigma)} - H_{\pi\left(\sigma|S_l^L\right)}. \qquad (4.162)$$

If structure S^L_l is determined by the binary relation of the strict order:

$$J_1 \rangle J_2 \rangle J_3 \rangle \ldots \rangle J_n; \ \sum_{k=1}^{L} \pi^-\left(I_k\right) = 1; \ \ \pi^-\left(I_k\right) = \frac{k}{S_L},$$

either set S^L_l contains the classes of equivalence or even wholly is the one class of equivalence $\pi^-\left(I_k\right) = \frac{1}{L}$, there is a possibility to calculate the information $J_\pm^1(S^L_l, \rho)$, connected with accounting of one additional imperative or rejection of one of them

$$H_{\pi\left(\sigma|S_j^L\right)} - H_{\pi\left(\sigma|S_j^{L\mp 1}\right)} = J_{\pm}^1\left(S_l^L, \rho\right). \tag{4.163}$$

This approach without the substantial changes can be attributed to the system of legal imperatives, culture imperatives. The solidly mastered habits, some fundamental theoretical positions can acquire the nature of imperatives.

If for example, the rationalistic part of education consists at the mastering by student of knowledge, skills, habits, and then training consist at the mastering of the certain ethics system and method of its realization on the sets S_a in different problem- resource situations.

Besides the entropy the subjective dispersion characterizes the preference distribution

$$D\left(S_l^L\right) = \frac{1}{L}\sum_{k=1}^{m}\left(\bar{r}_k - \bar{r}\right)^2 n_k, \tag{4.164}$$

where $\bar{r} = \frac{1}{L}\sum_{n=1}^{m}\bar{r}_k n_k$, $\sum_{n=1}^{m} n_k = 1$.

We see that in the case of the limited equivalence, when $\bar{r}_k = \bar{r}$ ($\forall k$), $D(S^L{}_l) = 0$. In the formulas given above the preference $\pi(I_k)$ are identified with the relative ranks:

$$\pi\left(I_k\right) = \bar{r}_k.$$

Let`s assume that two subjects i and j are "the carriers" of the ethics systems ($S_l^{L_i}, \rho_i$); ($S_l^{L_j}, \rho_j$) and let these systems are related to one and the same set of alternatives S_a. More precise the sets of alternatives S_{ai} and S_{aj} are identical and possibility of solution of the problem P_i: ($\sigma_0^{(i)} \rightarrow \sigma_k^{(i)} \in S_{ai}$) by the subject i dose not exclude solution of the problem P_j: ($\sigma_0^{(j)} \rightarrow \sigma_k^{(j)} \in S_{aj}$) by the subject j even if $\sigma_k^{(i)}$ and $\sigma_k^{(j)}$ are identical. The rate of correspondence or difference of two ethics systems of two different subjects - "carriers" can be evaluated by the correlation indices of distributions $\pi_i(I_k)$ and $\pi_j(I_k)$. In this case it is expedient to supplement S_l^{Li} or S_l^{Lj} with fictitious imperatives so that both sets would have the identical dimensionality

$$L = \max\left(L_i, L_j\right), \tag{4.165}$$

and the preferences of fictitious imperatives we consider equal to zero.

Then the Pearson`s correlation coefficient is determined from the formula

$$\rho_J\left(\pi_i,\pi_j\right)=\frac{\sum\limits_{k=1}^{L}\left(\pi_i\left(I_k\right)-\frac{1}{L}\right)\left(\pi_j\left(I_k\right)-\frac{1}{L}\right)}{\sqrt{\sum\limits_{k=1}^{L}\left(\pi_i\left(I_k\right)-\frac{1}{L}\right)^2\sum\limits_{k=1}^{L}\left(\pi_j\left(I_k\right)-\frac{1}{L}\right)^2}},\tag{4.166}$$

where the conditions are satisfied:

$$\sum_{k=1}^{L}\pi_i\left(I_k\right)=1;\qquad \sum_{k=1}^{L}\pi_j\left(I_k\right)=1,\tag{4.167}$$

and average values are calculated by the formulas:

$$\bar{\pi}_i=\frac{1}{L}\sum_{k=1}^{L}\pi_i\left(I_k\right)=\frac{1}{L};\quad \bar{\pi}_j=\frac{1}{L}.\tag{4.168}$$

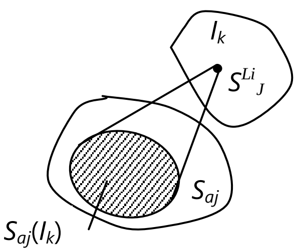


Fig.4. 23

Correlation coefficient $\rho_J(\pi_i,\ \pi_j)$ is not the complete characteristic of the degree of the similarity of two ethical systems. It is necessary also to study the topologies of two factor- sets M_{ai} and M_{aj}, formed by the subsets as S_{ai} and S_{aj}, to which the corresponding imperatives $I_k\in S_l^{L_i}$ and $I_k\in S_l^{L_j}$. are projected.

Besides the Pearson`s correlation coefficient the degree of the similarity of the ethical preferences distributions can be estimated with the aid of the rank correlation coefficient, for example, Spearmen's criterion

$$\rho_{JSp}\left(\pi_i,\pi_j\right)=1-\frac{6\sum d_k^2}{L\left(L^2-1\right)},\tag{4.169}$$

where $d_k = r_{ik} - r_{jk}$ is difference in the ranks those "appropriated" by two "carriers" to one and the same imperative.

If the ranks imperatives of the same name of I_k coincide $\rho_{JSp}=1$, . We have the same result, if sets S_{ai} and S_{aj} are one class of equivalence each. In this case

$$d_k=\frac{1}{L}-\frac{1}{L}=0;\ \forall k\in 1,\,L.$$

Let`s return to the model (4.155-4.157). Since the preferences $\pi(I_k)$ are considered the given ones, problem consists in obtaining of the models of canonical conditional distribution $\pi(\sigma_i|\ I_k)$. Let`s examine some forms of entropy:

$$H_{\pi(\sigma)} = -\sum_{k=1}^{N} \pi(\sigma_i) \ln \pi(\sigma_i); \tag{4.170}$$

$$H_{\pi(\sigma|I_k)} = -\sum_{k=1}^{N} \pi(\sigma_i \mid I_k) \ln \pi(\sigma_i \mid I_k); \tag{4.171}$$

$$\begin{gathered} H_{\pi(\sigma,I_k)} = -\sum_{i=1}^{N}\sum_{k=1}^{L} \pi(\sigma_i, I_k) \ln \pi(\sigma_i, I_k) = \\ = -\sum_{i=1}^{N}\sum_{k=1}^{L} \pi(I_k)\pi(\sigma_i \mid I_k) \ln \pi(I_k)\pi(\sigma_i \mid I_k) = \\ = -\left(\sum_{k=1}^{L} \pi(I_k) \ln \pi(I_k) + \sum_{k=1}^{L} \pi(I_k) \sum_{i=1}^{N} \pi(\sigma_i \mid I_k) \ln \pi(\sigma_i \mid I_k) \right) = \\ = H_{\pi(\sigma)} + \sum_{k=1}^{L} \pi(I_k) H\pi(\sigma \mid I_k); \end{gathered} \tag{4.172}$$

$$H_{\pi(\sigma|S_l^L)} = -\sum_{i=1}^{N} \pi(\sigma_i \mid S_l^L) \ln \pi(\sigma_i \mid S_l^L). \tag{4.173}$$

Let`s present entropy $H_{\pi(\sigma|S_j)}$ in more details. Since:

$$\pi(\sigma_i \mid S_l^L) = \sum_{n=1}^{L} \pi(I_k)\pi(\sigma_i \mid I_k), \tag{4.174}$$

then

$$H_{\pi(\sigma|S_l^L)} = -\sum_{i=1}^{N} \left(\sum_{k=1}^{L} \pi(I_k)\pi(\sigma_i \mid I_k) \right) \ln \left(\sum_{k=1}^{L} \pi(I_k)\pi(\sigma_i \mid I_k) \right). \tag{4.175}$$

The variational problems, which use different entropies, lead to different distributions. In the case (4.170) let`s accept the criterion of optimality in the form

$$\Phi_\pi = -H_{\pi(\sigma|I_k)} + \varepsilon\left(\pi(I_k), U_k, L_k \ldots\right) + N_{\pi(\sigma|I_k)}, \tag{4.176}$$

where ε(…) is the function of effectiveness, N(…) is the term, that corresponds to the normalization condition. In order to receive distribution $\pi(\sigma_i| I_k)$ depended on imperatives, function ε(…)must depend on $\pi(I_k)$.

Let`s introduce the reduced utility

$$\tilde{U}(\sigma_i \mid I_k) = \left(1 - \chi(\sigma_i \mid I_k)\pi^-(I_k)\right)U(\sigma_i). \tag{4.177}$$

Here $U(\sigma_i)$ is the rationalistic utility , understood in the usual sense, $\pi^-(I_k)$ — is the preference or value of imperative I_k, which has with respect to the alternatives

$\sigma_i \in S_a$ the limiting (prohibiting) sense. $\chi(\sigma_i| I_k)$ has the following sense: it is assigned on the direct product of sets S_a and S^L_I: $S_a\times S^L_I$ — set of all order pairs (σ_i, I_k), and is equal to 1, if I_k is "affined" to σ_i, and 0, if I_k is "not affined" to σ_i. "Affinity" is understood so that I_k in the meaningful sense is connected with σ_i and its presence can have an effect on the value of preference σ_i and, vice versa I_k "non- affined" σ_i, if it in semantic sense is not connected with σ_i.

We here certainly must say that this link I_k with σ_i depends in the Generally case on initial state σ_0, in which the system is located, i.e., from "the point of view". Thus for the time being in order not to complicate theory we will taciturn implies that all further considerations are conducted from one and the same "point of view". The setting, when " affinity" of certain imperative I_k is studied not with "final" state σ_i, but with the problem $P(\sigma_0 \to \sigma_i)$: i.e., with ordered pairs (σ_i, σ_j) which are being elements of direct product $S_a\times S_a$, is more Generally and correspond to the Generally sense.

Having in mind this observation, we omit subsequently the reference of state σ_0. Reduced utility $\tilde{U}$ is to the known degree the artificial structure, from "the common sense" and the reflecting not only objective utility but also the subjectivism, concentrated in this case in the distribution $\pi^{-}(I_k)$. As it was in all cases earlier, we again emphasize that the ethics system together with the distribution $\pi^{-}(I_k)$ is the property of subject. It has individual carriers and, on the whole, there exists as the attribute of individual and group consciousness. If this ethics system is universally recognized or acknowledged in the certain group (association), this means that the individual "projections" of this system are somehow similar. The developed approach is, apparently, capable to propose the certain criteria of similarity and, thus, the apparatus for the deeper structurization of the ethics study.

Let`s examine the criterion, which is based on the assumption that the subject is capable to study differentially entire set of object alternatives S_a from the point of view of the standard, connected with the given imperative I_k:

$$\Phi_{\pi(\sigma|I_k)} = -\sum_{i=1}^{N}\pi(\sigma_i \mid I_k)\ln\pi(\sigma_i \mid I_k) + +\beta\sum_{i=1}^{N}\pi(\sigma_i \mid I_k)\tilde{U}(\sigma_i \mid I_k) + \gamma\sum_{i=1}^{N}\pi(\sigma_i \mid I_k). \quad (4.178)$$

Canonical distribution $\pi(\sigma_i| I_k)$ has the form:

$$\pi(\sigma_i \mid I_k) = \frac{e^{\beta(1-\chi(\sigma_i|I_k)\pi^{-}(I_k))U(\sigma_i)}}{\sum_{j=1}^{N} e^{\beta(1-\chi(\sigma_j|I_k)\pi^{-}(I_k))U(\sigma_j)}}. \quad (4.179)$$

If all $\chi(\sigma_i| I_k) = 0$, in this trivial case of the ethics system is in no way connected with of set S_a. It can be seen from (4.179) in this case $\pi(\sigma_i| I_k)$ do not depend on $\pi^{-}(I_k)$ and are equal the "initial" rational (or utilitarian) preferences $\pi(\sigma_i| I_k) = \pi^0(\sigma_i)$. If all $\chi(\sigma_i| I_k)$ are identical, the $\pi(\sigma_i| I_k)$ also coincides with of utilitarian preferences $\pi^0(\sigma_i)$.

All $\pi^{-}(I_k)$ in view of normalization cannot be simultaneously equal to 1. However, if for certain $\pi^{-}(I_k) = 1$, every correspondent $\chi(\sigma_i | I_k) = 1$ for $\forall i \in \overline{1,N}$, then the distribution $\pi(\sigma_i | I_k)$ is uniform on i:

$$\pi(\sigma_i | I_k) = N^{-1}; \ (\forall i \in \overline{1,N}). \tag{4.180}$$

In other words, if all reduced utilities for given I_k are equal to zero, then the entropy $H_{\pi(\sigma_i | I_k)} = \ln N$ is maximum, independent on the values of the particular utilities $U(\sigma_i)$.

"Integral" preferences $\pi\left(\sigma_i \mid S_l^L\right)$, which are expressed with formula (4.179) in the first case also coincide with of utilitarian (rational) preferences

$$\pi\left(\sigma_i \mid S_l^L\right) = \sum_{k=1}^{L} \pi^{-}\left(I_k\right) \frac{e^{\beta U(\sigma_i)}}{\sum_{j=1}^{N} e^{\beta U\left(\sigma_j\right)}} = \frac{e^{\beta U(\sigma_i)}}{\sum_{j=1}^{N} e^{\beta U\left(\sigma_j\right)}} = \pi^0\left(\sigma_i\right). \tag{4.181}$$

In the second case

$$\pi\left(\sigma_i \mid S_l^L\right) = \frac{1}{N}$$

and the entropy

$$H_{\pi\left(\sigma_i | S_l^L\right)} = H_{\max}^L = \ln N.$$

Another form of distribution is obtained, if the function of effectiveness is selected in the form:

$$\varepsilon\left(\pi\left(I_k\right), U_k, \ldots\right) = \sum_{i=1}^{N}\left(\alpha \ln\left(1 - \chi\left(\sigma_i \mid I_k\right)\pi^{-}\left(I_k\right)\right) + \beta U\left(\sigma_i\right)\right). \tag{4.182}$$

Following distribution corresponds to this function of effectiveness:

$$\pi\left(\sigma_i \mid I_k\right) = \frac{\left(1 - \chi\left(\sigma_i \mid I_k\right)\pi^{-}\left(I_k\right)\right)^{\alpha} e^{\beta U(\sigma_i)}}{\sum_{j=1}^{N}\left(1 - \chi\left(\sigma_j \mid I_k\right)\pi^{-}\left(I_k\right)\right)^{\alpha} e^{\beta U\left(\sigma_j\right)}}. \tag{4.183}$$

This distribution has the following property: if all $\chi(\sigma_i | I_k)$ for $\forall i \in 1, N$ are identical, and in particular can be equal zero, then

$$\pi\left(\sigma_i \mid S_l^L\right) = \pi\left(\sigma_i \mid I_k\right) = \pi^0\left(\sigma_i\right).$$

This also means that $\pi(\sigma_i | I_k) = \pi(\sigma_i | I_q)$ for $\forall k, q \in \overline{1,L}$.

Presenting utility $U(\sigma_i)$ in the form $U(\sigma_i) = \delta\ln V(\sigma_i) + \beta W(\sigma_i)$ we will obtain the distribution

$$\pi\left(\sigma_i \mid I_k\right) = \frac{\left(1-\chi\left(\sigma_i \mid I_k\right)\pi^{-}\left(I_k\right)\right)^{\alpha} V\left(\sigma_i\right)^{\delta} e^{\beta W\left(\sigma_i\right)}}{\sum_{j=1}^{N}\left(1-\chi\left(\sigma_j \mid I_k\right)\pi^{-}\left(I_k\right)\right)^{\alpha} V\left(\sigma_j\right)^{\delta} e^{\beta W\left(\sigma_j\right)}}.$$

If the reduced utility is selected in the form

$$\tilde{U}\left(\sigma_i \mid I_k\right) = -\frac{\alpha}{1-\chi\left(\sigma_i \mid I_k\right)\pi^{-}\left(I_k\right)} + \beta U\left(\sigma_i\right), \tag{4.184}$$

distribution $\pi(\sigma_i| I_k)$ has the form:

$$\pi\left(\sigma_i \mid I_k\right) = \frac{e^{-\frac{\alpha}{1-\chi\left(\sigma_i \mid I_k\right)\pi^{-}\left(I_k\right)}} e^{\beta U\left(\sigma_i\right)}}{\sum_{j=1}^{N} e^{-\frac{\alpha}{1-\chi\left(\sigma_i \mid I_k\right)\pi^{-}\left(I_k\right)}} e^{\beta U\left(\sigma_j\right)}}. \tag{4.185}$$

Here, if all $\chi(\sigma_i| I_k) = 0$, which corresponds to the nonparticipation of this ethics system to the given set of alternatives, then

$$\pi(\sigma_i| I_k) = \pi^0(\sigma_i).$$

The same result occurs, if all $\chi(\sigma_i| I_k) = 0$ and $\pi^{-}(I_k) = \text{idem}(k)$, i.e., all are identical ($\pi^{-}\left(I_k\right) = \frac{1}{L}$). In particular, when S_a consists of only one alternative σ_1 ($N = 1$), then $\pi(\sigma_1| I_k) = \pi^0(\sigma_1) = 1$.

Summarizing the properties of the examined distributions, let`s formulate the following simple rule: if differential conditional distribution $\pi(\sigma_i| I_k)$ coincide with of utilitarian preferences $\pi^0(\sigma_i)$ and the ethics system is complete, i.e., $\sum_{k=1}^{L}\pi^{-}\left(I_k\right) = 1,$ then the distribution of the integral preferences $\pi\left(\sigma_i \mid S_I^L\right) = \pi^0\left(\sigma_i\right)$ coincides with of utilitarian distribution and ethics on the whole does not influence this distribution.

If S_a contains two alternatives σ_0, σ_1 ($N = 2$) and subject is in state σ_0, i.e. there exist, two possibilities:

1) he remains in the state σ_0;

2) he converts to state σ_1;

and let the corresponding utilities are equal to U_1 and U_2.

Suppose that the ethics system consists of two postulates I_1 and I_2, moreover $\pi^{-}(I_1) = \pi^{-}(I_2) = 0{,}5$ and matrix χ takes the form:

$$\chi_{(ik)} = \begin{bmatrix} \chi(\sigma_1 \mid I_1) & \chi(\sigma_1 \mid I_2) \\ \chi(\sigma_2 \mid I_1) & \chi(\sigma_2 \mid I_2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I}_{2\times 2}. \tag{4.186}$$

This type of matrix $\chi_{(ik)}$ corresponds to the situation, when imperative I_1 "acts" against the alternative σ_0 (to remain in σ_0), and I_2 — against the alternative σ_1 (to pass in σ_1).

Then, using, for example, distribution (4.183), we find matrix $\Pi(\sigma_i| I_k) = \Pi_{(ik)}$

$$\Pi_{(ik)} = \begin{bmatrix} \pi(\sigma_1 \mid I_1) & \pi(\sigma_1 \mid I_2) \\ \pi(\sigma_2 \mid I_1) & \pi(\sigma_2 \mid I_2) \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}; \tag{4.187}$$

$$\pi(\sigma_1 \mid S_I^2) = 0{,}5; \quad \pi(\sigma_2 \mid S_I^2) = 0{,}5\,.$$

If we use functional, based on the entropy $H_{\pi(\sigma,\, I)}$ and search for optimum (canonical) distributions $\pi(\sigma_i| I_k)$ from the condition

$$\frac{\partial \Phi_{\pi(\sigma|I)}}{\partial_{\pi(\sigma_i|I_k)}} = 0, \tag{4.188}$$

then the same distributions as above will be obtained.

Substantially different result occurs, if the variational problem will be formulated relative to the "integral" distributions $\pi(\sigma_i \mid S_I^L)$, i.e., such, which consider all ethical standards contained in S_I^L.

Let`s write the correspondent functional:

$$\Phi_{\pi(\sigma|S_I^L)} = -\sum_{i=1}^{N} \pi(\sigma_i \mid S_I^L) \ln \pi(\sigma_i \mid S_I^L) + \beta \sum_{i=1}^{N} \pi(\sigma_i \mid S_I^L) \tilde{U}(\sigma_i \mid S_j^L) + \gamma \sum_{i=1}^{N} (\sigma_i \mid S_I^L). \tag{4.189}$$

In order that the canonical distribution would not coincide with of utilitarian distribution $\pi_0(\sigma_i)$, it is necessary that the reduced utility would depend in any manner on "values" $\pi^-(I_k)$. In this case let's select the reduced utility in the form:

$$\tilde{U}(\sigma_i \mid S_I^L) = \prod_{k=1}^{L} \left(1 - \chi(\sigma_i \mid I_k)\pi^-(I_k)\right) U(\sigma_i). \tag{4.190}$$

Here, if all $\chi(\sigma_i| I_k)$ and $\pi^-(\sigma_k) = \dfrac{1}{N}$ are identical, then

$$\tilde{U}\left(\sigma_i \mid S_l^L\right) = \left(1 - \chi^0 \frac{1}{N}\right)^L U\left(\sigma_i\right); \ \ \chi^0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \tag{4.191}$$

if distribution $\pi^-(I_k)$ is singular:

$$\pi^-(I_q) = 1; \ \pi^-(I_k) = 0,$$

$$\tilde{U}\left(\sigma_i \mid S_l^L\right) = \begin{cases} U\left(\sigma_i\right), \ \texttt{if} \ \ \chi\left(\sigma_i \mid I_q\right) = 1, \\ 0, \ \ \texttt{if} \ \ \chi\left(\sigma_i \mid I_q\right) = 0. \end{cases} \tag{4.192}$$

Distribution $\pi\left(\sigma_i \mid S_l^L\right)$ has the form

$$\pi\left(\sigma_i \mid S_l^L\right) = \frac{\exp\left(\beta \prod_{k=1}^{L}\left(1 - \chi\left(\sigma_i \mid I_k\right)\pi^-\left(I_k\right)\right)U\left(\sigma_i\right)\right)}{\sum_{j=1}^{N}\exp\left(\beta \prod_{k=1}^{L}\left(1 - \chi\left(\sigma_i \mid I_k\right)\pi^-\left(I_k\right)\right)U\left(\sigma_i\right)\right)}. \tag{4.193}$$

This distribution is arranged in such a way that if in all products is at least one cofactor equal to zero, then the distribution $\pi\left(\sigma_i \mid S_l^L\right)$ coincides with of utilitarian $\pi^0(\sigma_i)$. This can occur only in one case, when distribution $\pi^-(I_k)$ is singular: $\pi^-(I_q) = 1$; $\pi^-(I_k) = 0$ with $k \neq q$ and the imperative I_q is affined to all alternatives $\sigma_i \in S_a$.
If the reduced utility $\tilde{U}\left(\sigma_i \mid S_l^L\right)$ is selected in the form

$$\tilde{U}\left(\sigma_i \mid S_l^L\right) = \alpha \ln \prod_{k=1}^{L}\left(1 - \chi\left(\sigma_i \mid I_k\right)\pi^-\left(I_k\right)\right) + \delta \ln V\left(\sigma_i\right) + \beta W\left(\sigma_i\right), \tag{4.194}$$

then instead of distribution (4.193) we will obtain the following distribution:

$$\pi\left(\sigma_i \mid S_l^L\right) = \frac{\left(\prod_{k=1}^{L}\left(1 - \chi\left(\sigma_i \mid I_k\right)\pi^-\left(I_k\right)\right)\right)^{\alpha} V^{\delta}\left(\sigma_i\right) e^{\beta W\left(\sigma_i\right)}}{\sum_{j=1}^{N}\left(\prod_{k=1}^{L}\left(1 - \chi\left(\sigma_i \mid I_k\right)\pi^-\left(I_k\right)\right)\right)^{\alpha} V^{\delta}\left(\sigma_i\right) e^{\beta W\left(\sigma_i\right)}}. \tag{4.195}$$

As earlier the product $\prod_{k=1}^{L}\ldots$ can become zero only if distribution $\pi^-(I_k)$ is singular and, if $\pi^-(I_q) = 1$, $\chi(\sigma_i| I_k) \neq 0$. In this connection $\pi(\sigma_i|S_j^L) = 0$ when some of $\chi(\sigma_i| I_k) \neq 0$ (at least for one $j \neq i$).

In connection with formula (4.195) it is possible to raise the question about the determination of differential preferences $\pi(\sigma_i| I_k)$, since the right side of the equality is linear combination of values $\pi(\sigma_i| I_k)$. Let`s designate right side (4.195) through $G(\sigma_i)$, then we can write to

$$\sum_{k=1}^{L}\pi^{-}\left(I_k\right)\pi\left(\sigma_i \mid I_k\right)=G\left(\sigma_i\right),\ (i\in\overline{1,N}). \tag{4.196}$$

This is the system of equations relative to N×L values $\pi(\sigma_i| I_k)$, where the coefficients $\pi^{-}(I_k)$and right sides $G(\sigma_i)$ are considered as the assigned values. Since to these equations L conditions of normalization for values $\pi(\sigma_i| I_k)$ should be added then in all we have $L + N$ equations. If $\nu = NL - N - L < 0$ system is over determined. If $\nu > 0$ system is indeterminate and if $\nu = 0$, then $N = L = 2$. Thus, if the optimality of the distribution $\pi\left(\sigma_i \mid S_I^L\right)$, is postulated then in the case of indeterminacy it is possible to introduce additional conditions, for example, the optimality of some differential preferences. If $L = 1$, i.e., there is only one imperative, then ν = - 1 and it is missing one additional condition. But in this case S^L_I: $I_1 \Rightarrow \pi(I_1) = 1$ and distribution $\pi(\sigma_i| I_k)$ coincides with of utilitarian distribution. Versions of conditional distribution examined above have the deficiency, that preferences can become zero only in the case of singular distribution $\pi^{-}(I_k)$. If distribution $\pi^{-}(I_k)$ is nonsingular: $\pi^{-}(I_k) \neq 1$, $\forall k\in\overline{1,L}$, distribution $\pi(\sigma_i| I_k)$ and $\pi\left(\sigma_i \mid S_I^L\right)$ do not become zero. , In order to attain that some preferences turn into zero it is necessary further modification of criteria $\Phi_{\pi(\dots)}$. The addition, which will be made below could be conditionally name "*the model of categorical imperative*".

Let`s write the following expression for the differential preferences:

$$\pi\left(\sigma_i \mid I_k\right)=\frac{\left(1-\chi\left(\sigma_i \mid I_k\right)\pi^{-}\left(I_k\right)\right)\prod_{q=1}^{L}\chi\left(\sigma_i \mid I_k\right)V^{\delta}\left(\sigma_i\right)e^{\beta W\left(\sigma_i\right)}}{\sum_{j=1}^{N}\left(1-\chi\left(\sigma_j \mid I_k\right)\pi^{-}\left(I_k\right)\right)\prod_{q=1}^{L}\chi\left(\sigma_j \mid I_k\right)V^{\delta}\left(\sigma_j\right)e^{\beta W\left(\sigma_j\right)}}. \tag{4.196a}$$

The coefficient $\prod_{q=1}^{L}\chi\left(\sigma_i \mid I_k\right)$, is here added, which *is equal to zero*, if at least one of the imperatives $I_k \in S^L_I$ "prohibits" σ_i. All terms in the denominator cannot be equal to zero in view of assumption about the completeness of the imperatives system, which *always must leave to subject "the output" from the situation*, which does not contradict this ethics system . Analogously, it is possible to construct distribution for $\pi\left(\sigma_i \mid S_I^L\right)$.

Characteristic function $\chi(\sigma_i| I_k)$ (in the more common case $\chi(S'_a \subset S_a| I_k)$) determine the type of ethics. The tables given below illustrate this circumstance.

Let function $\chi(\sigma_i| I_k)$, given on the direct product $S^N_a \times S^L_I$, is determined by table

(L = 5; N = 4) (tabele.6).

Tabele 6

I_k / σ_i	I_1	I_2	I_3	I_4	I_5
σ_1	0	0	0	0	1
σ_2	0	0	1	0	0
σ_3	1	0	0	1	0
σ_4	0	0	0	0	0

In this case the ethics is *injective*: any imperative either "serves" not one alternative or "serves" only one of them.

Tabele 7 represents an example of the functional ethics

Tabele 7

I_k / σ_i	I_1	I_2	I_3	I_4	I_5
σ_1	1	0	0	0	0
σ_2	0	0	1	0	0
σ_3	0	0	0	0	0
σ_4	1	0	0	0	0

When each alternative σ_i either is not "served" by any $I_k \in S^L{}_I$ or it is served only by one of them.

"*Surjective*" ethics can be represented as an example by tabele.8.

Tabele 8

I_k / σ_i	I_1	I_2	I_3	I_4	I_5
σ_1	1	0	1	1	1
σ_2	0	0	0	0	0
σ_3	1	1	1	0	0
σ_4	1	0	0	0	1

Finally, "everywhere defined" ethics is represented by tabele 9:

Tabele 9

I_k / σ_i	I_1	I_2	I_3	I_4	I_5
σ_1	1	0	0	0	0

σ_2	1	1	1	1	1
σ_3	0	0	1	0	0
σ_4	0	0	1	1	1

Between all given cases, only ethics, represented by tab.9 is compatible with the distribution $\pi(\sigma_i| I_k)$ of type (4.196), contains "categorical imperative" (having nothing common with the Kant categorical imperative), since there is alternative σ_2, for which

$$\prod_{k=1}^{5}\chi\left(\sigma_2 \mid I_k\right)=1 .$$

The distributions proposed in this chapter, take into account is creation degree, the influence of imperatives - the steady postulates of different nature, including ethical once. The special feature of approach lies in the fact that imperatives are considered as a priori given ones and are not subject to determination on the basis of some principle of higher level.

In conclusion let`s examine a simple example of the calculation of entropy and information, connected with of ethics system.

Let there are two alternatives $N = 2$ in S_a: σ_1 and σ_2, and utilities respectively are $U(\sigma_1) = 2$; $U(\sigma_2) = 1$. Utilitarian (rationalistic) preference is

$$\pi^0\left(\sigma_1\right)=\frac{e^2}{e^1+e^2}=0{,}7310;\;\; \pi^0\left(\sigma_2\right)=\frac{e^1}{e^1+e^2}=0{,}2689 .$$

Entropy $H_{\pi^0(\sigma)}=0{,}5822$.

Assume that now the subject is subordinated to the system of the ethics of that containing only imperative I_1, which is affined to σ_1: $\chi(\sigma_1| I_1) = 1$ and non-affined to σ_2: $\chi(\sigma_2| I_1) = 0$ imperative I_1 is "prohibiting", and, in view of normalization $\pi^-(I_k) = 1$, then

$$\pi\left(\sigma_1 \mid S_I^1\right)=\pi^-\left(I_1\right)\pi\left(\sigma_1 \mid I_1\right)=1\frac{\left(1-1\cdot 1\right)e^2}{\left(1-1\cdot 1\right)e^2+\left(1-0\cdot 1\right)e^1}=0;$$

$$\pi\left(\sigma_2 \mid S_I^1\right)=\pi^-\left(I_1\right)\pi\left(\sigma_2 \mid I_1\right)=1\frac{\left(1-0\cdot 1\right)e^1}{\left(1-1\cdot 1\right)e^2+\left(1-0\cdot 1\right)e^1}=1 .$$

Entropy $H_{\pi\left(\sigma|S_I^L\right)}=-0\ln 0-1\ln 1=0$. Consequently, "the introduction" of imperative I_1 decreases the entropy

$$I\left(S_I^1,\rho\right)=H_{\pi^0(\sigma)}-H_{\pi\left(\sigma|S_I^1\right)}=0{,}5822\ldots>0 .$$

Let`s now L = 2, there are two imperatives I_1 and I_2 in S_j^2 and, correspondingly, their significance $\pi^-(I_1)$ = 0,2, $\pi^-(I_2)$ = 0,8, and $\chi(\sigma_1| I_1)$ = 1, $\chi(\sigma_2| I_1)$ = 0, $\chi(\sigma_1| I_2)$ = 1, $\chi(\sigma_2| I_2)$ = 1.

Then

$$\pi\left(\sigma_1 \mid S_I^2\right) = \pi^-\left(I_1\right)\pi\left(\sigma_1 \mid I_1\right) + \pi^-\left(I_2\right)\pi\left(\sigma_1 \mid I_2\right) =$$

$$= 0,2\frac{\left(1-1\cdot 0,5\right)e^2}{\left(1-1\cdot 0,2\right)e^2 + \left(1-1\cdot 0,5\right)e^1} + 0,8\frac{\left(1-1\cdot 0,8\right)e^2}{\left(1-1\cdot 0,8\right)e^2 + \left(1-1\cdot 0,8\right)e^1} = 0,72186;$$

$$\pi\left(\sigma_2 \mid S_j^2\right) = \pi^-\left(I_1\right)\pi\left(\sigma_2 \mid I_1\right) + \pi^-\left(I_2\right)\pi\left(\sigma_2 \mid I_2\right) =$$

$$= 0,2\frac{\left(1-1\cdot 0,2\right)e^1}{\left(1-1\cdot 0,2\right)e^2 - \left(1-0\cdot 0,2\right)e^1} + 0,8\frac{\left(1-1\cdot 0,8\right)e^1}{\left(1-1\cdot 0,8\right)e^2 + \left(1-1\cdot 0,8\right)e^1} = 0,2781\,.$$

Entropy in this case $H_{\pi\left(\sigma|S_I^1\right)} = 0,59117$ and, consequently

$$I\left(S_I^1, \rho\right) = H_{\pi\left(\sigma|S_I^1\right)} - H_{\pi\left(\sigma|S_I^2\right)} = -0,00897\,.$$

This means that with the expansion of the set of imperatives from one to two "the production" of entropy in the system occurs, certainly for "the case of affinity", assigned by matrix χ, and "the significances" of imperatives $\pi^-(I)$:

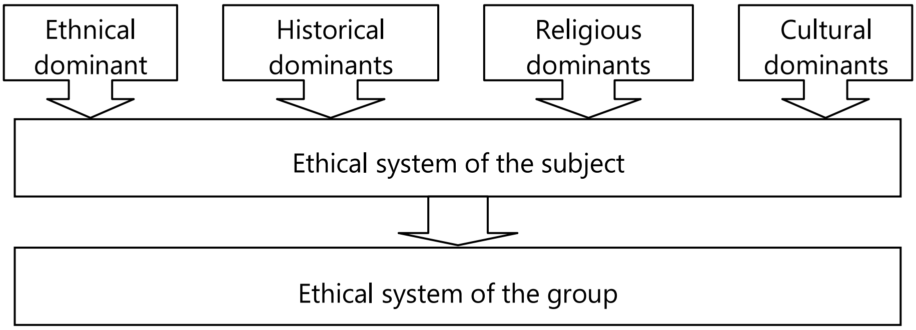
$$\chi = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}; \quad \pi^-\left(I\right) = \begin{bmatrix} 0,2 \\ 0,8 \end{bmatrix}.$$

4.12. One more time about "virtual subject".

Throughout the elongation of this chapter we repeatedly touched on a question about "virtual subject" as the factor connecting the group of subjects. The author completely realizes, that some even particular, but final model creation was not a success. As a whole the problem was only outlined. In the present paragraph we will make an attempt to move somewhat in the direction of the construction of such a model and examine several additional constructions.

One of the fundamental questions, which must be solved on this way, is the calculation of ethical factors. The common questions, which are concerned ethics, and some models of preferences, including the calculation of ethical factors have been discussed above.

In the common case it is possible to speak about four non-utilitarian components, which together form the ethical system of each subject and, if this concept is permissibly, the ethical system of the group: ethnic, historical, religious, cultural [117].



Ethical system seemingly integrates, makes, connects up in unified whole the ethnic, historical, religious, cultural features of this social (group) as the system of the ethical imperatives, which have an effect on the forming of the preferences distributions of the I and II kinds.

The idea about "virtual subject" (collective reason) is not absolutely new. It is possible to refer, for example, to american sociologist monograph J.Turner [201], where we can read the following: "*It can seem that the leader makes decisions more daringly than group as integrity, but in actuality common solution of group is considerably more substantiated and responsible, than that made as individual (Kogan, Wallach, Janis). There are two reasons for this phenomenon: first, when man is forced himself to make a decision, fear of error accomplishment inclines him to the caution, whereas committee decision divides responsibility between the members of group, which leads to the manifestation of the larger resoluteness: in the second place, in the total group, its members, frequently abstain from the mutual criticism, as a result of which they can also be less critical with respect to the more daring proposals, which leads to the fact that the group acts more decisively*".

And further the author of the mentioned work draws the conclusion: "*This process frequently leads to that what is determined by term "the group thinking" (Janis, 1982), which lies in the fact that the individual members of group, suppress their individual views to such an extent, that they cease to focus attention on detail and begin to feel and to behave more daring*".

Phenomenon which J. Turner calls "*group thinking*", we designate by term "*virtual subject*".

In the same work we can find the confirmation of the fact that *the religion is the group creating factor*. However, this is obvious without any literary confirmation.

Interesting and important is thesis about the fact that *the real inequality* not only divides social, but it serves also as uniting factor, since ensures dependence of some on others. The greater inequality, the greater dependence of the salesman on his labor (worker) on the buyer (employer). The first has, what to sell - his labor, the second has, for what to purchase - money. Here J. Turner refers to K. Marks and M. Veber.

In any akt of buying and selling relation "buyer- seller" *is antagonistic*. The society, in which there is large real inequality it cannot be free, and, in this case both a buyer and a salesman are restricted. In this society the sseller (worker) does not possess external freedom. The more he strives for the internal freedom. We already spoke about the phenomenon "of compensation", when slave transferring his "soul" to god partially becomes free from the power of earthy owner.

In the society, burdened by deep real inequality, it cannot be spoken about the freedom, and therefore about democracy. In such a society formal democracy is converted at "the authority of money", but not people.

As an example let`s give information from the book of J. Turner, characterizing the level of material inequalities in the USA. Table 10 gives the data, which reflect the non-uniformity of the distribution of property between five groups of American citizens at 1962 and 1983 yr. The source of these data is the ministry of Federal Reserve's of the USA.

Tabele 10

Group	Percentage of the common property	
	1962	1983
Upper 20 %	76,0	74,7
II 20 %	15,0	14,2
III 20 %	6,2	6,9
IV 20 %	2,1	3,0
Lower 20 %	0,2	0,1

Table 11 illustrates the non-uniformity of the distribution of incomes in the American society according to the data of Bureaus population calculation of the USA.

Tabele 11

Groups	1960	1970	1980	1990
Upper 20 %	42,0	43,3	44,2	46,6
II 20 %	23,6	23,5	24,8	24,0
III 20 %	17,6	14,4	16,8	15,9

IV 20 %	12,0	10,8	10,2	9,6
Lower 20 %	4,3	4,1	4,1	3,9

Coefficient of the property inequality of Lorenz, calculated according to the first table for 1983 year $K \approx 0,65$. This index is arranged in such a manner that it is equal to 0 in the case of the complete equality and equal to 1 for the case of "complete" inequality ("some has everything, rest - nothing").

From the point of view of the purposes of our study essential is the fact that the real inequality is the group-generating factor. It is assumed that this circumstance can be considered using mutual utilities. The greater the inequality and the greater the mutual utilities in the absolute calculation, the more strongly group are "soldered"; however, the fewer roles in "*the consciousness of virtual subject*" play the poorest members of group. Following these reasoning's it is possible to assume that in the group with the complete equality although mutual utilities can be strong (technological cooperation, for example), but "virtual subject", if it may be expressed that way, is invested by all members of group in even propertions.

Let`s attempt to interpret these reasoning's in the terms of subjective analysis. Since the optimality of the preference distribution is the basic postulate of subjective analysis, the question arises: who in this case is "*the author*" of the optimization, where, so to speak, "*virtual subject*" is located in the formal sense, who builds functional and solves the appropriate variational problem?

We will consider that "virtual subject" (or "collective reason") is those parts of individual consciousness and psyches, which function "in unison", they have total set of alternatives S'_a as the intersection of individual alternative subsets. Moreover, some of the alternatives S'_a can exist only as the corporative once since corresponding required resources exceed the possibilities of each individual members of group, and they require for their realization the consolidation of both passive, and active individual resources.

"Virtual subject" - this is no formal chief or hierarch, but formal leader under certain conditions can lean for his support during control of group. If between the formal leader and "virtual subject" is a divergence "in the views", then conflict appears, and control becomes ineffective. As the quantity indicator of agreement or divergence "in the views" can serve some measure of correlations (Pearson's coefficient, the rank correlation coefficients Spearmen`s, Kendal`s and others), expressed through the preferences distributions. *Multiple correlation coefficients* can play certain role.

Two different approaches to molding of the "virtual subject" appearance are possible. *The first* lies in that each individual subject extremalizes his own criterion (functional) and manufactures his individual preference distribution taking into account the presence of corporate problems and transfer of resources for the purpose of their consolidation.

Additionally to the fact that already was told about the difference between the passive and active resources let`s note that the distinctive special feature *of active resources* is the impossibility of their transfer from one individual subject to another, while on the transfer of the passive resources the fundamental prohibitions it is not superimposed.

The second approach to molding of the "virtual subject" appearance assumes that this subject exists and has equal rights with the remaining real subjects as M + 1 subject and "occupies" the part of the consciousness of each real subject. In this case there is a necessary information contact, which ensures the functioning of this information essence as whole. In this version "virtual subject" is "the carrier" of united functional and solves M + 1 variation problem for the purpose of the production of collective, acknowledged by all preference distribution on chosen common the subset of alternatives S'_a.

Let S_{aj} ($j \in \overline{1,M}$) is individual set of utilitarian (or object) alternatives. Using the method of construction of the covering set, let`s assume that this set

$$S_a = \bigcup_{j=1}^{M} S_{aj}.$$

Let`s pick out the set of corporate alternatives

$$S'_a = \bigcap_{j=1}^{M} S_{aj}.$$

In this case it is assumed that the possibility of the consolidation of individual resources was caused previously, so that in S_{aj} those alternatives, which require the consolidation of resources also are included. It is obvious that among the alternatives S'_a can be such once, which do not require the consolidation of resources. Finally, we can examine the intersections of set S_{aj} of different subgroups of subjects with dimensionality $M - 1$, $M - 2$, $M - 3$,...

Let`s further, $S_{Jj}$ is individual set of ethical imperatives. Let`s form the set

$$S'_I = \bigcap_{j=1}^{M} S_{Ij},$$

being the set of those ethical imperatives, which are " recognized" by all members of group? Pair $\{S'^{n}_{I}, \pi(I)\}$ we will consider as the "ethics of group". It is assumed more over that the individual preferences $\pi_j(\sigma_k)$ of alternatives $\sigma_k \in S'_a$, and also the individual characteristics of the imperatives significance $\pi_j(I_s)$ on the set S'_I differ little from each other, "they are not distinguished from without":

$$\pi_j(\sigma_k) \approx \pi_i(\sigma_k);$$

$$\sigma_k \in S'_a;\ \forall i, j \in \overline{1, M};$$

$$\pi_j\left(l_s\right) \approx \pi_i\left(l_s\right);$$

$$l_s \in S'_l;\ \forall i, j \in \overline{1, M}.$$

If we use, as this was done above, distribution $\pi_j(\sigma_k, l_s)$ on the product $S' = S'_a \times S'_l$, then it is necessary to assume that

$$\pi_j(\sigma_k, l_s) \approx \pi_i(\sigma_k, l_s);\ \forall j, i \in \overline{1, M}.$$

The degree of the proximity of distributions can be estimated quantitatively. Let`s refer pair to S', if

$$|\pi_j(\sigma_k, l_s) - \pi_i(\sigma_k, l_s)| \le \rho^*,$$

where ρ^* is low value (0,1; 0,05; 0,01; ...).

In this case the members of group can perceive themselves on set S' as something "united", sovereign "virtual subject" appears. For this subject in our lexicon we have a word "we". This collectivized part of the consciousness forces each real subject to act together in the composition of group.

Three different models will be examined. The first model corresponds to the least autonomous and solitary "virtual subject". It is based on the following assumptions:

1. Each of N of individual subjects optimizes his eigenfunctional.

2. For each subject j his set of alternatives is separated on two subsets S'_{aj} and S_{am}: $S_{aj} = S'_{aj} \cup S_{am}$, where S_{am}, contains alternatives common (in particular - corporate) for all subjects, S'_{aj} is the set of exceptionally personal alternatives of j. They can enter into the intersections of smaller dimensionality.

3. Function of the effectiveness's includes addend, in which utilitarian components do not depend on the number of subject. This occurs for the alternatives $\sigma_s \in S_{am}$.

Let each individual functional take the form:

$$\Phi_{\pi j} = -\sum_{k=1}^{N'_j} \pi_j\left(\sigma_k\right) \ln \pi_j\left(\sigma_k\right) - \sum_{k=1}^{N'_j} \pi_{jm}\left(\sigma_s\right) \ln \pi_{jm}\left(\sigma_k\right) \pm \tag{4.197}$$

$$\pm \beta_{1j} \sum_{k=1}^{N'_j} \pi_j\left(\sigma_k\right) F_j\left(\sigma_k\right) \pm \beta_{2j} \sum_{k=1}^{N'_j} \pi_{jm}\left(\sigma_s\right) F_m\left(\sigma_k\right) + \gamma\left(\sum_{k=1}^{N'_j} \pi_j\left(\sigma_k\right) + \sum_{S=N'_j+1}^{N} \pi_{jm}\left(\sigma_s\right)\right).$$

Note that each subject has the individual endogenous parameters β_{1j} and β_{2j}.

The distributions of preferences $\pi_j(\sigma_k)$ and $\pi_{jm}(\sigma_s)$ take the form:

$$\pi_j\left(\sigma_k\right)=\frac{e^{\pm\beta_{ij}F_i\left(\sigma_k\right)}}{\sum\limits_{p=1}^{N'_j}e^{\pm\beta_{1j}F_i\left(\sigma_p\right)}+\sum\limits_{s=N'_j+1}^{N}e^{\pm\beta_{2j}F_m\left(\sigma_s\right)}};\qquad(4.198)$$

$$\pi_{jm}\left(\sigma_k\right)=\frac{e^{\pm\beta_{2j}F_m\left(\sigma_s\right)}}{\sum\limits_{p=1}^{N'_j}e^{\pm\beta_{1j}F_j\left(\sigma_p\right)}+\sum\limits_{s=N'_j+1}^{N}e^{\pm\beta_{2j}F_m\left(\sigma_s\right)}}.\qquad(4.199)$$

In this case the preferences of corporate alternatives $\sigma_s \in S_{am}$ preserve their individuality, that is, they depend on the number of subject *j*. They are connected with of non-corporative preferences with formula:

$$\pi_{jm}\left(\sigma_s\right)=\pi_j\left(\sigma_k\right)\frac{e^{\pm\beta_{2j}F_m\left(\sigma_s\right)}}{e^{\pm\beta_{1j}F_j\left(\sigma_k\right)}}.\qquad(4.200)$$

In these formulas through F_j and F_m either utilities U_j, U_m or harmfulness as L_j, L_m are designated.

The second model is based on the following assumptions:

1. Group functions as a whole. Therefore variational problem is formulated for the entire group with the aid of the united functional.

2. Individuality of the subjects - the members of group nevertheless remains. Each of them has personal preference distribution $\pi_j(\sigma_k)$ in “the outskirts” $S_{aj}\backslash S'_a$ and acts in these “outskirts” independently. Distributions $\pi_j(\sigma_k)$ depends; nevertheless, on the preferences of $M + 1$ “virtual subject” on S'_a: $\pi_{M+1}(\sigma_s)$.

Functional for the entire group is taken in the form:

$$\begin{aligned}\Phi_\pi=&-M^{-1}\sum_{j=1}^{M}\sum_{k=1}^{N'_j}\pi_j\left(\sigma_k\right)\ln\pi_j\left(\sigma_k\right)-\sum_{s=N'+1}^{N}\pi_{M+1}\left(\sigma_s\right)\ln\pi_{M+1}\left(\sigma_s\right)\pm\\&\pm M^{-1}\sum_{j=1}^{M}\beta_{1j}\sum_{k=1}^{N'}\pi_j\left(\sigma_k\right)F_j\left(\sigma_k\right)\pm\beta_2\sum_{s=N'+1}^{N}\pi_{M+1}\left(\sigma_s\right)F_{M+1}\left(\sigma_s\right)+\\&+\gamma\left(\frac{1}{M}\sum_{j=1}^{M}\sum_{k=1}^{N'}\pi_j\left(\sigma_k\right)+\sum_{s=N'+1}^{N}\pi_{M+1}\left(\sigma_s\right)\right).\end{aligned}\qquad(4.201)$$

Here β_{ij} is individualized endogenous parameters, β_2 is the endogenous parameter, which relates to “virtual subject”.

Coefficient M^{-1} in the first, third and last forms plays the role of the weighting factor, which balances the role of the real members of group and “virtual subject”. If we do not introduce this coefficient, then with the large numbers of group *M*, even with small *N*', the first term will suppress the second term, which reflects the contribution to the entropy of “virtual subject”.

The number N' in this case is determined as follows: $S_a = \bigcup_{j=1}^{M} S_{aj}$ is "covering" set of alternatives, $S'_a = \bigcap_{j=1}^{M} S_{aj}$ is the set of alternatives, with which "virtual subject" operates, distributions $\pi_j(\sigma_k)$ are augmented in such a way that on the alternatives, which are contained at $S_a \backslash S_{aj}$ correspondent $\pi_j(\sigma_k) = 0$.

The presence in formulas of two signs "+" and "-" emphasizes the circumstance that $F_j(\sigma_k)$ can be utility or harmfulness. The preference distributions are determined by the formulas:

$$\pi_j\left(\sigma_k\right) = \frac{e^{\pm\beta_{1j}F_j\left(\sigma_k\right)}}{\frac{1}{M}\sum_{p=1}^{M}\sum_{k=1}^{N'} e^{\pm\beta_{1j}F_j\left(\sigma_p\right)} + \sum_{q=N'+1}^{N} e^{\pm\beta_{2j}F_{M+1}\left(\sigma_q\right)}}; \tag{4.202}$$

$$\pi_{M+1}\left(\sigma_s\right) = \frac{e^{\pm\beta_2 F_{M+1}\left(\sigma_s\right)}}{\frac{1}{M}\sum_{p=1}^{M}\sum_{k=1}^{N'} e^{\pm\beta_{1j}F_j\left(\sigma_p\right)} + \sum_{q=N'+1}^{N} e^{\pm\beta_2 F_{M+1}\left(\sigma_q\right)}}. \tag{4.203}$$

Values $F_j(\sigma_k)$, $F_{M+1}(\sigma_s)$, and, therefore, $\pi_j(\sigma_k)$ and $\pi_{M+1}(\sigma_s)$ are determined with taking into account the influence of ethical imperatives I_e. Here we for the purpose of simplification of the formulas record influence of I_e is not explicitly separated.

In the given model "virtual subject" is isolated as separate "essence", which is found in interaction with the remaining, real members of group. Preferences $\pi_{M+1}(\sigma_s)$ do not depend on index j.

Common entropy can be represented as the entropy of the entire group, which consists of $M + 1$ subject. This refloats by first two terms in (4.201). In this model "virtual subject" exists as isolated: each real subject deals with his own problems (to $S_{aj} \backslash S'_a$) (Fig. 4.24, *b*). Fig. 4.24, illustrates the first model, in which "virtual subject" is not detached as separate essence"

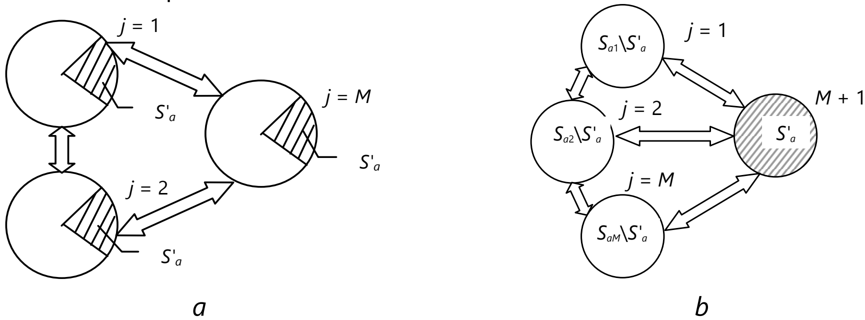


Fig. 4.24

The connections between the distributions are achieved through the normalization, the influence of ethical imperatives, mutual utilities or harmfulness.

In order to write the entropies of each M + 1 subjects, it is necessary to carry out the renormalization:

$$\overline{\pi}_j\left(\sigma_k\right)=\frac{\pi_j\left(\sigma_k\right)}{\sum\limits_{p=1}^{N'}\overline{\pi}_j\left(\sigma_p\right)};\quad \sum_{k=1}^{N'}\overline{\pi}_j\left(\sigma_k\right)=1;$$

$$\overline{\pi}_{M+1}\left(\sigma_k\right)=\frac{\pi_{M+1}\left(\sigma_k\right)}{\sum\limits_{q=1}^{N'}\overline{\pi}_{M+1}\left(\sigma_q\right)};\quad \sum_{q=N'+1}^{N}\overline{\pi}_{M+1}\left(\sigma_s\right)=1\,.$$

Let`s calculate the correspondent entropies by formulas:

$$H_{\pi j}=-\sum_{k=1}^{N'}\overline{\pi}_j\left(\sigma_k\right)\ln\overline{\pi}_j\left(\sigma_k\right),\ (\,j\in\overline{1,M}\,);$$

$$H_{\pi M+1}=-\sum_{q=N'+1}^{N}\overline{\pi}_{M+1}\left(\sigma_q\right)\ln\overline{\pi}_{M+1}\left(\sigma_q\right).$$

Values $F_{M+1}(\sigma_q)$ depend on the consolidated resources, and also, possibly, from the more intensive “support” or “opposition” of ethics.

Assuming that, all β_{2j} are identical, as well as all N'_j are identical, we can come to conclusion that the sum of all M functional (4.197) will differ from functional (4.201) only regarding presence of coefficient M in the second and the fourth terms. This version can be considered as the third intermediate model.

Let`s examine a special case. Let all β_{1j} and $F_j(\sigma_k)$ are identical for $\forall j$ and $\forall_k$, and let all $F_{M+1}(\sigma_s)$ are identical too. Designate

$$e^{\pm\beta_{1j}F_j\left(\sigma_s\right)}=A;\ \ e^{\pm\beta_2F_{M+1}\left(\sigma_s\right)}=B.$$

For model (4.201) the entropy of entire group (first two terms in the formula (4.201)) can be written in the form

$$H_\pi=-\frac{N'A}{N'A+\left(N-N'\right)B}\ln\frac{N'A}{N'A+\left(N-N'\right)B}-$$

$$-\left(\left(N-N'\right)\frac{B}{N'A+\left(N-N'\right)B}\ln\frac{B}{N'A+\left(N-N'\right)B}\right).$$

Let $B = mA$, then

$$H_\pi=\ln\left(N'+\left(N-N'\right)m\right)-\frac{\left(N-N'\right)m}{N'+\left(N-N'\right)m}\ln m. \qquad (4.204)$$

If m = 1, i.e., $A = B$, then $H_\pi = \ln N$.

In this case the entropy of entire group does not depend on the size of group. Let`s carry out calculations for the case $N = 2$ ($N' = 1$); m = 2; 3; 4 (see Fig. 4.25).

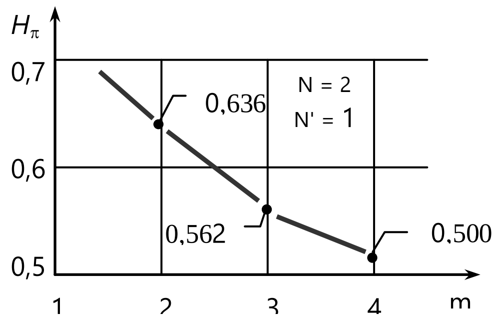


Fig.4.25

If the normalizing weighting factor M^{-1} in functional (4.201) is absent, in the qualitative sense this means that "virtual subject" is received as "series" group member. In this case with the same simplifying assumptions, which are made in the previous example, formula for the entropy takes the form:

$$H_\pi = -MN' \frac{A}{MN'A + (N - N')B} \ln \frac{A}{MN'A + (N - N')B}.$$

Assuming again $B = mA$, after simplifications we find:

$$H_\pi = \ln\left(MN' + (N - N')m\right) - \frac{(N - N')m}{MN' + (N - N')m} \ln m. \qquad (4.205)$$

From (4.205) we can see that with $N - N' = 1$ (one corporate problem) and $m = M$ (consolidated resources are M time more than individual, where M is the size of group)

$$H_\pi = \ln N + \left(1 - \frac{1}{N}\right) \ln M.$$

If M = 1, (group includes one subject), we obtain $H_\pi = \ln N$, i.e., usual value for the entropy of the preferences of N equivalent alternatives.

Fig.4.26 shows the results of calculation according to the formula (4.205) when M = 3 (group consists of three subjects), N = 2, (N' =1); m = 2; 3; 4.

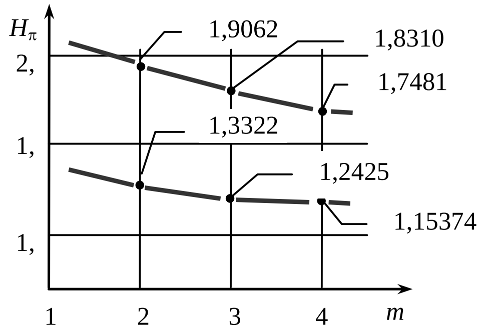


Fig.4.26

In formula (4.205) a dependence of entropy on the size of group presents, and we have:

$$\frac{\partial H_\pi}{\partial M} = \frac{N'}{MN'_-(N-N')m} + \frac{(N-N')N'm\ln m}{\left(MN' + (N-N')m\right)^2}.$$

Since $N - N' \geq 0$, assuming $m \geq 1$, we find that

$$\frac{\partial H_\pi}{\partial M} > 0,$$

whence it follows that the entropy H_π grows with an increase of group size.

Let`s define how H_π (other conditions are equal,) depends on the coefficient m, which shows in what times the available resources of "virtual subject" differ from the individual resources of the real members of group (if $m > 1$, then occurs "consolidation" of resources).

$$\frac{\partial H_\pi}{\partial m} = \frac{N-N'}{MN' + (N-N')m} -$$

$$-\frac{(N-N')(\ln m + 1)\left(MN' + (N-N')m\right) - (N-N')^2 m\ln m}{\left(MN' + (N-N')m\right)^2}.$$

Derivative $\frac{\partial H_\pi}{\partial m} = 0$, (1) when $N = N'$ and (2) when

$$1 - \frac{(\ln m + 1)\left(MN' + (N-N')m\right) - (N-N')m\ln m}{MN' + (N-N')m} = 0.$$

Last equality is converted to the equality $\ln m = 0$. Hence it follows that

$$H_\pi(m > 1) < H_\pi(m = 1).$$

Other models of “virtual subject” are not excluded. Entire previous content of chapter 4 refers direct to this problem. Different versions of the canonical distribution referred above, including of those aggregated, can be used for constructing of the “virtual subject” models. Extremely important is the calculation of the influence of ethical imperatives, and also the calculation of rating distributions. Finally, the study of the dynamics of group preferences is a persistent need. In chapters 5 quantitative examples are given.

It is seemed that the analysis of dynamics will make it possible to model cumulative effects in the groups, advancement and assertion of leaders, to trace the processes of group’s structurizing, including appearance and the disintegration of hierarchic structures. The problem of constructing the models of information traffics inside the group, the redistribution of resources, dynamics of individual and group active and passive resources is promising. In particular, the question requires answer: does “virtual subject” possess active resources, and, if yes, then by what?

The available resources of each subject R_j^{disp} consist of two parts: R_{jj}^{disp} isthe non-consolidated individual part and R_{jM}^{disp} is the part of the resources of those transferred by subjects to the consolidated fund. Total consolidated resources

$$R_M^{disp} = \sum_{j=1}^{M} R_{jM}^{disp} .$$

This fund cannot be used by individual subject *j* for the solution of his particular problems on the set $S_{aj} \backslash S'_a$. The withdrawal of the part of resources at the consolidated fund can lead to the fact that the part of the subject *j* problems on $S_{aj} \backslash S'_a$ will prove to be not realized and they will be removed from this subset. From the other side, the appearance of large volume of the consolidated resources will make it possible to include in the number of alternatives of set S'_a more “expensive” alternatives.

As has already been spoken, the transfer of resources from one subject to the other concerns only passive resources.

Let`s recall as it was assumed above that a) the realization of active resources requires the expenditures of the certain quantity of passive resources and, vice versa b) the realization of passive resources requires each time of expense of active resources. There are limit proportions, when some procedures become not realizable. Active resources are not transferred from one subject to the other, but can they become passive resources?

Answer, apparently, is reduced to the following: active resources can become passive simultaneously with a change in status of their owner. The Subject- possessor of active resources must change his own status and cease to be subject in this group and with respect to these set of alternatives. Thus, the transfer of active resources is possible simultaneously with their transformation in the passive resources of other subject, who can use them for the solution of his problems. Person, who possesses, but does not manage his own active resources, conditionally speaking, he is "slave", and his own active resources are the passive resources of his owner. It is possible to say that the moment of passive resources of subject ($R_{pj}^{disp} \to 0$) into zero coincides with the moment of changing of his status: it ceases to be subject, since "the object" of the application of active resources disappears. His individual set of alternatives are turned at the empty set ($S_{aj} \to \varnothing$).

Strictly speaking, there are always two additional "regular" alternatives: the total curtailment of activity in this association (group, σ_m is output from the group) or the acquisition of status of "slave" — (σ_s). The preference distribution on these alternatives can be only singular and consequently, "final" entropy $H'_\pi = 0$. These extreme possibilities present always, but in the normal situation their preferences are negligibly small.

An essential deficiency of the models of the preference distribution (4.198), (4.199), (4.202), (4.203) given above lies in the fact that the influence of "collective reason" is achieved exclusively through the normalization. It is seemed that the more direct mutual influence of the preferences of different subjects in the group must occur also. Remaining in the limits of hypothesis about the variational principle of forming of all preferences both of the I and II kind, we must to assume that the mutual influence of preferences in the group should be realized through the forming of the corresponding functionals.

Let`s examine for example, following model problem. Suppose that the aggregated preference of alternative σ_k

$$\pi^\Sigma(\sigma_k) = \sum_{j=1}^{M} \xi(j)\pi_j(\sigma_k),$$

where $\xi(j)$ is integral ratings, and $\pi_j(\sigma_k)$ is the preference of subject j.

And let "virtual subject" solves two variational problems:

1) integral ratings $\xi(j)$ determines, in accordance with the functional:

$$\Phi_\xi^\Sigma = -\sum_{j=1}^{M} \xi(j)\ln\xi(j) + \beta_1\sum_{j=1}^{M} \xi(j)R_{cj}^{disp} + \gamma_1\sum_{j=1}^{M} \xi(j), \tag{4.206}$$

where R_{cj}^{disp} is part of the available resources, which subject j transfers in common fund, referred to the total consolidated resources $R^{\Sigma disp}$, and

2) selects the distribution of object preferences on S_a using the functional:

$$\Phi_\pi^\Sigma = -\sum_{k=1}^{T} \pi^\Sigma(\sigma_k)\ln\pi^\Sigma(\sigma_k) + \beta\sum_{k=1}^{N} \pi^\Sigma(\sigma_k)\overline{r}^d(\sigma_k) + \gamma\sum_{k=1}^{N} \pi^\Sigma(\sigma_k), \tag{4.206a}$$

where $\bar{r}^d(\sigma_k) = \dfrac{R^{\Sigma disp} - R^{req}(\sigma_k)}{R^{\Sigma disp}} = 1 - \bar{r}^r$, $R^{\Sigma disp}$ — total consolidated resources.

The solution of the first variational problem takes the form:

$$\xi(j) = \frac{e^{\beta_1 R_{cj}^{disp}}}{\sum_{q=1}^{M} e^{\beta_1 R_{cq}^{disp}}}. \tag{4.207}$$

For $\pi^{\Sigma}(\sigma_k)$ we obtain

$$\pi^{\Sigma}(\sigma_k) = \sum_{j=1}^{M} \xi(j)\pi_j(\sigma_k) = \frac{e^{\beta \bar{r}^d(\sigma_k)}}{\sum_{s=1}^{N} e^{\beta \bar{r}^d(\sigma_s)}}. \tag{4.208}$$

Here $\pi_j(\sigma_k)$ is the individual preferences. Relationship (4.208) can be considered as the limitation, superimposed on the selection of functions $\pi_j(\sigma_k)$. There are N such limitations. Furthermore, M conditions the normalization s $\sum_{k=1}^{N} \pi_j(\sigma_k) = 1$, are carried out.

Thus, on NM values $\pi_j(\sigma_k)$ $N + M$ connections are superimposed. Consequently, not all individual preferences can be selected independently in accordance with the solution of individual extreme problem.

The number $NM - (N + M)$ can be conditionally named "the number of subjective freedom degrees" of subjects in the group. The number of freedom degrees is equal to zero, if $NM - (N + M) = 0$. Solution in the integers is $N = 2$, $M = 2$.
Forming in a special manner functional, we can obtain, for example, the following distribution of the individual preferences in the group:

$$\pi_j(\sigma_k) = \frac{e^{\beta \sum_{q=1}^{M} U_{jq}(\sigma_k)\pi_q(\sigma_k)}}{\frac{1}{M}\sum_{p=1}^{M}\sum_{s=1}^{N} e^{\beta \sum_{q=1}^{M} U_{pq}(\sigma_s)\pi_q(\sigma_s)}}, \tag{4.209}$$

where $U_{jq}(\sigma_k)$ are the mutual utilities, introduced above.

Here we have substantially nonlinear dependence between the object preferences. The problem of determination $\pi_j(\sigma_k)$ can be, linearized, if we introduce the process, which is developed in the time and examine the functions of time $\pi_j(\sigma_k, t)$. Then it is possible to convert (4.209) at the nonlinear differential equations with the discrete time.

One additional version appears, if the following functional is undertaken as the group functional (belonging to the "virtual subject"):

$$\Phi_{\pi}^{\Sigma} = -\sum_{i=1}^{N}\sum_{j=1}^{M}\left(\frac{1}{M}\sum_{q=1}^{M}\bar{U}_{jq}\left(\sigma_i\right)\pi_q\left(\sigma_i\right)\right)\ln\left(\frac{1}{M}\sum_{q=1}^{M}\bar{U}_{jq}\left(\sigma_i\right)\pi_q\left(\sigma_i\right)\right)+ \tag{4.210}$$

$$+\beta\sum_{i=1}^{N}\sum_{j=1}^{M}\left(\frac{1}{M}\sum_{q=1}^{M}\bar{U}_{jq}\left(\sigma_i\right)\pi_q\left(\sigma_i\right)\right)U_j\left(\sigma_i\right)+\gamma\sum_{i=1}^{N}\sum_{j=1}^{M}\left(\frac{1}{M}\sum_{q=1}^{M}\bar{U}_{jq}\left(\sigma_i\right)\pi_q\left(\sigma_i\right)\right),$$

where the normalized mutual utilities $\bar{U}_{jq}\left(\sigma_i\right)$ in the group are such, that for $\forall q \in \overline{1,M}$

$$\sum_{j=1}^{M}\bar{U}_{jq}\left(\sigma_i\right) = 1, \tag{4.211}$$

The appropriate canonical distribution can be presented in the form:

$$\frac{1}{M}\sum_{q=1}^{M}\bar{U}_{jq}\left(\sigma_i\right)\pi_q\left(\sigma_i\right) = \frac{e^{\beta U_j\left(\sigma_i\right)}}{\sum_{s=1}^{M}\sum_{k=1}^{N}e^{\beta U_s\left(\sigma_k\right)}}. \tag{4.212}$$

$U_j(\sigma_i)$ is the utility of subject j for the group, expressed, for example, through his contribution to the consolidated resources.

We see that here normalization conditions for $\pi_q(\sigma_i)$ are satisfied, if condition (4.211) is satisfied. The number of relationships (4.212) is equal NM, i.e., it coincides with of number of the individual preferences. Consequently, the number of subjective freedom degrees of the individual members of group equal to zero. They are completely subordinated to the common problems.

Together with the entropy of group preferences, the characteristic, which testifies about "virtual subject" existence can serve, for example, "the coefficient of subjective multiple correlation"

$$R = \sqrt{1-\frac{\det\mathbf{M}}{\det\mathbf{S}}},$$

where S is matrix of the paired correlation coefficients of the preferences distributions of the real subject's r_{jq}, and M is the bordered matrix:

$$\mathbf{M} = \left[\begin{array}{c|c} 1 & R_m^{\mathrm{T}} \\ \hline R_m & \mathbf{S} \end{array}\right],$$

where R_m is column vector of the coefficients r_{jm} of the distributions π_j correlation with the distribution π_m. As r_{jq} and r_{jm} the Pearson coefficients or the corresponding rank correlation coefficients, if ordinal distributions are examined can be undertaken.

4.13. Laws of supply and demand as the result of the action of the entropy principle of optimality

Let`s assume that demand at the point of the certain goods σ_i is connected with the user preferences $\pi_c(\sigma_i)$. Let there exist N different goods $\sigma_i(i \in \overline{1,N})$ and on the set S_a the prices p_i are assigned. M subjects achieve purchases: $j \in \overline{1,M}$. Let`s designate through $D_j$ the individual budget of subject $j$. Preferences $\pi_j(\sigma_i)$ of subject $j$ are distributed on the set $S_a$ (identical for all subjects). Let`s make the assumption that the budget D_j is divided proportional to the preferences:

$$D_j(\sigma_i) = D_j\pi_j(\sigma_i), \tag{4.213}$$

and demand at the point of the goods σ_i is equal

$$X_j(\sigma_i) = \frac{D_j(\sigma_i)}{p_i} = D_j\frac{\pi_{cj}(\sigma_i)}{p_i}. \tag{4.214}$$

Thus, D_j is the budget, which the subject uses completely. It is possible to determine the mean price, which the subject j pays,

$$\bar{p}_j = \frac{D_j}{X_j},$$

where $$X_j = \sum_{i=1}^{N} X_j(\sigma_i) = \sum_{i=1}^{n}\frac{D_j\pi_{cj}(\sigma_i)}{p_i}.$$

We find that

$$\bar{p}_j = \frac{1}{\sum_{i=1}^{N}\pi_{cj}(\sigma_i)p_j^{-1}}. \tag{4.215}$$

Thus, in this model the mean price, which the subject pays, is defined as harmonic mean of particular prices p_i with the weights equal to preferences $\pi_j(\sigma_i)$. Demand at the point of the goods for the entire group

$$X(\sigma_i) = \frac{1}{p_i}D(\sigma_i),$$

where $D(\sigma_i)$ the partial budget of group, in reference to the alternative σ_i.

Let`s examine the following case: for all goods, except one the prices remain valid. Let for the goods σ_1, σ_2, , σ_{N-1} prices are constant and equal p_1, p_2, ..., $p_{N-1.}$. Reduced price for these goods can be found from the formula:

$$p^* = \frac{1}{\sum_{i=1}^{N-1} \frac{\pi_c(\sigma_i)}{p_i}}. \tag{4.216}$$

The price p_N of goods σ_N changes. Index "*j*" we will omit, thus there is one subject or the set of identical subjects, the overall budget constraint for which takes the form:

$$X^* p^* + X_N p_N = \bar{P}D, \tag{4.217}$$

where $X^* = D\frac{\pi^*}{p^*}$; $X_N = D\frac{\pi_N}{p_N}$.

It is obvious that $\pi^* + \pi_N = 1$.

Let the preferences π^* and π_N are determined by the formulas:

$$\pi_c^* = \pi_c(\sigma *) = \frac{e^{-\beta_c p^*}}{e^{-\beta_c p^*} + e^{-\beta_c p_N}}; \ \ \pi_{cN}^* = \pi_{cN}(\sigma_N) = \frac{e^{-\beta_c p_N}}{e^{-\beta_c p^*} + e^{-\beta_c p_N}}. \tag{4.218}$$

In order to obtain from the entropy principle of optimal distribution (4.218) it is sufficient, as it was repeatedly shown earlier to take the function of the effectiveness of user in the form

$$\varepsilon_c = -\beta_c \sum_{i=1}^{N} \pi_{ci} p_i.$$

Budget is divided in the proportion:

$$D^* = D\pi^*; \ \ D_N = D\pi_N. \tag{4.219}$$

Total demand (in the comparable units of goods) $\bar{X} = X + X_N$.

Fig. 4.27 sows the result of calculation, executed for following initial data: β_c = 1, p^* = 1, p_n: 0,5; 1,0; 2,0; 3,0; 4,0; 5,0 (……) units.

Curve 1: $X_N(p_N)$ the dependence of the volume of the demand of goods σ_N on the price, curve 2: $\bar{X}$ the volume of overall demand, curve 3: $\bar{p}$ - the dependence of mean price on the price p_N of goods σ_N.

Curve 1: $X_N(p_N)$ takes the form typical for demand curve. We see that the redistribution of demand occurs. With an increase in the price σ_N the demand X_N sharply falls, while demand at the point of entire remaining goods (remaining part "of basket") rises. Overall demand first falls, then it begins to increase and tends asymptotically to the maximally possible demand "of product" with constant price. In this case the preference distribution determines separation of complete budget on the part D^* and D_N. Mean price $\bar{p}$ first grows, and then it is lowered and has a maximum at that point, where the total demand has a minimum.

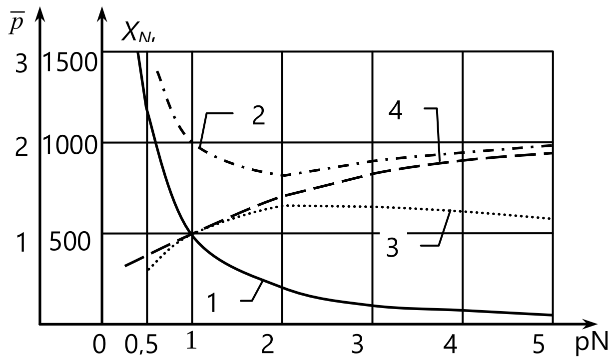


Fig. 4.27

The distribution, used in the first case, relates to the distributions of first type, which have monotonic nature. In this case it reflects the installation: the more expensive goods, the less it is desirable, although the cheeps goods, possessing certain utility , can be purchased. This distribution corresponds to such psychological type, for which the considerations of prestige do not play role.

Let`s conduct analogous calculations for distribution of second type:

$$\pi_c(\sigma_i) = \frac{p_i^{\alpha_c} e^{-\beta_c p_i}}{\sum_{j=1}^{N} p_j^{\alpha_c} e^{-\beta_c p_j}}; \quad (\alpha_c > 0,\ \beta_c > 0). \tag{4.220}$$

Distribution (4.220) has a maximum with $p_i = \frac{\alpha_c}{\beta_c}$ and with $\alpha_c = 0$ passes at distribution of first type. This distribution corresponds to the psychological type of the subject, who does not attempt to buy (with other conditions being equal,) both the cheap goods as well as the very expensive goods, but are selected goods, whose price corresponds to his wealth and public position. For this subject the matter of prestige plays essential role.

Fig. (4.28) shows the results of calculation with the use of the distributions given below.

$$\pi_c(\sigma^*) = \frac{p^{*\alpha} e^{-\beta_c p^*}}{p^{*\alpha} e^{-\beta_c p^*} + p_N^{\alpha_c} e^{-\beta_c p_N}}; \tag{4.221}$$

$$\pi_c\left(\sigma_N\right)=\frac{p_N^{\alpha_c}e^{-\beta_c p_N}}{p^{*\alpha_c}e^{-\beta_c p^*}+p_N^{\alpha_c}e^{-\beta_c p_N}}.$$

It is supposed that $\alpha_c = \beta_c = 1$, $p^* = \text{const} = 1$, $p_N = 0{,}5;\ 1{,}0;\ 1{,}5;\ 2{,}0;\ 3{,}0;\ 5{,}0$.

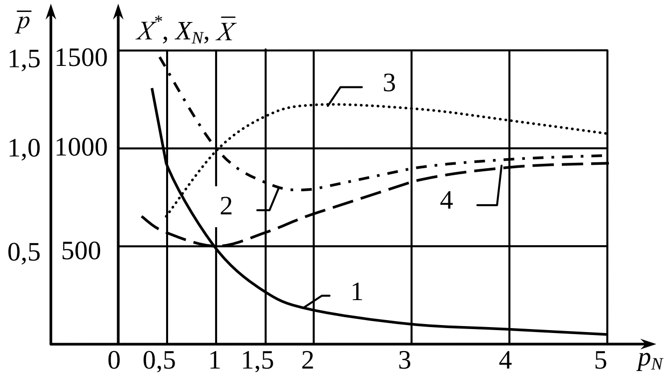


Fig. 4.28.

Dependences X_N (curve 1), $\bar{X}$ (curve 2), $\bar{p}$ (curve 3), X^* (curve 4) from the variable price p_N of goods σ_N

As a whole the nature of the dependence is the same as to fig.4.27 with exception curved $X^* = X^*(p_N)$.

In this case in contrast to fig.4.27 joint demand at the point of the goods of constant price has a minimum. The nature of the dependence of demand at the point of the goods, whose price changes takes the traditional form.

Speaking about one subject , we apparently can have in mind the group of subjects, with identic set of alternatives and the very close preference distributions. In this case value D is joint budget.

Let there is a group of unassuming subjects, who most of all worries the savings of means and for whom is, therefore, characteristic the preferences distribution of the first type (4.218) and a group of subjects, for whom the considerations of prestige are significant, such that form of their preferences corresponds to the distribution (4.220) (relation α/β plays the role of psychological type index in the discussed sense). Let`s assume that these groups have identical joint budgets D. The number of the first M_1 can be large ("poor"), the number of the second $M_2 < M_1$ (them it is possible to name "rich"). Fig.4.29 shows the joint values $\bar{X}^* = X_1^* + X_2^*$; $\bar{X}_N = X_{N1} + X_{N2}$; $\bar{X} = \bar{X}_1 + \bar{X}_2$. Curves are built on the data, given in the table.

p_N	0,5	1,0	1,5	2,0	3,0	5,0
X^*_1	377,6	500	622,5	731,1	880,8	7902,1
X_{N1}	1244,8	500	251,7	134,5	36,7	3,6
$\bar{X}_1$	1622	1000	874,2	865,6	920,5	985,7
X^*_2	548,1	500	523,6	576,1	711,2	916,1
X_{N2}	903,8	500	317,6	211,9	96,3	16,8
$\bar{X}_2$	1451,9	1000	841,2	787,0	807,5	933,0
$\bar{X}^* = X_1^* + X_2^*$	1451,9	1000	1146	1363,1	1518,7	1898,2
$\bar{X}_N = X_{N1} + X_{N2}$	2118,6	1000	569,3	346,4	136,0	28,4
$\bar{X} = \bar{X}_1 + \bar{X}_2$	3074,3	2000	1706,8	1662,6	1728	1919

The given results attest to the fact that both in the groups uniform relative to the type of the preference distribution and in the mixed groups, in which there are subjects with different types of the preference distribution, the nature of the dependence of demand on the price proves to be identical. In this case it corresponds to known macroeconomic dependences.

Let`s show now that the typical dependence of supply on the price can be also obtained as the consequence of the entropy principle of optimality. The competence of the forming of supply belongs to producer. His preferences are formed on the basis of the effectiveness function different from the user function. Let`s select the function of effectiveness, generates preferences distribution of the producer, from which the dependence of proposal on the price will follow, and qualitatively coincides with correspondent macroeconomic dependences.

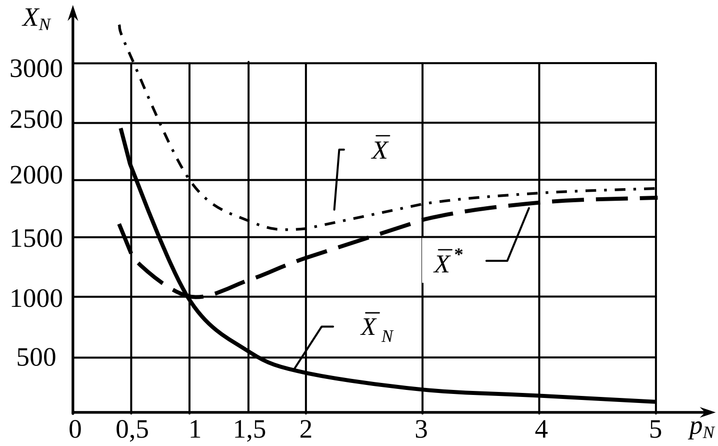


Fig. 4.29

Let p_i the price, and q_i is expenditure for the production of the unit of goods σ_i. Then profit from the realization of the unit of goods $\Delta_i = p_i - q_i$. Let`s select the function of the effectiveness of producer in the form

$$\varepsilon_{pr} = -\beta_{pr} \sum_{i=1}^{N} \pi_{pr}\left(\sigma_i\right) \frac{1}{\Delta_i}.$$

Here the value $\frac{1}{\Delta_i} = \frac{1}{p_i - q_i} = \lambda\left(\sigma_i\right)$ plays formally the role of "function of harm": the more price p_i with the fixed expenditures q_i the less "harm" is brought to producer. The entropy principle of optimality places in the correspondence to this function of effectiveness the canonical distribution:

$$\pi_{pr}\left(\sigma_i\right) = \frac{e^{-\beta_{pr} \frac{1}{\Delta_i}}}{\sum_{j=1}^{N} e^{-\beta_{pr} \frac{1}{\Delta_j}}}. \tag{4.222}$$

As in the case the calculation of demand, let`s assume that there is the group of goods $\sigma_1, \sigma_2, \ldots, \sigma_{N-1}$ prices of which do not change. Replace this group with one reduced goods $\sigma^*$, whose reduced price is $p^*$. The price of goods $\sigma_N - p_N$ changes. Let`s trace, how the release of this goods will change in this case. Suppose as it was made above, that his own budget B the producer divides into two parts proportional to the preferences

$$B^* = B\pi^*_{pr}; \quad B_N = B\pi_{pr}N, \tag{4.223}$$

moreover $\pi^*_{pr} + \pi_{prN} = 1$.

In a model example was accepted $q^* = qN = 0{,}8$ (c.u.); $\beta_{pr} = 1$. It is obvious that the production of goods σ_N and σ^* occurs only with the condition $p^* > q^*$; $p_N > q_N$.

The result of calculations is shown on fig.4.30. Obtained supply curve takes the characteristic form, which practically coincides with of form of empirical supply curves.

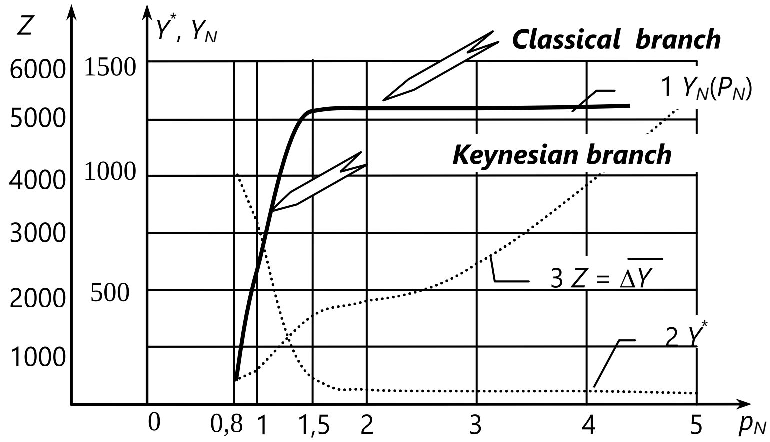


Fig.4.30. Dependence of supply on the price p_N, of goods σ_N.

The graph of supply (1) $Y_N(p_N)$ has two characteristic sections: the ascending section, called usually "Keynesian branch" corresponds to the presence of free resources and almost horizontal section, "classical", that is characterized by the exhaustion of the resources, when an increase in the price does not lead to an increase in the release.

Let`s note that both in the first and in the second problem alternative σ^* does not compulsorily indicate a certain goods, which has consumer demand.

This can be the postponed demand in the form of accumulations, the direction of means to an increase of capital and so forth

The executed calculations bear conditional nature, since they rest on hypothetical numerical data. The established fact of the possibility of obtaining the dependences of supply and demand, qualitatively very close to the real, from a priori theoretical considerations is unconditional. Further analysis in this direction is represented as street with the two-way traffic. From one side the theoretical comprehension of empirical facts, and from the other side on refinement and explanation of the applied sense of theoretical models, which follow from the entropy principle of optimality, in which, by the author's opinion, stable components are the subjective entropy and term which determines normalizing condition. The function of effectiveness allows variation over wide limits. Its structure and sense depend on the situation, in which the subject is situated.

Thus, variety and range of the spectrum of possible distributions are determined first of all by the possibilities of the selection of the effectiveness functions. In the present paragraph two simplest functions of effectiveness underwent testing, this led to the success in the form of the macroeconomic dependences of supply and demand that correspond to empirical data. Let`s note in conclusion that further analysis can be connected with taking into account the ratings of the subjects, for example, the integral ratings ξ (*j*) (see chap.4).

If we use the group of preference functions

$$\bar{\pi}_c^{\Sigma}(\sigma_i) = \sum_{j=1}^{M} \pi_{cj}(\sigma_i)\xi(j), \tag{4.224}$$

where $\xi(j)$ is the integral ratings of subjects, the fractionating of total budget D can be achieved in accordance with the formula

$$D(\sigma_i) = D\bar{\pi}_c^{\Sigma}(\sigma_i), \tag{4.225}$$

and demand at the point of the goods σ_i will be defined as

$$X_c(\sigma_i) \triangleq D\frac{\sum_{j=1}^{M_c} \pi_{cj}(\sigma_i)\xi(j)}{p_i}. \tag{4.226}$$

Assuming that each of M_c the group members will acquire only one unit of goods (for example, an automobile of this type), then demand can be determined with formula

$$X_c(\sigma_i) \triangleq M_c\bar{\pi}_c^{\Sigma}(\sigma_i). \tag{4.227}$$

Mean price for the joint demand

$$X_c = \sum_{i=1}^{N} X_c(\sigma_i)$$

can be defined as harmonic mean of the form:

$$\bar{p}_c = \frac{1}{\sum_{i=1}^{N} \frac{\sum_{j=1}^{M} \pi_{cj}(\sigma_i)\xi(j)}{p_i}}.$$

In formulas (4.224) - (4.227) the index "*c*" indicates reference to the user (consumer).

4.14. Cost of subjective information

In the information theory a question of the cost of information is determine traditional. It`s possible to find the appropriate determinations, for example, in Neumann [129], and in Stratonovich`s [137, 138]. There are set approaches to the determination of this concept. Everyone understands well that information in today's world has probably the greatest value. That's way the intensive search for the method of determining the information cost is in progress. Therefore the approach which directly follows from the developed theory will not be superfluous. In [137], as well as in the other works value of information is determined trough comparison of the effectiveness functions before and after obtaining its specific portion of information.

In the present investigation proves to be possible the direct determination of the cost *of the subjective information*, expressed in the units of various resources, whose change (translation) leads it a change of the preferences distribution of subject.

In contrast to other approaches the discussion here in deals with *the subjective information*, expressed through the preferences of subject, and it`s not connected with any probability distribution straight. The cost of information in this case is quite an individual index, since the other values and concepts of subjective analysis always have the relation to the certain individual carrier.

It is possible to tell about the cost of information, connected witch a change in the object preferences and the cost of information, caused by the modification of rating preferences. It is convenient to name the latter "cost of social information".

In the problems of social dynamics [209] this value can play an essential role, especially when the competition of ideas is studied or estimate of the effectiveness of political economy is produced. Spiking in general terms, it`s desirable to have any method to calculate "*how much necessary pay to a politic – technologist and what for*".

The questions examined here and in the row of other sections of the book refer to the problems represented for example in [205, 206, and 207].

In the chapters 2, 3, 4 above a large number of different variants of the canonical distributions of preferences was examined. It is possible to connect the appropriate cost with each of them. We will confine just few versions and with the aid of simulation will demonstrate the dependence of cost on the endogenous and exogenous parameters of distribution.

Reconstruction of the preferences distributions occurs as a result of exogenous events and endogenous changes. The first and the second are possible to connect with the expenditures of specific resources. Without defining concretely the form of resources, let`s determine the cost of information by the following formula

$$C_{subj} = \frac{\delta R}{J(|\delta R)}, \qquad (4.228)$$

where δR are the expenditure in the units accepted; $J(|\delta R)$ is the subjective information, caused by expenditures δR. The subjective information in the case of object preferences is determined by formulas (3.23), (3.24) or (3.25), (3.26).

In the case described by formula (3.29), we will imply δR as the resources, spent on the transfer of system from the state σ_s to the state σ_r. The determination (4.228) can be easily extrapolated to more complex distributions, such as compositions and ways.

Taking into account (3.29) we can transform (4.228) to the form:

$$C_{i\pi}(\sigma_s \to \sigma_r) = \frac{\delta R(\sigma_s \to \sigma_r)}{-\sum_{i=1}^{N}\left(\pi(\sigma_i \mid \sigma_s)\ln\pi(\sigma_i \mid \sigma_s) - \pi(\sigma_i \mid \sigma_r)\ln\pi(\sigma_i \mid \sigma_r)\right)}. \qquad (4.229)$$

In conformity with rating preferences, the cost of information answers the question: what the certain mean cost of information which ensures the assigned or desired change in the distribution of the rating subject in the group. Depending on the form of rating distribution information is defined subjectively. For the absolute or integral ratings $\xi(j)$ determine the information, connected with the event A (in particular, with the translation of resources δR) by the formula:

$$J_{subj\xi}(A) = H(\xi(j)) - H(\xi(i \mid A)). \qquad (4.230)$$

"The carrier" of this data is "collective reason", either "virtual subject" or the operator inside or out of the group.

If distributions $\xi\ (j|\ i)$, where $i \in \overline{1,M}$ are studied, then

$$J_{subj\xi_i}(A) = H(\xi(j \mid i)) - H(\xi(j \mid i \mid A)), \qquad (4.231)$$

and it is possible to speak about the "information vector" containing M of quantities of information. In the case of the differential distribution $\xi(j \mid i, \sigma_k)$ referred to the object alternative σ_k, appears the matrix of information quantities $J_{sub\xi\, ik}(A)$ by size $M \times N$.

A condition that political processes in society have a goal to change rating distribution of individual characters or political parties and for this purpose significant resources have to be spent, the method of the quantitative assessment of the certain cost of the obtained information and comparison of effectiveness in the expenditures produced is proposed.

Above we already postulated existence of the entropy thresholds H_π^*, H_ξ^*, which separate the region of discussion (or "the reign of freedom") from "the reign of need", below upper boundary of which the solutions, prepared in the region of discussion start.

If, for example, $H(\pi_0) > H^*(\pi)$, i.e., subject is available and "the reign of freedom", the achievement of the boundary $H^*(\pi)$ is connected with obtaining by its amount of information

$$J_{subj}\left(\pi_0 \to \pi^*\right) = H\left(\pi_0\right) - H^*\left(\pi\right). \tag{4.232}$$

In particular, if $\pi_0\left(\sigma_i\right) = \dfrac{1}{N}$, which corresponds to maximum uncertainty, then

$$J^*_{subj}\left(\frac{1}{N} \to \pi^*\right) = \ln N - H^*\left(\pi\right). \tag{4.233}$$

Analogous formulas can be written, for example, for the rating distribution $\xi(j)$:

$$J_{subj}\left(\xi_0 \to \xi^*\right) = H\left(\xi_0\right) - H^*\left(\xi\right); \tag{4.234}$$

$$J_{subj}\left(\frac{1}{M} \to \xi^*\right) = \ln M - H^*\left(\xi\right). \tag{4.235}$$

Fig. 4.31 shows conditionally the partition of the entropy scale as the regions, which correspond to the different states of group.

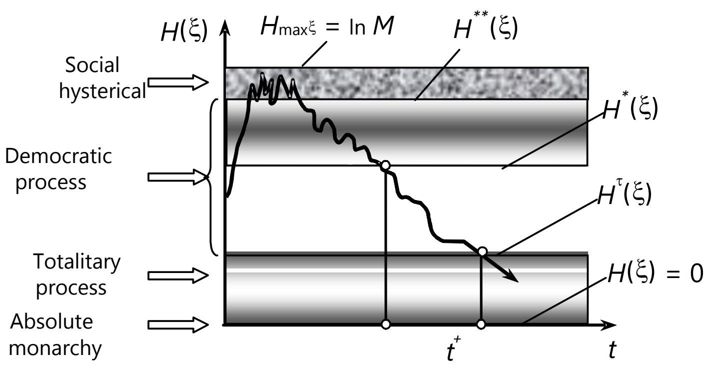


Fig. 4.31

Curve on Fig. 4.31 reflects the nature of the dependence, given in the training course ICAO (the International Civil Aviation Organization) p. 123/of air of live/of section A "Flight of deck of crew".

Four regions are separated. The upper region, when $H_{\max\xi} \leq H(\xi) \leq H^{**}(\xi)$ is characterized as the zone of "social hysteria": emotions whip reason, "emotional overheating" occurs. Selection in this region is impossible; social is not capable to be self-oriented in the political preferences rating. The state "of hysterics" cannot continue for long, since the incandescence of passions requires the significant expenditure of physiological energy. After a comparatively short time the subject falls in the region of rational discussion. Here emotions play smaller role. The entering of additional information brings subject to the boundary $H^{*}(\xi)$, lower of which the solution can be accepted.

Supposedly there is one more entropy rating threshold. Near this threshold rating distribution tends to singular, when the social is separated with the small group of subjects (or even one subject) with maximally high ratings, and the control region is approached to "totalitarian".

As it was noted, subjective entropy approaches zero, if when the endogenous parameter β tends to infinity. The "emotional super cooling" takes place.

Region $H^{**}(\xi) \leq H(\xi) \leq H^{T}(\xi)$ is conditionally speaking the "region of democracy", which, contains two sub regions: the "reign of freedom" and the "reign of need".

The retention of entropy $H(\xi)$ in this region ensures the retention "of democracy", which is accompanied by the periodic passages through the threshold $H^{*}(\xi)$.

"Totalitarian region":

$$H^{T}(\xi) > H_{\xi} \geq 0$$

as a rule is region "without the output". Output from this region is possible via qualitative change, "scrap of system" only. Such "scraps" bear spasmodic, catastrophic nature.

It is possible to speak about "storage capacity of the region of democracy":

$$J_{subj}(D) = H^{**}(\xi) - H^{T}(\xi). \tag{4.236}$$

Then "storage capacity of the region of totalitarianism"

$$J_{subj}(T) = H^{T}(\xi). \tag{4.237}$$

The given determinations and terminology are very conditional.

It is necessary to make two additional observations.

1. The boundaries (thresholds) $H^{**}(\xi)$, $H^{*}(\xi)$, $H^{T}(\xi)$ most likely change in the course of time.
2. They are subjective so they can essentially differ for the different subjects. We have the motley carpet of the entropy space of the subjects in social, coloring of with changes in the course of time.

The distribution *of rating preferences* depends on individual *object preferences* for sure. This makes picture of entropy space and its time structure even of more complex. The application of discussed approach, from the use of subjective entropy and canonical distributions of preferences, gives possibility to conduct quantitative analysis, including simulate the dynamics of preferences, to study the dependence of fundamental characteristics on the exogenous and endogenous parameters.

We speak about the cost of subjective information, as one of the essential characteristics of the processes of proceeding on the subjective level.

As a numerical example, let`s examine the case, when S_a contains two alternatives only, and object preferences take the form:

$$\pi_0\left(\sigma_i\right)=\frac{e^{-\beta x_i}}{e^{-\beta x_1}+e^{-\beta x_2}}.$$

Here $x_i = R_i^{req}$ is required resources for the realization of alternative σ_i. Reduction in the required resources, for example, by reduction in the costs of goods can be treated as the expenditures "of operator". Let`s assume that reduction in the required resources is proportional to initial cost, i.e.,

$$\delta R_i^{req} = r\frac{x_i}{x_1+x_2}.$$

Here $\delta R_1^{req} + \delta R_2^{req} = r$ is a total change in the required resources.

The distribution of preferences after reduction in the costs takes the form:

$$\pi\left(\sigma_i \mid \delta R_i^{req}\right)=\frac{e^{-\beta x_i\left(1-\frac{r}{x_1+x_2}\right)}}{e^{-\beta x_1\left(1-\frac{r}{x_1+x_2}\right)}+e^{-\beta x_2\left(1-\frac{r}{x_1+x_2}\right)}}. \quad (4.238)$$

Information is calculated from the formula:

$$J_{subj}(\pi) = H(\pi_0) - H(\pi|\, \delta R^{req}) = H(\pi_0) - H(\pi|r), \quad (4.239)$$

the certain cost of information, about the formula:

$$C_{subj}\left(\pi\right)=\frac{r}{H\left(\pi_0\right)+H\left(\pi \mid r\right)}. \quad (4.240)$$

The results of the preferences calculations, entropy, information and cost of information depending on the endogenous parameter r and the exogenous parameter β are shown on Fig. 4.32.

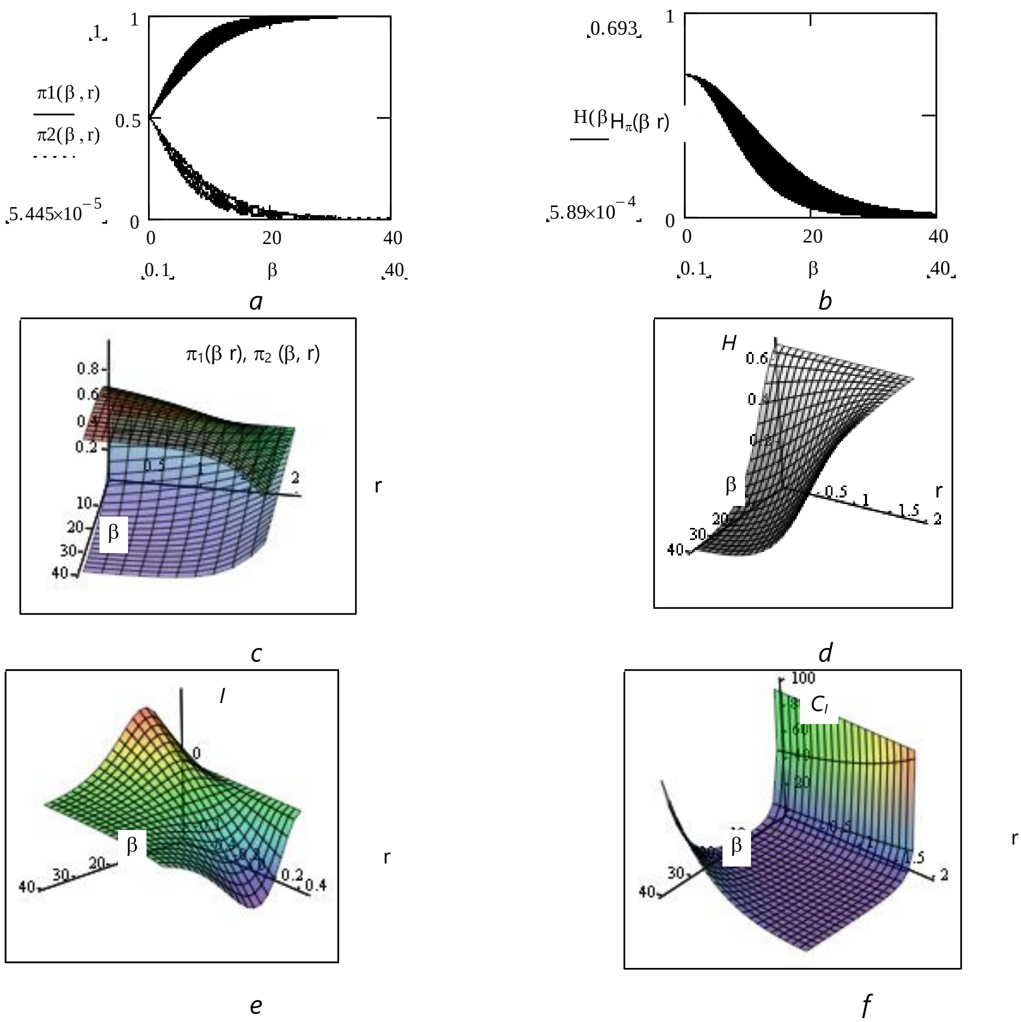


Fig. 4.32

It is evident from formula (4.238) that, if a priori "costs" are equal: x_1 = x_2, then with any r of preference they preserve their value and, therefore, subjective information is equal to zero in this case initial entropy it is maximum, and the cost of information is infinitely great. In other words with the large a prior uncertainty and a synchronous cost change of both "goods" the cost of information is great and the higher the nearer a prior preferences distribution to the uniform. However, if x_1 = x_2, the synchronous cost change will change distribution.

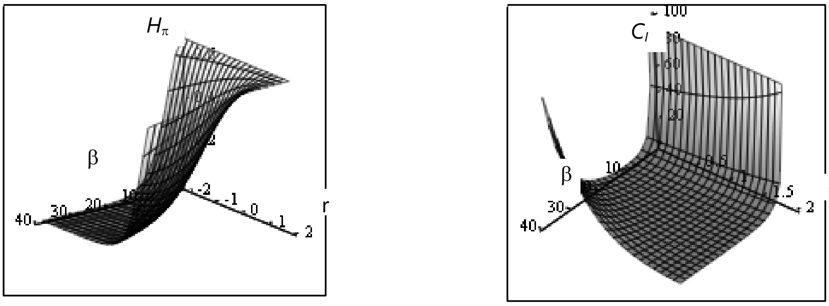
The parameter r was varied within the limits [- 0,5; 0,5]. The endogenous parameter β was alternated over wide limits: β [0,1; 40]. The parameters x_1 and x_2 are constant and different x_1 = 1,0, x_2 = 1,2. Fig. 4.32, *a*—*d* the beams of curves, which correspond to different values r, as function of β shows. We see that for all values r an increase β leads it the decrease of entropy. The cost of information has a minimum. With the tendency of β to zero it rapidly grows. Cost also grows with the big. Fig. 4.32, *d—f* illustrate the same dependences in the form of three-dimensional graphs in the axes β, r.

On the assumption that the cost of only one "goods" changes: $x_1' = x_1 + r$, new preferences are determined from the formulas

$$\pi_1\left(|\, r\right) = \frac{e^{-\beta(x_1+r)}}{e^{-\beta(x_1+r)} + e^{-\beta x_2}}.$$

$$\pi_2\left(|\, r\right) = \frac{e^{-\beta x_2}}{e^{-\beta(x_1+r)} + e^{-\beta x_2}}.$$

Fig. 4.33 shows the dependence of entropy and cost of subjective information on the exogenous parameter r and the endogenous parameter β (x_1 = 1,0; x_2 = 1,1; $r \in$ [0; 1,0], $\beta \in$ [0,1, 70].



a *b*

Fig. 4.33

Calculation of entropies and cost of information are represented on Fig. 4.34. Here also the dependence on the exogenous parameter x and the endogenous parameter β, which could be connected with so called " emotional temperature" $T = \beta^{-1}$ is explained.

Analogy of the value $\beta^{-1} = T$ with the temperature in the thermodynamic problems is from the formal similarity of canonical distribution $\pi(\sigma_i)$ and the Gibbs distribution. When $\beta \to 0$ or $T \to \infty$, distribution $\pi(\sigma_i)$ tends to uniform, and subjective entropy H_π tends to its maximum value ln N.

In the previous problems with a change of the resources on the certain finite quantity mean cost was determined. It is obvious that, if resources change on the low (infinitely small) value, then the corresponding information will be infinitely small. Let`s examine the distribution:

$$\pi\left(\sigma_i\right)=\frac{e^{-\beta\frac{R^{req}\left(\sigma_i\right)}{R^{disp}}}}{e^{-\beta\frac{R^{req}\left(\sigma_1\right)}{R^{disp}}}+e^{-\beta\frac{R^{req}\left(\sigma_2\right)}{R^{disp}}}}. \tag{4.241}$$

The value of required resources depends on the number of alternative, and the available resources are universal.

Let`s designate: $R^{req}(\sigma_i) = r_i$, and $R^{disp} = x$.

Then:

$$\pi\left(\sigma_i\right)=\frac{e^{-\beta\frac{r_i}{x}}}{e^{-\beta\frac{r_1}{x}}+e^{-\beta\frac{r_2}{x}}}. \tag{4.242}$$

Let`s determine "the speed" of a change in the preferences with a change in the available resources. We have

$$\frac{d\pi\left(\sigma_1\right)}{dx}=\frac{\beta}{x^2}\left(r_1-r_2\right)\pi\left(\sigma_1\right)\pi\left(\sigma_2\right); \tag{4.243}$$

$$\frac{d\pi\left(\sigma_2\right)}{dx}=\frac{\beta}{x^2}\left(r_2-r_1\right)\pi\left(\sigma_1\right)\pi\left(\sigma_2\right);$$

Then derivative of entropy is determined by the formula

$$\frac{dH_\pi}{dx}=-\frac{\beta}{x^2}\left(r_1-r_2\right)\pi\left(\sigma_1\right)\pi\left(\sigma_2\right)\ln\frac{\pi\left(\sigma_1\right)}{\pi\left(\sigma_2\right)}. \tag{4.244}$$

Let`s determine the instantaneous cost of the information:

$$C_{subj}^{inst}=\frac{dx}{dH_\pi}=\frac{1}{\frac{dH_\pi}{dx}}. \tag{4.245}$$

It is possible to notice that the cost C_{subj}^{inst}, is always positive.

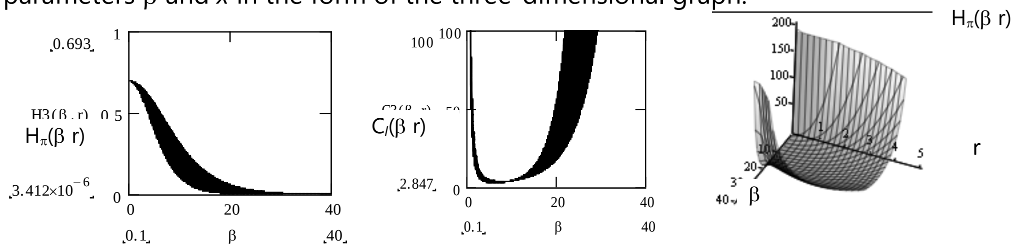
Actually, let $r_1 > r_2$, then $\pi(\sigma_1) < \pi(\sigma_2)$ and $\ln\frac{\pi\left(\sigma_1\right)}{\pi\left(\sigma_2\right)}<0$, $\frac{dH_\pi}{dx}>0$ and, consequently $C_{subj}^{inst}>0$. But if $r_2 > r_1$, then $\pi(\sigma_1) > \pi(\sigma_2)$, and $\ln\frac{\pi\left(\sigma_1\right)}{\pi\left(\sigma_2\right)}>0$ and $\frac{dH_\pi}{dx}>0$. Consequently, again $C_{subj}^{inst}>0$.

Fig. 4.34 represent the dependences on β and x "instantaneous cost of information" if the preferences distribution is assigned by formula (4.242). In this case instead of x in the exponents of exponential curves figures value $R^{disp} + x = R + x$. It is assumed that there are certain initial available resources, which then increase on the x value. The parameter β varies within the limits of $0,1 \le \beta \le 40$, and x —the limits: 0,1 20. Naturally r_1 and r_2 must be less than R.

It is accepted that $r_1 = 1,0$, $r_2 = 1,2$, the initial available resources R = 1,2.

Like in the previous cases, we see that the cost with very small β (high "emotional temperature"), proves to be very large.

It is evident from Fig. 4.34 *a*, *b* that the instantaneous cost also has a minimum with the certain value of the parameter β, and entropy with the increase β decreases and tends to zero. Fig. 4.34, *b* shows the dependence of instantaneous cost parameters β and x in the form of the three-dimensional graph.



a *b* *c*

Fig. 4.34

Let`s examine problem when there are two subjects and two alternatives. The transfer of certain part of the available resources from one subject to another (without loss "along the road") in such a way that after the transfer available resources of that removing person would be not less than the greatest required resources.

Preferences are determined with the formulas:

$$\pi_1(\sigma_i) = \frac{e^{-\beta_1 \frac{r_1(\sigma_i)}{x+R_1}}}{e^{-\beta_1 \frac{r_1(\sigma_1)}{x+R_1}} + e^{-\beta_1 \frac{r_1(\sigma_2)}{x+R_1}}};$$

$$\pi_2(\sigma_i) = \frac{e^{-\beta_2 \frac{r_2(\sigma_i)}{-x+R_2}}}{e^{-\beta_2 \frac{r_2(\sigma_1)}{-x+R_2}} + e^{-\beta_2 \frac{r_2(\sigma_2)}{-x+R_2}}}.$$

Fig.4.35, *a* show, correspondingly, the dependence $H_1(\beta_1 x)$ and $H_2(\beta_2 x)$, the final information, which corresponds to the transmitted quantity x

$$I_1(\beta_1 x) = H_1(\beta_1, 0) - H_1(\beta_1, x);$$

$$I_2(\beta_2 x) = H_2(\beta_{12}, 0) - H_2(\beta_2, x);$$

For the following conditions: the initial available resources are identical R_1 = R_2 = 3,0; required resources are constant for both subjects and equal $r_1(\sigma_1)$ = 0,5; $r_2(\sigma_2)$ = 1,5, the transferred part x varies within the limits [0; 1,49], the endogenous parameters

β_1 = 2; β_2 = 1 ("temperature" are different).

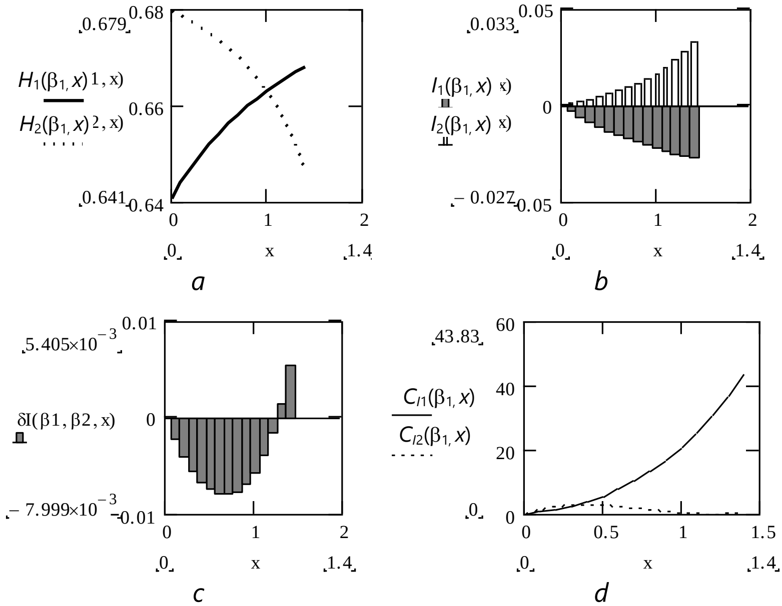


Fig. 4.35

Fig. 4.35, *b* and *c*, shows two information and "deficit" of the subjective information

$$\delta I = I_2(\beta_2, x) - I_1(\beta_1, x)$$

If is evident; in particular, that the deficit is not equal to zero, in other words, the total information as a result of the resources translation does not remain. Fig. 4.35, *d* shows the costs of the information C_1 and C_2 for both subjects participating in the translation. We can see that the entropy of the subject returning resources increases with an increase in the volume of the returned resources. The entropy of the subject obtaining additional resources decreases. The information, determined through the entropies of one returning is negative, the information of one obtaining is positive. It evident from Fig. 4.35, *c* that the cost of information for giving subject is much lower than the cost of information for receiving subject.

4.15. Aggregation of preferences in crew of the two pilots.

Let`s examine aggregation of the pilots preferences of the binomial aircraft crew as an application of the scheme of the cardinal preferences aggregation.

It is known, from the practice of flight operations surprising at first glance fact: when the appearance special situations in flight and when a number of alternatives presents the probability of adoption by the chief pilot of the erroneous solution proves to be higher when ratings of both pilots are nearly identical (close to each other), for example, if on the spot of copilot checking is available - as a rule having more high official rank than chief pilot and, correspondingly, in the eyes of chief pilot - higher rating.

This fact bears empirical nature. Let`s try to give substantiation, relying on assumptions and theoretical ideas, developed in this book.

Assume that group consists of two subjects (pilots) one of which - chief pilot makes decisions; however, in this case in a certain degree the opinion of copilot is considered.

The situation, when both subjects analyze one and the same set alternatives, which include in the simplest case two alternatives σ_1 and σ_2, is examined. Let π_{11} and π_{12} are a priori preferences of chief pilot, respectively alternatives σ_1 and σ_2, π_{21} and π_{22} are a priori preferences of copilot. The role of pilots in the composition of crew is not only in the separation of functions during the control process of vessel, but also the exchange of information special situations appearance. Copilot, thus, can have an effect on the adoption of command decision of ship. It is possible to assume that this influence proportional to the rating of copilot "in the eyes" of chief pilot. Let`s accept the following model of the aggregated preferences, generated as a result of "discussion".

Determine the aggregated preferences of chief pilot by the formula

$$\pi_1^{\Sigma}(\sigma_i) = \pi_1(\sigma_i)\xi(1\,|\,1) + \pi_2(\sigma_i)\xi(2\,|\,1), \ (i \in \overline{1,2}). \tag{4.246}$$

For the copilot respectively:

$$\pi_2^{\Sigma}(\sigma_i) = \pi_1(\sigma_i)\xi(1\,|\,2) + \pi_2(\sigma_i)\xi(2\,|\,2), \ (i \in \overline{1,2}). \tag{4.247}$$

Here $\xi(j|k)$ is the rating of subject j "the eyes" of subject k.

Entering these formulas preferences are subordinated to normalizing conditions:

$$\pi_j(\sigma_1) + \pi_j(\sigma_2) = 1 \text{ for } \forall j \in \overline{1,2};$$

$$\xi(1\,|\,k) + \xi(2\,|\,k) = 1 \text{ for } \forall k \in \overline{1,2};$$

Each of the pilots determines the rating of associate, comparing with his own rating.

In accordance with model (4.246), (4.247), the aggregated preference is composed of a priori preferences, considered with the weights equal to the conditional ratings of the carriers of priori preferences. In order to reduce the number of variables, designate:

$$\mu_1 = \frac{\pi_1(\sigma_1)}{\pi_1(\sigma_2)}; \quad \mu_2 = \frac{\pi_2(\sigma_1)}{\pi_2(\sigma_2)};$$

$$\eta_1 = \frac{\xi(1|1)}{\xi(2|1)}; \quad \eta_2 = \frac{\xi(1|2)}{\xi(2|2)}.$$

In these formulas ξ(1|1) and ξ(2|2) are ratings “assigned” after subject to itself. In the new variables formula (4.246) gives

$$\left.\begin{aligned} \pi_1^{\Sigma}(\sigma_1) &= \frac{\mu_1}{1+\mu_1}\frac{\eta_1}{1+\eta_1} + \frac{\mu_2}{1+\mu_2}\frac{1}{1+\eta_1}; \\ \pi_1^{\Sigma}(\sigma_2) &= \frac{1}{1+\mu_1}\frac{\eta_1}{1+\eta_1} + \frac{1}{1+\mu_2}\frac{1}{1+\eta_1}; \end{aligned}\right\} \tag{4.248}$$

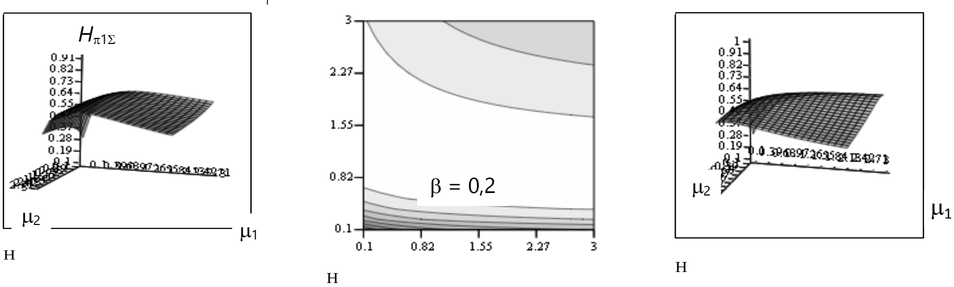
we obtain from formula (4.297):

$$\left.\begin{aligned} \pi_2^{\Sigma}(\sigma_1) &= \frac{\mu_1}{1+\mu_1}\frac{\eta_2}{1+\eta_2} + \frac{\mu_2}{1+\mu_2}\frac{1}{1+\eta_2}; \\ \pi_2^{\Sigma}(\sigma_2) &= \frac{1}{1+\mu_1}\frac{\eta_2}{1+\eta_2} + \frac{1}{1+\mu_2}\frac{1}{1+\eta_2}; \end{aligned}\right\} \tag{4.249}$$

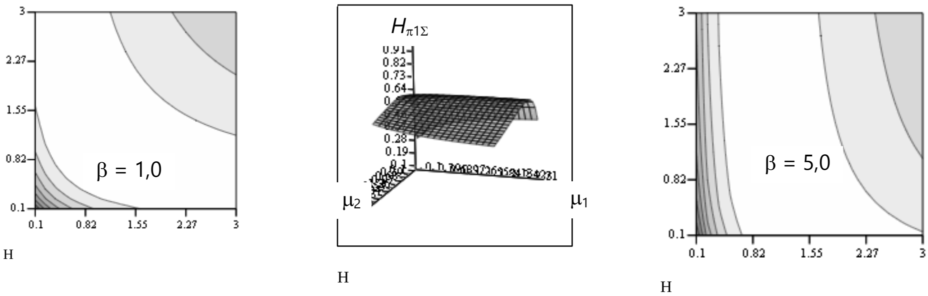
It is easy to verify that the conditions are satisfied

$$\pi_j^{\Sigma}(\sigma_1) + \pi_j^{\Sigma}(\sigma_2) = 1, \ (j \in \overline{1,2}).$$

Fig. 4.36 shows the results of calculation of the entropy and aggregated preferences of the chief pilot $H_{\pi_1}^{\Sigma}$ depending on the variables μ_1, μ_2, η_1.



a *b* *c*



d *e* *f*

Fig. 4.36

The high level of entropy, as it was spoken earlier, hampers the solution.

Maximum entropy H_{max} = ln 2 ≅ 0,693. Fig. 4.36, *b*, *d*, *f* bright region (for the range of values η_1 = 0,2; 1,0; 5,0) correspond to the cases, when $H_{\pi_1}^{\Sigma} \in [0,6;0,693]$, the degree of uncertainty is high and decision making is extremely difficult. In the adjacent regions the degree of uncertainty decreases, decision making simplifies and, respectively decreases required time in proportion to decrease of entropy.

To construct diagrams is possible, when a priori preferences of both subjects are determined and their a priori conditional ratings are known.

Similar rough estimates can prove to be useful with the development of the procedures of the crew's selection of, as well as during the investigation of aviation incidents. The calculation of the dependence of a priori object preferences and rating preferences on the exogenous and endogenous factors is from the models of canonical distributions, including the aggregated preferences.

It is necessary in each case to select the form of canonical distribution from the arsenal of the distributions, examined earlier, included with the content of certain objectives. In special cases this will be made in chapter 6, dedicated to the problem of safety of the active systems, including flight safety, where it will be shown that use of the version of subjective analysis developed here gives the alternative possibility of the account "human factor" in the problems of flight safety.

5. DYNAMICS AND INTER INFLUENCE OF PREFERENCES

5.1. Problems of the analysis of dynamics and mutual influence of the preferences

Problem resource situation, whether it is connected with one subject, or with the group of subjects, represents itself as a dynamic object. In general case all elements, characterizing it are changing in the course of time: problem sets S_{aj}, available, required, and expected resources, in a more general formulation - utility, preferences of I and II type, finally, the composition of the group S_{ξ}.

The reasons for temporized evolution are some changes in the ambient conditions: a change in market, prices (and, therefore, required resources), political situation, laws ("rules of game"), natural and public phenomena (including - catastrophes, revolutions) and so forth, as well as the spontaneous changes, stimulated "from within" of an active system.

The latter is the determining quality of the active system: the ability to generate "its own" problems, independently forming sets of problems, determining strategy of interaction with the external world, to undergo some age dependent changes.

It is possible to say with confidence that there are steady (slowly changing) imperatives of behavior with the mastery and generalization, historical experience, and also following from the systems nature and existing at "genetic" level - a priori preferences.

The public nature of each individual attests to the fact that there is a reciprocal effect of the individual preferences in the group.

Priori preferences are consequences of training, cultural "circle"; education got by the experience of solution some previous problems, religious ideas and political persuasions.

The substantial part of these circumstances could be combined under the term of "ethical standards" or "ethical imperatives".

A question of the evolution, connected with the extremity "of the life cycle" of any active system and the corresponding change in the composition of sets S_{aj}, S_{ξ} as well as preferences distributions is important.

The fact that a study in particular, the preferences distributions problem resource situation dynamics is acted, follows from the entropy determining role in this case, subjective entropy H_{π} or H_{ξ} and their change.

This role of entropy is in particular, in the fact that decision making - the selection on the set of alternatives is supposedly connected with the value of subjective entropy. This assumption is a very realistic. There are threshold values of entropies $H_\pi^{*\pm}$ and $H_\xi^{*\pm}$, which determine the moments of the appearance the "decision making" is necessary conditions. Most likely, the thresholds $H_\pi^{*\pm}$ and $H_\xi^{*\pm}$ are individualized and, therefore, they can also serve as the characteristics of individual psyche.

The collective component ("collective reason") realizes through "general preferences" or "general imperatives" - common for all members of group (association). There can be several imperatives of such a kind. They are the connecting material. We have already shown above that independently on the fact what the object of a subject's interest is: the positive or negative results of its election from S_{aj} and of his following activity, the subjective entropy H_π takes the maximum value (on the appropriate variety, determined by limitations).

For example, as it has already been said, if some numbers of medicines, prescribed for a given disease, are alternatives, then in the descriptions we study, in the first place, indications and expected effectiveness of each of them, second information about the contra-evidence and the side effects, i.e., about the expecting us dangers. The first stage of the analysis has a positive nature and it is possible to say about "the utility" $U(\sigma_k)$, the second has a negative nature and it must be formalized (in our context) as "the harmfulness" $L(\sigma_k)$. During the first stage the entropy H^+_π occurs smaller, i.e., the subject acts under the conditions of a relative *psychological comfort*. In the second case the entropy H^-_π is higher than, in the previous case and, therefore, one has a relative *psychological discomfort*.

The fact that in the open active systems the entropy can both grow and decrease follows from the general theoretical views. Let us refer to the monograph by Ebeling V., Engel A., Faystel R. [55]:

"Entropy is a key physical value when it's going about description of self-organizing. It serves as the measure of the value of energy being contained in the system and as the measure of disorder… The entropy of a system can decrease if the system exports entropy and if the export per a unit of time exceeds the appropriate production of the entropy inside the system…" and further "the export of entropy… happens not spontaneously, but on "entropy pump" is required,…, "we distinguish between active and passive form - structuring systems… active systems… they contain entropy pumps inside themselves…".

In our case "energy" is associated with the product of utility and preferences; moreover it is possible to speak about the available subjective energy and the required subjective energy, and entropy - with the subjective entropy. Apparently, by analogy, it could be said about "production" and "export" of subjective entropy.

If we will designate through $\frac{d_e H}{dt}$ - the speed of the entropy export, and through $\frac{d_i H}{dt}$ - the speed of its production, then in accordance with Ebeling export exceeds the production in the successfully developing active system

$$\left(-\frac{d_e H}{dt}\right) > \frac{d_i H}{dt} > 0, \tag{5.1}$$

moreover most clearly this circumstance is manifested «...*with the embryogenesis, in the childhood with the curing of wounds and regeneration of organisms*...».

Particularly we could say that inequality (5.1) must have place, if training in HEI - Higher Education Institute is successful. In the work [64] we already partially touched on this question. In particular it was noted that if the so-called problematic instruction is being realized then a change of entropy has a periodic nature: during the creation of problem situation the total entropy $H = H_e + H_i$ increases, but in the stage of the solution it decreased due to the prevailing entropy export.

We already brought the numerical example, when subjective entropy have been decreased with the «return» of resources.

From the aforesaid it is evident that the dynamics of preferences is key issue during the study of active systems. This circumstance bears the fundamental character. The second circumstance, which also specifies the study of the dynamics of active system, is in an essential part technological.

A whole vent of problems cannot be satisfactorily solved, if we remain within the framework of stationary theory. To the problems relate, for example, to the accounting of the mutual influence of the individual preferences in the group. As it will be shown, in a stationary version a reciprocal effect of subjects, almost always reduces to the solution of essentially nonlinear algebraic system of equations.

Upon transfer to the dynamic version, which more adequately reflects the essence of the matter - the real nature of the studied processes, we may succeed in going around of these mathematical difficulties. Conditionally speaking "linearization".

In particular, in chapter 7 in this book, we in particulate repeatedly turn ourselves to such a categories as "social justice" and "freedom". It is impossible to give the satisfactory definition of these categories within the framework of "static" theory as the characteristics of the system or subject state in the given moment of time. These categories are dynamic. In other words: *"social justice" is process, "freedom" is process*.

There is influence at prehistory on the individual preferences distribution (preferences of the I type) a priori preferences, tastes, beliefs, habits,..., the influence of the save factors on the individual preferences of the II type (ratings). As the one of the important problems of the "ethical imperatives" accounting, which refers to the conditions of the market relations forming and regulating comes out.

Since one of the education problems is the assertion in the consciousness of trained person the specific system of "ethical imperatives". It is obvious, that the result of solution of the indicated problem can be useful in the correspondent of development of effective training systems and programs. Generally the application of subjective analysis and, in particular, entropy approach to the problems of education makes it possible to built the more simple logic scheme and design sequence of the instructional systems.

The important tools of subjective analysis are elasticity and stiffness of preferences including their changeability. Therefore the part, which concerns of these characteristics, is introduced in present chapter. The explicit accounting of the preferences dynamics in the economic dynamics problem essentially influence on the time dependence on the economic parameters. This circumstance is illustrated within the framework of Walras - Leontiew dynamic model, together with the model of the preferences dynamics.

It is obvious that any preferences distribution at the given moment of time is the result of the specific processes and, at the same time - is initial one for the subsequent changes. The questions, which actually require the approach, with the dynamics attitude – the evolution in time, include:

1. Forming of problem alternative sets S_{aj}, sets of ratings the subjects in group S_{ξ}, corporate problems, and hierarchic structures.

2. Influence on the preferences of all forms existing at the given moment of a priori preference prehistory, tastes, beliefs, habits…

3. Mutual influence of the preferences of subjects belonging to the same group with the different conditions of mutual knowledge ability.

4. Accounting of "ethical component" - very sTabele, weakly depending on the time for both "static" and dynamics settings.

5. Development of the preferences models change during "life" cycle of active system or its subject.

Besides the enumerated questions there are a number of problems for investigations of the active systems qualitative properties and their subjective characteristics: Lyapunov stability and Hadamard stability, the formation of attractors on the preferences distributions, control of the preferences, including optimum control and so forth.

The questions about reversibility and irreversibility of subjective processes, about the entropy stability, the bifurcational phenomena, caused by the nonlinearity of mathematical models, or as a result of deliberate ("power") inclusion of attractors in the model naturally arise.

There exist a separate problem of structure and study of the dynamics models of the I and II type canonical preferences distributions with specialized active and passive resources, taking into account of interdependence and mental stipulation as well.

The Gumilev „passionarity push" theory [41] in the view of the author can be interpreted in the terms of the developed formalism. In any case, it could be imagine real to structure the "frame model" with the use of canonical distributions of the I and II type and entropy - information analysis, if it will be possible to model the cumulative effects on the sets S_a and S_ξ.

The sharp decrease of rating entropy simultaneously with the decrease of the entropy of object preferences and the growth of intersubject correlation can be the condition of the „passionarity push" appearance; and as a consequence - hierarchization of socium, the nonformal passage of the large volume of imperious authorities in the hands of the small group of passionarities, corporatization of several alternatives.

For the part of the enumerated problems, the quantitative models are proposed. In other cases we will limit ourselves to the discussion only. *The problem, solved in this chapter is the construction of line of the models, which ensure the accounting of special features declared above.*

It any case several such models could be proposed. Their effectiveness and acceptability for decrypting the corresponding effect must be subsequently established taking into account the experimental data.

Here as it was noticed above there can be two ways: first, a qualitative evaluation of the obtained conclusions, their correspondence „to the common sense" and to experience and, secondly and numerical, identification on the basis of statistic study and thus inclusion the empirical component into the model.

5.2. Basic problems types of the preferences dynamics

There are, at least, two approaches to the study of the dynamics of preferences.The first of them is connected with the assumption, that the alternatives σ_k do not change (the „drift" of alternatives is absent). In this case a change („drift") of preferences can be caused by the factors, about which it was already said: the influence of „prehistory", i.e., past preferences, natural („spontaneous") change as a result, for example, of the age variability change in the needs, tastes, the influence of sTabele and variable „ethical imperatives" and other similar factors, which bear the internal (endogenous) nature.

The dynamics of preferences, caused by these factors, can be named *„endogenous dynamics*".

It has the wide variety of the manifestations in the form of variety of contributing factors.

It is naturally to distinguish *„endogenous dynamics"* of the preferences of *„isolated individual"* and individual *„endogenous dynamics"* of the subject as a member of the group, the dynamics of group („weighed") preferences, as well as endogenous dynamics of ratings. Connecting with the group analysis the mutual influence of individual preferences should be included in the number of endogenous factors, since this factor look like internal one for the group.

Another class of the dynamic problems of subjective analysis is connected with the study of the preferences distribution change, caused by *„external"* factors - the exogenous *„drift"* of alternatives. It means that the alternatives are characterized by the qualitative and quantitative attributes, which change as a result of the *„external" „exogenous"* factors action, for example, by the economic situation - prices, inflation, and bank percentage; change in the climatic conditions and so forth.

The essential feature of active systems is their ability not only to adaptation, but also, under certain conditions, to act on the external circumstances, including on the „exogenous factors". One of the examples appears the action on the „prices" and the „releases" in the Walras - Leontev economic model through the influences of the I type preferences on the value of final demand [108].

Let us name the *„exogenous dynamics"* the dynamics of preferences, caused by exogenous factors. In both cases the presence of the model of dynamics, the canonical preferences distributions as well as the phenomenological equations, which describe changes in the endogenous or exogenous factors is assumed. This additional system of equations can be deterministic or stochastic.

5.3. Exogenous dynamics of the I type preferences

Let us examine the absolute preferences of the I type $\pi(\sigma_k)$ on the set S_a. All normalized canonical functions of preferences have the following structure

$$\pi\left(\sigma_k\right) = \frac{f\left(\sigma_k\right)}{\sum_{j=1}^{N} f\left(\sigma_j\right)}. \tag{5.2}$$

Let us assign the quantitative sense to symbol σ_k. This can be utilities, the resources of the specific form, price ..., and, in connection with this we assume that σ_k is the continuous variable.

In reality, as it was accepted earlier, σ_k indicates the individualized alternative (kind of good, geographical point, ...). Alternative σ_k is characterized by the set of the quantitative parameters (sizes, cost, distance...) - $x_k = (x_{k1}, x_{k2}, ..., x_{k3})$. In order not to introduce additional designations, we identify σ_k with the appropriate parameters.

We see from (5.2) that $\pi(\sigma_k)$ depends on all σ_j ($j \in \overline{1,N}$) as a result of condition. Let us determine the marginal (limiting) preferences $\frac{\partial \pi(\sigma_k)}{\partial \sigma_j}$, we will find

$$\frac{\partial \pi(\sigma_k)}{\partial \sigma_k} = \pi(\sigma_k)\left(1 - \pi(\sigma_k)\right)\left(\ln f(\sigma_k)\right)'_{\sigma_k}.$$

For $j \neq k$:

$$\frac{\partial \pi(\sigma_k)}{\partial \sigma_j} = -\pi(\sigma_k)\pi(\sigma_j)\left(\ln f(\sigma_j)\right)'_{\sigma_j}.$$

Complete time derivative of $\pi(\sigma_k)$ is

$$\frac{d\pi(\sigma_k)}{dt} = \pi(\sigma_k)\left[\frac{f'(\sigma_k)}{f(\sigma_k)}\dot{\sigma}_k - \sum_{j=1}^{N}\pi(\sigma_j)\frac{f'(\sigma_j)}{f(\sigma_j)}\dot{\sigma}_j\right]. \qquad (5.3)$$

Actually, this is evident from the following:

$$\frac{d\pi(\sigma_k)}{dt} = \frac{\partial \pi(\sigma_k)}{\partial(\sigma_1)}\dot{\sigma}_1 + \ldots + \frac{\partial \pi(\sigma_k)}{\partial(\sigma_k)}\dot{\sigma}_k + \ldots + \frac{\partial \pi(\sigma_N)}{\partial(\sigma_N)}\dot{\sigma}_N;$$

$$\frac{\partial \pi(\sigma_k)}{\partial \sigma_k} = \frac{f_k'\left(\sum\ldots\right) - f_k f_k'}{\left(\sum\ldots\right)^2} = \frac{f_k'}{f_k}\pi(\sigma_k) - \frac{f_k'}{f_k}\pi^2(\sigma_k) = \pi(\sigma_k)\left(1 - \pi(\sigma_k)\right)\frac{f_k'}{f_k}.$$

Multiplying partial derivatives on $\dot{\sigma}_k$, $\dot{\sigma}_j$ and adding them we will obtain equation (5.3).

From normalization condition $\sum_{j=1}^{N}\pi(\sigma_j) = 1$ it follows, that

$$\sum_{j=1}^{N}\frac{d\pi(\sigma_j)}{dt} = 0. \qquad (5.4)$$

It is possible to verify that with the use of formula (5.3) condition (5.4) is identically satisfied.

Actually, suppose that there are only two alternatives σ_1 and σ_2. Then the condition must be satisfied:

$$\frac{d\pi(\sigma_1)}{dt} + \frac{d\pi(\sigma_2)}{dt} = 0.$$

Designating $\pi(\sigma_1) = \pi_1$, $\pi(\sigma_2) = \pi_2$, we will find according to (5.3):

$$\pi_1(1-\pi_1)\frac{f_1'}{f_1}\dot{\sigma}_1 - \pi_1\pi_2\frac{f_2'}{f_2}\dot{\sigma}_2 + \left(-\pi_1\pi_2\frac{f_1'}{f_1}\dot{\sigma}_1 + \pi_2(1-\pi_2)\frac{f_2'}{f_2}\dot{\sigma}_2\right) =$$

$$= \frac{f_1'}{f_1}\dot{\sigma}_1\left(\pi_1 - \pi_1^2 - \pi_1\pi_2\right) + \frac{f_2'}{f_2}\dot{\sigma}_2\left(\pi_2 - \pi_2^2 - \pi_1\pi_2\right) = 0.$$

Bracketed expressions are equal to zero in view of normalization condition: $\pi_1 + \pi_2 = 1$.

Using formula (5.3), let us find time derivative of the entropy:

$$\frac{dH_\pi}{dt} = -\sum_{k=1}^{N}\left(\ln\pi(\sigma_k)+1\right)\pi(\sigma_k)\left(\frac{f'(\sigma_k)}{f(\sigma_k)}\dot{\sigma}_k - \sum_{j=1}^{N}\pi(\sigma_j)\frac{f'(\sigma_j)}{f(\sigma_j)}\dot{\sigma}_j\right). \tag{5.5}$$

Information flow

$$q(I) = -\frac{dH_\pi}{dt}.$$

Sign „-" means that „positive", entering information decreases the entropy, and „negative" - outgoing information increases it.

Functions $\sigma_k = \sigma_k(t)$ either are assigned in the final form or they are the solutions of a certain system of equations, which is in this case supposed to be as given one. In particular, this can be system of differential equations form:

$$\frac{d\sigma_k}{dt} = g_k\left(\sigma_1, \sigma_2, \ldots, \sigma_N, b, t\right), \tag{5.6}$$

where b - vector of the structural parameters of system, whose dynamics is described by equations (5.6). Certainly, the variety of systems dynamic models is so great, as great is variety of the real systems, which function together with the active system subject: the technical systems, economic systems, weapon systems and so forth. System (5.6) is a very special case. In the simplest case this is the system of the deterministic differential equations, with the fixed parameters b, which is „governed" by initial conditions only. If real system functions in the random conditions and (or) it contains the sources of stochastic „inside itself", then the system of equations of type (5.6) is the system of the stochastic differential equations, whose right sides contain both the random parameters and random functions of time.

In other cases, if exogenous component of active system has „memory", exogenous equations prove to be integrodifferential.

Exogenous model (type (5.6.)) can describe the dynamics of other active systems, which interact with given system, but which are appearing „external" with respect to it.

In this case the model can contain the integrodifferential equations, which make possible to include in examination the effect of „memory", the influences of „track", as this is done, for example, in some problems of nonstationary aerodynamics [65]. The case, when „exogenous system" appears to be controlled, some parameters β can itself represent control and be functions of time.

In this situation the question immediately arises: who appears to be controller, who forms „external" controls? So the positioning or self-guidance schemas of control are possible, when *b* depends on the state of system.

The independent optimality criterion *of exogenous system* can exist, different from Φ_π. This criterion can interest the subject of active system, but it does not mean that the problem is bi-criterial at all. Let us emphasize again, that the principle of optimality, which generates the canonical functions of subjective preferences lies in the fact, that *the optimality of the psyche manifestation in the form of subjective preferences is (according to hypothesis taken) the objective fact*. In *this* and only in this *sense* the psyche of the subject is not manipulated from without. It, however, is completely manipulated by exogenous actions. Let system (5.6) takes the form:

$$\frac{d\sigma_k}{dt} = g_k\left(\sigma_1, \sigma_2, \ldots, \sigma_N, u, b, t\right), \tag{5.7}$$

where through *u* the parameters, which play the role of the controlling variables are designated. In the problems of the programmed control $u = u(t)$, in the problem of positional control $u = u(\sigma, \pi)$.

In the latter case the differential equations model can be represented in the form:

$$\frac{d\sigma_k}{dt} = g_k\left(\sigma_1, \sigma_2, \ldots, \sigma_N, \pi_1, \pi_2, \ldots, \pi_N, b, t\right), \tag{5.8}$$

where b preserve the sense of the structural parameters. Model remains exogenous with respect to the forming of the preferences, but preferences, in their turn have an influence on behavior of exogenous component, through of the dependence right sides from the preferences. Finally, if there is external - exogenous „controller", which can form the programmed control u (t) then, he can govern the preferences of this subject.

In this case, if model (5.8), is accepted the complete model *of the dynamics of active system with „exogenous shell"* is composed with two groups of the differential equations: the equations, which describe the evolution of preferences and equations, which describe the evolution of the *„exogenous shell":*

$$\frac{d\pi_k}{dt} = \pi_k \left(\frac{f'_k}{f_k} g_k - \sum_{j=1}^{N} \pi_j \frac{f'_j}{f_j} g_j \right), \tag{5.9}$$

$$\frac{d\sigma_k}{dt} = g_k \left(\sigma_1, \sigma_2, ..., \sigma_N, \pi_1, ..., \pi_N, b, t \right), \tag{5.10}$$

If under σ_k we imply the utility of alternative $U_k(x_{k1}, ..., x_{ks})$, which depends on several parameters $x_{k1}, ..., x_{ks}$, then.

$$\frac{d\sigma_k}{dt} \sim \sum_{q=1}^{s} \frac{\partial U_k}{\partial x_q} g_{kq} \left(x_{k1}, x_{k2}, ..., x_{ks}, \pi_1, ..., \pi_N, b, t \right), \tag{5.11}$$

where

$$\frac{dx_{kq}}{dt} = g_{kq} \left(x_{k1}, x_{k2}, ..., x_{ks}, \pi_1, ..., \pi_N, b, t \right). \tag{5.12}$$

In the particular case utilities are the functions of the resources of different types, and x_{qs} can be these resources, either prices or other essential parameters. Let us look how the concrete form of the preference functions $\pi(\sigma_k)$ is reflected on the structure of model (5.9), (5.10). Earlier it was shown that depending on the postulated functional the different structures of the preference functions can be obtained. The simplest models took the form:

$$\pi^- \left(\sigma_k \right) = \frac{e^{-\beta x_k}}{\sum_{j=1}^{N} e^{-\beta x_j}}; \quad \pi^+ \left(\sigma_k \right) = \frac{e^{\beta x_k}}{\sum_{j=1}^{N} e^{\beta x_j}} \tag{5.13}$$

and

$$\pi^- \left(\sigma_k \right) = \frac{x_k^\alpha e^{-\beta x_k}}{\sum_{j=1}^{N} x_j^\alpha e^{-\beta x_j}}; \quad \pi^+ \left(\sigma_k \right) = \frac{x_k^\alpha e^{\beta x_k}}{\sum_{j=1}^{N} x_j^\alpha e^{\beta x_j}}. \tag{5.14}$$

For case (5.13) $\frac{f'_k}{f_k} = -\beta$ or $\frac{f'_k}{f_k} = \beta$. For case (5.14)

$$\frac{f'_k}{f_k} = \alpha \frac{1}{x_k} - \beta, \tag{5.15}$$

or

$$\frac{f'_k}{f_k} = \alpha \frac{1}{x_k} + \beta. \tag{5.16}$$

Let us make a notice

1. Equations for the preferences are nonlinear and even if the „exogenous shell" model is linear, the complete model of the active system dynamics is nonlinear.

2. In [64] and in the chapter 2 of this work the versions of other preferences distributions of the I type, which generalize „absolute" distribution $\pi(\sigma_k)$, are given. For example, we can print the preferences distribution of one step ways $\pi(\sigma_i, \sigma_j)$ (passages $i \to j$), the distribution of conditional preferences $\pi(j|\ i)$ and others. The models of dynamics can also be written for these functions.

3. The approach presented above applies easily to the function of preferences π^+, π^-, υ^+, υ^-, ..., $\pi^+(j|\ i)$, ..., $\upsilon^+(\sigma_j|\ \sigma_i)$,and also different structures of functions $f(\sigma_i, ...)$.

4. The model of dynamics is enlarged substantially, if the functions of the preference of the I and II type are examined simultaneously and the active system, which includes the group of subjects, is the object of analysis.

Let us examine the simplest version, when the absolute preferences of the I type $\pi_j(\sigma_k)$ and absolute ratings $\xi_j(\sigma_k)$ (functions of the preferences of the II type), differentiated on the alternatives are studied, and set S_a of alternatives is supposed to be common for all group subjects. In this case it is naturally to assume, that „exogenous shell" of one subject differs from the such one of another subject.

We need to pass from the formal identification of alternative σ_k to its characteristics utility indices, resources of the specific form...

The utility $U_{jk}(x^k{}_1, x^k{}_2, ..., x^k{}_s)$ is supposed to be individualized, where j is the number of subject, and k is the number of alternative. The equations of the shell dynamics for j take the form

$$\left.\frac{dx_i^k}{dt}\right|_j = g_j^k\left(x_1^k, x_s^k, \pi_{11}, \pi_{12}, ..., \pi_{MN}, \xi_{11}, \xi_{12}, ..., \xi_{MN}, b, t\right). \tag{5.17}$$

It is evidently from the latest record that dependence of index (factor) x_i^k rate of change on all preferences of the I and II type in the group is allowed. The model of the dynamics of the active system, which includes the group of subjects appears in this case as follows:

$$\frac{d\pi_j\left(\sigma_k\right)}{dt}=\pi_j\left(\sigma_k\right)\times\left(\frac{f'_{jk}}{f_{jk}}\sum_{q=1}^{S_j^k}\frac{\partial U_j\left(\sigma_k\right)}{\partial x_q^k}g_{jq}^k-\sum_{p=1}^{N}\pi_j\left(\sigma_p\right)\frac{f'_{jp}}{f_{jp}}\sum_{q=1}^{S_j^k}\frac{\partial U_j\left(\sigma_p\right)}{\partial x_p^k}g_{jq}^p\right); \tag{5.18}$$

$$\left.\frac{dx_q^k}{dt}\right|_j=g_{jq}^k\left(x_1^k,x_s^k,\pi_{11},\pi_{12},...,\pi_{MN},\xi_{11},\xi_{12},...,\xi_{MN},b_j^k,t\right), \tag{5.19}$$

$$(\, j\in\overline{1,M},\ \ k\in\overline{1,N}\,);$$

$$\frac{d\xi_j\left(\sigma_k\right)}{dt}=\xi_j\left(\sigma_k\right)\times\left(\frac{h'_{jk}}{h_{jk}}\sum_{m=1}^{L}\frac{\partial\bar{U}_j\left(\sigma_k\right)}{\partial y_m^k}d_{jm}^k-\sum_{r=1}^{N}\xi_j\left(\sigma_r\right)\frac{h'_{jr}}{h_{jr}}\sum_{n=1}^{S_j^r}\frac{\partial\bar{U}_j\left(\sigma_r\right)}{\partial y_n^r}d_{jn}^r\right); \tag{5.20}$$

$$\left.\frac{dy_m^k}{dt}\right|_j=d_{jm}^k\left(y_1^k,y_s^k,\pi_{11},\pi_{12},...,\pi_{MN},\xi_{11},\xi_{12},...,\xi_{MN},b_j^m,t\right). \tag{5.21}$$

In this system of equations the values U and $\bar{U}$ - utilities, — the functions, having different meaning in the expressions for $\pi_j(\sigma_k)$ and $\xi_j(\sigma_k)$, what is marked in the second case by top dash, h_{jk} is analog of function f in formula (5.2). Indices x^k_q and y^r_m coincide, or differ both in their meaning as well as in their number. Accordingly, it was accounted, that in the equations (5.19) and (5.21) the right sides differ from each other in the general case. These groups of equations represent itself as a version of the model of system „*exogenous shell*". The necessity for individualizing right sides in the system (5.19) is connected yet with the fact, that the initial conditions, in which the members of group are situated differ, in the general case. In the general case, the limitations, superimposed on the indices x_j^k and y_j^k, are different.

As an example of such limitations the natural requirement of non-negativity of „issues" and prices in the Walras - Leontev dynamic model, in the economy can serve; the limitations, superimposed on the motion parameters of the flight vehicle and so on.

Let us notice that if the indices x_j^r, y_j^k, σ_k, ... do not change, which is

$$\frac{dx_j^k}{dt}=\frac{dy_j^k}{dt}=0;\ \ \frac{d\sigma_k}{dt}=0,\ ..,$$

the preferences do not change as well: $\pi_j(\sigma_k)$ = const, $\xi_j(\sigma_k)$ = const.

Thus, in the exogenous dynamics the only reason for the evolution of preferences are the alternations which occur in the exogenous shell, which is manifested through a change in the indices x, y (or σ).

In turn, as we have already told, alternation in the preferences can act on exogenous „shell". The activity of the system is manifested in the fact that it, founding on the extreme nature of preferences, forms the structure of their models independently of the „shell".

Everywhere before we assume that the values of the parameters (indices) are reliably known to subject, which are characterizing the problem - resource situation. In reality, only some estimations of these parameters $\hat{x}$, $\hat{R}$,... can be known to him. In this situation, the models of the exogenous dynamics of active system given above, should be supplemented with the model of „observer", which would make possible to form these estimations. At present the author cannot propose any such plausible and substantiated models. Some examples are known. So in the works of Barucha [10] the Kalman filter is supposed to be as „subjective observer". This version, however, does not contains any specific peculiarities, reflecting the subjective nature of the information perception.

If function (5.14) is supposed to be as the model of the function of preference, instead of equations (5.3) and (5.4) we must use the following equations:

$$\frac{d\pi(\sigma_1)}{dt} = \pi(\sigma_1)\pi(\sigma_2)\left[\left(\alpha p_1^{-1} - \beta\right)g_1 - \left(\alpha p_2^{-1} - \beta\right)g_2\right]; \tag{5.22}$$

$$\frac{d\pi(\sigma_2)}{dt} = \pi(\sigma_1)\pi(\sigma_2)\left[-\left(\alpha p_1^{-1} - \beta\right)g_1 + \left(\alpha p_2^{-1} - \beta\right)g_2\right], \tag{5.23}$$

where, for example,

$$g_1 = \frac{1}{n_1}\left(a_{11}p_1 + a_{12}p_2 + c_1(\pi_1) - p_1\right); \quad g_2 = \frac{1}{n_2}\left(a_{21}p_1 + a_{22}p_2 + c_2(\pi_2) - p_2\right).$$

Here g_1 and g_2 are the right sides of the version of the Walras - Leontev model of economic dynamics, where p_1 and p_2 - the price of commodities units (see further in this chapter), $c_1(\pi_1)$, $c_2(\pi_2)$ — function of the final demand, which in this case depends on the price only.

We can see, that, of course, $\frac{d\pi(\sigma_1)}{dt} = -\frac{d\pi(\sigma_2)}{dt}$.

One of the possibilities of final demand submission happen to be the expression of the product of „natural" preference π^*_i and a certain function $\varphi(p_i)$ of the price p_i, which corresponds to the following requirements: if p_i exceeds a certain „mean" price p^*, then $\varphi(p_i) < 1$ and with $p_i \to \infty$ $\varphi(p_i) \to 0$.

If $p_i < p^*$, then $\varphi(p_i) > 1$ and with $p_i \to 0$ $\varphi(p_i) \to \infty$. Such conditions satisfies the function

$$\varphi(p_i) = \frac{p^*}{p_i}.$$

If we exclude limiting cases from the examination, then function $\varphi(p_i)$ can be taken in the form:

$$\varphi\left(p_i\right) = 1 + k\frac{p^* - p_i}{p_i}; \quad k < 1. \tag{5.24}$$

If we assume that the empirical function of preference can be represented as the product:

$$\pi_i = \pi_i^*\left(1 + k\frac{p^* - p_i}{p_i}\right) \tag{5.25}$$

and normalization condition is satisfied

$$\sum_{i=1}^{N} \pi_i = 1,$$

then mean price is harmonic mean of the form

$$p^* = \frac{1}{\sum_{i=1}^{N}\frac{\pi_i^*}{p_i}}.$$

Thus, it is possible to assume

$$c_i\left(\pi_i\right) = c_i\pi_i^*\left(1 - k\frac{p^* - p_i}{p_i}\right).$$

Function (5.25) was examined in the publication [65].

5.4. Reciprocal effect of the individual preferences in the group

At first we will examine the models, which consider the mutual influence of preferences in the group of subjects in the stationary setting. We see many real examples of this influence from very weak to maximally strong, when the discussion deals, for example, with the group, which is like the army subdivision, where the preferences of commander (one subject) absolutely prevail, or the „crowd effect", where this influence is also prevailing.

It is necessary to make some assumptions, which will be gradually supposed to be in the proposed models:

1. Accounting of mutual influence of preferences can be realized with the aid of discussed in this work entropy information technology, via postulation of specific variation settings. In this case at any moment of time we will obtain the model of the function of preferences distribution, which we call „*canonical*". Different way consists in forming of certain dependences without use of variation principle, but on the dose of speculative contemplations to a considerable degree or by analogy with already known theories. An example can be some demographic models (of Lotka - Volterra type and other [55]).

The corresponding models are phenomenological. At last, combination of the first and second ways and corresponding class of models - semi-phenomenological makes sense.

2. In the problems supposed to be below it is assumed that all members of group „play on the one and the same field", i.e., have identical alternatives sets: $S_{ai} = S_{aj}\ \ \forall i, j \in \overline{1,M}$. If in reality this is not so, then by expanding each set S_{aj} to the sum $\bigcup_{j=1}^{N} S_{aj}$ and the extension of determination of preferences distributions with zeros on differences $S_{aj} \setminus S_{ai}$ it is possible to accomplish the assumption made.

3. If $\pi_i(\sigma_k)$ is of alternative σ_k preference for the subject i, that is assumed, that only „similar" preferences $\pi_j(\sigma_k)$ of subject j can influence on it, but not $\pi_j(\sigma_q)$, where $q \neq k$. This assumption somewhat decreases the generality of assignment.

4. The complete and instantaneous knowledge ability of subject i about the preferences of all other subjects $j \in \overline{1,M}$ am allowed. This assumption can be subsequently modified, eliminate for example, the requirement of instantaneousness. It is not excluded, that the subject j itself informs i about his preferences in some manner (result will depend on the degree of the „truthfulness" of subject j). We will assume that the mutual influence could be is described by odd function of „diver-

gence on the opinions" $\Delta_{ij} = \pi_i - \pi_j$, or logarithmic „divergence" (tender influence) $S_{ij} = \ln\pi_i - \ln\pi_j = \ln\frac{\pi_i}{\pi_j}$.

5. let us assume that the influence of „opinion" j on „opinion" i depends on the rating „j in the eyes i": $\xi(j|\ i) = \xi_{ji}$ in such a way, that, the greater ξ_{ji}, the greater the influence j on i.

In this case it is supposed to be that ξ_{ji} are defined as the normalized values

$$\sum_{j=1}^{M} \xi(j \mid i) = 1 .$$

The „self rating" ξ_{ii} enters into the number of ξ_{ji}.

Let us examine some versions of the models of the individual preferences of subjects in the group.

Let us assign the „built-in" criterion of optimality to each subject $i \in \overline{1,M}$. In this criterion the individual ratings of subjects in the group must be accounted, moreover there could be ratings of different types: ξ_j is integral ratings irrespectively to the issue and „point of view", from which the subject j is examined; $\xi(j|\ i) = \xi_{ji}$ is integral conditional ratings, depending on the „point of view"

$$i \overset{\xi(j|i)}{\Rightarrow} j$$

$\xi(j,\ i|\ \sigma_k)$ is differential conditional ratings (referred to the specific alternative).

In all cases ratings are normalized:

$$\sum_{j=1}^{M} \xi(j) = 1; \ \sum_{j=1}^{M} \xi(j \mid i) = 1; \ \sum_{j=1}^{M} \xi(i, j \mid \sigma_k) = 1 .$$

It is assumed that all members of group are in the state of information contact, that those exists a „tet -a-tet " information, or information through the mediator, on model of the advertizing campaign models type and so forth.

Let the subject i be the *„carrier"* of alternatives set S_{ai}, preferences distribution $\pi_i(\sigma_k)$ on S_{ai}: $\sigma_k \in S_{ai}$ and the optimality criterion $\Phi_{\pi i}$. In this case, since i is the member of group, then $\Phi_{\pi i}$ consists of two parts $\Phi^{(A)}{}_{\pi i}$ and $\Phi^{(B)}{}_{\pi i}$, where $\Phi^{(A)}{}_{\pi i}$ reflects the individual factors, and $\Phi^{(B)}{}_{\pi i}$ reflects interaction with the other members of the group:

$$\Phi_{\pi i} = \Phi^{(A)}{}_{\pi i} + \Phi^{(B)}{}_{\pi i}.$$

It can be said, that $\Phi^{(B)}{}_{\pi i}$ is caused by to participation i in the group and, consequently, reflects the collectivistic component of psyche, whereas $\Phi^{(A)}{}_{\pi i}$ is the reflection of individualistic component.

Should it be always the sum, when complete criterion considers both circumstances additively? Answer to this question is inevident.

Let

$$\Phi^{(A)}_{\pi_i} = -\sum_{k=1}^{N}\xi_{ii}\pi_i(\sigma_k)\ln\pi(\sigma_k) + \beta_i\sum_{k=1}^{N}\xi_{ii}\pi_i(\sigma_i)U_i(\sigma_k) + \gamma_i\sum_{k=1}^{N}\pi_i(\sigma_k) \quad (5.26)$$

and

$$\Phi^{(B)}_{\pi_i} = \sum_{j=1}^{M}\sum_{k=1}^{N}\xi_{ji}\pi_i(\sigma_k)\ln\frac{\pi_i(\sigma_k)}{\pi_j(\sigma_k)} = \sum_{j=1}^{M}\sum_{k=1}^{N}\xi_{ji}\pi_i(\sigma_k)\left(\ln\pi_i(\sigma_k) - \ln\pi_j(\sigma_k)\right). \quad (5.27)$$

If $i = j$, the corresponding term in $\Phi^{(B)}{}_{\pi i}$ becomes zero, and it is possible to write

$$\Phi^{(B)}_{\pi_i} = +\sum_{j=1}^{M}\sum_{k=1}^{N}{}'\xi_{ji}\pi_i(\sigma_k)\ln\frac{\pi_i(\sigma_k)}{\pi_j(\sigma_k)}, \quad (5.28)$$

where „prime” means, that in the sum on j there is no term with $j = i$.
Complete criterion takes the form:

$$\Phi_{\pi i} = \sum_{k=1}^{N}\pi_i(\sigma_k)\Big[-\xi_{ii}\ln\pi_i(\sigma_k) + \beta_i\xi_{ii}U_i(\sigma_k) + \quad (5.29)$$

$$+\sum_{j=1}^{M}{}'\xi_{ji}\left(\ \ln\pi_i(\sigma_k) - \ln\pi_j(\sigma_k)\ \right) + \gamma_i\Big].$$

The presence of coefficient ξ_{ii} in the term, which contains utility $U_i(\sigma_k)$ means, that the following condition must be satisfied: when $\xi_{ii} = 1$ all ξ_{ji} ($j \neq i$) become zero and subject i does not experiences influence from the remaining members of group, and the criterion takes the form:

$$\Phi_{\pi i} = \sum_{k=1}^{N}\pi_i(\sigma_k)\left[-\ln\pi_i(\sigma_k) + \beta_i U_i(\sigma_k) + \gamma_i\right]..$$

On the other hand, if $\xi_{ii} \neq 1$, the role of individual utility $U_i(\sigma_k)$ in the structure of the preference function $\pi_i(\sigma_k)$ decreases. If coefficient ξ_{ii} would be excluded from the term with $U_i(\sigma_k)$, then this corresponds to the fact that the role of individual utility $U_i(\sigma_k)$ is supposed to be by subject i without the „caution” on ratings distribution inside the group.
When in the structure of functional $\Phi_{\pi i}$ the integral absolute ratings ξ_j are used, then it is possible to speak about the completeness of the conformism of all subjects of group with respect to the recognition of such ratings and, consequently, about the higher level of the collective component of the psyche:

$$\Phi_{\pi i} = \sum_{k=1}^{N}\pi_i(\sigma_k)\left[-\xi_i\ln\pi_i(\sigma_k) + \beta_i\xi_i U_i(\sigma_k) + \sum_{j=1}^{M}\xi_i\left(\ln\pi_i(\sigma_k) - \ln\pi_j(\sigma_k)\right) + \gamma_i\right]. \quad (5.30)$$

Criterion (5.29) leads to the model of the preference function of the form

$$\pi_i\left(\sigma_k\right)=\frac{\exp\left\{\beta_i U_i\left(\sigma_k\right)+\sum_{j=1}^{M}{}'\xi_{ji}\ln\pi_j\left(\sigma_k\right)\right\}}{\sum_{q=1}^{N}\exp\left\{\beta_i U_i\left(\sigma_q\right)+\sum_{j=1}^{M}{}'\xi_{ji}\ln\pi_j\left(\sigma_q\right)\right\}}, \tag{5.31}$$

which let us rewrite in the form?

$$\pi_i\left(\sigma_k\right)=\frac{\prod_{j=1}^{M}{}'\pi_j\left(\sigma_k\right)^{\xi_{ji}}\,e^{\beta_i U_i\left(\sigma_k\right)}}{\sum_{q=1}^{N}\prod_{j=1}^{M}{}'\pi_j\left(\sigma_q\right)^{\xi_{ji}}\,e^{\beta_i U_i\left(\sigma_q\right)}}. \tag{5.32}$$

This function corresponds to the principle of „unanimity" or the „veto" law of each subject in the group, if at least one of the subjects has zero preference σ_k (one of the multiplier $\pi_j(\sigma_k) = 0$ in the numerator), then the preference of subject also becomes zero (all terms in the denominator cannot be a zero of normalization conditions).

We see that, if all utility are equal to zero: $U_i(\sigma_k) = 0$, than this preference is determined by the preferences of other group members only.

Let in particular, let it be $\xi_{ji}=\frac{1}{M}$; $\pi_j\left(\sigma_k\right)=\frac{1}{N}$ for $\forall k\in\overline{1,N}$ and $\forall j\in\overline{1,M}$, then

$$\pi_i\left(\sigma_k\right)=\frac{\left(\frac{1}{N}\right)^{\frac{M-1}{M}}}{N\left(\frac{1}{N}\right)^{\frac{M-1}{M}}}=\frac{1}{N}.$$

The same have place. When $\prod_{j=1}^{M}{}'\pi_j\left(\sigma_k\right)^{\xi_{ji}}=f_i$ arbitrary integer (< 1), since in this case

$$\pi_i\left(\sigma_k\right)=\frac{f_i}{Nf_i}=\frac{1}{N}.$$

But then it is obvious, that all $\pi_i\left(\sigma_k\right)=\frac{1}{N}$ and $f_i=\left(\frac{1}{N}\right)^{\frac{M-1}{M}}$.

The different version appears, if we select the criterion $\Phi_{\pi i}$ in the form:

$$\Phi_{\pi i} = -\sum_{k=1}^{N} \pi_i(\sigma_k) \ln \pi_i(\sigma_k) - \frac{1}{2} \sum_{k=1}^{N} \sum_{j=1}^{M} \xi_{ji} \left(\pi_i(\sigma_k) - \pi_j(\sigma_k) \right)^2 + \tag{5.33}$$

$$+\beta_i \xi_{ii} \sum_{k=1}^{N} \pi_i(\sigma_k) U_i(\sigma_k) + \gamma_i \sum_{k=1}^{N} \pi_i(\sigma_k).$$

The corresponding canonical preference function takes the form:

$$\pi_i(\sigma_k) = \frac{\exp\left\{ -\sum_{j=1}^{M} \xi_{ji} \left(\pi_i(\sigma_k) - \pi_j(\sigma_k) \right) + \beta_i \xi_{ii} U_i(\sigma_k) \right\}}{\sum_{q=1}^{N} \exp\left\{ -\sum_{j=1}^{M} \xi_{ji} \left(\pi_i(\sigma_k) - \pi_j(\sigma_k) \right) + \beta_i \xi_{ii} U_i(\sigma_q) \right\}}. \tag{5.34}$$

Here we obtained essentially the system of nonlinear equations relative the preferences $\pi_j(\sigma_k)$. The solution of this system relative $\pi_j(\sigma_k)$ represents by itself complex problem. It is possible to search for it by iteration technique, or to assume that process realizes in time, then on the left side there will be the values of preferences in the moment $\pi_j(\sigma_k, t+1)$, and on the right side the preferences will correspond to moment t and the problem, conditionally speaking, linearizes.

Function (5.31) possesses the following properties:

1. When $\xi_{ji} = 0$ for $\forall j \neq i$ it takes the form

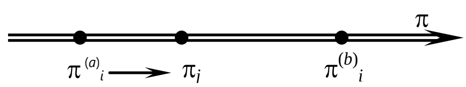
$$\pi_i(\sigma_k) = \frac{e^{\beta_i U_i(\sigma_k)}}{\sum_{q=1}^{N} e^{\beta_i U_i(\sigma_q)}},$$

namely it is determined only by utility. It was taken into account here, that from normalization condition follows $\xi_{ii} = 1$.

2. With fulfillment of the conditions of the preferences identity: $\pi_i(\sigma_k) = \pi_j(\sigma_k)$ for $\forall j \neq i$ the same result occurs as in the 1st case.

3. If $\xi_{ii} = 0$ (subject takes himself for nothing) and additionally $\pi_i(\sigma_k) = \pi_j(\sigma_k)$ for all $\forall j \neq i$, then $\pi_i = \frac{1}{N}$, and the distribution entropy possesses its maximum value.

Sign „-” in front of the sum in numerator and denominator is determined from the following conditions:



with $\pi_i^{(a)} < \pi_j$ pulling $\pi_i^{(a)}$ to π_j from left is ensured,
with $\pi_i^{(b)} > \pi_j$ pulling $\pi_i^{(b)}$ to π_j from right is ensured,
with $\pi_i = \pi_j$ for $\forall j \neq i$, as it has already been spoken π_i depends only on rational utility.

We see that as a result of normalization conditions the preference $\pi_i(\sigma_k)$ depends on complete spectrum of preferences of all group members. It is important fact that in the preferences distributions of the first type in this model the preferences of the second type - rating multiplier ξ_j or $\xi(j|\ i)$ or $\xi(i,\ j|\ \sigma_k)$ are mixed. This way must lead to forming of more general common criterion, which would make it possible to obtain both the preferences of the I and II type in the common scheme. However, the question arises: who is personally the carrier of this consolidated criterion?

Let us examine the particular, but very important case, when there is a leader in the group, who possesses indisputable authority $j = R$ and each subject divides his preferences in such a way that

$\xi_{ii} \neq 0$; $\xi_{ri} = 1 - \xi_{ii}$; $\xi_{ji} = 0$ for $j \neq i, j \neq r$.

Then basing on formula (5.32) we obtain

$$\pi_i\left(k\right)=\frac{\pi_r\left(k\right)^{1-\xi_{ii}}\ e^{\beta_i\xi_{ii}U_i\left(\sigma_k\right)}}{\sum_{q=1}^{N}\pi_r\left(\sigma_q\right)^{1-\xi_{ii}}\ e^{\beta\xi_{ii}U_i\left(\sigma_q\right)}}. \tag{5.35}$$

As it has been told already, in reality, given relationships for the preferences represent equations, or to be more precise, in the general case systems of nonlinear equations, whose analytical solution is most often impossible. So, the preference $\pi_i(\sigma_k)$ is contained both in the left and in the right sides of the formula (5.34).

Let us as an example examine the group of two subjects $M = 2$, when the total set S_a consists of two alternatives ($N = 2$): σ_1 and σ_2.

Rating matrix

$$Z=\begin{Vmatrix}\xi_{11} & \xi_{12}\\ \xi_{21} & \xi_{22}\end{Vmatrix}.$$

Let us designate $y_i(k) = \ln\pi_i(\sigma_k)$, then equations, from which the functions $\pi_i(\sigma_j)$ have to be sought it can be written in the form

$$\begin{aligned}y_1(1) - \xi_{21}y_2(1) &= \beta_1\xi_{11}U_1(1) + \gamma_1 - 1;\\ y_1(2) - \xi_{21}y_2(2) &= \beta_1\xi_{11}U_1(2) + \gamma_1 - 1;\\ -\xi_{12}y_1(1) + y_2(1) &= \beta_2\xi_{12}U_2(1) + \gamma_2 - 1;\\ -\xi_{12}y_1(1) + y_2(2) &= \beta_2\xi_{22}U_2(2) + \gamma_2 - 1.\end{aligned}$$

Unknown's $y_1(1)$, $y_1(2)$ and $y_2(1)$, $y_2(2)$ are determined from the system of equations:

$$\begin{bmatrix}1 & -\xi_{21}\\ -\xi_{12} & 1\end{bmatrix}\begin{bmatrix}y_1\left(1\right)\\ y_2\left(1\right)\end{bmatrix}=\begin{bmatrix}f_{11}\\ f_{21}\end{bmatrix}, \tag{5.36}$$

$$\begin{bmatrix} 1 & -\xi_{21} \\ -\xi_{12} & 1 \end{bmatrix} \begin{bmatrix} y_1(2) \\ y_2(2) \end{bmatrix} = \begin{bmatrix} f_{12} \\ f_{22} \end{bmatrix}, \tag{5.37}$$

where $f_{ik} = \beta_i \xi_{ii} U_i(k) + \gamma_i - 1$.
The solution of systems (5.36) and (5.37) exists

$$y_1(1) = \ln \pi_1(\sigma_1) = \frac{\begin{vmatrix} f_{11} & -\xi_{21} \\ f_{21} & 1 \end{vmatrix}}{1 - \xi_{12}\xi_{21}};$$

$$y_1(2) = \ln \pi_1(\sigma_2) = \frac{\begin{vmatrix} 1 & f_{11} \\ -\xi_{21} & f_{21} \end{vmatrix}}{1 - \xi_{12}\xi_{21}}.$$

Hence

$$\pi_1(\sigma_1) = \exp\left(\frac{f_{11} + \xi_{21} f_{21}}{1 - \xi_{12}\xi_{21}}\right) = \tag{5.38}$$

$$= \exp\frac{\beta_1 \xi_{11} U_1(1) + \xi_{21} \beta_2 \xi_{22} U_2(1)}{1 - \xi_{12}\xi_{21}} \exp\frac{\gamma_1 - 1 + \xi_{21}(\gamma_2 - 1)}{1 - \xi_{12}\xi_{21}};$$

$$\pi_2(\sigma_1) = \exp\left(\frac{f_{21} + \xi_{12} f_{11}}{1 - \xi_{12}\xi_{21}}\right) =$$

$$= \exp\frac{\beta_2 \xi_{22} U_2(1) + \xi_{12} \beta_1 \xi_{11} U_1(1)}{1 - \xi_{12}\xi_{21}} \exp\frac{\gamma_2 - 1 + \xi_{12}(\gamma_1 - 1)}{1 - \xi_{12}\xi_{21}}.$$

Analogously, we will find $\pi_1(\sigma_2)$ and $\pi_2(\sigma_2)$

$$\pi_1(\sigma_2) = \exp\frac{\beta_1 \xi_{11} U_1(2) + \xi_{21} \beta_2 \xi_{22} U_2(2)}{1 - \xi_{12}\xi_{21}} \exp\frac{\gamma_1 - 1 + \xi_{21}(\gamma_2 - 1)}{1 - \xi_{12}\xi_{21}}; \tag{5.39}$$

$$\pi_2(\sigma_2) = \exp\frac{\beta_2 \xi_{22} U_2(2) + \xi_{12} \beta_1 \xi_{11} U_1(2)}{1 - \xi_{12}\xi_{21}} \exp\frac{\gamma_2 - 1 + \xi_{12}(\gamma_1 - 1)}{1 - \xi_{12}\xi_{21}}.$$

If we designate

$$c_1 = \exp\frac{\gamma_1 - 1 + \xi_{21}(\gamma_2 - 1)}{1 - \xi_{12}\xi_{21}};$$

$$c_2 = \exp\frac{\gamma_2 - 1 + \xi_{12}(\gamma_1 - 1)}{1 - \xi_{12}\xi_{21}},$$

that (5.38) and (5.39) takes the form:

$$\pi_i(k) = \pi_i'(k)c_i,$$

where $\pi_i'(k)$ - the multiplier, which depend on the utilities. Normalization conditions: $\pi_1(1) + \pi_1(2) = 1$; $\pi_2(1) + \pi_2(2) = 1$

let us find

$$c_1 = \left(\pi_1'(1) + \pi_1'(2)\right)^{-1}; \qquad c_2 = \left(\pi_2'(1) + \pi_2'(2)\right)^{-1},$$

The formula gives

$$\pi_j(k) = \frac{\pi_i'(k)}{\pi_i(1) + \pi_i(2)}; \ (i \in \overline{1,2}). \tag{5.40}$$

In the detailed record, for example, $\pi_1(1)$ has the form:

$$\pi_1(1) = \frac{\exp\dfrac{\beta_1\xi_{11}U_1(1) + \xi_{21}\xi_{22}\beta_2U_2(1)}{1-\xi_{12}\xi_{21}}}{\exp\dfrac{\beta_1\xi_{11}U_1(1) + \xi_{21}\xi_{22}\beta_2U_2(1)}{1-\xi_{12}\xi_{21}} + \exp\dfrac{\beta_1\xi_{11}U_1(2) + \xi_{21}\xi_{22}\beta_2U_2(2)}{1-\xi_{12}\xi_{21}}}.$$

We see that, in the final analysis, preferences are determined by utilities only. If this means that in given model, the preferences bear exceptionally rational nature. This result is intelligible and is explained by the fact that in the examined scheme the factors of „ethical" nature are, or any others, not With the utility newer present. Let us notice that the condition $U_1(1) = 0$ is insufficient, in order to turn $\pi_1(1)$ to zero. Additionally one should require, in order that $\xi_{21} = 0$, so it is, may be the rating of the second subject „in the eyes of the first" would be equal to zero. But if $\xi_{21} \neq 0$, then even in the case when individual utility $U_1(1) = 0$, $\pi_1(1) \neq 0$, if given alternative possesses nonzero utility for the second subject. It is possible to interpret this fact as the manifestations of the collectivist component of psyche.

It is not difficult to find the multiplier γ_1 and γ_2 in explicit form:

$$\gamma_1 = \frac{c_1' - c_2'\xi_{21}}{1-\xi_{12}\xi_{21}}; \quad \gamma_1 = \frac{c_2' - c_1'\xi_{12}}{1-\xi_{12}\xi_{21}}.$$

Let us examine one more possible model of the preferences distribution, which is characterized by the fact that in the right side function $\pi_i(_k)$ is not contained, but the correcting difference is determined with respect to a certain fixed (with perceptions of subject i) level $\pi^*_i(k)$. This level can be differentiated on alternatives on S_a and not necessarily connected with the ethical standards. Let

$$\pi_i(k) = \frac{\exp\left[\sum_{j=1}^{M}{}' \xi_{ji}\left(\pi_j(k) - \pi_i^*(k)\right) + \beta_i \xi_{ii} U_i(k)\right]}{\sum_{q=1}^{N} \exp\left[\sum_{j=1}^{M}{}' \xi_{ji}\left(\pi_j(q) - \pi_i^*(q)\right) + \beta_i \xi_{ii} U_i(q)\right]}. \quad (5.41)$$

The prime above the sum means that there is no term with sub index $j = i$ in it. The excess of $\pi_j(k)$ above $\pi^*_i(k)$: $\pi'_j(k) > \pi^*_i(k)$ increases $\pi^*_i(k)$ and vice versa. Rating multiplier $\xi_{ji}(\xi_{ji}, \xi(j|i, \sigma_k)\ldots)$ reflect the distribution of subjective ideas about the „dignities" of the group members. The accounting only of these multiplier in models of the individual preferences of the I type, generally speaking, can prove to be insufficient. If subject possesses i of rating preferences $\xi(j|i)$ it does not mean yet, that he is ready to take them into account when the distributing of his preferences on the set S_a.

The model of these preferences will be more flexible and more „rich" in terms of reflecting features of the psyche functioning, if we introduce into it the complementary factor, responsible for the propensity to integrate rating preferences. Let us designate this factor through α and name it „tendency to the conformism".

Functional, taking into account the factor of conformism

$$\Phi\pi_i = \left(1 - \alpha\left(1 - \xi_{ii}\right)\right)\left[-\sum_{k=1}^{N} \pi_i(\sigma_k) \ln \pi_i(\sigma_k) + \beta \sum_{k=1}^{N} \pi_i(\sigma_k) U_i(\sigma_k)\right] + \quad (5.42)$$
$$+\alpha \sum_{k=1}^{N} \sum_{j=1}^{M}{}' \xi_{ji} \pi_i(\sigma_k) \ln \pi_j(\sigma_k) + \gamma \sum_{k=1}^{N} \pi_i(\sigma_k).$$

This functional possesses the following properties:

1. If $\xi_{ii} = 1$, then $\xi_{ji} = 0$ for $\forall j \in \overline{1,M}$, $j \neq i$ in view of normalization condition, then

$$1 - \alpha(1 - \xi_{ii}) = 1$$

the term containing $\pi_j(\sigma_k)$ $(j \neq i)$ disappears, and as a result of that the accounting of other group members preferences disappears in the functional.

2. If $\xi_{ii} \neq 1$ and $\xi_{ji} \neq 0$, but $\alpha = 0$, meaning there is no tendency to the conformism, then, as in the preceding case,

$$1 - \alpha(1 - \xi_{ii}) = 1,$$

also there is no term with $\pi_j(\sigma_k)$ $(j \neq i)$, and functional takes the same form as in the preceding case.

3. Finally, if $\xi_{ii} \neq 1$ and $\xi_{ji} \neq 0$, but $\alpha = 1$, then

$$1 - \alpha(1 - \xi_{ii}) = \xi_{ii},$$

and functional can be reduced to the form:

$$\Phi\pi_i = -\sum_{k=1}^{N} \xi_{ii} \pi_i(\sigma_k) \ln \pi_i(\sigma_k) + \sum_{k=1}^{N} \sum_{j=1}^{M}{}' \xi_{ji} \pi_i(\sigma_k) \ln \pi_j(\sigma_k) + \quad (5.43)$$

$$+\beta_i\sum_{k=1}^{N}\xi_{ji}\pi_i\left(\sigma_k\right)U_i\left(\sigma_k\right)+\gamma_i\sum_{k=1}^{N}\pi_i\left(\sigma_k\right).$$

Functional (5.42) generates the following canonical preferences distribution.

$$\pi_i\left(\sigma_k\right)=\frac{\prod_{j=1}^{M}{}'\,\pi_j\left(\sigma_k\right)^{\frac{\alpha\xi_{ji}}{1-\alpha\left(1-\xi_{ii}\right)}}e^{\beta_i U_i\left(\sigma_k\right)}}{\sum_{q=1}^{M}\prod_{j=1}^{M}{}'\,\pi_j\left(\sigma_k\right)^{\frac{\alpha\xi_{ji}}{1-\alpha\left(1-\xi_{ii}\right)}}e^{\beta_i U_i\left(\sigma_q\right)}}. \qquad (5.44)$$

When α = 1

$$\pi_i\left(\sigma_k\right)=\frac{\prod_{j=1}^{M}{}'\,\pi_j\left(\sigma_k\right)^{\frac{\xi_{ji}}{\xi_{ii}}}e^{\beta_i U_i\left(\sigma_k\right)}}{\sum_{q=1}^{M}\prod_{j=1}^{M}{}'\,\pi_j\left(\sigma_k\right)^{\frac{\xi_{ji}}{\xi_{ii}}}e^{\beta_i U_i\left(\sigma_q\right)}}. \qquad (5.45)$$

Correspondingly distributions (5.34) and (5.41) can be modified.

5.5. A priori and a posteriori preferences of the I type. "The chains of distributions"

5.5.1. Accounting of the influence of a priori preferences

Examining preferences of subject at given instant t_k , we must be prepared with the fact that they can depend

1) on „a priori" preferences prevailing in the previous period (t – 1, t – 2, ...), which in majority of cases reflect ethical standards;

2) on a certain „spontaneous" component, connected with indeterminacy of human psyche, a spontaneous change in the preferences generally, that may depend on a change in the tastes, physiological needs due to, in particular, by the changes depending on age, changes of subject social position, ethical imperatives and so forth;

3) on preferences distribution of other subjects, interacting (connected) by any means with given subject (through the general resources, the corporate problems, ...).

The questions, which relate to the third observation, are studied in 4th chapter, dedicated to aggregated preferences of the I type and II type.

In the present paragraph the problems of elaborating of models the preferences distributing change in time are examined (p. 1). We will limit to distributions of the π^+, π^- type bearing in mind, that theory for υ^+ and υ^- is analogous to a considerable extent. We will neglect the marks „+" and „-". We will begin examination basing on the simplest problems.

Let us designate by $\pi(\sigma_k, t)$ distribution of absolute preferences on S_a at the moment of time t and suppose that distributing at the moment of time $t + 1$: is $\pi(\sigma_k, t + 1)$. Subject forms preferences $\pi(\sigma_k, t)$ in such way that certain functional reaches extreme value.

Let this functional takes the following form:

$$\Phi_\pi(t+1,t) = -\sum_{k=1}^{N}\pi(\sigma_k,t+1)\ln\pi(\sigma_k,t+1) \pm \qquad (5.46)$$

$$\pm\beta\sum_{k=1}^{N}\pi(\sigma_k,t+1)\ln\pi(\sigma_k,t) + \gamma\sum_{k=1}^{N}\pi(\sigma_k,t+1).$$

One additional assumption, adopted here is, that from the one step to the next step alternatives set S_a does not change. In this case we do not define concretely, what the sense if „step" is, it namely what is the meaningful side of the passage $t \to t + 1$. Let us notice, that if we dealt above with passage $\sigma_k \to \sigma_j$, then in the particular problem examined here the temporary passage occurs with retention of alternative number.

The second term in the functional $\Phi_\pi(t + 1, t)$ is written as the weighted sum old „old" entropy quotients which „new" preferences. The reason for temporary changes in the preferences is „hidden" partially in the coefficient β. Writing, as earlier the necessary condition of extremum (assuming thus the extremality of the new selection of preferences):

$$\frac{\partial\Phi_\pi}{\partial\pi(\sigma_k,t+1)} = 0,$$

let us find relationship connecting $\pi(\sigma_k, t + 1)$ with $\pi(\sigma_k, t)$ (for the „+"sign before the second term):

$$\pi(\sigma_k, t + 1) = c_t(\pi(\sigma_k, t))^\beta,$$

c_k — is found from the condition for normalization for $\pi(\sigma_i, k + 1)$:

$$c_t = \left(\sum_{k=1}^{N}\left(\pi(\sigma_k,t)\right)^\beta\right)^{-1}.$$

In particular we see, that if $\beta = 1$ and for each t the normalization condition is satisfied

$$\sum_{k=1}^{N}\pi(\sigma_k,t) = 1,$$

then c_t = 1 for $\forall t \in \overline{t_0, \infty}$. This means that with these conditions the preferences distribution on S_a does not change

$$\pi(\sigma_k, t+1) = \pi(\sigma_k, t), \ \ \forall t \in \overline{t_0, \infty}.$$

When $\beta \neq 1$, then for the cases $\beta < 1$ and $\beta > 1$ the solutions are different.Let us examine the example, when $\beta = 2$ and S_a contains two alternatives σ_1 and σ_2 only and we assume that at the moment k = 1 $\pi_1(t = 1) = 0{,}3$, $\pi_2(t = 1) = 0{,}7$. The values of $\pi(\sigma_k, t)$ in t = 1,2, 3 are presented in the tabele

k	$\pi(\sigma_1, k)$	$\pi(\sigma_2, k)$
1	0,3	0,7
2	0,15517	0,84482
3	0,03263	0,967395

We see that in the course of time the smaller preference decreases, and larger - increases, i.e. the heterogeneity of the preferences distribution grows.
If $\beta = 0{,}5 < 1$, then the analogous tabele

k	$\pi(\sigma_1, k)$	$\pi(\sigma_2, k)$
1	0,3	0,7
2	0,39563	0,60436
3	0,44723	0,55276

tells that the initial heterogeneity of distribution decreases and distribution strives for uniform with $t \to \infty$.

In the first case when $\beta = 2$ (> 1) the entropy $H_\pi(t)$ in the course of time diminishes, which completely corresponds to the growth of the preferences distribution heterogeneity (see Fig. 5.1).

If we consider a change in the entropy as information, then in the first case, when decrease of entropy (decrease of uncertainty) occurs, subject *obtains* the information, which makes possible to increase the certainty of selection.

On the contrary, in the second case ($\beta = 0{,}5$) uncertainty (entropy) grows.

We will conditionally consider that the subject „loses" information. On Fig. 5.2 for the example in question are shown the values of information. In formula (5.46) the sign „+"can be accepted, since $\pi(\sigma_i, t) \geq 0$.

Actually the first two members in the criterion Φ_π represent itself the Kulbak "information" [151].

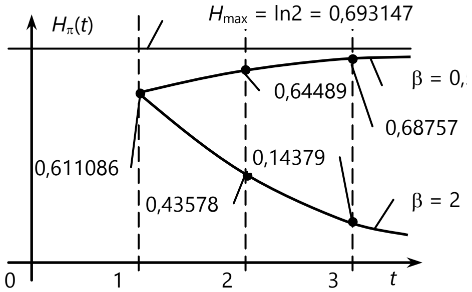


Fig.5.1

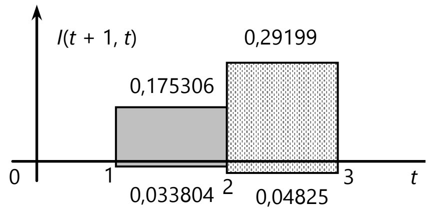


Fig.5.2

Let us examine the following functional:

$$\Phi_\pi(t+1,t)=\sum_{k=1}^{N}\Big[-\pi(\sigma_k,t+1)\ln\pi(\sigma_k,t+1)+\alpha\pi(\sigma_k,t+1)\ln\pi(\sigma_k,t)-$$

$$-\beta\pi(\sigma_k,t+1)\pi(\sigma_k,t)+\gamma\pi(\sigma_k,t+1)\Big]=$$

$$=\sum_{k=1}^{N}\pi(\sigma_k,t+1)\Big[-\ln\pi(\sigma_k,t+1)+\alpha\ln\pi(\sigma_k,t)-\beta\pi(\sigma_k,t)+\gamma\Big].$$

Hence we find optimum values $\pi(\sigma_i,\ t + 1)$

$$\pi(\sigma_k,t+1)=c_t\pi(\sigma_k,t)^{\alpha}\,e^{-\beta\pi(\sigma_k,t)}. \tag{5.47}$$

where $c_t=\sum_{j=1}^{N}\pi(\sigma_k,t)^{\alpha}\,e^{-\beta\pi(\sigma_k,t)}$.

It was shown earlier, that the extremum of distribution (5.47) is located at the point $\sigma_k^*=\frac{\alpha}{\beta}$.

Let us give the accountings, which show, now preferences change in the course of time in dependence on the numerical values of the parameters α and β. As an example, as it is above, let us consider the case, of two alternatives (N = 2), and designate $\pi(\sigma_i,\ t) = \pi_{it}$, then:

$$\pi_{1,t+1}=\frac{\pi_{1,t}^{\alpha}e^{-\beta\pi_{1,t}}}{\pi_{1,t}^{\alpha}e^{-\beta\pi_{1,t}}+\pi_{2,t}^{\alpha}e^{-\beta\pi_{2,t}}}=\frac{1}{1+\left(\dfrac{\pi_{2,t}}{\pi_{1,t}}\right)^{\alpha}e^{-\beta(\pi_{2,t}-\pi_{1,t})}}.$$

We can see, that independently on values α and β the initial uniform distribution $\pi_{1,0} = \pi_{2,0}$ = 0,5 remains with t > 0. Singularly distribution $\pi_{1,0}$ = 1, $\pi_{2,0}$ = 0, as distribution $\pi_{1,0}$ = 0, $\pi_{2,0}$ = 1 also remain.

Let $\pi_{1,0}$ = 0,7, $\pi_{2,0}$ = 0,3, α = 1, β = 1, then $\pi_{1,1}$ = 0,609997, $\pi_{2,1}$ = 0,390001 and respectively entropy

$$\left.\begin{aligned} H_{\pi,0} &= 0{,}61087 \\ H_{\pi,1} &= 0{,}66875 \end{aligned}\right\} \Rightarrow H_{\pi,1} > H_{\pi,0}.$$

With the selected values α and β the flattening of distribution and entropy increase occurs.

Let us assume, that with the same initial values $\pi_{1,0}$ and $\pi_{2,0}$ α = 2, β = 0,5. We find, that $\pi_{1,1}$ = 0,816767, $\pi_{2,1} = 1 - \pi_{1,1}$ = 0,183235 and $H_{\pi,1}$ = 0,476264. In this case the distribution tends to singular type, and entropy decreases. It we assume that α = 0,5, β = 2, then

$$\pi_{1,1} = \frac{1}{1\left(\dfrac{0{,}3}{0{,}7}\right)^{0{,}5} e^{-2(0{,}3-0{,}7)}} = 0{,}255499;$$

$$\pi_{2,1} = 1 - \pi_{1,1} = 0{,}744500.$$

A change in preferences and entropy is shown in tabele (with α = 0,5, β = 2):

t	$\pi_{1,\,t}$	$\pi_{2,\,t}$	$H_{\pi,\,t}$
0	0,7	0,3	0,61087
1	0,255499	0,744500	0,568299
2	0,60901	0,390988	0,669188
3	0,64873	0,341267	0,641873
4	0,72387	0,276128	0,589260

As we see, in this case the entropy changes no monotonically (Fig. 5.3).

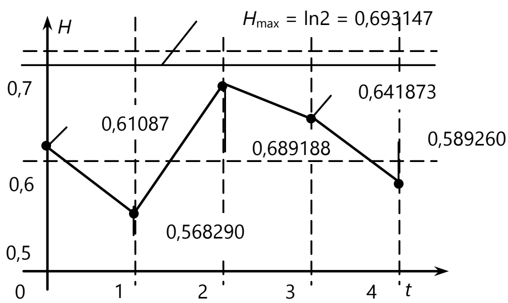


Fig. 5.3

It is possible to speak about stability or instability of the preferences distributions. So it is evident on the of base Fig. 5.1, that if β = 0,5 the uniform distribution with the maximum entropy is sTabele, while if β = 2 the singular distribution with zero entropy is sTabele.

Relationship (5.47) can be supposed to be as „differential equation" for preferences distributing on S_a, that expresses *a posteriori* preferences through *a priori*, that forms the *„heredity"* of preferences. Examined variational problems and canonical distributions do not consider the influence of exogenous factors and reflect only the nature of evolution in the times of the preferences distribution, depending on the endogenous parameters: β - in the first case and α and β - in the second.

Here the important point is the observation of stability of the preferences distributions in the Lyapunov sense depending on the endogenous parameters α, β,..., which can be conditionally called the „endogenous stability".

Apparently, this can be reflected in the theory of subjects' mental stability.

Let us continue the studies of the time-sequential routines of preferences but this time, however, taking into account the exogenous factors: the functions of utility $U(\sigma_i)$, the functions of harmfulness $L(\sigma_i)$, their quotients represented through the resources, or through the probabilities of achieving the result.

Let us select functional in the form:

$$\begin{aligned}\Phi_{\pi}^{+}\left(t+1,t\right)=-\sum_{i=1}^{N}\pi^{+}\left(\sigma_k,t+1\right)\ln\pi^{+}\left(\sigma_k,t+1\right)+\\ +\beta^{+}\sum_{k=1}^{N}\pi^{+}\left(\sigma_k,t+1\right)\pi^{+}\left(\sigma_k,t\right)+\delta^{+}\sum_{k=1}^{N}\pi^{+}\left(\sigma_k,t+1\right)\ln\pi^{+}\left(\sigma_k,t\right)+\\ +\gamma^{+}\sum_{k=1}^{N}\pi^{+}\left(\sigma_k,t+1\right)U\left(\sigma_k,t\right)+\eta^{+}\sum_{i=1}^{N}\pi^{+}\left(\sigma_k,t+1\right).\end{aligned} \tag{5.48}$$

Here, as we see, the functions of the absolute preferences $\pi^{+}\left(\sigma_k,t+1\right)$ and $\pi^{+}\left(\sigma_k,t\right)$ are used, the preferences distribution at the approaching moment t + 1 depend on the preferences distribution in the previous moment t and not depend on distributions at all previous moments T-1, t - 2,... Thus, in this sense the „Markov behavior" occurs. Besides the „heredity" of preferences functional reflects dependence on „absolute" utility U(σ_i, t + 1). The canonical preferences distribution, which corresponds to criterion (5.48) takes the form:

$$\pi^{+}\left(\sigma_k,t+1\right)=C_{t,t+1}^{+}\pi^{+}\left(\sigma_k,t\right)^{\beta^{+}}e^{\delta^{+}\pi^{+}\left(\sigma_k,t\right)}e^{\gamma^{+}U\left(\sigma_k,t+1\right)}, \tag{5.49}$$

where

$$C^{+}_{t,t+1}=\left(\sum_{j=1}^{N}\pi^{+}\left(\sigma_k,t\right)^{\beta^{+}}\ e^{\delta^{+}\pi^{+}\left(\sigma_k,t\right)+\gamma^{+}U\left(\sigma_k,t+1\right)}\right)^{-1}.\qquad(5.50)$$

A drawback in distribution (5.49) is the circumstance, that, if at a certain moment of time t the preference $\pi(\sigma_k,\ t)=0$, then subsequently $t+1$, $t+2$,... the preference of this alternative remains equal to zero.

From other side, if $\pi(\sigma_k,\ t)\neq 0$, then it subsequently can never be reduced to zero (e^{X} - certainly, if $x\neq-\infty$). The normalizing coefficient on each step has its new value. It is possible to obtain analogously the canonical distributions for π^{-}, υ^{+}, υ^{-} absolute preferences, that depends both on $U(\sigma_k,\ t+1)$, so on $L(\sigma_k,\ t+1)$.

It is natural to suppose that future distribution of $\pi^{+}(\sigma_k,\ t+1)$ type depends not on the previous distribution of the same type, i.e. from $\pi^{+}(\sigma_k,\ t)$ only, but also on the previous distributions of another type, for example, on $\pi^{-}(\sigma_k,\ t)$ or $\upsilon^{-}(\sigma_i,\ t)$.
In this case it is necessary to track in parallel the history of forming of these preferences in time and, therefore, if the distribution π^{-} is selected as factor influencing on π^{+}, then the following examined functional can be taken:

$$\begin{aligned}\Phi^{-}_{\pi}\left(t+1,t\right)=&-\sum_{k=1}^{N}\pi^{-}\left(\sigma_k,t+1\right)\ln\pi^{-}\left(\sigma_k,t+1\right)+\qquad(5.51)\\&+\beta^{-}\sum_{k=1}^{N}\pi^{-}\left(\sigma_k,t+1\right)\ln\pi^{-}\left(\sigma_k,t\right)+\delta^{-}\sum_{k=1}^{N}\pi^{-}\left(\sigma_k,t+1\right)\pi^{-}\left(\sigma_k,t\right)+\\&+\mu^{+}\sum_{k=1}^{N}\pi^{-}\left(\sigma_k,t+1\right)\ln\pi^{+}\left(\sigma_k,t\right)+\upsilon^{+}\sum_{k=1}^{N}\pi^{-}\left(\sigma_k,t+1\right)\pi^{+}\left(\sigma_k,t\right)-\\&-\gamma^{-}\sum_{k=1}^{N}\pi^{-}\left(\sigma_k,t+1\right)L\left(\sigma_k,t+1\right)+\eta^{-}\sum_{k=1}^{N}\pi^{-}\left(\sigma_k,t+1\right).\end{aligned}$$

The corresponding canonical distribution takes the form:

$$\pi^{-}\left(\sigma_k,t+1\right)=C^{-}_{t,t+1}\left(\pi^{-}\left(\sigma_k,t\right)\right)^{\beta^{-}}\left(\pi^{+}\left(\sigma_k,t\right)\right)^{\mu^{+}}\times e^{\delta^{-}\pi^{-}\left(\sigma_k,t\right)+\upsilon^{+}\pi^{+}\left(\sigma_k,t\right)}e^{-\gamma^{-}L\left(\sigma_k,t+1\right)}\qquad(5.52)$$

with the normalizing constant

$$\bar{C}_{t,t+1}=\left[\sum_{j=1}^{N}\left(\pi^{-}\left(\sigma_j,t\right)\right)^{\beta^{-}}\left(\pi^{+}\left(\sigma_j,t\right)\right)^{\mu^{+}}e^{\delta^{-}\pi\left(\sigma_j,t\right)+\upsilon^{+}\pi\left(\sigma_j,t\right)}e^{-\bar{\gamma}L\left(\sigma_j,t+1\right)}\right]^{-1}.$$

Additional terms, which contain $\pi^{-}(\sigma_k,\ t)$ must be included in functional (5.48), and distribution (5.49) will depend on $\pi^{-}(\sigma_k,\ t)$. In this case equations (5.49) and (5.52) form the system of two distributions $\pi^{+}(\sigma_k,\ t)$ and $\pi^{+}(\sigma_k,\ t)$. Thus, on each step two new distributions π^{+} and π^{-} are formed. In this case the information to be, both about the utility and about the harmfulness of alternatives is supposed. This process, as it is represented, can prove to be the useful model in the studies of the cognitive disoned development [79]. In more detailed way this problem is examined beside 5.6.

The different version appears, when with generation of the „*history*" of the preferences distribution to „accept" one or another alternative $\pi^+(\sigma_i, t)$, is supposed to be the preferences distribution „to reject" existing in the previous step: $\upsilon^-(L_i)$. We will not write here the appropriate functionals, let us simply notice that both obtained canonical distributions π^+ and υ^- maximize „their" appropriate entropies.

Abundance and unwieldiness of formulas must not, in our opinion, confuse the reader. The real manifestations of subject psyche are extremely complex and different. The author is convinced, that the given schemas for obtaining and analysis of preferences are the very simplified models of reality.

First of all they give the „guiding thread", which makes it possible to plan, to carry out sensible psychometric experiment and then to carry out quantitative processing of obtained data.

The previous formulations of variational problems were related to the distributions of the absolute preferences on S_a, i.e. such preferences, which do not depend (or depend very weakly) on the initial state of system σ_i in which it is found at the moment t.

Now we will reject this simplifying assumption.

Let us assume that at the moment t the distribution of the absolute preferences on S_a is known and all alternatives are inconsistent pairs.

Let us examine the functional

$$\Phi^+_{\pi_i} = -\sum_{j=1}^{N} \pi^+\left(\sigma_j, t+1 \mid \sigma_i, t\right) \ln \pi^+\left(\sigma_j, t+1 \mid \sigma_i, t\right) + \tag{5.53}$$

$$+\beta \sum_{j=1}^{N} \pi^+\left(\sigma_j, t+1 \mid \sigma_i, t\right) \ln \pi^+\left(\sigma_j, t\right) + \delta \sum_{j=1}^{N} \pi^+\left(\sigma_j, t+1 \mid \sigma_i, t\right) U\left(\sigma_j \mid \sigma_i\right) +$$

$$\gamma \sum_{j=1}^{N} \pi^+\left(\sigma_j, t+1 \mid \sigma_i, t\right).$$

Here $U(\sigma_j|\sigma_i)$ – the conditional utility of alternative σ_j when the system is in state σ_i.

The function of the distribution of the conditional preferences corresponding to criterion $\Phi^+_{\pi_i}$ is:

$$\pi^+\left(\sigma_j, t+1 \mid \sigma_i, t\right) = C^+_{t,t+1,i} \pi^+\left(\sigma_j, t\right)^{\beta} e^{\delta U\left(\sigma_j \mid \sigma_i\right)}, \tag{5.54}$$

with the normalizing constant

$$C^+_{t,t+1,i} = \left(\sum_{q=1}^{N} \pi^+ \left(\sigma_q, t\right)^\beta e^{\delta U\left(\sigma_q | \sigma_i\right)} \right)^{-1} .$$

This constant depends on index i. Distribution $\pi^+(\sigma_j|\ t)$, as it was already said, is supposed to be as known.

In order to determine a posteriori absolute distribution $\pi^+(\sigma_j|\ t + 1)$, it is possible to use a formula similar to the full probability formula. This formula is strictly valid, if alternatives from inconsistent pairs. In the subjective analysis its analog has a , most likely, approximate nature, since just rarely it is possible to isolate the complete group of mutually inconsistent alternatives and to construct set S_a as set of elementary alternatives.

Let us accept, nevertheless, that

$$\pi^+\left(\sigma_j, t+1\right) = \sum_{i=1}^{N} \pi^+\left(\sigma_i, t\right) \pi^+\left(\sigma_j, t+1 \mid \sigma_i, t\right) = \tag{5.55}$$

$$= \sum_{i=1}^{N} \pi^+\left(\sigma_i, t\right) \frac{\pi^+\left(\sigma_j, t\right)^\beta e^{\delta U\left(\sigma_j | \sigma_i\right)}}{\sum_{q=1}^{N} \pi^+\left(\sigma_q, t\right)^\beta e^{\delta U\left(\sigma_q | \sigma_i\right)}}.$$

We see that if at the moment t $\pi^+(\sigma_j, t) = 0$ that subsequently for $t + 1$ also $\pi^+(\sigma_j, t + 1) = 0$ and so forth - for all subsequent moments of time.

Somewhat different sense acquires the problem of obtaining the canonical preferences distribution of one-step ways $\pi^+(\sigma_i, \sigma_j)$, which under the assumption about the factorization possibility is represented in the form:

$$\pi^+(\sigma_i, \sigma_j) = \pi^+(\sigma_i)\pi^+(\sigma_j|\ \sigma_i). \tag{5.56}$$

Entropy in this case is given by the formula:

$$H^+_\pi = -\sum_{i=1}^{N} \sum_{j=1}^{N} \pi^+\left(\sigma_i, \sigma_j\right) \ln \pi^+\left(\sigma_i, \sigma_j\right). \tag{5.57}$$

Let us designate for the reduction $\pi^+(\sigma_i) = \pi^+_i$; $\pi^+(\sigma_i, \sigma_j) = \pi^+_{i,j}$; $\pi^+(\sigma_j|\ \sigma_i) = \pi^+_{j|i}$. It is easy to find the formula:

$$H^+_\pi = H^+\left(\pi^+_i\right) + H^+\left(\pi^+_{j|i}\right). \tag{5.58}$$

If state σ_i occurs at the moment t, and the state σ_j is assumed at the moment $t + 1$, the following criterion must lead to the selection of the best way $\sigma_i \to \sigma_j$ with the optimum selection both the „sending point" σ_i, and the „destination station" σ_j.

$$\Phi_{\pi}^{+}=-\sum_{i=1}^{N}\sum_{j=1}^{N}\pi_{i,j}^{+}\ln\pi_{i,j}^{+}+\beta\sum_{i=1}^{N}\sum_{j=1}^{N}\pi_{i,j}^{+}U_{i,j}^{+}+\gamma\sum_{i=1}^{N}\sum_{j=1}^{N}\pi_{i,j}^{+}. \tag{5.59}$$

Here $U(i, j)$ is the utility of way $\sigma_i \to \sigma_j$, that includes the utility of stay in the state σ_i and σ_j and also passage $\sigma_i \to \sigma_j$.

Obviously, there is a wide variety of functionals and canonical distributions, connected with the resources of different types. Study of other possible formulations - matter of future. Here we designated the way only, along which it is possible to continue the motion.

The following question is connected with the representation of the utility and harmfulness functions through the „speeds" of the resources conversion.

5.5.2. The canonical distributions of the I type preferences expressed through the „speeds" of the resources conversions

It is assumed that „physical" essence of alternatives is such that not only the complete required resources are determined, but also the required rates of the resources usage. It is possible to give many examples, when precisely the rate of the resources consumption, play the main role but not resources itself. Man must consume from 1800 to 2200 kcal in a 24 hour period - this is the necessary (required) rate of processing the resources consumption $V_R^{req}=\frac{dR^{req}}{dt}$. Let us also determine accurately the rate of the usage of the available resources $V_R^{disp}=\frac{dR^{disp}}{dt}$, the rate of the new resources obtaining $V_R^{\exp}=\frac{dR^{\exp}}{dt}$.

In this case forming his own preferences the subject is oriented not on the final volumes of resources, but on the „rate" of their consumption or expense. In the more overall meaning the discussion deals about the „rate" of benefit obtaining and speed of „losses" is the harm obtaining:

$$V_U^r=V_U^{req}=\frac{dU(\sigma)}{dt};\; V_L^r=V_L^{req}=\frac{dL(\sigma)}{dt}.$$

Variational problem can be formed as follows

$$\Phi=-\sum_{i=1}^{N}\pi(\sigma_i)\ln\pi(\sigma_i)+\beta\sum_{i=1}^{N}\pi(\sigma_i)V_U^r(\sigma_i)+\gamma\sum_{i=1}^{N}\pi(\sigma_i) \tag{5.60}$$

and then

$$\pi^{+}(\sigma_i)=\frac{e^{\beta V_U^r(\sigma_i)}}{\sum_{j=1}^{N}e^{\beta V_U^r(\sigma_j)}}. \tag{5.61}$$

Together with the minimum required speed — V_U^{req} of „benefit obtaining", and the maximum permissible speed of „harm obtaining" — $V_L^{permissible} = V_L^p$, the maximally possible (available) speeds of expenditure of the available resources, and also smallest possible speeds of new resources V_R^{exp} obtaining are introduced.

By analogy with (5.61), we will find:

$$\pi^-(\sigma_i) = \frac{e^{-\beta V_L^p(\sigma_i)}}{\sum_{j=1}^{N} e^{-\beta V_L^p(\sigma_i)}}. \tag{5.62}$$

Here it is possible to propose also the large number of the diverse variants of canonical distributions. Which of them are realized in actuality - this is the question, the psychometric experiment can give the answer.

The sense of the examination of a maximally broad spectrum of canonical distributions lies in the fact that they must be supposed to be as the objects of testing when the psychometric studies are conducting. The significance of the examined distributions class, which express preferences depending on „speeds" lies in the fact that, as it was said above, in the number of complex alternatives the compositions C^k are included, which foresee simultaneous „motion" in the direction of several simple alternatives.

Amount of the available resources may be insufficient, in order to cover „at the same moment" all required resources for the subset $S'_a \subset S_a$ of alternatives.

However, the parallel „supply" of resources (with determined speeds) in several directions can be organized taking into account the fact, that in the process of gradual resolution of several problems the available resources will be renewed from the specific moment due to the „return" – the running profit.

5.6. Models of the endogenous dynamics of the active systems

In accordance with the accepted in this work terminology, the endogenous dynamics of active systems, in narrower sense, the endogenous dynamics of preferences, is caused by the internal factors: ethical imperatives, influence of the previous preferences (retrospective), a change in the preferences, caused age of subject, by the mutual influence of subjects (the „crowd effect" - for example), a change in the endogenous parameters (α, β...).

Separate problem is the accounting of the interrelations between preferences distributions of the different types, including distributions of the type π^+, π^-, υ^+, υ^-, introduced above, detection of the corresponding entropies and information flows.

Within the framework of endogenous dynamics it is expedient to examine also the results, which follow from the hypothesis about the presence (objective existence) of the so-called „collective reason" (or „collective psyche").

This hypothesis is consonant with the theories, which consider associations not as the sum of individuals, but as the certain „living organism" (or „information essence"), which exists and is developed according to the laws different from the development laws of individual.

Here we will present the elements of the endogenous dynamics of preferences in a maximally simplified form, without going, where it is possible, into details, which do not influence on the clear idea of basic concept.

In point 5.5 we proposed the schemes for the accounting of „retrospective" for the preferences of the I type to one step „back". It is certainly possible to imagine that the retrospective is supposed to be on the deeper distance in the past, when for the distributions at the given moment $\pi(t + 1)$ are subjected with the influence of distributions at the moments of time t, $t - 1$, $t - 2$, ..., $t - k$, moreover this influence can be supposed to be with different weights. In 3.6 and 3.7 the discussion did not deal with the reciprocal effect of the distributions of different types π^+, π^-, υ^+, υ^-. Let us now begin examination with this question.

Assume that at the moment $t + 1$ positive preferences $\pi^+(\sigma_k, t + 1)$ depend not only on utilities $U(\sigma_k, t + 1)$, but also on a certain function F aggregating positive π^+ and negative π^- preferences, which occurred in the previous step, i.e. at the moment t. One of the possibilities of the of negative preferences calculation consists in the following: preferences $\pi^-(\sigma_k, t)$ have to be compared with the value of the indifference preferences of the $\pi^{\sim}\left(\sigma_k,t\right)=\dfrac{1}{N}$. Then function F is represented as function from $\pi^-(\sigma_k, t)$ and $\pi^{\sim}\left(\sigma_k,t\right)=\pi^{\sim}=\dfrac{1}{N}$:

$$F = F\left(\pi^-\left(\sigma_k,t\right),\pi^{\sim}\right).$$

Designation $F(.)$ used here, represent the function which not always can be identified as the effectiveness.

It must possess a sufficient sensitivity with respect to the difference $\pi^{-} - \pi^{\sim} = \pi^{-} - \frac{1}{N}$. In this case, if $\pi_k^{-} > \frac{1}{N}$ the accounting of preference π^{-}_k must lead to the decrease of the corresponding positive preference π^{+k}, vice versa if $\pi_k^{-} < \frac{1}{N}$, that accounting π^{-}_k either leads to an increase of π^{+k} or π^{-}_k is generally not supposed to be. Let us notice that the „threshold" for the negative preferences is not compulsorily equal $\pi^{\sim} = \frac{1}{N}$. It can swing for the different subjects in the limits from 0 to 1. Let us designate „threshold" through π^{-} and we will examine the function

$$F = F(\pi^{-}(\sigma_k, t), \pi^{-}).$$

In the particular case $\pi^{-} = \pi^{\sim} = \frac{1}{N}$. If $\pi^{-} = 0$, then any negative estimations are considered. Generally

$$0 \le \pi^{-} \le 1.$$

The process of subjective analysis can by represented in the form of the chain

$$\ldots \to \pi^{-}_{t} \to \pi^{+}_{t+1} \to \pi^{-}_{t+2} \to \ldots \tag{5.63}$$

It depends on the properties of individual psyche. With what estimation nega-tive or positive this process begins. The careful subject inclined to the thorough accounting of risks, negative consequences, dangers, will begin analysis With the estimation of negative preferences and will establish the lower threshold $\pi^{*-} < \frac{1}{N}$.

If first step in chain (5.63) is the estimation of negative preferences, they can be formed, according to general concept as the result of the functional extremalization

$$\Phi_{\pi^{-},t=1} = -\sum_{i=1}^{N} \pi^{-}\left(\sigma_k,1\right)\ln \pi^{-}\left(\sigma_k,1\right) - \tag{5.64}$$
$$-\beta^{-}\sum_{k=1}^{N} \pi^{-}\left(\sigma_k,1\right)F\left(\pi_k^{-},\pi^{*-}\right) - \gamma^{-}\sum_{k=1}^{N} \pi^{-}\left(\sigma_k,1\right)L\left(\sigma_k,1\right) + \delta^{-}\sum_{k=1}^{N} \pi^{-}\left(\sigma_k,1\right).$$

The second term in this formula is included formally, because function $F\left(\pi_k^{-},\pi^{*-}\right)$ possesses such property, that $F\left(\pi^{-},\pi^{*-}\right) = 0$.

Then

$$\pi^{-}(\sigma_k,1)=\frac{e^{\gamma^{-}L(\sigma_k,1)}}{\sum_{j=1}^{N}e^{\gamma^{-}L(\sigma_k,1)}} \tag{5.65}$$

and it is determined only through such „rational” function of harmfulness, as it is seemed to the subject at the moment t = 1.

Next step is the estimation of positive preferences $\pi^{+}(\sigma_k,\ t = 2)$. Here t is understood rather not as astronomical time, but as the „operating” time, related with the stages of analysis. It is possible to identify it with the stage number. Transition to „physical” time requires additional assumptions. For the determination $\pi^{+}(\sigma_k,\ 2)$ let us form the functional:

$$\Phi_{\pi^{+},t=2}=-\sum_{i=1}^{N}\pi^{+}(\sigma_k,2)\ln\pi^{+}(\sigma_k,2)-\beta^{+}\sum_{k=1}^{N}\pi^{+}(\sigma_k,2)F\left(\pi^{-}(\sigma_k,1),\pi^{*-}\right)+ \tag{5.66}$$
$$+\gamma^{+}\sum_{k=1}^{N}\pi^{+}(\sigma_k,2)U(\sigma_k,2)+\delta^{+}\sum_{k=1}^{N}\pi^{+}(\sigma_k,2).$$

For the function F the following is used:

$$\mathrm{F}=\varphi(\pi^{\pm})-\varphi(\pi^{*\pm}), \tag{5.67}$$

where $\varphi(\pi)\geq 0$. In particular it can be $\varphi(\pi)=\ln\pi$ or $\varphi=a\pi^{\alpha}$ ($a>0$, $\alpha\in R$) or $\varphi=ae^{\lambda\pi}$... Suppose that $\varphi(\pi)=\ln\pi$, then the canonical distribution $\pi^{+}(\sigma_k,\ 2)$ take the form (t = 2):

$$\pi^{+}(\sigma_k,2)=\frac{\left(\pi_k^{-}\right)^{-\beta}e^{\gamma U(\sigma_k,2)}}{\sum_{j=1}^{N}\left(\pi_j^{-}\right)^{-\beta}e^{\gamma U(\sigma_j,2)}} \tag{5.68}$$

In order to explain the sense of this result, let us examine an example.

There are be only two alternatives and

$$U(\sigma_1,\ 2)=U_1\ \ U(\sigma_2,\ 2)=U_2.$$

Let us designate $\pi^{+}(\sigma_k,\ 2)=\pi^{+}{}_k$, then for β = 1 and k = 1 we have:

$$\pi_1^{+}=\frac{\frac{1}{\pi_1^{-}}e^{\gamma U_1}}{\frac{1}{\pi_1^{-}}e^{\gamma U_1}+\frac{1}{\pi_2^{-}}e^{\gamma U_2}}=\frac{e^{\gamma U_1}}{e^{\gamma U_1}+\frac{\pi_1^{-}}{\pi_2^{-}}e^{\gamma U_2}}. \tag{5.69}$$

We see that if $\pi_1^- < \pi_2^-$, then то $\pi_1^+ > \pi^+_{10}$, where π^+_{10} — positive preference of alternative without taking into account negative preferences.

Let now $\varphi(\pi) = a\pi$. Then

$$F = a\left(\pi - \frac{1}{N}\right) \tag{5.70}$$

and the canonical function of preferences $\pi^+(\sigma_k, 2)$ takes the form:

$$\pi^+\left(\sigma_k, 2\right) = \frac{e^{-\beta a\left(\pi_k^- - \frac{1}{N}\right) + \gamma U\left(\sigma_k, 2\right)}}{\sum_{j=1}^{N} e^{-\beta a\left(\pi_j^- - \frac{1}{N}\right) + \gamma U\left(\sigma_j, 2\right)}} = \frac{e^{-\beta a\pi_k^- + \gamma U\left(\sigma_k, 2\right)}}{\sum_{j=1}^{N} e^{-\beta a\pi_j^- + \gamma U\left(\sigma_j, 2\right)}}. \tag{5.71}$$

We see that if all $\pi_k^- = \dfrac{1}{N}$, than $\pi^+(\sigma_k, 2)$ depends only on the available utility ($e^{-\beta a\left(\pi_j^- - \frac{1}{N}\right)} = 1$, $\forall j \in \overline{1, N}$), if $\pi_k^- > \dfrac{1}{N}$ then $\pi_k^+ < \pi^+_{k0}$.

Let us examine the case mentioned above, when negative preferences are supposed to be only, if $\pi_k^- > \dfrac{1}{N}$. This circumstance can be expressed formally after accepting the expression for the function F following expression.

$$F = \theta\left(\pi_k^- - \frac{1}{N}\right)\left(\varphi\left(\pi_k^-\right) - \varphi\left(\frac{1}{N}\right)\right), \tag{5.72}$$

where $\theta(.)$ — Heaviside's function:

$$\theta\left(x\right) = \begin{cases} 0, -\infty < x < 0; \\ 1, x \geq 0. \end{cases}$$

In this case let us find from normalization condition:

$$\pi^+\left(\sigma_k, 2\right) = \frac{e^{-\beta\theta\left(\pi_k^- - \pi^{*-}\right)\left[\varphi\left(\pi_k^-\right) - \varphi\left(\pi^{*-}\right)\right] + \gamma U\left(\sigma_k, 2\right)}}{\sum_{j=1}^{N} e^{-\beta\theta\left(\pi_j^- - \pi^{*-}\right)\left[\varphi\left(\pi_j^-\right) - \varphi\left(\pi^{*-}\right)\right] + \gamma U\left(\sigma_j, 2\right)}}. \tag{5.73}$$

On the third step of chain the influence of the previous positive preferences on the value of new negative preferences $\pi^-(\sigma_k, 3)$ ($t = 3$) is supposed to be analogously and so forth.

In actuality it is possible to assume that process is limited to 2 or 3 steps, along the chain (if in this time it is possible to lower entropy H^+_π and H^-_π to the values of the smaller threshold values H_π^{*+}, H_π^{*-}, which guarantee the possibility of solution acceptance). It is possible to show that in this case the ultimate result depends on estimations (preferences) which were the results of the first step: positive or negative. One more possibility of chain „organization" lies in the fact, that on each step the preferences $\pi^+(\sigma_k, t)$ and $\pi^-(\sigma_k, t)$ are compared and function F takes the form:

$$F = \varphi(\pi^-) - \varphi(\pi^+). \tag{5.74}$$

Canonical distribution is represented in the form

$$\pi^+\left(\sigma_k, t+1\right) = \frac{e^{-\beta\left[\varphi\left(\pi_k^-\right)-\varphi\left(\pi_k^+\right)\right]+\gamma U\left(\sigma_k, t+1\right)}}{\sum_{j=1}^{N} e^{-\beta\left[\varphi\left(\pi_j^-\right)-\varphi\left(\pi_j^+\right)\right]+\gamma U\left(\sigma_j, t+1\right)}}. \tag{5.75}$$

If, for example, $\varphi(\pi) = \ln\pi$, then

$$\pi^+\left(\sigma_k, t+1\right) = \frac{\left(\pi^+\left(\sigma_k, t\right)\right)^{\beta}\left(\pi^-\left(\sigma_k, t\right)\right)^{-\beta} e^{\gamma U\left(\sigma_k, t+1\right)}}{\sum_{j=1}^{N}\left(\pi^+\left(\sigma_j, t\right)\right)^{\beta}\left(\pi^-\left(\sigma_j, t\right)\right)^{-\beta} e^{\gamma U\left(\sigma_j, t+1\right)}}. \tag{5.76}$$

This distribution possesses the following properties: if $\pi^+(\sigma_j, t) = \pi^-(\sigma_j, t)$, ($\forall j \in \overline{1,N}$), then $\pi^+(\sigma_k, t)$ is determined on utilities only, existing at the given moment (t + 1). If moreover all utilities are identical, then every $\pi^+\left(\sigma_k, t+1\right) = \frac{1}{N}$, ($\forall k \in \overline{1,N}$). If $\pi^+(\sigma_j, t) > \pi^-(\sigma_j, t)$, then the calculation of negative preferences "strengthens" $\pi^+(\sigma_j, t+1)$, and vice versa, when $\pi^+(\sigma_j, t) < \pi^-(\sigma_j, t)$, calculation of $\pi^-(\sigma_j, t)$ "weakens" (decreases) preference $\pi^+(\sigma_j, t+1)$ for $\forall\beta$. Furthermore, it is obvious, that the calculation of previous π^+ and π^- leads down to the fact that the distribution of utilities in the retrospection is considered through them. A similar scheme can be realized for the preferences "to reject" alternative υ^+ and υ^-. In the particular case preference υ^+ or υ^- can be obtained as far as the conversion through the preferences π_k, by assuming for example:

$$\upsilon_k^+ = \frac{1}{N-1}\left(1-\pi_k^+\right). \tag{5.77}$$

It is seen that normalization condition $\sum_{k=1}^{N}\upsilon_k = 1$, is satisfied. However, the distribution function υ^+ (or υ^-) exists in the subjective sense independently and is not necessarily is rigidly connected π^+(or π^-), by the relationship (5.77) for example.

Then we should assume that there is a criterion Φ_υ, the result of extremalization of which is distribution υ^+ (or υ^-), and, correspondingly, the entropy of the alternatives elimination $H_{\upsilon+}$ or $H_{\upsilon-}$.

Without going into details of forming and sequential calculation of distributions υ^+ and υ^-:

$$\ldots \to \upsilon_t^- \to \upsilon^+_{t+1} \to \upsilon^-_{t+2} \to \ldots \tag{5.78}$$

notice that the possibility of realization of the mixed series

$$\ldots \to \pi_t^- \to \upsilon^+_{t+1} \to \pi^-_{t+2} \to \ldots \tag{5.79}$$

or similar is problematic.

The assumption that the series, composed with π^+, π^- and υ^+, υ^- are formed in parallel (without intersecting) appears more realistic, for example:

$$\left.\begin{array}{l} \ldots \to \pi_t^- \to \pi_{t+1}^+ \to \pi_{t+2}^- \to \ldots \\ \ldots \to \upsilon_t^- \to \upsilon_{t+1}^+ \to \upsilon_{t+2}^- \to \ldots \end{array}\right\} \tag{5.80}$$

and interaction is accomplished through the threshold values of the corresponding entropies and the threshold values of preferences.

We will follow, in the course of time the entropies $H^+_\pi(t)$, $H^-_\pi(t)$, $H^+_\upsilon(t)$, $H^-_\upsilon(t)$ alternative. Let for the assigned subject there be threshold values of entropies H_π^{+*}, H_π^{-*}, H_υ^{+*}, H_υ^{-*}.

It is assumed that, if the value of the running entropy at the determined moment of time becomes less than the corresponding threshold value, an abrupt change in the problem – resourced situation occurs: *an abrupt change of preferences distributing , the redistribution of resources; the event, for called above „the target selection", i.e., one or several alternatives and, correspondingly, the problems, to solution of which the resources are directed, the transfer of utilities, if subject is the member of group and so forth.* Most likely, there are specific threshold values of entropies for each of the enumerated events. Thus, for instance, the condition

$$H^+_\pi(t) < H_\pi^{+*}, \tag{5.81}$$

corresponds to the alternative acceptance as "the series", on the basis of positive analysis, whereas the condition

$$H^-_\pi(t) < H_\pi^{-*}, \tag{5.82}$$

corresponds to the solution in the form of a certain alternative acceptance on the basis of negative analysis.

Conditions

$$H^+_\upsilon(t) < H_\upsilon^{+*} \text{ and } H^-_\upsilon(t) < H_\upsilon^{-*} \tag{5.83}$$

Are „signals" about possible solution „to reelect" certain alternatives. Let us make a note that, if we work on the basis of assumption about extended multiple alternatives, then the latter means that the preference of the rejected alternative *becomes zero spasmodically*.

Let us make a note also that both in the case of the joint examination of conditions (5.81) and (5.82) and conditions (5.83) the most preferable (accepted as the chain) alternatives not coincide. Moreover, in the general case they almost for sure do not coincide. Necessity for finding a compromise appears.

Conditions (5.81), (5.82) and (5.83) do not solve the problem of selection on the set S_a, they only determine the moment, when this selection can occur. These conditions should be examined only as "necessary" conditions of decision making. Fulfilling of the inequalities, shown above, begins not simultaneously. We can assume that choice of the alternative as the target or rejection of alternative occurs on the basis of that distribution of preferences, for which the corresponding inequality is fulfilled earlier. In any case, analysis and the validity of the selection are not complete, if they are achieved by taking into account only positive or only negative estimations.

Let us examine the situation, shown on Fig. 5.4.

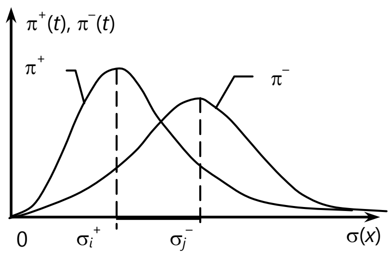


Fig. 5.4

In this case σ_i^+ is the best alternative on the basis of positive analysis, σ_j^- is the best alternative from the point of view of negative analysis.

Let in this case conditions (5.81) and (5.88) are satisfied. How in this case select trade-off settlement σ^*: $\sigma_i^+ < \sigma^* < \sigma_j^-$ can be selected? One of the possible schemes of the compromise uses the same method, which we used during aggregation of preferences in the group. Let for some subject $\xi_1 = \alpha$ is "rating" of the results of positive analysis, and $\xi_2 = 1 - \alpha$ is "rating" of the results of negative analysis.

Then we use the aggregated preference in the form

$$\pi(\sigma_k, t) = \alpha\pi^+(\sigma_k, t) + (1 - \alpha)\pi^-(\sigma_k, t). \quad (5.84)$$

Let for example, there are 3 alternatives: σ_1, σ_2, σ_3 and preferences $\pi^+(\sigma_k, t)$ and $\pi^-(\sigma_k, t)$ are shown in tabele, $\alpha = 0,3$; $1 - \alpha = 0,7$.

	σ_1	σ_2	σ_3	Σ
π^+	0,3	0,4	0,3	1,0
π^-	0,6	0,3	0,1	1,0
$\pi = 0,3\pi^+ + 0,7\pi^-$	0,51	0,33	0,16	1,0

Here as the compromise subject will select alternative σ_1.

Scheme of the gradual adaptation of preferences π^+ and π^- as a result of sequential analysis shown earlier differ from version (5.84).

Besides the entropy criterion in the form of given inequalities we should assume that in each case the preference of the best alternative, must also overcome the specific barriers, i.e., conditions must be fulfilled accordingly:

$$\pi^+(\sigma_k, t) > \pi^{+*};$$
$$\pi^-(\sigma_k, t) > \pi^{-*};$$
$$\upsilon^+(\sigma_k, t) > \upsilon^{+*};$$
$$\upsilon^-(\sigma_k, t) > \upsilon^{-*},$$

where values of π^*, υ^* are close to unity. It means that in a view of normalization conditions, the preferences of other alternatives will be close to the value of zero. The thresholds for the preferences mentioned earlier, as well as thresholds for the entropies can be considered as the characteristics of the subject individual psyche.

We will examine some models of the preferences of the first kind endogenous dynamics as an example. If we, as it was done, will identify σ_k with the quantitative exogenous characteristics of alternatives, then the equations of exogenous dynamics (5.10), or, equations of type (5.17) contain preferences $\pi(\sigma_k, \alpha, \beta, \ldots)$ on their right sides. The latter, in their turn, depend on the endogenous parameters α, β,... It we say about *model of endogenous dynamics* we have in mind the specific system of equations, which determines a change of these parameters in the course of time. "The common sense" prompts us, that in the majority of the cases the endogenous parameters vary slowly, that they can be linked, for example, with change in the priorities dependant on age, evolutionary change in the political persuasions, the ethical principles. The exception comprise the cases of the Force Majored situations, when radical solution is being made by subject itself, the cases related with the spontaneous instability of psyche. It is necessary to assume, that the endogenous structural parameters do not depend straightly on preferences. However, they for sure depend on the exogenous factors, which in view of equations of type (5.10), (5.17), in their turn, depend on preferences. Taking into account the aforesaid, it is convenient to give to the model of endogenous dynamics the form of the system of differential equations. It gives the possibility to vary easily the structure of the right sides of the equations, their initial conditions, to trace influence of the numerical values of the structural parameters on the course of solution.

For the canonical distribution, which depends on one endogenous parameter β:

$$\pi_i = \pi(\sigma_i) = \frac{e^{\pm\beta x_i}}{\sum_{j=1}^{N} e^{\pm\beta x_i}}$$

we have

$$\frac{d\pi_i}{dt} = \pm\beta\left(\dot{x}_i - \sum_{j=1}^{N}\dot{x}_j\pi_j\right)\pi_i \pm\dot{\beta}\left(x_i - \sum_{j=1}^{N}x_j\pi_j\right)\pi_i. \tag{5.85}$$

Exogenous variables are described by the same equations as have been used in model (5.17).

Let us assume that the parameter β changes in the course of time in accordance with the equation

$$\frac{d\beta}{dt} = h(\beta, x_1, \ldots, x_N, t). \tag{5.86}$$

Equations (5.85) ensure the fulfillment of normalization conditions and, in particular, condition $\sum_{i=1}^{N}\dot{\pi}_i = 0$ is identical. In case examined, the system of equations of the active system dynamics can be written down in the form:

$$\frac{d\pi_i}{dt} = \pm\pi_i\left[\left(x_i - \sum_{j=1}^{N}x_j\pi_j\right)h(\beta, x_1, \ldots, x_N, t) + \right. \tag{5.87}$$

$$\left. +\beta\left(f_i(x_1, \ldots, x_N, \ldots, \pi_1, \ldots, \pi_N, t) - \sum_{j=1}^{N}\pi_j f_j(x_1, \ldots, x_N, \ldots, \pi_1, \ldots, \pi_N, t)\right)\right];$$

$$\frac{dx_i}{dt} = f_i(x_1, \ldots, x_N, \ldots, \pi_1, \ldots, \pi_N, t); \tag{5.88}$$

$$\frac{d\beta}{dt} = h(\beta, x_1, \ldots, x_N, t). \tag{5.89}$$

For the canonical distribution, which depends on two endogenous parameters α and β:

$$\pi_i = \pi(\sigma_i) = \frac{x_i^{\alpha} e^{\pm\beta x_i}}{\sum_{j=1}^{N} x_i^{\alpha} e^{\pm\beta x_i}}$$

we find the following equation:

$$\frac{d\pi_i}{dt} = \pi_i \left[\left(\ln x_i - \sum_{j=1}^{N} \ln x_j \pi_j \right) \dot{\alpha} + \left(\dot{x}_i x_i^{-1} - \sum_{j=1}^{N} \dot{x}_j x_j^{-1} \pi_j \right) \alpha - \right.$$

$$\left. \pm \left(x_i - \sum_{j=1}^{N} x_j \pi_j \right) \dot{\beta} \pm \left(\dot{x}_i - \sum_{i=1}^{N} \dot{x}_j \pi_j \right) \beta \right] (x_i > 0).$$

Let us write down the system of equations describing the active system dynamic and taking into account time dependence of the endogenous parameters (in this case α and β), in the form:

$$\frac{d\pi_i}{dt} = \pi_i \left[\left(\ln x_i - \sum_{j=1}^{N} \ln x_j \pi_j \right) g\left(\alpha, \beta, x_1, \ldots, x_N, t\right) \pm \right. \tag{5.90}$$

$$\pm \left(x_i - \sum_{j=1}^{N} x_j \pi_j \right) h\left(\alpha, \beta, x_1, \ldots, x_N, t\right) + \left(x_i^{-1} \alpha \pm \beta \right) f_j \left(x_1, \ldots, x_N, \ldots, \pi_1, \ldots, \pi_N, t \right) -$$

$$\left. - \sum_{j=1}^{N} \left(x_i^{-1} \alpha \pm \beta \right) \pi_j f_j \left(x_1, \ldots, x_N, \ldots, \pi_1, \ldots, \pi_N, t \right) \right];$$

$$\frac{dx_i}{dt} = f_i \left(x_1, \ldots, x_N, \ldots, \pi_1, \ldots, \pi_N, t \right); \tag{5.91}$$

$$\frac{d\alpha}{dt} = g\left(\alpha, \beta, x_1, \ldots, x_N, t\right); \tag{5.92}$$

$$\frac{d\beta}{dt} = h\left(\alpha, \beta, x_1, \ldots, x_N, t\right). \tag{5.93}$$

In the particular case where $N = 2$: $Sa \to (\sigma_1, \sigma_2)$ this system of equations takes the form:

$$\frac{d\pi_1}{dt} = \pi_1 \pi_2 \left[\left(\ln x_2 - \ln x_1 \right) g + \left(x_1 - x_2 \right) h - \left(x_2^{-1} \alpha \pm \beta \right) f_2 + \left(x_1^{-1} \alpha \pm \beta \right) f_1 \right];$$

$$\pi_2 = 1 - \pi_1;$$

$$\frac{dx_1}{dt} = f_1 \left(x_1, x_2, \pi_1, \pi_2, t \right); \quad \frac{dx_2}{dt} = f_2 \left(x_1, x_2, \pi_1, \pi_2, t \right);$$

$$\frac{d\alpha}{dt} = g\left(\alpha, \beta, x_1, x_2, t\right); \quad \frac{d\beta}{dt} = h\left(\alpha, \beta, x_1, x_2, t\right).$$

Both for the exogenous parameters x_i and endogenous parameters other models can be used. For example, α and β could be the coordinates of attractors, in particular, Lorenz's attractors, which would make it possible to simulate the excited states of psyche.

The possibility of the fact that the dynamics of the endogenous parameters can depend on individual characteristics, subjective entropy, correlation coefficients of preferences is not excluded. Change α and β within the limits $[0, +\infty)$ can lead to the uniform distribution with the maximum entropy, or to the singular distribution, when only preference is equal to 1, and every other are equal to zero.

If $\alpha \to 0$ and $\beta \to 0$, then distribution $\pi(\sigma_i)$ strives for uniform, i.e., to the state of complete indifference - set S_a turns into one class of equivalence, if $\alpha \to 0$, $\beta \to \infty$ or $\alpha \to \infty$, $\beta \to 0$, then distribution $\pi(\sigma_i)$ with the condition that not all x_i are equal to each other, when not all x_i are equal to each other aims into singularity, when all preferences except one are equal to zero, and only value of this is equal to one. If all x_i are identical exactly, distribution is uniform; however, it is unsTabele with respect to any small disturbance of equality x_i to each other.

The structural parameters α, β, ..., in accordance with the accepted hypothesis about the genesis of canonical preferences from the variation principle, we have right to consider them as the characteristics of psyche. As it was already said, in the calm conditions these parameters experience the slow temporary drift, which is caused by the changes dependant on age, or the spontaneous changes as a result of the action of such factors as “fatigue”, “habituation”, “forgetting” and others.

In other circumstances “explosive” changes in the structural parameters are possible, which lead down to bifurcation type phenomena.

If external (exogenous) situation is stable, which conditions are possible to express in the form x_i = const, then from equations (5.90) - (5.93) shown above result the equations of endogenous dynamics strictly, i.e., the dynamics of preferences, caused by a change of the exceptionally endogenous factors. Let us include in the number of arguments of function $\pi(\sigma_i)$ additionally to the already said, the quantitative measures of ethical imperatives $\pi(I_s)$.

Let $\pi(\sigma_i) = \pi(x_i, \alpha, \beta, \ldots, \pi(I_s), \ldots)$; $I_s \in S_I^L$. Exogenous measure can realize itself as the function of resources, utility,..., for example

$$x_i = \frac{\bar{r}_i}{1-\bar{r}_i}; \quad \bar{r}_i = \frac{R^{req}\left(\sigma_i\right)}{R^{disp}\left(\sigma_i\right)};$$

In the examined models of preferences there is no direct dependence on the uncertainty level, tension of conflict and other general characteristics of situation; however, their mediated influence through the dependence of the endogenous parameters on them cannot be excluded.

Assume that the exogenous parameters x_i are fixed, then

$$\frac{d\pi(\sigma_i)}{dt} = \frac{\partial\pi(\sigma_i)}{\partial\alpha}\frac{d\alpha}{dt} + \frac{\partial\pi(\sigma_i)}{\partial\beta}\frac{d\beta}{dt} + \ldots + \frac{\partial\pi(\sigma_i)}{\partial\pi(I_s)}\frac{d\pi(I_s)}{dt} + \ldots \tag{5.94}$$

If subject has "solid persuasions", then one should consider that $\pi(I_s)$ = const. In the conditions of intensive manipulation of consciousness, as "target" precisely "sTabele" imperatives appear (ethical, political, and cultural). Leaving this theme for future reference, let us assume first, that

$$\frac{d\pi(I_s)}{dt} = 0.$$

Based on (5.90) - (5.94) we will obtain the version of the endogenous dynamics model, not considering the effects, related to the change of "sTabele imperatives":

$$\frac{d\pi_i}{dt} = \pi_i\left[\left(\ln x_i - \sum_{j=1}^{N}\ln x_j\pi_j\right)g(\alpha,\beta,x_1,\ldots,x_N,t) \pm\right. \tag{5.95}$$

$$\left.\pm\left(x_i - \sum_{j=1}^{N}x_j\pi_j\right)h(\alpha,\beta,x_1,\ldots,x_N,t)\right];$$

$$\frac{d\alpha}{dt} = g(\alpha,\beta,x_1,\ldots,x_N,t); \tag{5.96}$$

$$\frac{d\beta}{dt} = h(\alpha,\beta,x_1,\ldots,x_N,t). \tag{5.97}$$

Equations (5.96), (5.97) as well as analogous equations in the previous models, bear phenomenological nature. At the present instant there is no possibility to propose any sufficiently general principle, which would make it possible to establish the substantiated structure of these equations. However, on the qualitative level it is possible to give the specific interpretation for the structural parameters. Thus, if sign "-" is accepted β in the expression for $\pi(\sigma_i)$, then the parameter β "answers" for the diminishing nature of preference with an increase "of the cost" x_i and, therefore, the predominance "of caution" above the prestige considerations. The parameter α, on the contrary, determines the increasing dependence π_i on x_i and the prestigious considerations. A compromise of two opposing tendencies, is expressed in the fact that the function $\pi_i(\alpha, \beta)$ has a maximum with $x_i^* = \frac{\alpha}{\beta}$. In such a case, if in the meaningful sense alternatives $\sigma_i \in S_a$ are related with the subject safety and the corresponding expenditures, then the parameters α and β reflect the competitive requirements of safety and savings.

If it is necessary to consider the influence of a change of the ethical imperatives (more precise - their significances $\pi(I_s)$), then instead of equation (5.95) one should use the equation:

$$\frac{d\pi_i}{dt} = \pi_i\left[\left(\ln x_i - \sum_{j=1}^{N}\ln x_j\pi_j\right)g \pm \left(x_i - \sum_{j=1}^{N}x_j\pi_j\right)h + \frac{\partial\pi_i}{\partial\pi(I_s)}\frac{d\pi(I_s)}{dt}\right]. \qquad (5.98)$$

We will examine some possible models, which determine change of the endogenous variables α, β,... The linear model is simplest:

$$\left.\begin{aligned}\frac{d\alpha}{dt} &= a_{11}\alpha + a_{12}\beta + v(x_1,\ldots,x_N,t),\\ \frac{d\beta}{dt} &= a_{21}\alpha + a_{22}\beta + w(x_1,\ldots,x_N,t).\end{aligned}\right\} \qquad (5.99)$$

The solution of the corresponding uniform system can be oscillatory, or aperiodic depending on eigenvalues of matrix

$$M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.$$

Influence of the external conditions is concentrated here in the forcing functions v and w. If we consider v and w as " disturbing forces", then by analogy with physics it is possible to expect the appearance of "beating" or "resonance" regimes.

Most simple model (5.99), using together with the equations for the preferences, makes possible to model a number of situations on the qualitative level. Thus, it is possible to imagine that an increase in the prestige of more expensive automobiles or the dwelling as the response of the individual psyche will entail an increase in the relation $\frac{\alpha}{\beta}$.

If the solution $\alpha(t) = 0$, $\beta(t) = 0$ is asymptotically stable, then with damping v and w, the distribution of preferences strives to the uniform one (but not to the indifference), which is characterized by the increased mental stress.

However, the possibilities of linear model are limited. The manifestations of psyche have clearly nonlinear nature; therefore it is natural to turn to the nonlinear mathematical objects as the tool of simulation. Such objects, in particular, are attractors, for example, well known "Lorenz attractor" and "Brusselator". Without going in the theoretical analysis of attractors and their classification [128] let us demonstrate basing on particular examples, as it is possible to use attractors for obtaining the number of the characteristic solutions, which can be applicable to the simulation of a change in the preferences.

Let us assume that the parameters α and β change in accordance with the equations

$$\left.\begin{aligned}\frac{dx_0}{dt} &= -a_1(t)x_0 + x_0x_1;\\ \frac{dx_1}{dt} &= -a_2(t)x_1 - b(t)x_0^2.\end{aligned}\right\} \tag{5.100}$$

Designations introduced here are $\alpha = x_0$; $\beta = x_1$. System of equations (5.100) is so-called "brusselator" [128]. In contrast to the standard form in equations (5.100) the dependence of the coefficients *of* a_1 and a_2 on the time is added and coefficient $b(t)$ is introduced. It is possible to simulate behavior $x_0(t)$ and $x_1(t)$ with the slow time drift of coefficients. Let us assume, that there are two alternatives: $N = 2$ and we designate $\pi(\sigma_1) = x_2$, $\pi(\sigma_2) = x_3$.

Equations for the preferences take the form:

$$\frac{dx_2}{dt} = x_2x_3\left[\left(\ln\bar{r}_2 - \ln\bar{r}_1\right)\left(-a_2x_1 - bx_0^2\right) + \left(\bar{r}_2 - \bar{r}_1\right)\left(-a_1x_0 + x_0x_1\right)\right]; \tag{5.101}$$

$$\frac{dx_3}{dt} = x_2x_3\left[\left(\ln\bar{r}_1 - \ln\bar{r}_2\right)\left(-a_2x_1 - bx_0^2\right) + \left(\bar{r}_1 - \bar{r}_2\right)\left(-a_1x_0 + x_0x_1\right)\right].$$

Assuming, for example, that $a_1 = 1 + \sin kt$; $k = 20$; $a_2 = 0{,}1$; $\bar{r}_1 = 0{,}01$; $\bar{r}_2 = 0{,}03$ and the initial conditions of $x_0 = 3$; $x_1 = 1$; $x_2 = x_3 = 0{,}5$ (state of equivalence), we obtain the solution, shown on Fig. 5.5.

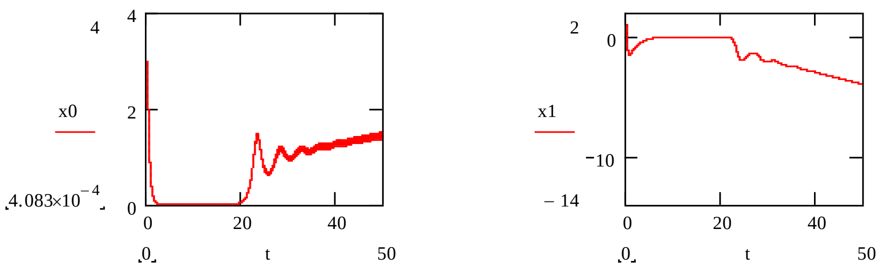


Fig. 5.5

We perceive from Fig. 5.5, that the bifurcation of the structural parameters α and β leads to the bifurcation of preferences.

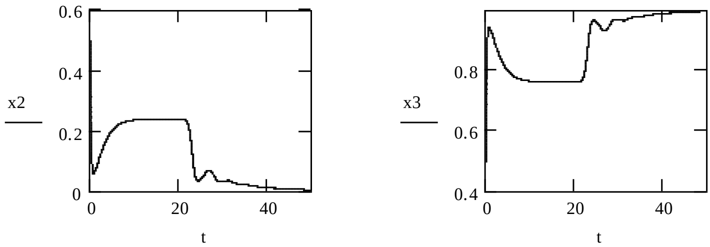


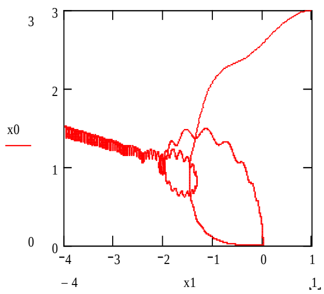
Fig. 5.6

Fig. 5.6 presents the phase portrait of variables $\alpha = x_0$ and $\beta = x_1$ is.

It follows, from the graph of the entropy of the preferences

$$H_\pi = -(x_2 \ln x_2 + x_3 \ln x_3)$$

that the entropy spasmodically changes in the region of $t = 20$, after which it decreases.



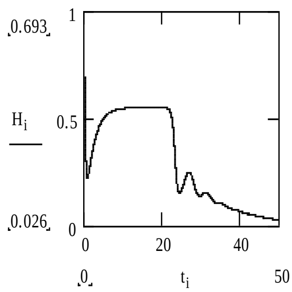


Fig. 5.7

Fig. 5.8

Perform the inversion of the structural parameters and interchange the position of α and β: $\alpha = x_1$, $\beta = x_0$.

Let now the model for α and β takes the form:

$$\frac{dx_1}{dt} = -a_1(t)x_1 + x_0 x_1; \tag{5.102}$$

$$\frac{dx_0}{dt} = -a_2 x_0 - b(t)x_1^2. \tag{5.103}$$

It is obvious that, since the first two equations compose the independent system, it would be possible to use the solution x_0, x_1 directly for enumerating the preferences $\pi(\sigma_1) = x_2$ and $\pi(\sigma_2) = x_3$ on the initial formulas. This becomes impossible, if in equations (5.102), (5.103) the preferences x_2 and x_3 are „mixed" in any manner.

Let us make a note that, of course, it would be possible to determine one of the preferences on normalization condition, for example,

$$x_3 = 1 - x_2,$$

however, in the represented form the system is symmetrical and with quantity of alternatives larger than $N = 2$, this simplification is unessential…

Let us assume: $a_1(\mathrm{t}) = 1 - 0{,}1t + 3\sin(k,\ t)$; $b(t) = 1 + 0{,}3a_1(t)$; $\bar{r}_1 = 0{,}01$; $\bar{r}_2 = 0{,}03$; $a_2 = 0{,}05$, $k = 20$.

The solution is represented on Fig.5.9, *a–e*. It is seen, that in the region t = 200 the significant fluctuations of preferences occur. This zone can be considered as the zone of uncertainty, in which subject "decision making" by subject is, apparently, excluded.

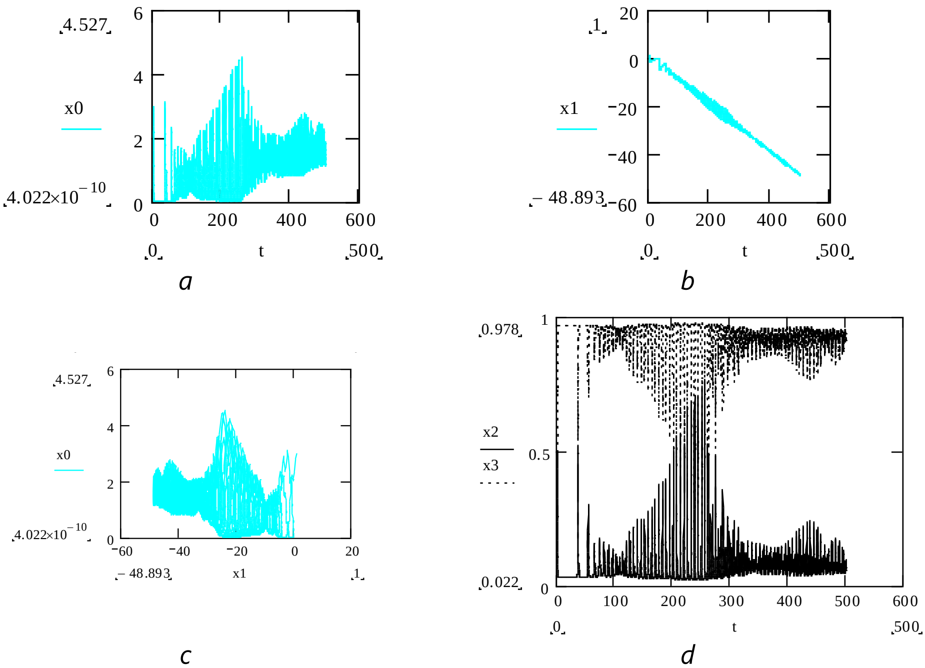


Fig. 5.9

The attraction of different mathematical objects as the models of endogenous dynamics can be seemed arbitrary. However, in view of the absence of reliable models on the macro level, it is permissible to use them in such way, when macro-models are selected a priori, and the results of simulation are compared with the observed phenomena. With the presence of qualitative agreement there always is the possibility to provide a sufficient similarity by variation of the internal (structural) parameters of the model, also in the quantitative level. We already used this way, when we postulated "built-in" the psyche variation principle. With respect to the endogenous parameters of canonical distributions (of type α and β) we cannot postulate any qualitative principle thus far. This principle for sure exists and it is most likely, variational, but it is hidden until a certain time, and situation does not leave another selection, except purely empirical search.

Therefore we move in this region "on by feel" relying on the common sense, qualitative reasoning's and hope for the obtainment of the results, which can be reasonably interpreted.

In the book [128] authors give the following considerations: *In all examined examples analysis is achieved in two stages. At first the specific analogies between the observations and the behavior of the "standard"... system are supposed. This makes it possible to establish the model type, which appears the most adequate for representation of the examined system. Next, the attempt to exceed the limits of simple analogies is made, in order to establish within the framework of chosen model the specific special features of each task and then to include them in the description of system. In the end, the results of this analysis are compared with the experience, and if, qualitative agreement is obtained, then they are used for further predictions*".

From the authors of [128] point of view in many instances the arena "of antagonism" and certainly, "of compromise" the chance appears on the "microlevel" inaccessible for the mathematical description, but ordering is assumed on the "macrolevel". "*The necessary prerequisite of all these phenomena serves nonlinear dynamics, which under specific conditions leads to the instability of motion, bifurcations*".

In the book [128] the "macro- model" is given, which describes the fight of immune system with the population of malignant cells x (t):

$$\frac{dx}{dt} = \lambda x\left(1 - \frac{x}{N}\right) - k_1 E_0 x; \tag{5.104}$$

$$\frac{dE_0}{dt} = -k_1 E_0 x + k_2 E_1.$$

Here E_0 - the density of the free cytotoxic cells, part of them encounters with the malignant cells, whose population x forms the complexes E_0x. The number of such "encounters" is proportional to the number E_0 and x, k_1 —coefficient determined by the probability of encounter (complex formation). As a result of complex disintegration cancerous cell is destroyed and retires from the population x.

The intensity of the cancerous cells decomposition is proportional to the product of the number of already existing population and the number of healthy cells on the same volume. Equations are similar to the equations, which describe advertizing campaign.

Let us try to interpret equations (5.104) as the model of the election campaign. Let $\alpha = \mu x$, where μ = const, a α — endogenous parameter in the distribution

$$\pi\left(\sigma_i\right) = \frac{Z_i^{\alpha} e^{-\beta Z_i}}{\sum_{j=1}^{N} Z_j^{\alpha} e^{-\beta Z_j}}, \tag{5.105}$$

where Z_i — specific quantitative model of alternative $\sigma_i \in S_a$. "Agitator" would want to modify the distribution of preferences on S_a. It has two ways:

1. To change the distribution of measure Z_i.

2. To influence on the endogenous parameters α and β by the means of suggestion ("consciousness manipulation"). An increase of α will bring displacement of the preferences peak to the right ([Fig.5.10).

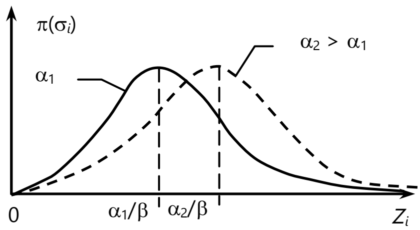


Fig. 5.10

Using the second method "agitator" will pack his resources in other to increase the parameter λ (probability of the encounter of "inverted" and "not inverted") and decrease of the parameter k_1. "Agitator" works in the opposition conditions consideration, when E_0 — number of "anti-agitation" acts, the condition when the sum of successful (E_0) and unsuccessful (E_1) acts $E_t = E_0 + E_1$ remains constant, for example, E_t - total number of copies of "anti-agitational" literature. It is possible, of course, to assume that E_t changes in time and look, , how in this case the fight for the life of "patient" will occur.

It is possible to imagine the reverse scheme, when x and E_0 are interpreted as the number of population, and "probabilities" λ and k depend on the distribution of preferences. Then equations (5.104) pass into discharge digit of the exogenous dynamics equations. Another nonlinear object, which can be used in the course of simulation of endogenous dynamics, is Lorenz's attractor.

The corresponding equations take the form:

$$\left.\begin{aligned}\frac{dQ_0}{dt} &= aQ_1 - bQ_0;\\ \frac{dQ_1}{dt} &= -Q_1 - Q_0Q_2 + cQ_1;\\ \frac{dQ_2}{dt} &= Q_0Q_1 - dQ_2.\end{aligned}\right\} \quad (5.106)$$

Here Q_0, Q_1, Q_2 — attractor's coordinates.

Below, the modified model will be used, which contains damping terms

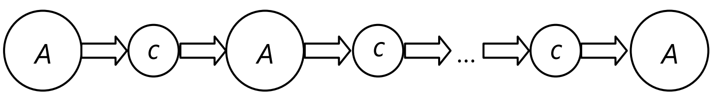
$$\left.\begin{aligned}\frac{dQ_0}{dt} &= aQ_1 - bQ_0 - hQ_0^2 + f(t);\\ \frac{dQ_1}{dt} &= -Q_1 - Q_0Q_2 + cQ_0 - mQ_1^2;\\ \frac{dQ_2}{dt} &= Q_0Q_1 - dQ_2 - nQ_2^2.\end{aligned}\right\} \quad (5.107)$$

Values *a*, *b*, *c*, *d*, *h*, *m*, *n* are considered constants, *f* (*t*) - external forcing action.

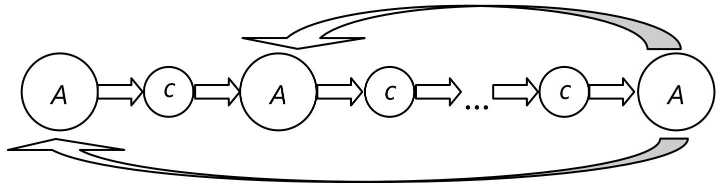
The possibilities of simulation of the endogenous dynamics can be enlarged, if we will use the structures, composed from the block-attractors as the models, linked in the specific order. As "kinematic" connections between the blocks, the linear links *of n-th* order can be used. Universal branched hierarchic structure, with feedback could be sufficient. Fig. 5.11 shows some possible structures, "collected" with the attractors and connecting links.

On Fig. 5.11, *a* we have the linear network of the attractors, connected by "kinematic" links *s.c.*In the case of (*b*) there are feedbacks in the linear chain. Fig. 5.11, *c* shows the fragment of the hierarchical system of attractors where, there can be "direct" interactions between the attractors at each level.

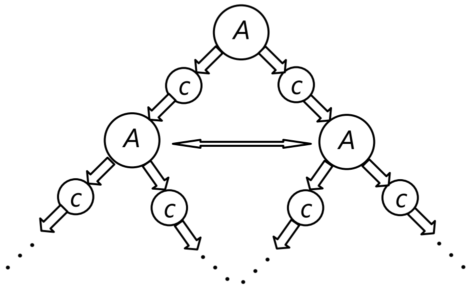
The attractors, composing the structure so the coupling links can certainly be individualized, relative to the numerical values of their structural parameters. Being included in the system, they preserve their characteristic individual dynamic properties . It opens the possibility to form deliberately desired effects.



a



b



c

Fig. 5.11

In this case the system of equations take the form:

$$D(t,Q)=\begin{bmatrix} aQ_1-bQ_0+f\sin(lt)-hQ_0^2; \\ -Q_1-Q_0Q_2+cQ_0-mQ_1^2; \\ Q_0Q_1-dQ_2-nQ_2^2; \\ Q_4; \\ \left(Q_2-2qQ_4-k^2Q_3\right)e^{-\left(\frac{\alpha}{t+\varepsilon}\right)^{\zeta}}; \\ aQ_6-bQ_5+f\sin(lt)-hQ_5^2+sQ_4; \\ -Q_6-Q_5Q_7+cQ_5-mQ_6^2; \\ Q_5Q_6-dQ_7-nQ_7^2; \\ Q_9; \\ \left(Q_7-2qQ_9-k^2Q_8\right)e^{-\left(\frac{\beta}{t+\varepsilon}\right)^{\zeta}}; \\ aQ_{11}-bQ_{10}+f\sin(lt)-hQ_{10}^2+rQ_9; \\ -Q_{11}-Q_{10}Q_{12}+cQ_{10}-mQ_{11}^2; \\ Q_{10}Q_{11}-dQ_{12}-nQ_{12}^2. \end{bmatrix} \qquad Q\big|_{t=0}=\begin{pmatrix} 1\cdot 10^{-9} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{5.108}$$

a = 8; b = 8; c = 24; d = 0,43; f = 0; h = 0,015; m = 0,015; n = 0,015;

q = 0,1; k = 0,3; s = 2; r = 2; α = 200; β = 400; ε = 0,0001; ζ = 4. Let us examine as an example the dynamics of the system, which consists of three series-connected Lorenz attractors with linear components of the second order (n = 2) as “kinematic” connections (Fig. 5.12).

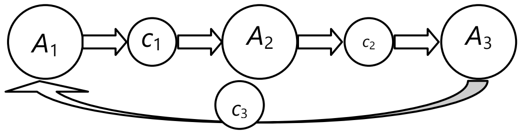


Fig. 5.12

In this system in the "kinematic" connections equations, the exponential functions are inserted as multipliers, what makes possible to simulate the time delays (lags). By the selection of the α and β parameters magnitudes, it is possible to regulate the value of lags. Initial conditions — are practically zero: only one coordinate Q_0 differs negligibly small from zero.

As we see from the graphs on Fig. 5.13 attractors present itself as the self-excited system. Different time delays (lags) for the excitation moments of the second and third attractors occur. External excitation was relied by zero (f = 0). Connection between the attractors is realized by inclusion of additive term in the first equation of the second and third attractors (sQ_4, rQ_9). Damping of the excitation is regulated by the value of the damping terms.

On the other side, Fig. 5.14 shows two-dimensional phase portraits from which it is seen that the attractors behave practically identically. The following variant where the feedback of the last attractor with the first one is realized, shown on Fig. 5.12 c_3. The corresponding system of equations takes the form:

$$D(t,Q)=\begin{bmatrix} aQ_1-bQ_0+f\sin(lt)-hQ_0^2+\pi Q_{12}e^{-\left(\frac{\gamma}{t+\varepsilon}\right)^{\xi}}; \\ -Q_1-Q_0Q_2+cQ_0-mQ_1^2; \\ Q_0Q_1-dQ_2-nQ_2^2; \\ Q_4; \\ Q_2-2qQ_4-k^2Q_3; \\ aQ_6-bQ_5+f\sin(lt)-hQ_5^2+sQ_4e^{-\left(\frac{\alpha}{t+\varepsilon}\right)^{\zeta}\cdot 10^2}; \\ -Q_6-Q_5Q_7+cQ_5-mQ_6^2; \\ Q_5Q_6-dQ_7-nQ_7^2; \\ Q_9; \\ Q_7-2qQ_9-k^2Q_8; \\ aQ_{11}-bQ_{10}+f\sin(lt)-hQ_{10}^2+rQ_9e^{-\left(\frac{\beta}{t+\varepsilon}\right)^{\eta}\cdot 10^2}; \\ -Q_{11}-Q_{10}Q_{12}+cQ_{10}-mQ_{11}^2; \\ Q_{10}Q_{11}-dQ_{12}-nQ_{12}^2; \end{bmatrix} \qquad Q\,|_{t=0}=\begin{pmatrix} 1\cdot 10^{-15} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (5.109)$$

a = 8; b = 8; c = 28; d = 0,43; f = 0; h = 0,016; m = 0,016; n = 0,016; q = 0,01;

k = 0,3; s = 0,01; r = 0,01; α = 100; β = 200; ε = 0,0001; ζ = 2; η = 3; π = 15.

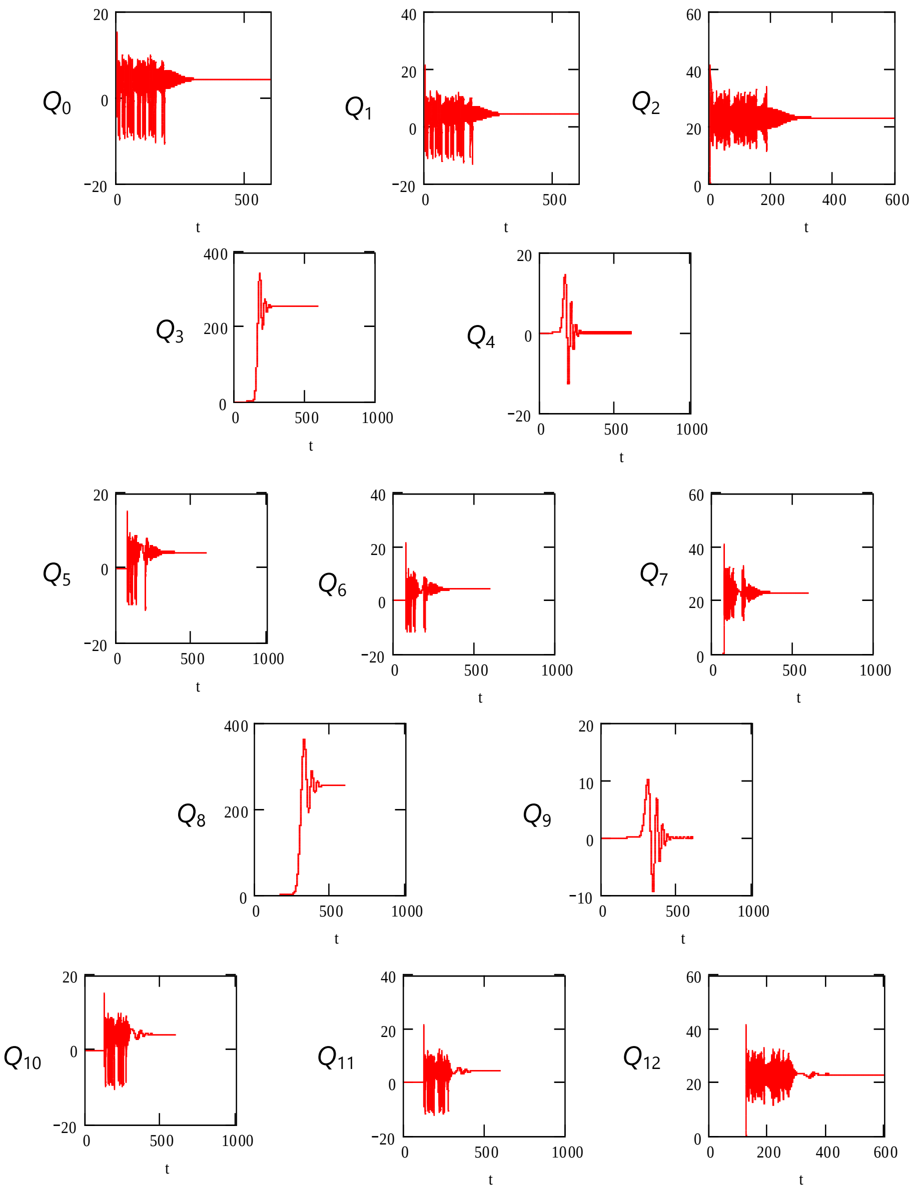


Fig. 5.13

Feedback is ensured by last term in the first equation. Difference from the previous model consists also in this, that the multipliers which create delays act not on "kinematic" connections, i.e., on the rate of change of components these connections, but directly on the components contained in the attractors equations.

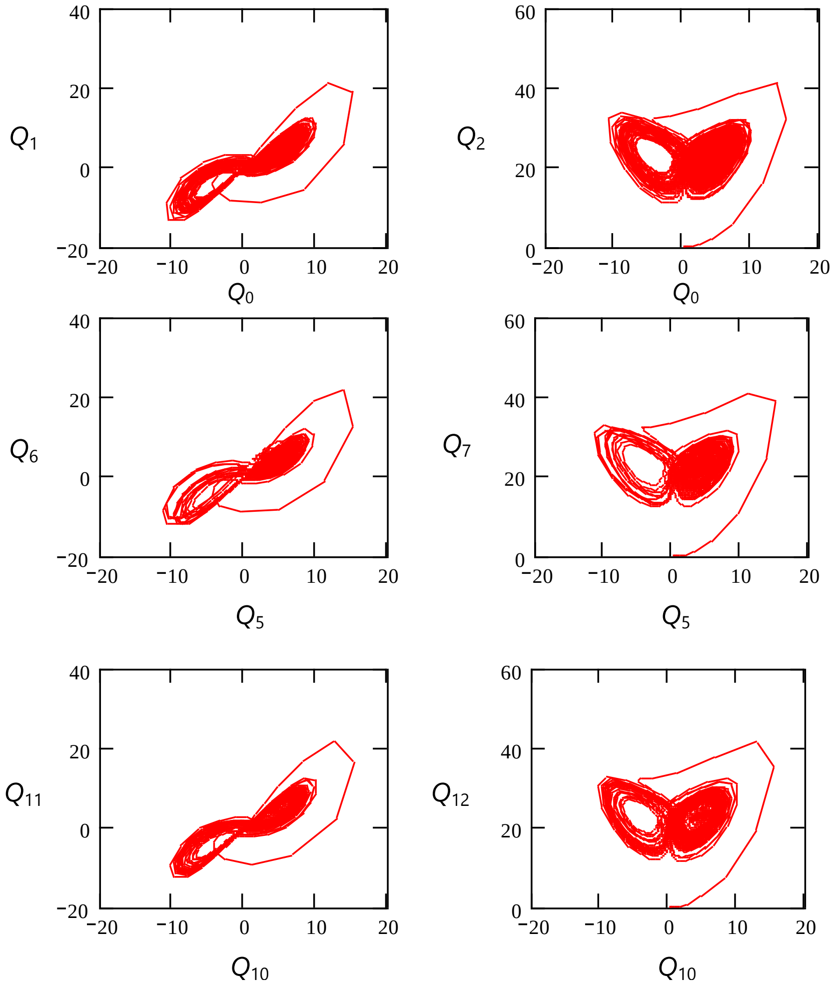


Fig. 5.14

Since in the "feedback" there is no transitional "kinematic component", delay is created through variable Q_{12}. Initial conditions are analogous that also gives the possibility to speak on the system self-excitation. Let us focus attention on the fact that the first attractor after a certain time "goes out". The development of process is presented on Fig. 5.15.

Graphs on Fig. 5.16 tell us about the similar behavior of attractors. The first attractor is exception on phase portraits of which the special feature appears, caused by the feedback manifestation.

The simulation of the attractor's moments of the excitation delay testifies about the possibility to simulate propagation of excitation "waves" in the systems of the interconnected attractors.

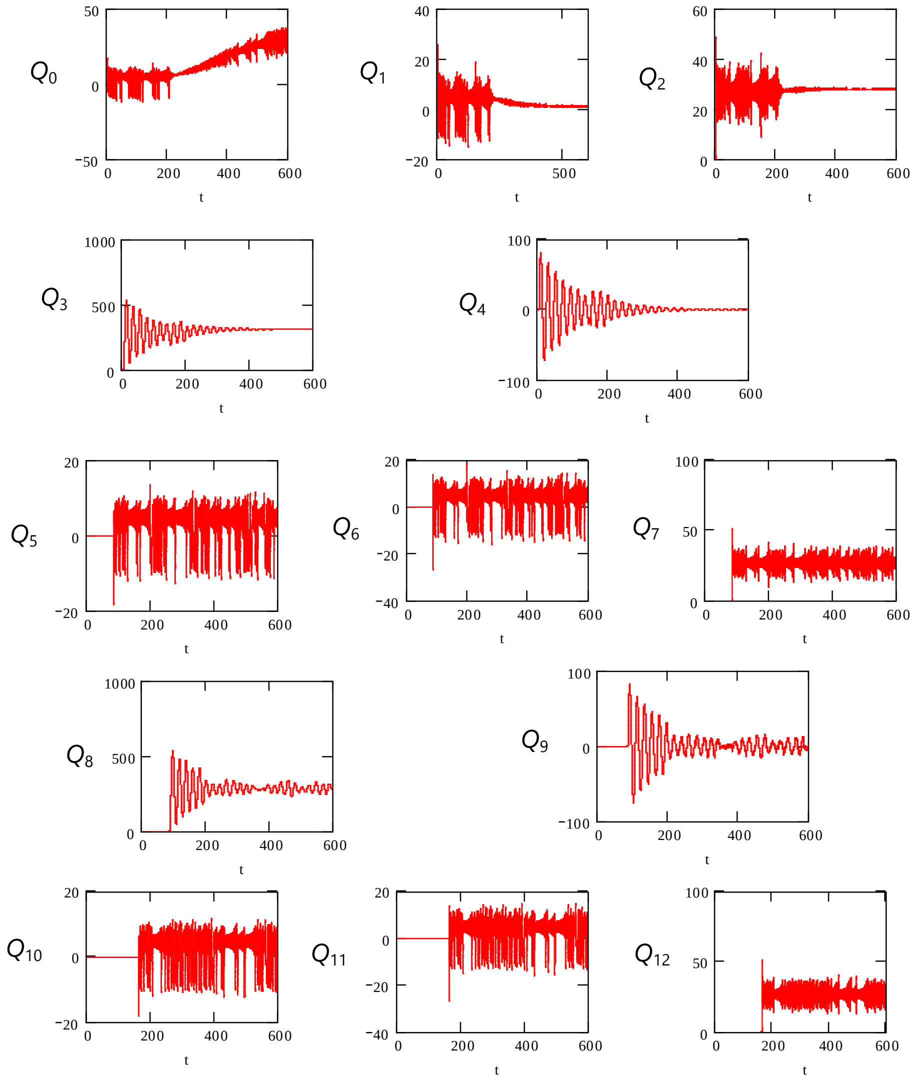


Fig. 5.15

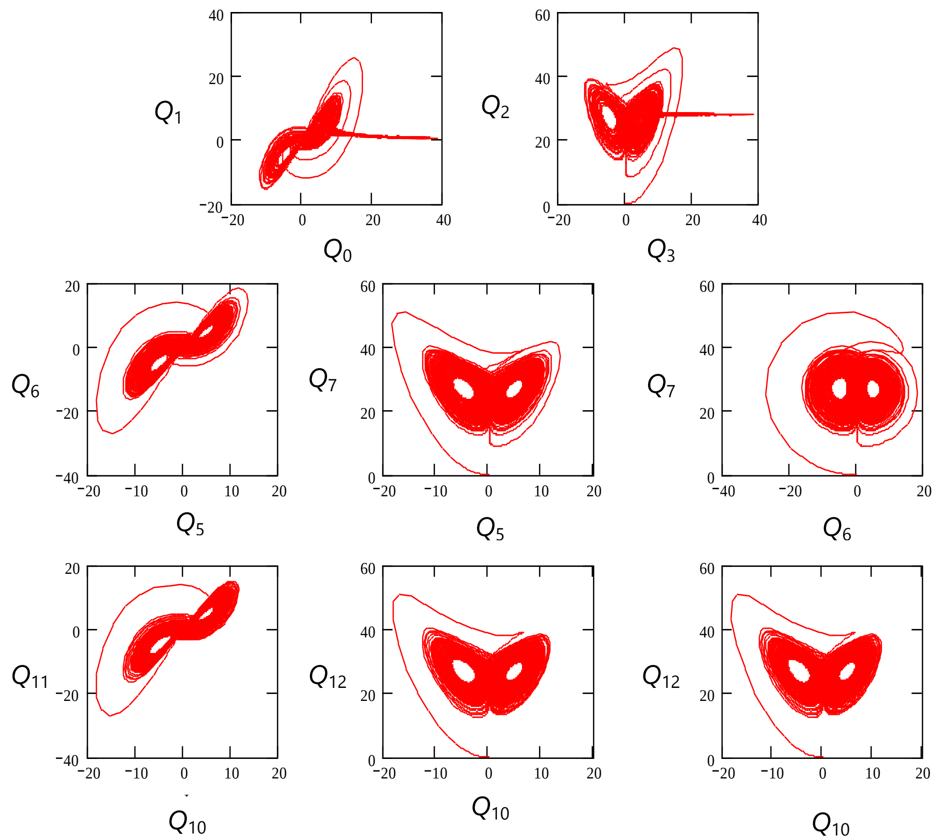


Fig. 5.16

Let us bring one additional variant of the delays simulation, based on the use of another method. In the following system of equations, in the "kinetic" connections model the functions are included, which assign the set of the high spike pulses of the unity height (in this case there are three pulses on each of two kinematic links with the different arrangement of pulses on the time axis). External perturbance is equal to zero (f = 0), and initial conditions are practically uniform. The structural parameters in all three attractors have identical values, the damping factors (h, m, n) are selected so that perturbances are "spilled" in various moments and they keep "awake" for limited time, after which their activity attenuates, but on the other, steady levels. This is — the very essential feature, which tells us about the possibility to simulate "quantum" nature of the psyche performance, moreover both on the time scale and on the "three-dimensional" scale.

$$D(t,Q)=\begin{bmatrix} aQ_1-bQ_0+f\sin(lt)-hQ_0^2+\theta Q_{12}; \\ -Q_1-Q_0Q_2+cQ_0-mQ_1^2; \\ Q_0Q_1-dQ_2-nQ_2^2; \\ Q_4; \\ \left(\mu Q_2-2qQ_4-k^2Q_3\right)\times \\ \times\left[e^{-(t-100)^2 10^{\eta}}+e^{-(t-200)^2 10^{\eta}}+{}^{-(t-300)^2 10^{\eta}}\right]; \\ aQ_6-bQ_5+f\sin(lt)-hQ_5^2+sQ_4; \\ -Q_6-Q_5Q_7+cQ_5-mQ_6^2; \\ Q_5Q_6-dQ_7-nQ_7^2; \\ Q_9; \\ \left(Q_7-2qQ_9-k^2Q_8\right)\times \\ \times\left[e^{-(t-200)^2 10^{\eta}}+e^{-(t-300)^2 10^{\eta}}+{}^{-(t-400)^2 10^{\eta}}\right]; \\ aQ_{11}-bQ_{10}+f\sin(lt)-hQ_{10}^2+rQ_9; \\ -Q_{11}-Q_{10}Q_{12}+cQ_{10}-mQ_{11}^2; \\ Q_{10}Q_{11}-dQ_{12}-nQ_{12}^2; \end{bmatrix} \qquad Q\,|_{t=0}=\begin{pmatrix} 10^{-9} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (5.110)$$

a = 8; b = 8; c = 20; d = 0,43; f = 0; h = 0,003; m = 0,003;
n = 0,003; q = 0,3; k = 0,3; s = 0,1; r = 0,1; η = 1; μ = 1; θ = 0,001.

The behavior of the system coordinates is presented on Fig. 5.16.

The phase portraits of attractors are qualitatively similar, but they differ in the quantitative sense (Fig. 5.17).

The very particular version of the system of interconnected attractors is examined and it can be shown that the insignificant modifications of their structure enlarge the possibilities of the simulation of the qualitative manifestations of psyche, with respect to the forming and change of preferences. At the same time it is necessary to recognize that in this case there are no "physiological" bases for the selection of one or other nonlinear object (attractor) as a model of the processes, proceeding in the brain of man. For justification of their use the same order considerations can serve by which one is guided using, for example, regression models.

Let us note that two methods of control for the active systems through the preferences are obvious:

1. Control with the aid of the resources (utilities, harmfulness).

This is, so to say, the direct control, which consists on the goal-directed change in the elements of exogenous situation.

2. Control through a change in the endogenous (structural) variables — mediated control by the action on the deep properties.

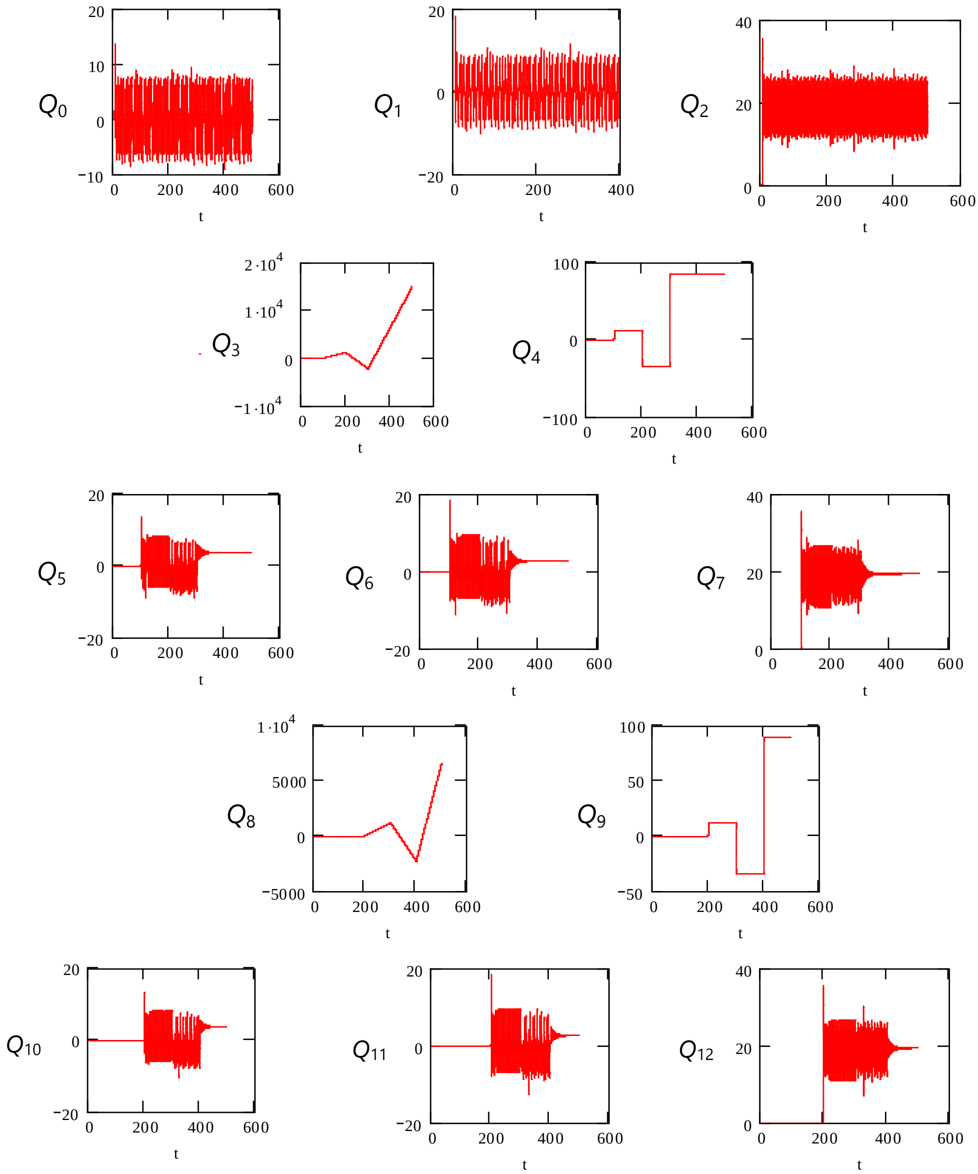


Fig. 5.17

A difference in these two methods in a certain sense is similar to the difference between the instruction (transfer of information resources) and the training (change in the personal properties of trainee).

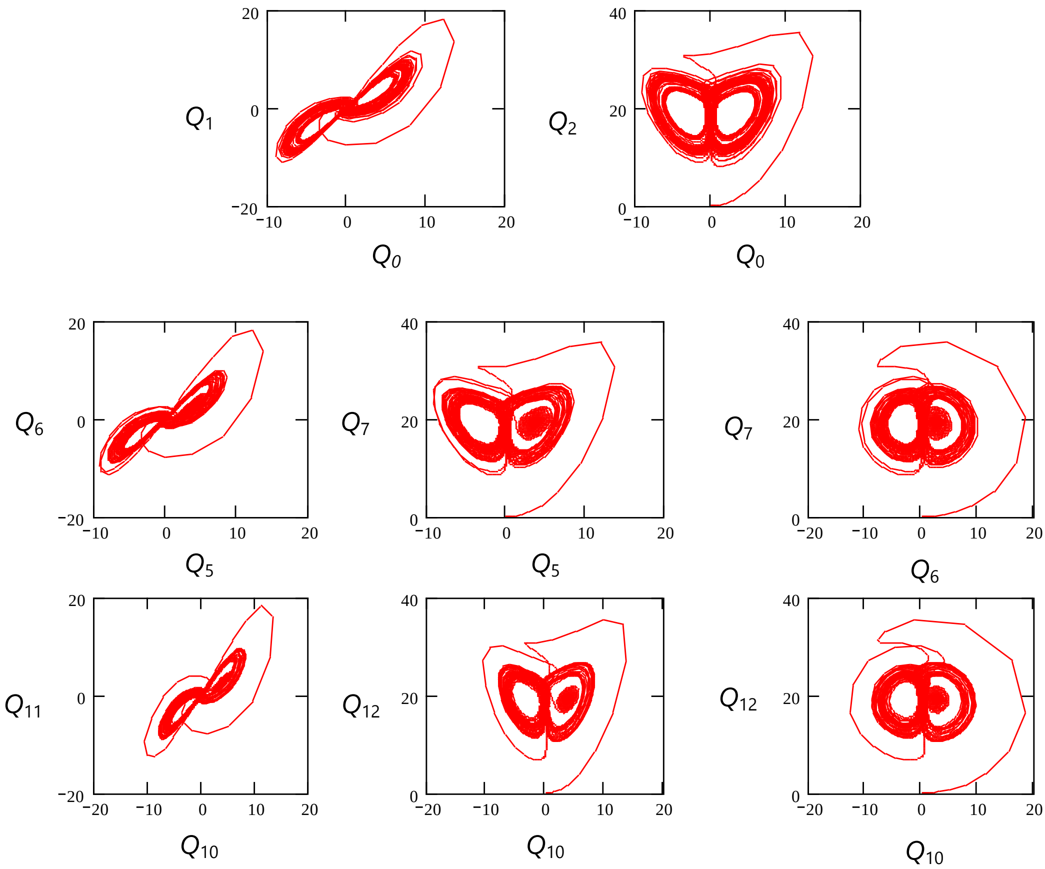


Fig. 5.18

5.7. Simulation of the preferences dynamics of the first kind and exogenous processes

5.7.1. Dynamics of the consumer's preferences by the exogenous model of Walras – Leontyev

Let us examine examples of the joint simulation of the preferences dynamics of the first kind and exogenous dynamics with the presence of mutual influence. As the first example let us take the active system, which includes the consumer as the subject. As "exogenous shell" comes out the economic system, described by the Walras-Leontyev dynamic model [108], for which the outputs of goods x_i and factors r_j are the variables, as well as prices p_i and v_j - respectively. Let x_i - total release of merchandise i, c_i - final demand for that merchandise, r_j - total proposal of factor j, p_i - price of the unit of merchandise x_i, v_j - price of the factor single quantity. The dynamic system of Walras-Leontyev takes the form:

$$\left.\begin{aligned} &H\frac{dp}{dt}=Ax+c\left(v,p,1\right)-x;\\ &K\frac{dv}{dt}=Bx-r\left(p,v,1\right);\\ &M\frac{dx^{\mathrm{T}}}{dt}=p^{\mathrm{T}}-p^{\mathrm{T}}A-Bv^{\mathrm{T}}-b_{m+1},\end{aligned}\right\}\qquad(5.111)$$

where $x = (x_1,..., x_n)$, *is n-th* the column vector of outputs, $p = (p_1, ..., p_n)$, n is the column vector of the prices of goods, $r = (r_1, ..., r_m)$ is production factors, $v = (v_1, ..., v_m)$ is the sector - column of the prices of factors, there is one additional factor r_{m+1} and its price $v_{m+1} = 1$, which is called the price money gauge. It is assumed to be a constant and equal to the value of one.

In connection with Walras - Leontyev model beside [108] a number of the assumptions are done, which concretely define the elements of model and brighten its meaning. In particular the matrix A of the specific expenditures of goods for reproduction is non-negative, not resolved and satisfying conditions of Hawkins - Simon theorem [108]: all principal minors of the determinant $\det(I - A)$, where I — unit matrix, are positive.

If moreover final demand $c \geq 0$, then this is necessary and sufficient condition, so that the solution x would be strictly positive in the statics.

In the dynamics it is insufficient for the positivity of the solution of system x, p, ...

In the task examined below, we will simplify the situation maximally. Let us refuse the separation of production on the goods and the factors. We will speak about the „outputs" x_i and prices p_i, the money gauge in this case can be neglected.

Let us assume that in the subject (buyer) attention field the two goods fell. Thus, there are two alternatives: σ_1 and σ_2 and preferences are determined only by the prices of goods (this corresponds to the assigned assumption, that utilities of both goods are identical).

Let us designate the price of a unit quantity of k merchandise through p_k, and let us examine two models of the function of the consumer (consumer) preferences:

$$\pi_{con}(\sigma_k) = \frac{e^{-\beta p_k}}{e^{-\beta p1} + e^{-\beta p2}} \quad \text{and} \quad \pi_{con}(\sigma_k) = \frac{p_k^{\alpha} e^{-\beta p_k}}{p_1^{\alpha} e^{-\beta p1} + p_2^{\alpha} e^{-\beta p2}}.$$

In the first case $\frac{f'(\sigma_k)}{f(\sigma_k)} = -\beta$, and in the second $\frac{f'(\sigma_k)}{f(\sigma_k)} = \alpha p_k^{-1} - \beta$ (see Chapter 3).

In the second case at the point $p_k = 0$ there is the singularity.

Since the functions $\pi(\sigma_k)$ are limited and, if the right sides of the exogenous shell equations are limited, then at the point $p_k = 0$ preferences rate of change becomes infinite. The second case is interesting, as it was already mentioned earlier, that fact it corresponds to the specific type of the psyche, when "cheap" good is preferred less, then good, which has the higher cost, appropriate to the subject possibilities, the attractiveness of very "expensive" goods in this case is reduced in proportion to an increase in the price.

There is "optimum" price, the subject select even dependence of prince on the objective utility. Thus, for instance, if you acquire medicine for the person close to you, and these are two medicines, of approximately identical effectiveness then with the availability of opportunity you will purchase more expensive.

The story is known, when in one of the Soviet stores in America a very good, children's toy was put out - doll on the very low for the Americans price - 5 dollars.

It practically was not sold. Administration invited local expert, and it advised to place the price of 50 dollars. Doll it was bought up in a short time. For the selection of more expensive good influence the considerations of prestige, mode and so forth.

Let us write down the model of active system in the special case in question. Under assumption that the second type canonical distribution is realized as more general, equations for the preferences are taking the form:

$$\frac{d\pi_1}{dt} = \pi_1 \pi_2 \left[\left(\alpha \frac{1}{p_1} - \beta \right) \dot{p}_1 - \left(\alpha \frac{1}{p_2} - \beta \right) \dot{p}_2 \right]; \tag{5.112}$$

$$\frac{d\pi_2}{dt} = \pi_1 \pi_2 \left[\left(\alpha \frac{1}{p_2} - \beta \right) \dot{p}_2 - \left(\alpha \frac{1}{p_1} - \beta \right) \dot{p}_1 \right]. \tag{5.113}$$

In the case when there are only two goods x_1 and x_2 and from standardization condition it follows that

$$\frac{d\pi(\sigma_2)}{dt} = -\frac{d\pi(\sigma_1)}{dt}.$$

and $\pi(\sigma_2) = 1 - \pi(\sigma_1)$, the value $\pi(\sigma_2)$ is possible to determine from this formula without integrating the equation (5.113).

We make the assumption that the final demand linearly depends on the preference:

$$c_1(\pi(\sigma_1)) = c_1\pi(\sigma_1);$$
$$c_2(\pi(\sigma_2)) = c_2\pi(\sigma_2),$$

where c_1 and c_2 — structural parameters, "leveling" dimensionality.

As the equations of the "exogenous shell" we take the equations of the Walras - Leontyev adapted model:

$$\frac{dp_1}{dt} = \frac{1}{h_1}\left(a_{11}x_1 + a_{12}x_2 + c_1\pi(\sigma_1) - x_1\right); \tag{5.114}$$

$$\frac{dp_2}{dt} = \frac{1}{h_2}\left(a_{21}x_1 + a_{22}x_2 + c_2\pi(\sigma_2) - x_2\right); \tag{5.115}$$

$$\frac{dx_1}{dt} = -\frac{1}{m_1}\left(a_{11}p_1 + a_{21}p_2 - p_1 + a_1w_1 + q_1\right); \tag{5.116}$$

$$\frac{dx_2}{dt} = -\frac{1}{m_2}\left(a_{12}p_1 + a_{22}p_2 - p_2 + a_2w_2 + q_2\right). \tag{5.117}$$

Here a_{ij} - matrix A elements; h_1, h_2 - matrix N elements; m_1, m_2 - matrix M elements; w_1, w_2 - wages of producer on the unit of the produced goods, q_1, q_2 - embedded profit from the realization of the unit of the produced goods. The sense of equations (5.114), (5.115) lies in the fact that, if the production of goods x_i exceeds its consumption per time unit, i.e., there is excess of goods, price on it falls. Equation (5.116), (5.117) reflect the fact that in the conditions, when expenditures for production exceed the price of the goods unit, a quantity of the produced goods must decrease.

The conditions of Hawkins - Simon theorem are sufficient in the statics, that every x_i and p_i would be non-negative values. In the dynamic version, in spite of satisfaction of these conditions, for the specific combinations of the structural parameters, , x_i and p_i can become negative at some moments of time, what contradicts to the „physical" sense of these values.

In order to correct this imperfection of Walras - Leontyev model, which is compensated in [108] by introduction, additional assumption 9 (p.49), „extra systemic" correction is proposed: if the release of goods becomes zero and in this case, its price is less than expenses, then release is relied to be further zero. This circumstance can be realized in the computational algorithm in the form of additional logical conditions. A similar method is, however, too „powerful". It would be more natural to modify the very system of equations in any manner so that the output x_i and p_i in the region of negative values is not allowed.

It is possible to attain this, introducing in Walras - Leontyev equation „the barrier functions", which create the impenetrable boundary for the variables. his function „reduces to zero" derivative (variable rate of change) in immediate proximity of the assigned boundary. There are many versions of barrier functions. The selection of specific function is connected with the condition, so that its influence on the behavior of the solution far from the boundary would be unessential.

This is not always possible to reach. Strictly speaking, barrier function must reflect the special features of the subject behavior in the thin „layer", which corresponds to very low prices, or very small outputs. The study of economic dynamics in the mentioned thin „boundary layer" apparently represents the independent task, which is not the object of this work. Our task consists in the fact, in order to demonstrate the influence of preferences.

Let us give some results of process simulation with the Walras - Leontyev equations (5.114) - (5.117) as the exogenous model and equations (5.112), (5.113) describing the dynamics of preferences. Predestinate the variables $\pi(\sigma_1) = Y_0$, $\pi(\sigma_2) = Y_1$, the price of the first goods Y_2, the price of the second goods Y_3, a quantity of first goods Y_4, a quantity of second goods Y_5. In the new variables the system of equations takes the form:

$$D(t,Y) = \begin{bmatrix} Y_0Y_1\left[\frac{1}{h_1}\left(\alpha\frac{1}{Y_2}-\beta\right)\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)-\frac{1}{h_2}\left(\alpha\frac{1}{Y_3}-\beta\right)\left(a_{21}Y_4+a_{22}Y_5+c_1Y_0-Y_5\right)\right] \\ Y_0Y_1\left[\frac{1}{h_2}\left(\alpha\frac{1}{Y_3}-\beta\right)\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)-\frac{1}{h_1}\left(\alpha\frac{1}{Y_2}-\beta\right)\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)\right] \\ \frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)\left(\frac{4}{\pi}atan\left(e^{4Y_2}\right)\right) \\ \frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)\left(\frac{4}{\pi}atan\left(e^{4Y_3}\right)\right) \\ -\frac{1}{m_1}\left(a_{11}Y_2+a_{21}Y_3-Y_2+a_1w_1+q_1\right)\left(\frac{4}{\pi}atan\left(e^{4Y_4}\right)\right) \\ -\frac{1}{m_2}\left(a_{12}Y_2+a_{22}Y_3-Y_3+a_2w_2+q_2\right)\left(\frac{4}{\pi}atan\left(e^{4Y_5}\right)\right) \end{bmatrix} \tag{5.118}$$

where D (t, Y) - the vector of derivatives of vector Y = (Y_0, Y_1, Y_2, Y_3, Y_4, Y_5).

Let us assume that the parameters, being contained on the right sides of the equations, posses the values:

a_{11} = 0,7; a_{22} = 0,3; a_{12} = 0,3; a_{21} = 0,1; m_1 = 100; m_2 = 100; h_1 = 10; h_2 = 10;

c_1 = 3; c_2 = 3, w_1 = 4; w_2 = 4; β = 0,03; q_1 = 1; q_2 = 2; a_1 = 1; a_2 = 1; α = of 0,01.
Let us select the following initial conditions: with t = 0 Y_0 = 0,5; Y_1 = 0,5 (the preferences at the initial moment are equal); Y_2 = 60; Y_3 = 60; Y_4 = 30; Y_5 = 30. At the initial moment the prices are identical, the outputs of goods are also identical.

In the equations for the prices and the outputs, the barrier functions in the right sides are introduced, which retain the correspondent variables in the region of the positive values: ($\frac{4}{\pi} atg\left(e^{4Y_i}\right)$). In accordance with the standardizations condition preferences π (σ_1) = Y_0 and π (σ_2) = Y_1 are the mirror images of each other, which is evidently shown on Fig.5.16.

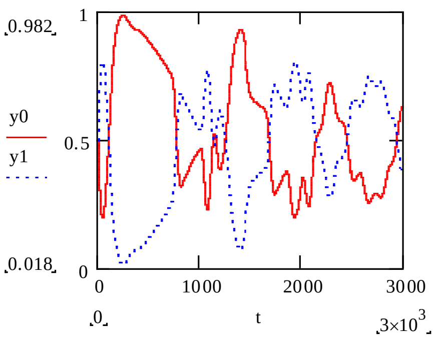


Fig. 5.19

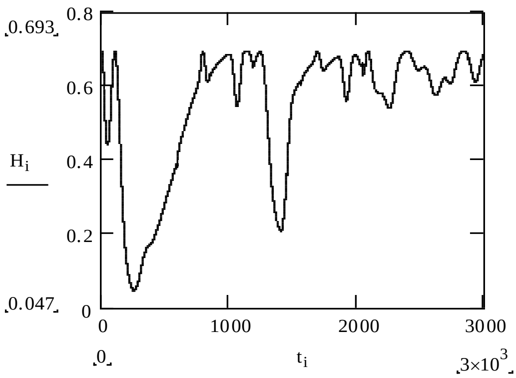


Fig. 5.20

The dependence of prices and outputs on time is shown on Fig.5.21. On the same figures the corresponding consumer preferences are given. We can see that also the prices and outputs are retained in the positive region. In some sections both the outputs and the prices become close to zero. Practically this it means that the production almost ceases. After a certain time the deficit leads to an exuberance in the prices, than follows an increase (or renewal) in the production, the outputs are lagging from the prices „on the phase", which completely answers general ideas.

Let us notice, that essential role in the distribution of prices and outputs the preferences play, which are considered in Walras - Leontyev equations through the final demand.

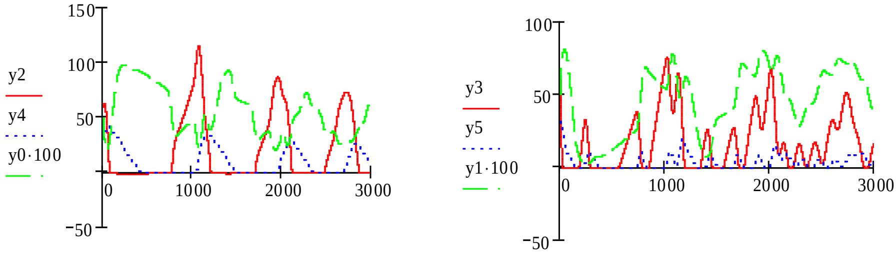


Fig.5.21

Fig.5.22 showed the trajectories in the phase space.

The following series of graphs shows that the introduction of barrier function is not the formal mathematical operation. An insignificant change in the structure of barrier function can entail the change in the solution. In this case a change in the barrier function (see the right sides of the equations) such, that "impenetrable" boundary seemingly is shifted upward from the axis - in the field of positive values, in order to exclude any possibility of the negative values appearance and, furthermore, to exclude „complete stoppage" of the goods production.

The value of the endogenous parameter α is preserved (α = of 0,01).

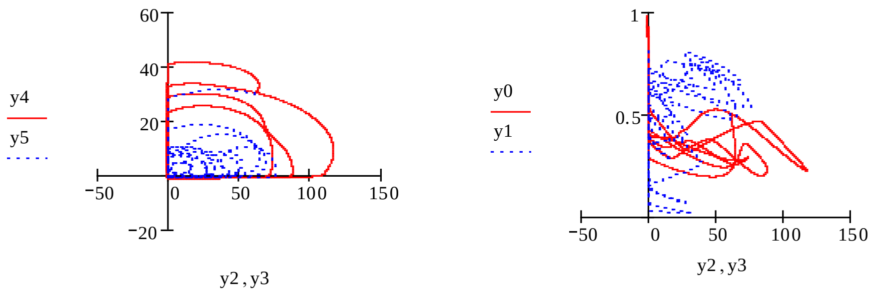


Fig. 5.22

System of equations with the modified barrier function takes the form:

$$D(t,Y)=\begin{bmatrix} Y_0Y_1\left[\frac{1}{h_1}\left(\alpha\frac{1}{Y_2}-\beta\right)\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)-\frac{1}{h_2}\left(\alpha\frac{1}{Y_3}-\beta\right)\left(a_{21}Y_4+a_{22}Y_5+c_1Y_0-Y_5\right)\right] \\ Y_0Y_1\left[\frac{1}{h_2}\left(\alpha\frac{1}{Y_3}-\beta\right)\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)-\frac{1}{h_1}\left(\alpha\frac{1}{Y_2}-\beta\right)\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)\right] \\ \frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)\left(\frac{2}{\pi}atan\left(e^{4Y_2-5}\right)\right) \\ \frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)\left(\frac{2}{\pi}atan\left(e^{4Y_3-5}\right)\right) \\ -\frac{1}{m_1}\left(a_{11}Y_2+a_{21}Y_3-Y_2+a_1w_1+q_1\right)\left(\frac{2}{\pi}atan\left(e^{4Y_4-5}\right)\right) \\ -\frac{1}{m_2}\left(a_{12}Y_2+a_{22}Y_3-Y_3+a_2w_2+q_2\right)\left(\frac{2}{\pi}atan\left(e^{4Y_5-5}\right)\right) \end{bmatrix} \quad (5.119)$$

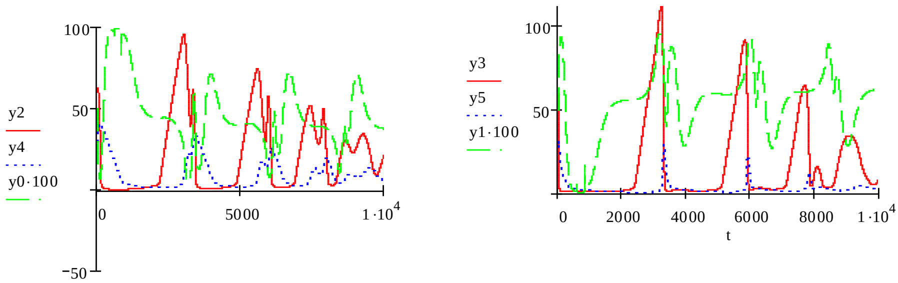


Fig.5.23

Phase portraits of preferences and entropy are shown on Fig.5.24.

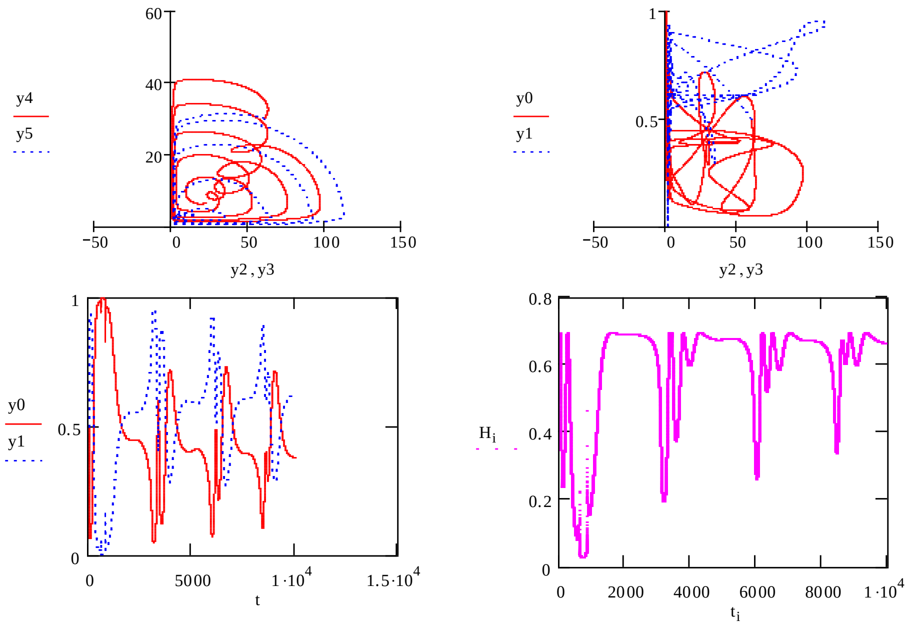
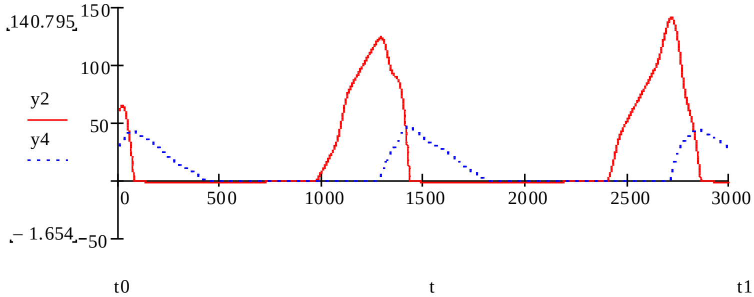


Fig. 5.24

Fig.5.25 and Fig.5.26 shows the behavior of prices and outputs, obtained as a result of the integration of Walras - Leontev equations without taking into account the influence of the dynamics of preferences.



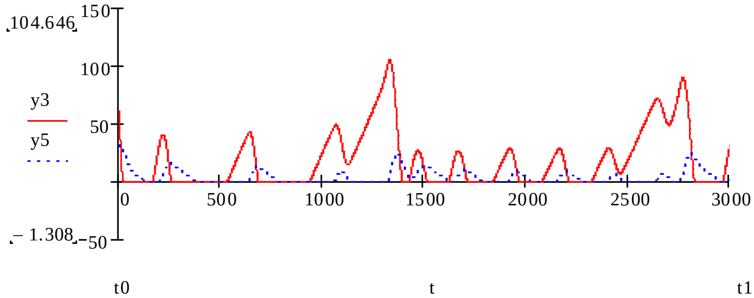


Fig. 5.25

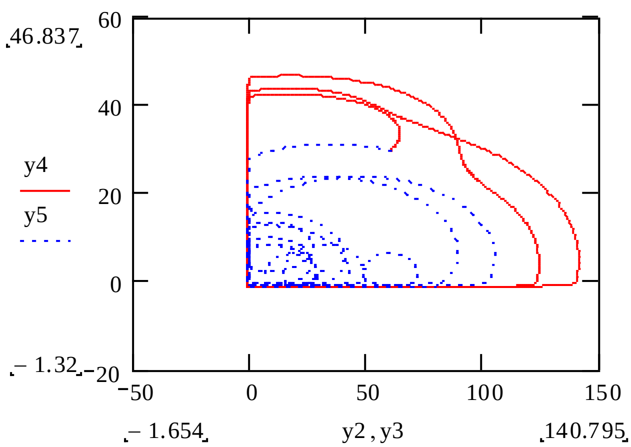


Fig. 5.26

It is easy to note essential differences from the previous results.

This speaks that, if suppositions about the preferences forming are right and canonical distributions correctly reflect the properties of psyche, then accounting of them in the economical dynamics is necessary and the economical forecasts, based on this technology, can differ significantly from the forecasts, which are obtained on the basis of „purely economical models", similar to Walras - Leontyev model. This conclusion has selective nature. Psychology of consumer and producer is considered and defined by example in many economic models. Thus, in the same Walras-Leontyev model psychological factor is considered through the assumption on the dependence of the prices and outputs rate of change on the „imbalance" of supply and demand. Nevertheless, the accounting of the dynamics of canonical preferences is explicitly a promising trend in development of studies on economical dynamics.

5.7.2. Dynamics of the preferences of consumer and producer with the Walras- Leontyev's exogenous model.

The following scheme is characterized by the fact, that the producer is introduced into the „game" along with the consumer. It is assumed that his preferences also depend on the distribution of the prices p_1 and p_2 (Y_2 and Y_3).

Actually system of two subjects is examined. They have the structurally different canonical distributions of preferences, and are located in „antagonistic" relations, which are realized with the aid of the connecting link - the „price- outputs" model of Walras-Leontyev. This model play the role of „kinematic constraint" (Fig.5.27).

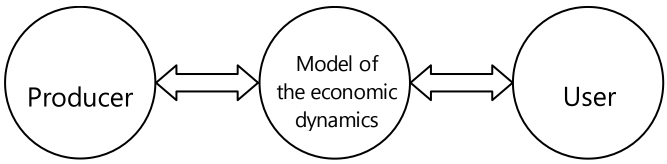


Fig. 5.27

Task becomes „symmetrical" and it is possible to say that on „exogenous field" what is the Walras-Leontyev system, two players play. Each of them investigates two alternatives:

S_a^{con}: σ_1, σ_2; S_a^{pr}: σ_1, σ_2 (con is consumer, pr is producer).

For each consumer σ_1 and σ_2 indicate the possibility of the acquisition of first or second type goods, for the producer σ_1 and σ_2 means the expedience of production the corresponding types of goods. The distributions of preferences ($\pi_{pr}(\sigma_1)$; $\pi_{pr}(\sigma_2)$) are determined on S_a^{pr} . Since tasks considered here bear illustrative nature, we will accept the simplifying assumption relative to this distribution. Let the expedience of the goods production is higher for the producer, the more price of it.

The preferences of producer can be proportional to the income, i.e., to the product of price and output: $p_i x_i$. It is clear that in the given version the optimization is transferred on the subjective level is of the optimum canonical distributions of preferences forming. The combined approach is realized, based on *the objective* optimization with the conditions of larger or smaller uncertainty, as well as the optimization combined with *the subjective* component, having an essential influence on decision making.

This is substantial for the small and average producer, when the degree of uncertainty is high and in the majority of cases there is no possibility of the mathematical setting and solving of the optimization problems, and the solutions are made frequently on the intuitive level. This decision making process could be named empirical. The models, examined here, correspond to the case, when there are a large number of almost identical consumers, and producer outputs such quantity of goods, that the single purchase vanishingly little influences on the total balance.

Furthermore, all assumptions of Walras-Leontyev model, of course, are carried out. In what „place" of Walras-Leontyev system is the influence of subjective preferences reflected on? Generally speaking, in the case of two „players" of buyer and producer, it the complete system of Walras-Leontyev equations (5.111) should be used and to count that the producer is preferences influence on process through the investment vector function $r(p, v, 1)$ in the proposal of these factors.

For simplicity we will, however, remain our self within the framework of model (5.112) and (5.113) as well as assumptions that $\pi_{pr}(\sigma_1)$ and $\pi_{pr}(\sigma_2)$ have an effect on the producer assumed levels of specific profit q_1 and q_2, expended in this case on the improvement of production. Let us accept hypothesis, that

$$q_1 = q_0\pi_{pr}(\sigma_1);\ q_1 = q_0\pi_{pr}(\sigma_2).$$

Suppose at first, that:

$$\pi_{pr}\left(\sigma_i\right) = \frac{e^{\beta_1 p_i}}{\sum_{j=1}^{N} e^{\beta_1 p_j}}. \tag{5.120}$$

This corresponds to the case, when „utility" $U(\sigma_i)$ is interpreted by producer as price of the goods produced by him and to the functional of the form

$$\Phi_{pr\pi} = -\sum_{i=1}^{N} \pi_{pr}\left(\sigma_i\right)\ln \pi_{pr}\left(\sigma_i\right) + \beta\sum_{i=1}^{N} \pi_{pr}\left(\sigma_i\right)p_i + \gamma\sum_{i=1}^{N} \pi_{pr}\left(\sigma_i\right). \tag{5.121}$$

If utility is considered income per unit time, then

$$\pi_{pr}\left(\sigma_i\right)=\frac{e^{\beta_1 p_i x_i}}{\sum_{j=1}^{N} e^{\beta_1 p_j x_j}}, \tag{5.122}$$

and in the functional second term is replaced with the term

$$\ldots+\beta\sum_{i=1}^{N}\pi_{pr}\left(\sigma_i\right)p_i x_i+\ldots$$

In the first case (5.120) differential equations for $\pi_{pr}(\sigma_1)$ and $\pi_{pr}(\sigma_2)$ take the form

$$\frac{d\pi_{pr}\left(\sigma_1\right)}{dt}=\beta_1\pi_{pr}\left(\sigma_1\right)\pi_{pr}\left(\sigma_2\right)\left[\dot{p}_1-\dot{p}_2\right]; \tag{5.123}$$

$$\frac{d\pi_{pr}\left(\sigma_2\right)}{dt}=\beta_1\pi_{pr}\left(\sigma_1\right)\pi_{pr}\left(\sigma_2\right)\left[\dot{p}_2-\dot{p}_1\right]. \tag{5.124}$$

Here instead of the derivatives $\dot{p}_i$ should be substituted the right sides of equations (5.112) and (5.113). In the second case

$$\frac{d\pi_{pr}\left(\sigma_1\right)}{dt}=\beta_1\pi_{pr}\left(\sigma_1\right)\pi_{pr}\left(\sigma_2\right)\left[\dot{p}_1x_1+p_1\dot{x}_1-\left(\dot{p}_2x_2+p_2\dot{x}_2\right)\right]; \tag{5.125}$$

$$\frac{d\pi_{pr}\left(\sigma_2\right)}{dt}=\beta_1\pi_{pr}\left(\sigma_1\right)\pi_{pr}\left(\sigma_2\right)\left[\dot{p}_2x_2+p_2\dot{x}_2-\left(\dot{p}_1x_1+p_1\dot{x}_1\right)\right]. \tag{5.126}$$

Here instead of $\dot{p}_i$ $\dot{x}_i$ are substituted the right sides of equations (5.114) - (5.117), moreover equations (5.116), (5.117) are taken in the form:

$$\frac{dx_1}{dt}=-\frac{1}{m_1}\left(a_{11}p_1+a_{21}p_2-p_1+a_1w_1+q_0\pi_{pr}\left(\sigma_1\right)\right); \tag{5.127}$$

$$\frac{dx_2}{dt}=-\frac{1}{m_2}\left(a_{12}p_1+a_{22}p_2-p_2+a_2w_2+q_0\pi_{pr}\left(\sigma_2\right)\right). \tag{5.128}$$

Having explicit analytical expressions for the function $\pi_{pr}(\sigma_i)$ (as for $\pi_{con}(\sigma_i)$) we can substitute these expressions directly in equations (5.114) - (5.117), and do not use the differential equations for the preferences, however, in a number of cases they represent independent interest. Furthermore, the task from a mathematical point of view appears more symmetrical. Instead of (5.124) and (5.128) in the case of two alternatives it is possible to use condition $\pi_{pr}(\sigma_2) = 1 - \pi_{pr}(\sigma_1)$.

The possibility appears to calculate the one additional useful characteristic - the coefficient of subjective correlation between the consumer and producer preferences. In the situation in question these subjects - not competitors, but they are in the antagonistic relations (opposite (antagonistic) relation to the price of goods). This contradiction is insolvable and therefore correlation coefficient is selected in the form:

$$\rho_{con,pr} = -\frac{\sum_{i=1}^{N}\left(\pi_{con}\left(\sigma_i\right)-\frac{1}{N}\right)\left(\pi_{pr}\left(\sigma_i\right)-\frac{1}{N}\right)}{\sqrt{\sum_{i=1}^{N}\left(\pi_{con}\left(\sigma_i\right)-\frac{1}{N}\right)^2\sum_{j=1}^{N}\left(\pi_{pr}\left(\sigma_i\right)-\frac{1}{N}\right)^2}}. \quad (5.129)$$

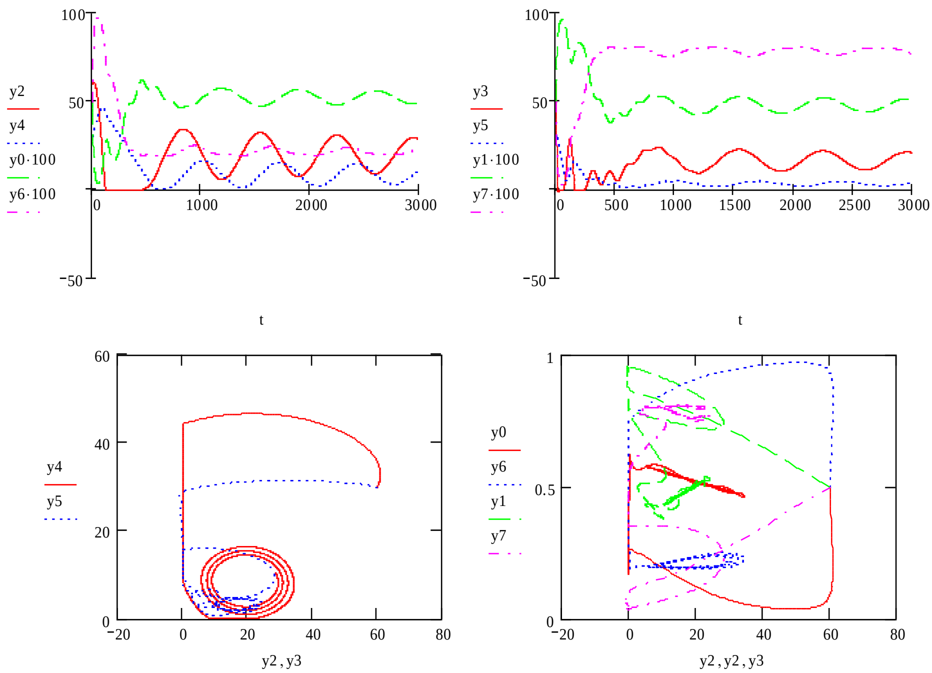
The said above is taken into account by „-" sign.

As earlier, with the numerical simulation the redesignation of variables is executed. All variables are designated as Y_i ($i \in \overline{0,7}$). The "physical" sense of variables Y_i can be easily established from the form of the equations system, given below.

$$D(t,Y)=\begin{bmatrix}
-\beta Y_0 Y_1\left[\frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)-\frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_1Y_0-Y_5\right)\right]\\
-\beta Y_0 Y_1\left[\frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)-\frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)\right]\\
\frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)\left[\frac{2}{\pi}atan\left[e^{nY_2}-0,5\left(1-\frac{\frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)}{\left|\frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)\right|}\right)\right]\right]\\
\frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)\left[\frac{2}{\pi}atan\left[e^{nY_3}-0,5\left(1-\frac{\frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)}{\left|\frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)\right|}\right)\right]\right]\\
-\frac{1}{m_1}\left(a_{11}Y_2+a_{21}Y_3-Y_2+a_1w_1+q_1\right)\left[\frac{2}{\pi}atan\left[e^{nY_4}-0,5\left(1-\frac{-\frac{1}{m_1}\left(a_{11}Y_2+a_{21}Y_3-Y_2+a_1w_1+q_1\right)}{\left|-\frac{1}{m_1}\left(a_{11}Y_2+a_{21}Y_3-Y_2+a_1w_1+q_1\right)\right|}\right)\right]\right]\\
-\frac{1}{m_2}\left(a_{12}Y_2+a_{22}Y_3-Y_3+a_2w_2+q_2\right)\left[\frac{2}{\pi}atan\left[e^{nY_5}-0,5\left(1-\frac{-\frac{1}{m_2}\left(a_{12}Y_2+a_{22}Y_3-Y_3+a_2w_2+q_2\right)}{\left|-\frac{1}{m_2}\left(a_{12}Y_2+a_{22}Y_3-Y_3+a_2w_2+q_2\right)\right|}\right)\right]\right]\\
\beta_1 Y_6 Y_7\begin{bmatrix}\frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)+Y_2\left[-\frac{1}{m_1}\left(a_{11}Y_2+a_{21}Y_3-Y_2+a_1w_1+q_1Y_6\right)\right]\ldots\\ -\frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)+Y_3\left[-\frac{1}{m_2}\left(a_{12}Y_2+a_{22}Y_3-Y_3+a_2w_2+q_2Y_7\right)\right]\end{bmatrix}\\
\beta_1 Y_6 Y_7\begin{bmatrix}\frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)+Y_3\left[-\frac{1}{m_2}\left(a_{12}Y_2+a_{22}Y_3-Y_3+a_2w_2+q_2Y_7\right)\right]\ldots\\ -\frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)+Y_2\left[-\frac{1}{m_1}\left(a_{11}Y_2+a_{21}Y_3-Y_2+a_1w_1+q_1Y_6\right)\right]\end{bmatrix}
\end{bmatrix} \quad (5.130)$$

$$Y_i\Big|_{t=0} = Y_0 = \begin{pmatrix} 0{,}5 \\ 0{,}5 \\ 60 \\ 60 \\ 30 \\ 30 \\ 0{,}5 \\ 0{,}5 \end{pmatrix}$$

In the economical block of the system of differential equations – Walras - Leontyev equations the modified barrier functions are additionally introduced, which don't „let out" the solution $Y_2(t)$, $Y_3(t)$, $Y_4(t)$, $Y_5(t)$ in the region of negative values.



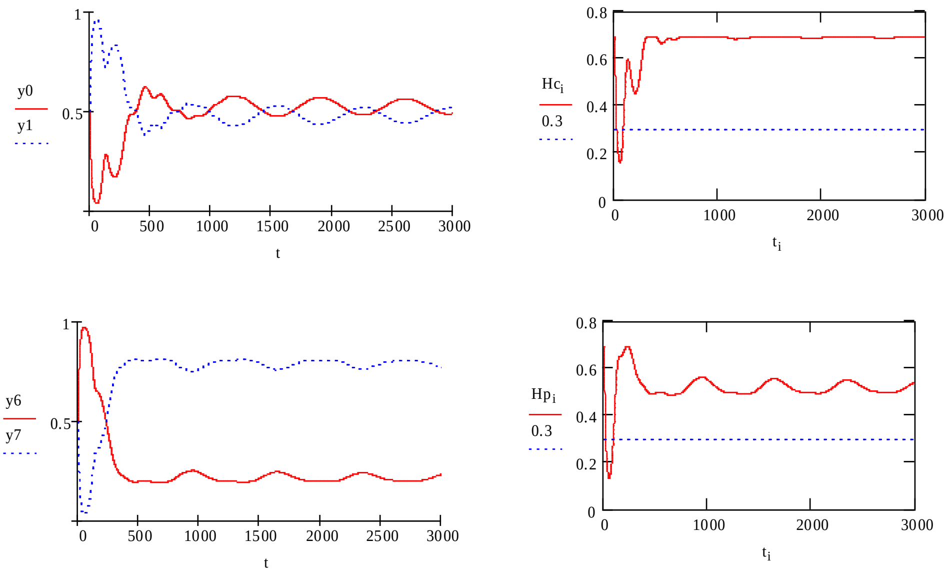


Fig. 5.28

Modification consists of adding in the argument the term equal to 0, if the corresponding variable grows even 1, if it decreases and make the transfer into zero the entire barrier function very quick.

This function reliably „dissipates" in the thin layer close to „impenetrable boundary" the derivatives $\frac{dY_i}{dt}$, if $\frac{dY_i}{dt} < 0$.

In the system of equations given below, two last equations reflect the dynamics of the preferences of producer. Preferences of both subjects Y_0, Y_1 and Y_6, Y_7 are distributed uniformly at the initial moment.

Fig. 5.28 shows the results of accounting for the following values of the structural parameters:

$a_{11} = 0{,}7$; $a_{22} = 0{,}3$; $a_{12} = 0{,}3$; $a_{21} = 0{,}1$; $m_1 = 50$; $m_2 = 50$; $\beta_1 = 0{,}001$; $\beta = 0{,}03$; $h_1 = 15$; $h_2 = 5$; $c_1 = 3$; $c_2 = 3$; $w_1 = 4$; $w_2 = 4$; $q_1 = 1$; $q_2 = 2$; $a_1 = 1$; $a_2 = 1$; $n = 10$.

The correlation coefficient of the preferences of producer and consumer in this case can take only two values: (- 1, +1), which is evident from Fig. 5.29.

$$\rho_{conprod i} = -\frac{\left(Y_{0i}-\frac{1}{2}\right)\left(Y_{6i}-\frac{1}{2}\right)+\left(Y_{1i}-\frac{1}{2}\right)\left(Y_{7i}-\frac{1}{2}\right)}{\sqrt{\left[\left(Y_{0i}-\frac{1}{2}\right)^2+\left(Y_{1i}-\frac{1}{2}\right)^2\right]\left[\left(Y_{6i}-\frac{1}{2}\right)^2+\left(Y_{7i}-\frac{1}{2}\right)^2\right]}}. \qquad (5.131)$$

In this particular case of the consumer preference acquire an oscillatory nature in the region of equilibrium value $\pi(\sigma_i) = 0{,}5$.

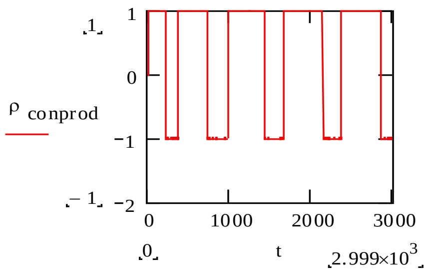


Fig. 5.29

The preferences of the producer were more or less determined; however, entropies of both subjects are sufficiently high.

On Fig. 5.30 the different version is given with the changed parameters β and β_1: $\beta = 0{,}5$; $\beta_1 = 0{,}01$. As we see after the initial period of oscillations of the consumer preferences they become more determined in comparison with the producer preferences. Accordingly the entropy of the first is stabilized on the sufficiently lows level: $H_c \sim 0{,}3$, while $H_p \sim 0{,}59$. The parameters β and β_1 are the endogenous parameters and apparently characterize the ability to react more energetically on changes of the exogenous parameters (in this case - prices).

Simulation with different combinations of the structural parameters values shows, that in many instances the accounting of preferences stabilizes the behavior of the economic parameters in comparison with their behavior, obtained by integrating Walras - Leontyev „naked" system.

For the comparison let us give the result of accounting on the basis of the same model of economic dynamics, but without taking into account the preferences of producer (Fig. 5.31). Let us accept $\beta = 0{,}03$ and the same initial conditions:

$$Y_i^{\mathrm{T}}\Big|_{t=0} = Y_0^{\mathrm{T}} = \left(0{,}5;0{,}5;60;60;30;30\right).$$

(Price of the unit of output of 60 u.e., release - 30 units.)

5.7.3. Model of the appearance of the stress states of the consumer

We examine the following task: in what way economical situation can influence on consumer the mental state of subject. Let us assume, as this was done above, that one of the possible models is the Lorenz attractor [83]. The certain meaning can be assigned, to the corresponding variables subsequently for example, it is possible to attempt to link them with the specific physiological parameters, which correlate with the mental condition of subject. Let us assume that attractor („subjective attractor") is excited when subjective entropy approaches the maximum value. In other words, let us attempt to model the stress state of „Buridan donkey", which is obviously linked with the high value of entropy. Let us introduce in the right sides of the attractor equations the elements, proportional to the relative entropy

$$\bar{H}_\pi = \frac{-\sum_{i=1}^{N} \pi_i \ln \pi_i}{\ln N}; \quad \bar{H}_\pi \in [0,1]..$$

If this function is taken with a high degree

$$\ldots + p\bar{H}_\pi^s,$$

where s ~ 100—200, then we will obtain „needle-shaped" disturbance in the narrow neighborhood of the value $\bar{H}_\pi = 1$. In the entire remaining range of values this function is in effect close to zero. The matching system, where Y_6, Y_7, Y_8 is the attractor variables, takes the form:

$$D(t,Y) = \begin{bmatrix} -\beta Y_0 Y_1 \left[\frac{1}{h_1}\left(\alpha \frac{1}{Y_2} - \beta\right)\left(a_{11}Y_4 + a_{12}Y_5 + c_1 Y_0 - Y_4\right) - \frac{1}{h_2}\left(\alpha \frac{1}{Y_3} - \beta\right)\left(a_{21}Y_4 + a_{22}Y_5 + c_1 Y_0 - Y_5\right) \right] \\ -\beta Y_0 Y_1 \left[\frac{1}{h_2}\left(\alpha \frac{1}{Y_3} - \beta\right)\left(a_{21}Y_4 + a_{22}Y_5 + c_2 Y_1 - Y_5\right) - \frac{1}{h_1}\left(\alpha \frac{1}{Y_2} - \beta\right)\left(a_{11}Y_4 + a_{12}Y_5 + c_1 Y_0 - Y_4\right) \right] \\ \frac{1}{h_1}\left(a_{11}Y_4 + a_{12}Y_5 + c_1 Y_0 - Y_4\right)\left(\frac{2}{\pi} atan\left(e^{4Y_2 - 10}\right)\right) \\ \frac{1}{h_2}\left(a_{21}Y_4 + a_{22}Y_5 + c_2 Y_1 - Y_5\right)\left(\frac{2}{\pi} atan\left(e^{4Y_3 - 10}\right)\right) \\ -\frac{1}{m_1}\left(a_{11}Y_2 + a_{21}Y_3 - Y_2 + a_1 w_1 + q_1\right)\left(\frac{2}{\pi} atan\left(e^{4Y_4 - 5}\right)\right) \\ -\frac{1}{m_2}\left(a_{12}Y_2 + a_{22}Y_3 - Y_3 + a_2 w_2 + q_2\right)\left(\frac{2}{\pi} atan\left(e^{4Y_5 - 5}\right)\right) \\ aY_7 - bY_6 + f\sin(kt) - hY_6^2 + \left[\frac{-\left(Y_0 \ln Y_0 + Y_1 \ln Y_1\right)}{\ln 2}\right]^s p \\ -Y_7 - Y_6 Y_8 + cY_6 - mY_7^2 + \left[\frac{-\left(Y_0 \ln Y_0 + Y_1 \ln Y_1\right)}{\ln 2}\right]^s p \\ Y_6 Y_7 - dY_8 - nY_8^2 + \left[\frac{-\left(Y_0 \ln Y_0 + Y_1 \ln Y_1\right)}{\ln 2}\right]^s p \end{bmatrix} \quad (5.132)$$

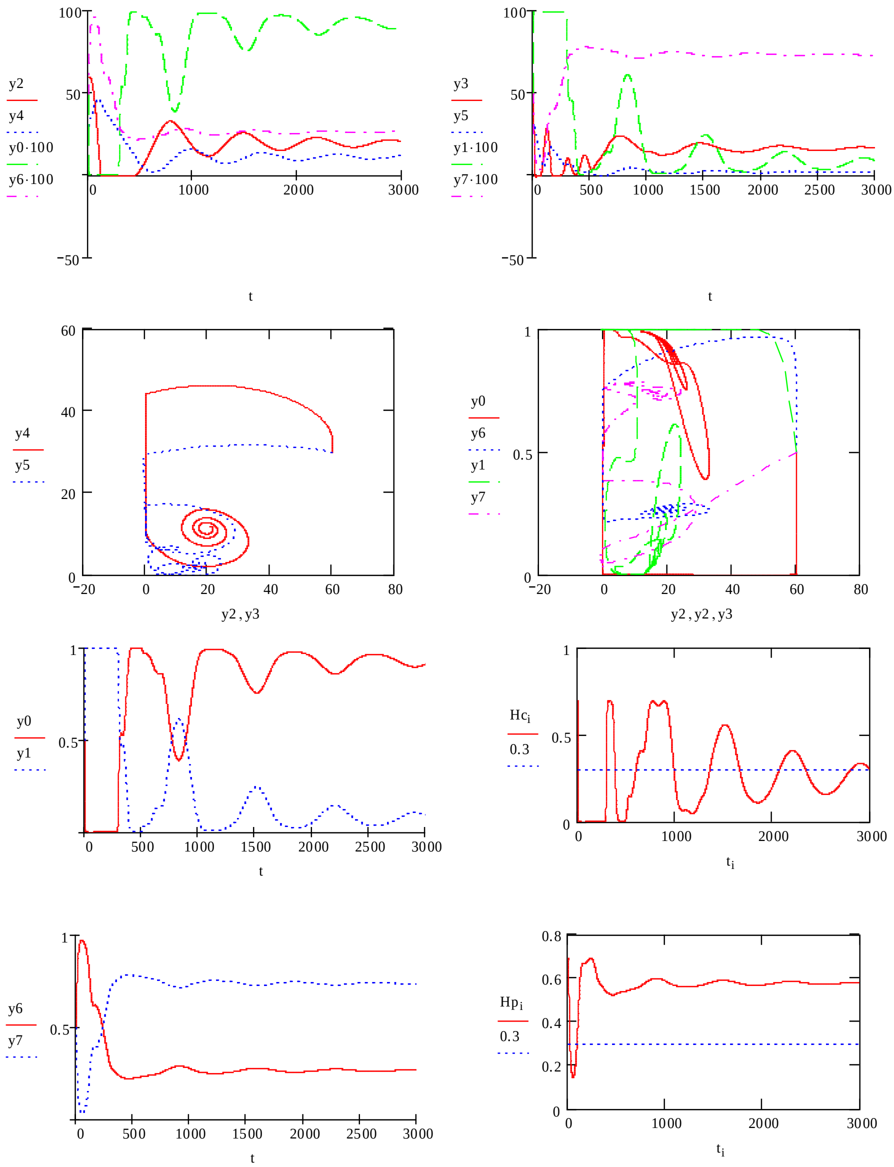

y2
y4
y0·100
y6·100
100
50
−50
0
1000
2000
3000
t
y3
y5
y1·100
y7·100
100
50
−50
0
500
1000
1500
2000
2500
3000
t
y4
y5
60
40
20
0
−20
0
20
40
60
80
y2 , y3
y0
y6
y1
y7
1
0.5
0
−20
0
20
40
60
80
y2 , y2 , y3
y0
y1
1
0.5
0
500
1000
1500
2000
2500
3000
t
Hc_i
0.3
1
0.5
0
0
1000
2000
3000
t_i
y6
y7
1
0.5
0
500
1000
1500
2000
2500
3000
t
Hp_i
0.3
0.8
0.6
0.4
0.2
0
0
1000
2000
3000
t_i

Fig. 5.30

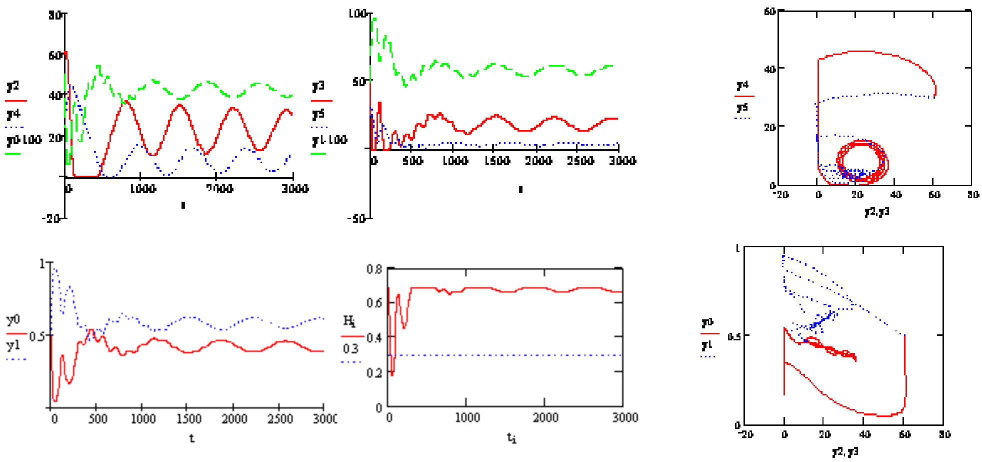


Fig. 5.31

Attractor contains the damping contribution components. The solution is shown below for the following values of the structural parameters:

a = 8; b = 8; c = 20; d = 0,43; f = 0,1; h = 0,3; k = 0,1; m = 0,0001; n = 0,00005; p = 100; ω = 0,1; s = 200; r_0 = 0,5; r_1 = 0,6; a_{11} = 0,7; a_{22} = 0,3; a_{12} = 0,3; a_{21} = 0,1; m_1 = 100; m_2 = 100; h_1 = 10; h_2 = 10; c_1 = 3; c_2 = 3; w_1 = 4; w_2 = 4; β = 0,03; q_1 = 1; q_2 = 2; a_1 = 2; a_2 = 1.

The simplified version of barrier functions is assumed. The results of accounting are given on Fig. 5.32.

It is evident on the base of Fig. 5.33 that at the moment, when entropy reaches maximum the appearance of the attractor „stress" state is simulated. It attenuates after a certain time and the „psyche" of attractor passes into the calm state. The process of „stress" appearance and damping is shown on the example of variable Y_6. Needle-shaped form of the function $\left(\bar{H}_\pi\right)^s$ it is shown on the Fig. 5.34. The phase portraits of attractor are shown on the Fig. 5.35. Let us ask the following question: is it possible to consider the influence of stress states on a change in the preferences? To answer this question it is necessary to make additional assumptions (sufficiently arbitrary) relative to the sense of the attractor variables.

Let the attractor variables Y_6 and Y_7 simulate the structural parameters α and β of the distribution of preferences. We utilize the model of endogenous dynamics, given in the paragraph 5.6.

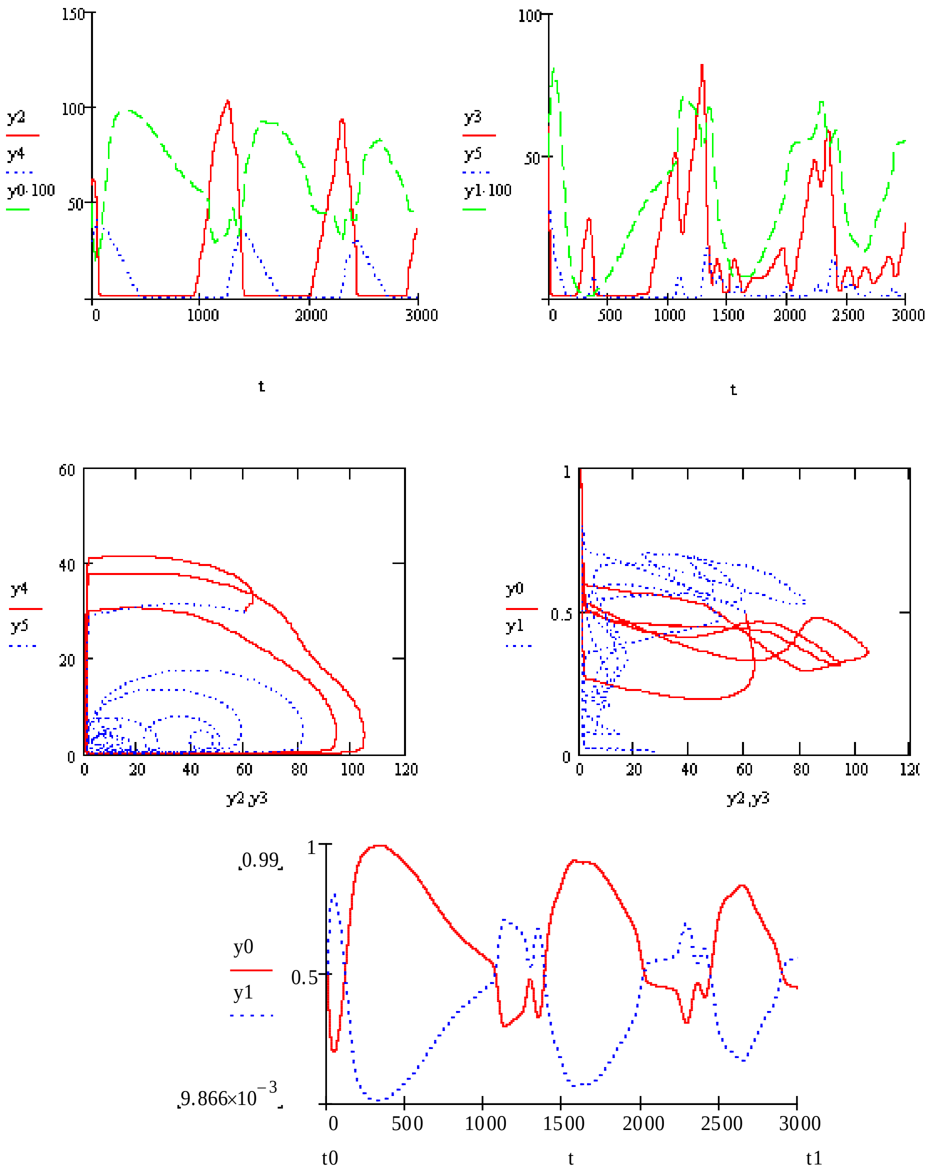

y2
y4
y0·100
y3
y5
y1·100
t
y4
y5
y2,y3
y0
y1
y2,y3
0.99
y0
y1
9.866×10^{-3}
t0
t
t1

Fig. 5.32

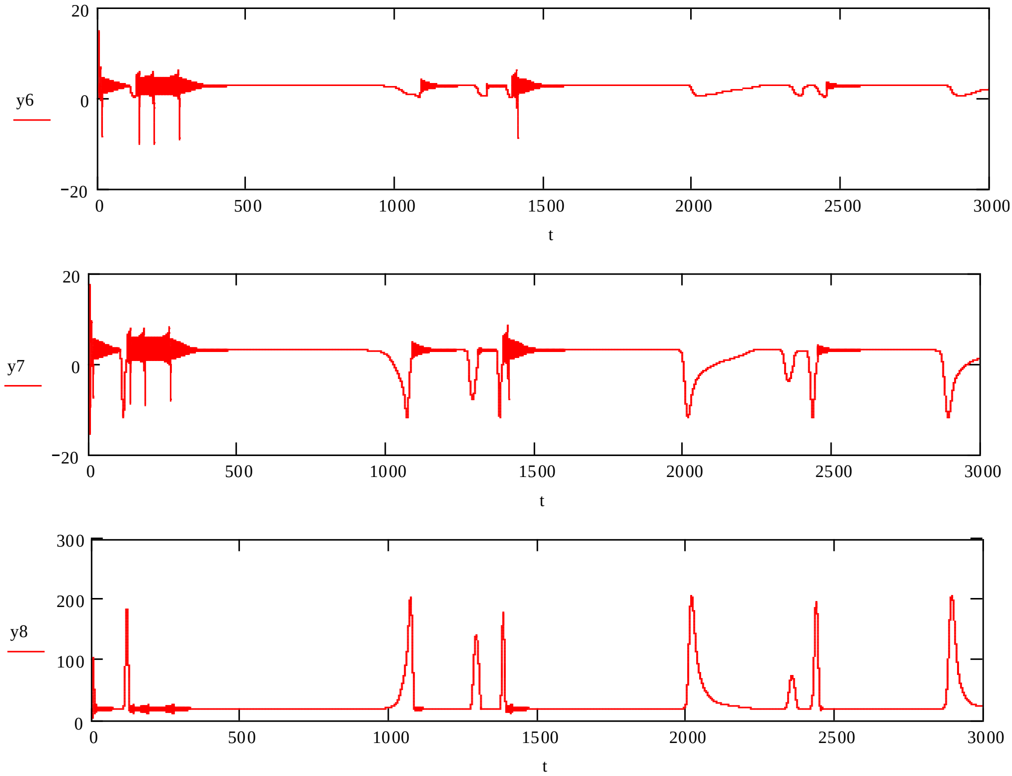


Fig. 5.33

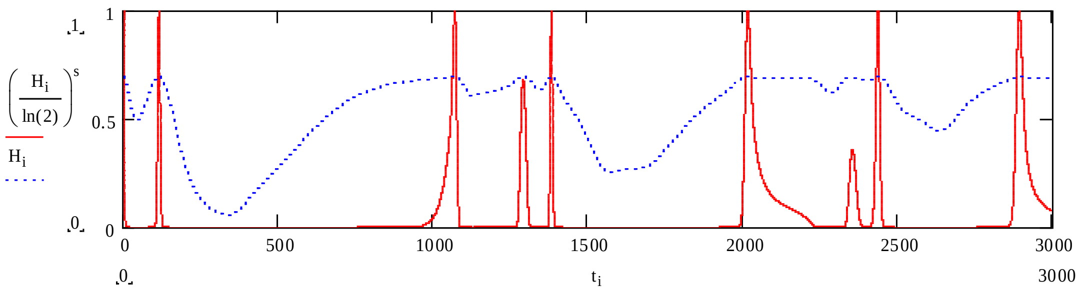


Fig. 5.34

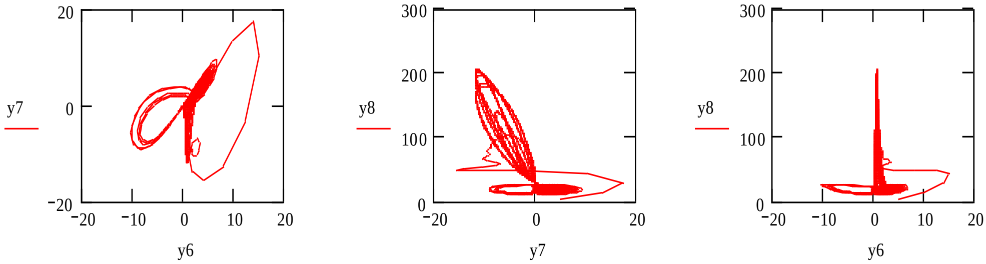


Fig. 5.35

Let us examine system of equations:

$$D(t,Y)=\begin{bmatrix} Y_0Y_1\begin{bmatrix}(\ln r_0-\ln r_1)\left[aY_3-bY_2+f\sin(kt)-hY_2^2+\left[\frac{-(Y_0\ln Y_0+Y_1\ln Y_1)}{\ln 2}\right]^s p\right]\ldots\\ (\ln r_0-\ln r_1)\left[-Y_3-Y_2Y_4+cY_2-mY_3^2+\left[\frac{-(Y_0\ln Y_0+Y_1\ln Y_1)}{\ln 2}\right]^s p\right]\end{bmatrix}\\ Y_0Y_1\begin{bmatrix}(\ln r_1-\ln r_0)\left[aY_3-bY_2+f\sin(kt)-hY_2^2+\left[\frac{-(Y_0\ln Y_0+Y_1\ln Y_1)}{\ln 2}\right]^s p\right]\ldots\\ (\ln r_1-\ln r_0)\left[-Y_3-Y_2Y_4+cY_2-mY_3^2+\left[\frac{-(Y_0\ln Y_0+Y_1\ln Y_1)}{\ln 2}\right]^s p\right]\end{bmatrix}\\ aY_3-bY_2+f\sin(kt)-hY_2^2+\left[\frac{-(Y_0\ln Y_0+Y_1\ln Y_1)}{\ln 2}\right]^s p\\ -Y_3-Y_2Y_4+cY_2-mY_3^2+\left[\frac{-(Y_0\ln Y_0+Y_1\ln Y_1)}{\ln 2}\right]^s p\\ Y_2Y_3-dY_4-nY_4^2+\left[\frac{-(Y_0\ln Y_0+Y_1\ln Y_1)}{\ln 2}\right]^s p \end{bmatrix} \qquad (5.133)$$

$$Y_i\Big|_{t=0}=\begin{pmatrix}0{,}95\\0{,}2\\0\\0\\0\end{pmatrix}.$$

With following values of the parameters given below

a = 8; b = 8; c = 20; d = 0,43; f = 0; h = 0,45; m = 0,01; n = 0,005;
k = 0,1; p = 10; ω = 0,1; s = 20; r_0 = 0,6

the solution takes the form, presented on the Fig. 5.36.

Fig. 5.37 shows the phase portraits of attractor.

Fig. 5.38 demonstrates a change in the relative entropy.

We see that the attractor perturbations extend to preferences, correspondingly, the latter also experience bifurcation.

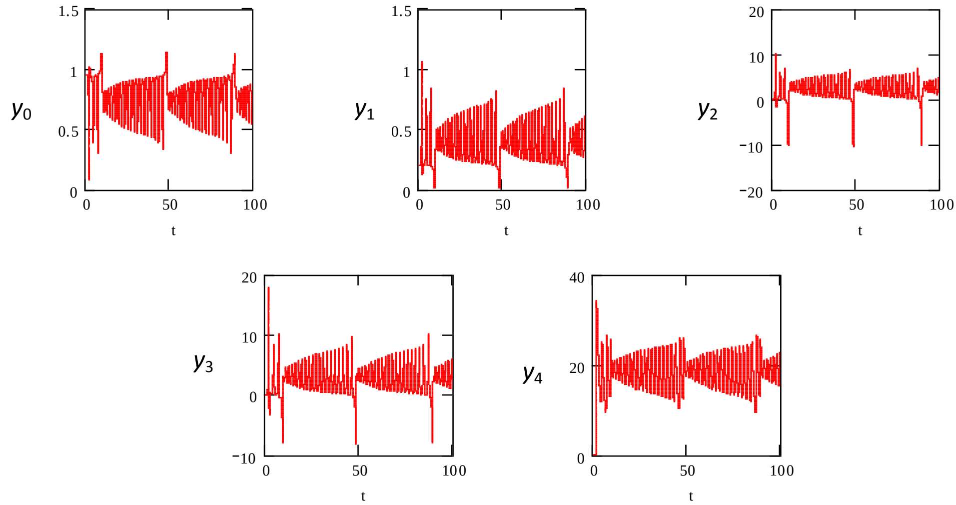


Fig. 5.36

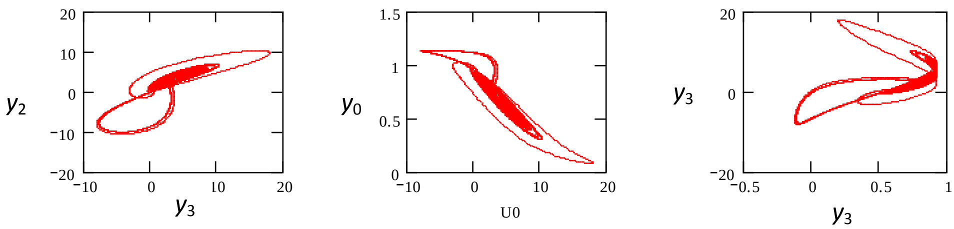


Fig. 5.37

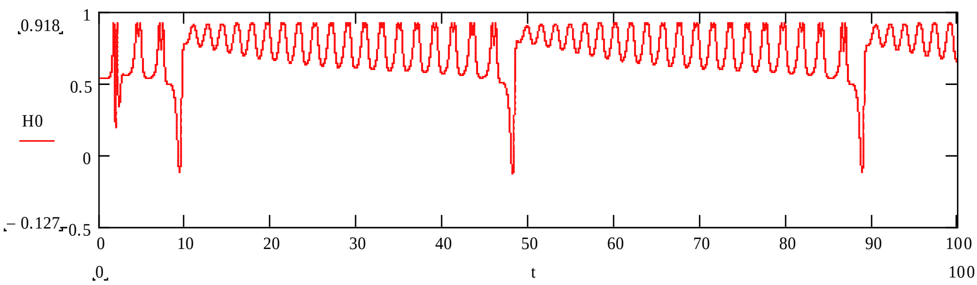


Fig. 5.38

In the present paragraph the possible approaches to the simulation of exogenous dynamics are outlined, as well as joint exogenous and endogenous dynamics. The author is completely aware, that a number of assumptions bear heuristic nature. Just as we have right to use different simplified models of the regression type, auto regression and similar others, which reflect the phenomenological described processes and are not based on „physical" considerations, so in this case the author has right to use more complex mathematical objects (for example, attractors) for the phenomenological description of some subjective processes, physics of which is unknown. This way is seemed promising to the author, especially, if it will be subsequently combined with the appropriate statistical experiment.

5.8. Elasticity and rigidity of preferences – elasticity and the rigidity of psyche. Sensitivity of preferences

In the present chapter the elasticity of the preferences of the I and II type are introduced. Like preferences, elasticity appears as the tool of subjective analysis. The property of psyche, which they characterize, can be named „the elasticity of psyche". The presence of the canonical distributions of preferences makes this concept and corresponding analysis, meaningful. With the aid of the elasticity it is possible to describe the specific types of psyche. It is assumed that this would be important and productive in such directions as the theory of instruction, security issue of active systems, in the economy, especially, in the theory of advertizing activity, in political science and in a number of other subject areas. As if is known, the elasticity of function y (x) with respect to the parameter x is determined by the formula

$$\varepsilon_y^x = \frac{y'}{y} x .$$

Elasticity is the limit of the quotient of the relative increment of function to the relative and increment of independent variable

$$\varepsilon_y^x = \lim_{\Delta x \to 0} \frac{\dfrac{\Delta y}{y}}{\dfrac{\Delta x}{x}}. \tag{5.134}$$

Following relationships are valid for the elasticity

$$\varepsilon_{yz}^x = \varepsilon_y^x + \varepsilon_z^x; \tag{5.135}$$

$$\varepsilon_{y/z}^x = \varepsilon_y^x - \varepsilon_z^x \tag{5.136}$$

Elasticity of complex function *y*(*z* (*x*))

$$\varepsilon_{y(z)}^x = \varepsilon_y^z \cdot \varepsilon_z^x. \tag{5.137}$$

The elasticity of preferences $\pi(\sigma_k)$ or $\xi(\sigma_k)$ and their different generalizations with respect to the quantitative indicators, which are contained in the canonical distributions, we will name *canonical subjective elasticity*.

If σ_k is understood as quantity indicator, then the elasticity of object preferences is the first type preferences are

$$\varepsilon_\pi^\sigma = \frac{\frac{\partial \pi}{\partial \sigma}}{\pi}\sigma.$$

In the particular situations as such index comes out the utility $U(\sigma_k)$, or the specific form of the resources: $R^{disp}(\sigma_k) = R^d(\sigma_k)$; $R^{req}(\sigma_k) = R^r(\sigma_k)$; $R_p{}^{disp}(\sigma_k)$; $R_p{}^{req}(\sigma_k)$; $R_a{}^{disp}(\sigma_k)$; $R_a{}^{req}(\sigma_k)$; ..., where the indices d, R, p, a designate those resources as: „located", „required", „passive", „active". For example,

$$\varepsilon_\pi^U = \frac{\frac{\partial \pi}{\partial U}}{\pi}U. \tag{5.138}$$

There is „the elasticity of preference with respect to the utilities". Elasticity π relatively the located and required resources are determined by the formulas

$$\varepsilon_\pi^d = \frac{\frac{\partial \pi}{\partial R^d}}{\pi}R^d; \tag{5.139}$$

$$\varepsilon_\pi^U = \frac{\frac{\partial \pi}{\partial R^r}}{\pi}R^r. \tag{5.140}$$

Ratings elasticity is determined by the analogous formulas:

$$\varepsilon_\xi^\sigma = \frac{\frac{\partial \xi}{\partial \sigma}}{\xi}\sigma; \tag{5.141}$$

$$\varepsilon_\xi^U = \frac{\frac{\partial \xi}{\partial U}}{\xi}U; \tag{5.142}$$

$$\varepsilon_\xi^d = \frac{\frac{\partial \xi}{\partial R^d}}{\xi}R^d; \tag{5.143}$$

$$\varepsilon_{\xi}^{r} = \frac{\frac{\partial \xi}{\partial R^{r}}}{\xi} R^{r}. \tag{5.144}$$

For the group of subjects appears the additional form of elasticity characterizing „the force" of the mutual influence of subjects in the group. The preferences elasticity of the first type of subject *i* with respect to the preferences of subject *j*:

$$\varepsilon_{\pi_j}^{\pi_i} = \frac{\frac{\partial \pi_j}{\partial \pi_i}}{\pi_j} \pi_i = \frac{\partial \pi_j}{\partial \pi_i} \frac{\pi_i}{\pi_j}, \tag{5.145}$$

where π_i and π_j is „similar" preferences of subjects *i* and *j* (i.e., the preference of the one and the same alternative σ_k). Ratings ξ_j, ξ_i mutual elasticity

$$\varepsilon_{\xi_j}^{\xi_i} = \frac{\frac{\partial \xi_j}{\partial \xi_i}}{\xi_j} \xi_i = \frac{\partial \xi_j}{\partial \xi_i} \frac{\xi_i}{\xi_j}. \tag{5.146}$$

As it was shown above, the preferences of the subject of first type in the group composition are functions of its rating and vice versa, ratings can be the functions of the „object" preferences. In connection with this the cross elasticity are introduced:

$$\varepsilon_{\pi}^{\xi} = \frac{\frac{\partial \pi}{\partial \xi}}{\pi} \xi = \frac{\partial \pi}{\partial \xi} \frac{\xi}{\pi}; \tag{5.147}$$

$$\varepsilon_{\xi}^{\pi} = \frac{\frac{\partial \xi}{\partial \pi}}{\xi} \pi = \frac{\partial \xi}{\partial \pi} \frac{\pi}{\xi}. \tag{5.148}$$

Finally, we can speak about the mutual elasticity of the individual (object) preferences of the first type

$$\varepsilon_{\pi(\sigma_k)}^{\pi(\sigma_i)} = \frac{\frac{\partial \pi(\sigma_k)}{\partial \pi(\sigma_i)}}{\pi(\sigma_k)} \pi(\sigma_i). \tag{5.149}$$

It is easy to notice that for the standardized canonical preferences, which have a structure, expressed by the formula

$$\pi\left(\sigma_k\right) = \frac{f\left(k\right)}{\sum_{j=1}^{N} f\left(j\right)}, \tag{5.150}$$

the following condition is satisfied

$$\varepsilon_{\pi(\sigma_k)}^{\pi(\sigma_i)} = \frac{\pi\left(\sigma_i\right)}{\pi\left(\sigma_k\right)}.$$

Also the specific general properties of elasticity emerge from standardization conditions. Since

$$\sum_{k=1}^{N} \pi\left(\sigma_k\right) = 1 \text{, then } \sum_{k=1}^{N} \frac{\partial \pi\left(\sigma_k\right)}{\partial \sigma_i} = 0.$$

Hence we obtain, that

$$\sum_{k=1}^{N} \varepsilon_{\pi(\sigma_k)}^{\sigma_i} \pi\left(\sigma_k\right) = 0, \;\; (i \in 1, N). \tag{5.151}$$

This means that *the vector of elasticity with respect to any alternative is orthogonal to the vector of preferences*.

In particular for S_a, that contains only two alternatives σ_1 and σ_2, we obtain

$$\frac{\varepsilon_{\pi_1}^{\sigma_1}}{\varepsilon_{\pi 2}^{\sigma_1}} = -\frac{\pi_2}{\pi_1}.$$

Since relationship (5.151) is correct for any alternative σ_i ($i \in \overline{1, N}$), then designating by ε the elasticity matrix $\left\| \varepsilon_{\pi_k}^{\sigma_i} \right\|$ and by $\pi^{\mathrm{T}} = (\pi_1, \pi_2, \ldots, \pi_N)$ is the vector of the preferences of the first type (subscript shows the alternative number), we can write the condition

$$\varepsilon\pi = 0. \tag{5.152}$$

Having in mind the functional form of preference (5.150), we obtain the following useful formula

$$\varepsilon_{\pi_k}^{\sigma_k} = \left(1 - \pi\left(\sigma_k\right)\right) \varepsilon_{f_k}^{\sigma_k}, \tag{5.153}$$

where $\varepsilon_{f_k}^{\sigma_k}$ is the elasticity of numerator in (5.150) based on the factor σ_k:

$$\varepsilon_{f_k}^{\sigma_k} = \frac{\dfrac{\partial f\left(\sigma_k\right)}{\partial \sigma_k}}{f\left(\sigma_k\right)} \sigma_k.$$

Elasticity $\varepsilon_{\pi_k}^{\sigma_j}$ with $j \neq k$ is determined from the formula

$$\varepsilon_{\pi_k}^{\sigma_j} = -\pi\left(\sigma_j\right)\varepsilon_{f_j}^{\sigma_j}. \tag{5.154}$$

The following formula occurs:

$$\varepsilon_{\varepsilon_\pi^\sigma}^t = \frac{\frac{d}{dt}\left(\varepsilon_\pi^\sigma\right)}{\varepsilon_\pi^\sigma} t = \varepsilon_{\pi'_\sigma}^t - \varepsilon_\pi^t + \varepsilon_\sigma^t. \tag{5.155}$$

where $\varepsilon_{\varepsilon_\pi^\sigma}^t$ is time elasticity of the preference elasticity by with respect to the factor ε_π^σ. Let us explicitly calculate flexibilities for the basic factor σ_k for two most characteristic preference functions:

$$\pi\left(\sigma_k\right) = \frac{e^{-\beta\sigma_k}}{\sum_{j=1}^{N} e^{-\beta\sigma_j}}; \quad \pi\left(\sigma_k\right) = \frac{\sigma_k^\alpha e^{-\beta\sigma_k}}{\sum_{j=1}^{N} \sigma_j^\alpha e^{-\beta\sigma_j}}. \tag{5.156}$$

Here through σ_k is designated the index, which determines preferences (utility U, the available resources R^d, required resources R^r, ..). Direct accounting gives us for the first function

$$\varepsilon_{\pi(\sigma_k)}^{\sigma_k} = \frac{-\beta e^{-\beta\sigma_k}\sum_{j=1}^{N} e^{-\beta\sigma_k} + \beta e^{-2\beta\sigma_k}}{\left(\sum_{j=1}^{N} e^{-\beta\sigma_j}\right)^2}\frac{\sigma_k}{\pi\left(\sigma_k\right)}.$$

We find after transformation:

$$\varepsilon_{\pi(\sigma_k)}^{\sigma_k} = -\beta\left(1 - \pi\left(\sigma_k\right)\right)\sigma_k. \tag{5.157}$$

For the case of the second function

$$\varepsilon_\pi^{\sigma_k} = \left(\alpha - \beta\sigma_k\right)\left(1 - \pi\left(\sigma_k\right)\right), \tag{5.158}$$

comparing with formulas (5.153) and (5.154), we see that in the first case $\varepsilon_{f_k}^{\sigma_k} = -\beta\sigma_k$, in the second case $\varepsilon_{f_k}^{\sigma_k} = \alpha - \beta\sigma_k$.

It follows from (5.153) that for the function of first type $\pi(\sigma_k)$ the elasticity $S_{\pi_k}^{\sigma_k}$ is negative, if $\beta > 0$, $\sigma_k > 0$ and grows in the absolute value with the increase of $\sigma_k(1 - \pi(\sigma_k) > 0)$, on the contrary, the elasticity $S_{\pi_k}^{\sigma_j}$ is positive and grows with the increase of σ_k. In the case function $\pi(\sigma_k)$ belong to the second type the elasticity $S_{\pi_k}^{\sigma_k}$ and $S_{\pi_k}^{\sigma_j}$ become zero at the point $\sigma_k = \frac{\alpha}{\beta}$, ($\sigma_j = \frac{\alpha}{\beta}$) and at this point reverses the sign, moreover, if $S_{\pi_k}^{\sigma_k} > 0$, that $S_{\pi_k}^{\sigma_j} < 0$ and vice versa.

We will use the obtained formulas in order to write down the differential equation, which describes a change of the elasticity with time.

Equation (5.3) can be rewritten in the form:

$$\frac{d\pi(\sigma_k)}{dt} = \pi(\sigma_k)\left[\varepsilon_{f_k}^{\sigma_k}\frac{\dot{\sigma}_k}{\sigma_k} - \sum_{j=1}^{N}\pi(\sigma_j)\varepsilon_{f_j}^{\sigma_j}\frac{\dot{\sigma}_j}{\sigma_j}\right]. \tag{5.159}$$

If we designate $\varepsilon_{\pi(\sigma_k)}^{t}$ elasticity $\pi(\sigma_k)$ relative to time, and $\varepsilon_{\sigma_k}^{t}$ – the elasticity of the parameter σ_k on the time, then basing on (5.159) we obtain:

$$\varepsilon_{\pi(\sigma_k)}^{t} = \varepsilon_{f_k}^{\sigma_k} S_{\sigma_k}^{t} - \sum_{j=1}^{N}\pi(\sigma_j)\varepsilon_{f_j}^{\sigma_j} S_{\sigma_j}^{t}. \tag{5.160}$$

This equation for the elasticity contains also preferences $\pi(\sigma_j)$ and, therefore, must be examined simultaneously with the equation, which determines the dynamics of preferences. Equation (5.160) is not a differential equation. It only indicates the connection between different flexibilities. Similar relationships have place for the rating preferences.

The study of sensitivity and elasticity of preferences with respect to the structural parameters of functions $\pi(\sigma_k)$, $\xi(\sigma_k)$,..., such, as α and β and others is presented is meter of interest. Let the sensitivity function is defined as:

$$S_{\pi_k}^{\beta} = \frac{\partial\pi(\sigma_k)}{\partial\beta}, \tag{5.161}$$

then basing on equation (5.3) it is possible to obtain the differential equation (for $\pi(\sigma_k)$ of the first type)

$$\frac{dS_{\pi_k}^{\beta}}{dt} = \left(\pi(\sigma_k) + \beta S_{\pi_k}^{\beta}\right)\sum_{j=1}^{N}{}'\dot{\sigma}_j = \left(\pi(\sigma_k) + \beta S_{\pi_k}^{\beta}\right)\sum_{j=1}^{N}{}' g_j. \tag{5.162}$$

However, it is possible to calculate the function of sensitivity directly from the formula:

$$S_{\pi_k}^{\beta} = \pi(\sigma_k)\left(-\sigma_k + \sum_{j=1}^{N}\pi(\sigma_j)\sigma_j\right). \tag{5.163}$$

A study of subjective elasticity and subjective sensitivities presents one of the possibilities of the mental processes analysis, connected with decision making on many alternatives.

The elasticity of preferences of type (5.156) with respect to the endogenous parameters α and β are determined by the formulas:

$$\varepsilon_{\pi_i}^{\beta} = \beta\left(\sigma_i - \sum_{j=1}^{N} \sigma_j \pi_j\right); \qquad (5.164)$$

$$\varepsilon_{\pi_i}^{\alpha} = \alpha^2\left(\sigma_i^{-1} - \sum_{j=1}^{N} \sigma_j^{-1} \pi_j\right). \qquad (5.165)$$

Here, as it was above, σ_i am understood as the quantitative characteristic (continuous) of the given alternative. If there are only two alternatives σ_1 and σ_2, then from (5.164) and (5.165) we can find that

$$\varepsilon_{\pi_1}^{\beta} = \beta\left(\sigma_1 - \sigma_2\right)\pi_2; \quad \varepsilon_{\pi_2}^{\beta} = \beta\left(\sigma_2 - \sigma_1\right)\pi_1;$$

$$\varepsilon_{\pi_1}^{\alpha} = \alpha^2\left(\frac{\sigma_2 - \sigma_1}{\sigma_1\sigma_2}\right)\pi_2; \quad \varepsilon_{\pi_2}^{\alpha} = \alpha^2\left(\frac{\sigma_1 - \sigma_2}{\sigma_1\sigma_2}\right)\pi_1.$$

Hence we can find, that in both cases

$$\frac{\varepsilon_{\pi_1}^{\beta}}{\varepsilon_{\pi_2}^{\beta}} = \frac{\varepsilon_{\pi_1}^{\alpha}}{\varepsilon_{\pi_2}^{\alpha}} = -\frac{\pi_2}{\pi_1}. \qquad (5.166)$$

This result from point of view of the „common sense" appears very realistically: the more I want something (the greater $\pi_1 > 0$), the less elasticity of my psyche with respect to this alternative. In this case, since $\pi_1 > 0$ and $\pi_2 > 0$, elasticity have different signs.

Easily let us find, that the sum of the flexibilities

$$\left.\begin{aligned} \varepsilon_{\pi_1}^{\beta} + \varepsilon_{\pi_2}^{\beta} &= \beta\left(\sigma_1 - \sigma_2\right)\left(\pi_2 - \pi_1\right); \\ \varepsilon_{\pi_1}^{\alpha} + \varepsilon_{\pi_2}^{\alpha} &= \alpha^2\left(\frac{\sigma_1 - \sigma_2}{\sigma_1\sigma_2}\right)\left(\pi_2 - \pi_1\right). \end{aligned}\right\} \qquad (5.167)$$

hence it follows, that with the equality of preferences ($\pi_1 = \pi_2$) the sum of elasticity must be equal to zero:

$$\varepsilon_{\pi_1}^{\alpha(\beta)} + \varepsilon_{\pi_2}^{\alpha(\beta)} = 0 \quad \text{or} \quad \varepsilon_{\pi_2}^{\alpha(\beta)} = -\varepsilon_{\pi_1}^{\alpha(\beta)}.$$

Let σ_i come out as the corresponding utility U_i. The previous relationship should be understood thus: if the utility of the first alternative is more than the utility of the second alternative: $U_1 > U_2$ that $\varepsilon_{\pi_1}^{\alpha(\beta)} > 0$, and $\varepsilon_{\pi_2}^{\alpha(\beta)} < 0$ and vice versa, $U_1 < U_2$, $\varepsilon_{\pi_1}^{\alpha(\beta)} < 0$ and $\varepsilon_{\pi_2}^{\alpha(\beta)} > 0$, finally, if utility are identical, the elastics for the first and second alternative are simultaneously equal to zero. In this case from (5.166) it follows, that

$$\lim_{U_1 \to U_2} \frac{\varepsilon_{\pi_1}^{\alpha(\beta)}}{\varepsilon_{\pi_2}^{\alpha(\beta)}} = -1, \text{ a } \pi_1 \to \pi_2.$$

The sum of elasticity, as it can be seen from (5.167) is also equal to zero, if $\pi_1 = \pi_2$. According to (5.156) this condition is satisfied simultaneously with the condition $\sigma_1 = \sigma_2$(or $U_1 = U_2$).

It is possible to assert, consequently, that if the entropy

$$H_\pi = H_{\pi\max} = \ln 2,$$

they the sum of subjective elasticity is equal to zero, and their relation is equal - 1.

The correspondence of these conclusions to the „common sense" can be seen also as indirect confirmation of the realism of the theory basic assumption about existence of the subjective principle of optimality. Let us separate the sensitivities and the elasticity of preferences on the two types, depending, on respect to what they are calculated.

The values, which are contained in the canonical distributions, are divided into two substantially different groups: *endogenous* characteristics and *exogenous* characteristics. In the sufficiently common form the canonical distribution of preferences can be represented in the form:

$$\pi(\sigma_i) = \pi(\alpha, \beta, I_q, e_i, \dots, U_i(R_i^r, R_i^d, R_i^e, \dots)). \tag{5.168}$$

Here α, β, I_q, e,... - *endogenous* characteristics (α, β - the structural parameters of distribution, I_q - ethical imperatives, e_i - the characteristics, which relate to the instruction process,...); R_i^r, R_i^d, R_i^e, ... - *exogenous* characteristics (resources required, available, expected,...

Accordingly, it is possible to tell about the exogenous elasticity, i.e., elasticity with respect to a change *in the exogenous* („external") characteristics, and endogenous elasticity with respect to *the endogenous* („internal") characteristics. The latter appear steadier as a result of the prolonged process of instruction, training, and also reflecting the physiological needs, the innate properties of psyche, which exist on the genetic level. Thus, for instance, exogenous elasticity can be connected with the receptivity of training information, and endogenous elasticity with the receptivity to the educational processes in the broad sense of this word.

The separation of the characteristics, which are contained in the preference functions models, on the two types proves to be highly useful in this sense.

The possibility to speak about two directions of the instruction appears: the forming of endogenous and exogenous characteristics and action through them on the of trainee distribution of the preferences.

Let us examine some special cases of the elasticity calculating. The simplest model of the canonical distribution of preferences depending on required resources R_i^r takes the form:

$$\pi(\sigma_i) = \pi_i = \frac{e^{-\beta R_i^r}}{\sum_{j=1}^{N} e^{-\beta R_j^r}}.$$

Elasticity $\pi(\sigma_i)$ on R_i^r is determined by the formula

$$\varepsilon_{\pi_i}^{R_i^r} = -\beta(1-\pi_i)R_i^r. \tag{5.169}$$

Since $\pi_i < 1$, $R_i^r > 0$, $\beta > 0$, than $\varepsilon_{\pi_i}^{R_i^r} < 0$. With an increase of the preference π_i the elasticity absolute value decreases and for $\pi_i = 1$ becomes zero. The greater is the elasticity absolute value, the greater are the required resources R_i^r with the condition, that $\pi_i \neq 1$. One of the preferences distribution model takes the form:

$$\pi(\sigma_i) = \frac{x_i^{\alpha} e^{-\beta x_i^{\mu}}}{\sum_{j=1}^{N} x_j^{\alpha} e^{-\beta x_j^{\mu}}}, \tag{5.170}$$

where x_i - given relative resources, determined by the formulas

$$x_i = \frac{\bar{r}_i}{1-\bar{r}_i}; \quad \bar{r}_i = \frac{R_i^r}{R_i^d}.$$

When $R_i^r \to R_i^d$, $x_i \to \infty$, with $R_i^r = 0$, $R_i^d \neq 0$, $x_i = 0$. Let us determine for distribution (5.170) elasticity on the required resources and elasticity on the available resources.

We have:

$$\frac{\partial \pi(\sigma_i)}{\partial R_i^r} = \left(\alpha x_i^{-1} - \beta\mu x_i^{\mu-1}\right)\pi(\sigma_i)\left(1-\pi(\sigma_i)\right)x_i^2 \frac{1}{\bar{r}_i^2}\frac{1}{R_i^d};$$

$$\frac{\partial \pi(\sigma_i)}{\partial R_i^d} = -\left(\alpha x_i^{-1} - \beta\mu x_i^{\mu-1}\right)\pi(\sigma_i)\left(1-\pi(\sigma_i)\right)x_i^2 \frac{1}{\bar{r}_i}\frac{1}{R_i^d}.$$

We see that $\frac{\partial \pi(\sigma_i)}{\partial R_i^d} = -\frac{\partial \pi(\sigma_i)}{\partial R_i^r}\bar{r}_i$.

Let us find the elasticity :

$$\varepsilon_{\pi_i}^{R_i^r} = \frac{\dfrac{\partial \pi(\sigma_i)}{\partial R_i^r}}{\pi(\sigma_i)} R_i^r = \left(\alpha x_i^{-1} - \beta\mu x_i^{\mu-1}\right)\left(1-\pi(\sigma_i)\right) x_i^2 \frac{1}{\overline{r}_i}; \tag{5.171}$$

$$\varepsilon_{\pi_i}^{R_i^d} = \frac{\dfrac{\partial \pi(\sigma_i)}{\partial R_i^d}}{\pi(\sigma_i)} R_i^d = \left(\alpha x_i^{-1} - \beta\mu x_i^{\mu-1}\right)\left(1-\pi(\sigma_i)\right) x_i^2. \tag{5.172}$$

Hence, similar to the previous, we can find the connection

$$\varepsilon_{\pi_i}^{R_i^r} = -\varepsilon_{\pi_i}^{R_i^d} \frac{1}{\overline{r}_i} = -\varepsilon_{\pi_i}^{R_i^d} \frac{R_i^d}{R_i^r}. \tag{5.173}$$

Consequently

$$\frac{\varepsilon_{\pi_i}^{R_i^r}}{\varepsilon_{\pi_i}^{R_i^d}} = -\frac{R_i^d}{R_i^r},$$

i.e., the elasticity $\varepsilon_{\pi_i}^{R_i^r}$ and $\varepsilon_{\pi_i}^{R_i^d}$ relate inversely proportional to the values of resources and have always different signs.

One additional version of the elasticity calculation relates to the case, when as the resources the probability of specific event P_i comes out. This can be the probability of the successful resolution of problem P: $\sigma_0 \to \sigma_i$. Let

$$\pi(\sigma_i) = \frac{e^{\beta P_i}}{\sum_{j=1}^{N} e^{\beta P_j}}.$$

Here $\pi(\sigma_i)$ depend on probabilities ($\pi_i = \pi(P_i)$): $p_1, p_2, \ldots, p_N$ and therefore it appears as random variable.
Elasticity $\varepsilon_{\pi_i}^{P_i}$ is expressed by the formula

$$\varepsilon_{\pi_i}^{P_i} = \beta\left(1-\pi(P_i)\right)P_i. \tag{5.174}$$

Along with the preferences elasticity we identify above with the elasticity of psyche and which reflects pliability, variability, adaptability of subject it is naturally to introduce in a certain sense the opposite characteristic, which, vice versa reflects hardness, stability, stability of desires and which can be named *the hardness of* *preferences* or *„the rigidity of psyche"*. Let us define the hardness of preferences as the value

$$\Re_{\pi}^{U} = \left(\varepsilon_{\pi}^{U}\right)^{-1}. \qquad (5.175)$$

In this case the minimum hardness, corresponds to the maximum elasticity and to the minimum elasticity – the maximum hardness. If $\varepsilon_{\pi}{}^{U} = 0$, then hardness is equal to the infinity. Since elasticity of preferences $\varepsilon_{\pi}{}^{U}$ on the utility (as the different versions of elasticity , examined above) can take both positive and negative values, rigidity can have both positive and negative value as well.

If for $y = f(x)$ the inverse function $x = \varphi(y)$ is determined, then

$$\varepsilon_{y}^{x} = \frac{\frac{df}{dx}}{y}x; \quad \varepsilon_{x}^{y} = \frac{\frac{d\varphi}{dy}}{x}y. \qquad (5.176)$$

and consequently $\varepsilon_{y}^{x} = \left(\varepsilon_{x}^{y}\right)^{-1}$. Then we see that the rigidity is equal to the inverse elasticity . When $y = f(x, t)$ is the function of two variables, is easy to notice that in the case of existence of the inverse functions

$$x = \varphi(y, t); \; t = \psi(y, x)$$

the relationships occur

$$\frac{\partial\varphi}{\partial y} = \frac{1}{\frac{\partial f}{\partial x}}; \quad \frac{\partial\psi}{\partial y} = \frac{1}{\frac{\partial f}{\partial t}},$$

then

$$\varepsilon_{x}^{y} = \frac{\frac{\partial\varphi}{\partial y}}{x}y = \frac{y}{\frac{\partial f}{\partial x}x} = \frac{1}{\varepsilon_{y}^{x}} = \Re_{y}^{x}; \qquad (5.177)$$

$$\varepsilon_{t}^{y} = \frac{\frac{\partial\psi}{\partial y}}{t}y = \frac{y}{\frac{\partial f}{\partial t}t} = \frac{1}{\varepsilon_{y}^{t}} = \Re_{y}^{t}. \qquad (5.178)$$

Consequently, the rigidities quotients are also inversely proportional to the corresponded elasticity quotients. Interesting is the detailed study *of elasticity* and *rigidity* of psyche, the construction of corresponded isoquantum, the dynamics of elasticity and rigidities. We see from the formulas given above, that the elasticity for the alternative σ_i becomes zero, and rigidity becomes infinity, if preference $\pi(\sigma_i) = 1$, i.e., if $H_{\pi} = 0$. Let us discuss briefly the concept *of elasticity with respect to the* *ethical imperatives*. This type of elasticity is exceptionally important, if the discussion deals with the training processes in the broad sense. The model presentation of this type of elasticity depends on the way how the model of preferences is built.

If I_q - ethical imperative and there is a model of preferences $\pi(\sigma_i, I_q, \ldots)$, the corresponding elasticity could be expressed by the formula

$$\varepsilon_{\pi}^{I_q}(\pi) = \frac{\mathrm{var}_I\, \pi(\sigma_i, I_q, \ldots)}{\pi(\sigma_i, I_q)}\theta(I_q). \tag{5.179}$$

Here $\mathrm{var}_I\, \pi$ means absolute change in the preferences during introduction or elimination of this imperative, and $\theta(I_q)$ is the Heaviside function, which reacts on the fact of imperative I_q calculation. The example given below illustrates, as the elasticity and rigidities alternatives with a change of the exogenous parameter. The following situation was simulated: the model of the I kind preferences distribution function takes the form:

$$\pi(\sigma_i) = \frac{x_i^{\alpha} e^{-\beta x_i^{\mu}}}{\sum_{j=1}^{N} x_i^{\alpha} e^{-\beta x_i^{\mu}}},$$

the set S_a contains three alternatives σ_1, σ_2, σ_3.

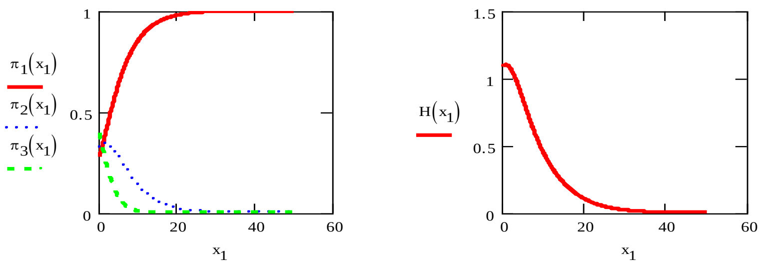
The exogenous parameters x_i represent related given resources: $x_i = \bar{r}_i \left(1 - \bar{r}_i\right)^{-1}$; $\bar{r}_i = \dfrac{R_i^{req}}{R_i^{disp}}$,. Let us assume that $\alpha = \beta = \mu = 1$, and the parameters x_1, x_2, x_3 are connected by the relationships $x_2 = cx_1$; $x_3 = dx_1$ and let x_1 changes within the limits of 0,1...50. These conditions can be interpreted thus as following: subject has the certain available resources R^{disp}, after appearance in the market of three different goods, which are of interest for the subject, his preferences are determined only by the relationship between the required resources (in this case - the price) and available resources. As the required resources approaches to the available „to the left" the parameters x_i grow and with $R_i^{req} \to R^{disp}$, $x_i \to \infty$.

We assume, that $c > 1$, $d > c$, therefore, the alternative σ_2 is more expensive than σ_1, and σ_3 is more expensive than σ_2. We want to trace as preferences $\pi(\sigma_1)$, $\pi(\sigma_2)$, $\pi(\sigma_3)$ will change, with the simultaneous proportional growth of the prices of alternatives (the goods). Further let us designate

$$\pi(\sigma_1) = \pi_1(x_1),\ \pi(\sigma_2) = \pi_2(x_1),\ \pi(\sigma_3) = \pi_3(x_1).$$

From Fig. 5.39, *a* we can see, that with the sufficiently large growth of prices the preferences „disperse", the preferences σ_2 and σ_3 are diminishing with different speed, and approach zero, and the preferability of the „cheapest" alternative σ_1 grows and it approaches the value of one. In this case (as for the examples, examined earlier) a simultaneous increase in the prices leads down to entropy reduction. As it is evident, however, at the certain point $\tilde{x}_1$ entropy reaches maximum, i.e., in the initial period of prices increase the entropy can increase, but then its drop occurs. If there is a critical value of the entropy H^*_π such, that with the occurrence of the inequality $H_\pi \le H^*_\pi$ the subject makes selection, then there exists a corresponding critical value x^*_1 (the price of the first alternative). Fig. 5.39, *b* shows the dependence of subjective entropy on x_1:

$$H_\pi(x_1) = -(\pi_1(x_1)\ln\pi_1(x_1) + \pi_2(x_1)\ln\pi_2(x_1) + \pi_3(x_1)\ln\pi_3(x_1)).$$



a *b*

Fig. 5.39

A change in the entropy is connected with the inflow or outflow of information. On Fig. 5.40 it is evident that the absolute inflow of information, computed on the base of the formula

$$I(x_1) = H_\pi(x_1) - H_\pi(x_1 + 0{,}1)$$

has maximum intensity, where entropy diminishes with the maximum speed and further approaches zero, when the change in the entropy ceases.

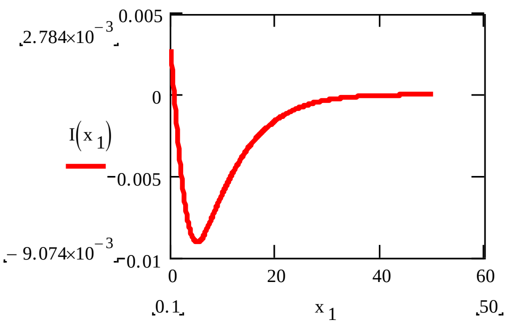
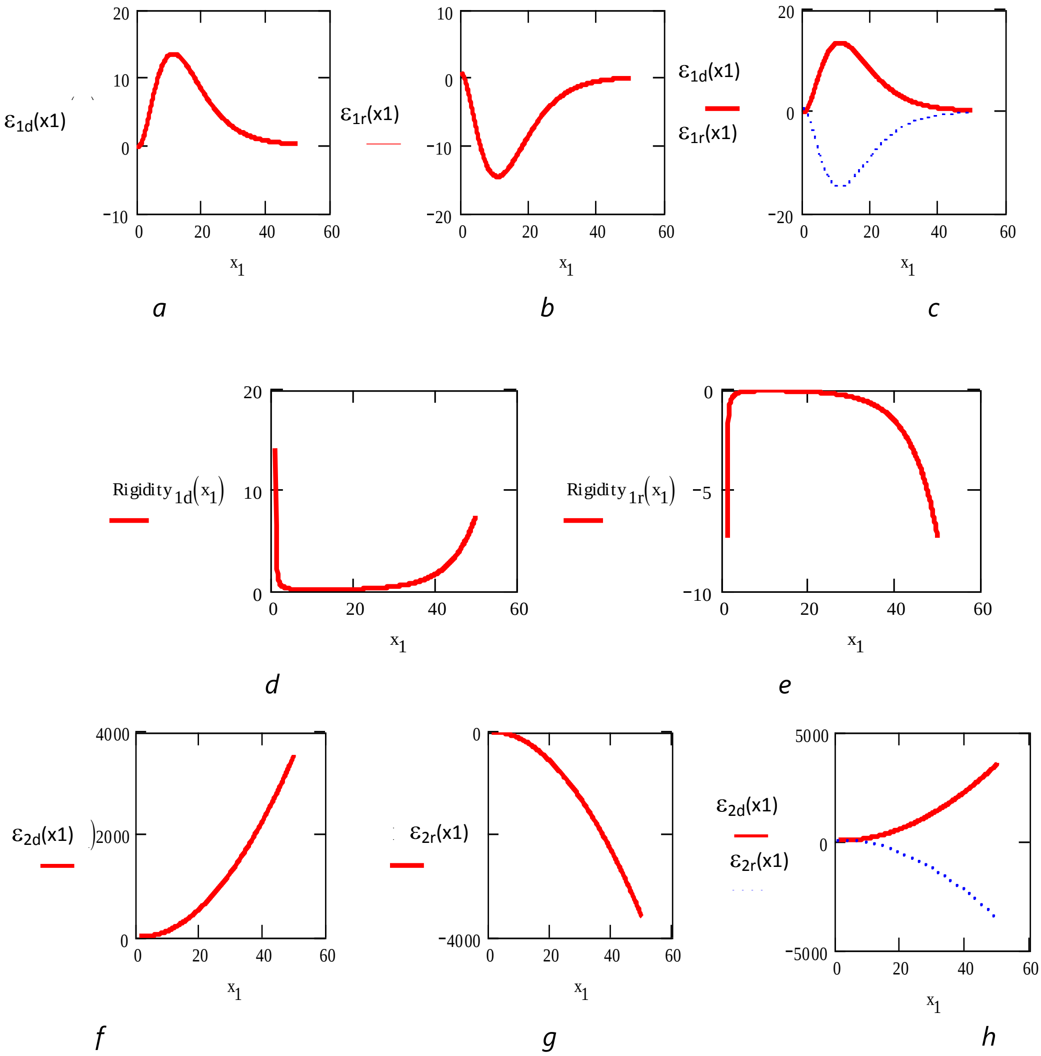
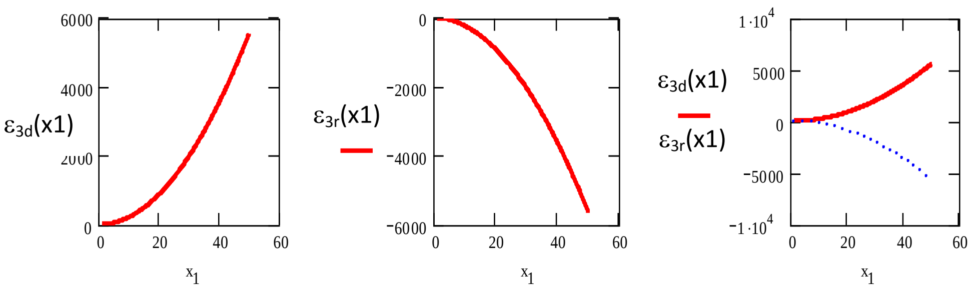


Fig. 5.40

On the following graphs (Fig. 5.41, *a*—*k*) show the elasticity and rigidity of subjective preferences changing. The designations are introduced for the elasticity relative to the required resources $\varepsilon_{ir}(x_i)$, relative to the available resources $\varepsilon_{id}(x_i)$, the rigidities of preferences are designated $Rigidity_{ir}(x_i)$ and $Rigidity_{id}(x_i)$ respectively (in the latter case the result is given only for the first alternative).





i *j* *k*

Fig. 5.41

We see that the elasticity $\pi(\sigma_1)$ relative to required resources is negative and has negative extremum, while rigidity with respect to the available resources is positive and has a maximum. As process damping both elasticity approach zero. This is completely obvious, because the preferences „were radiated" completely in this case and the conditions of total certainty arose ($H_\pi \cong 0$). The corresponding elasticity for $\pi(\sigma_2)$ and $\pi(\sigma_3)$ behave different (in the inspected range x_1). They do not have extreme and in the absolute value they grow. We see that the rigidities $Rigidity_{1r}(x_1)$ и $Rigidity_{1d}(x_1)$ approach infinity and they are close to zero, when preferences rapidly change, it is obvious, that for $\pi(\sigma_2)$ and $\pi(\sigma_3)$ (i.e. - for the rejected alternatives) the absolute value of rigidities will approach zero with an increase of x_1 (increase of the prices).

In the present chapter we only mentioned about the rigidity of the second kind preferences is the ratings, and also we completely did not mention a question about the elasticity of group preferences π^Σ.

5.9. Alternation of the problem- resource situation in the course of decision making on the self of alternatives

In this chapter we are going to study the dynamics of preferences, connected with decision making. It is assumed that individual alternatives set S_a evolve as a result of alternatives exception or addition. In this case the distribution of preferences also changes. These changes occur in the evolutionary, or revolutionary way. The changes in the exogenous shell and endogenous shifts are the reasons for changes.

One of the criteria, which can serve for determining the abrupt changes appears to be the entropy H_π. Let us introduce hypothesis, that in each situation the critical value of the entropy H^* is such, one that as soon as the following condition will be satisfied

$$H_\pi(S_a) < H^*,$$

subject makes decision about „retargeting" of the available resources. In a very special case he selects one of the alternatives $\sigma_i \in S_a$ and the corresponding problem P: $(\sigma_0 \to \sigma_i)$ is converted into aim. In this case immediately the part of the available resources $R^{disp}(R^{disp}(\sigma_i))$ have to be, equal to the required resources $R^{req}(\sigma_i)$ is separated. In the more general case several alternatives are selected immediately and the necessary required resources are canalized in several directions, or the redistribution of the already realizable resources flows occurs. The entropy H^*makes it possible to establish the moment t^* of decision making about selection as the aim σ_i:

$$t_i^*: H_\pi(S_a, t_i^*) = H^*$$

and

$$\left.\frac{dH_\pi}{dt}\right|_{t_i^*} < 0.$$

After that as the selection occurs, the structure of resources spasmodically changes, accordingly, the preferences and standardization conditions change. The latter circumstance is connected with the fact that selected alternative σ_i can be excluded from S_a. Formally it is possible to preserve S_a dimensionality, but to assume, that $\pi(\sigma_i) = 0$.

If the group of subjects is studied, then in the general case the moments of decision making are different, and, when all t_i^* belong to the small time interval δ, this means, that individual entropies reach the critical values approximately simultaneously.

In this case the threshold values for different individuals are not obligingly identical. If all t_i^* are different, the group effect in this sense is absent. Until now, the dynamics was studied preferably in the „continuous" version. This corresponds to the situation, when there are many „identical" subjects (with the identical collections of alternatives S_a, the identical distributions of preferences and the available resources), and also the sufficient amount of units „of goods" so that the single acquisition would not cause an abrupt change (essential) in the function of preference. Let us examine now the one isolated subject with the set of alternatives S_{aN}, available resources R_N^{disp} and distribution $\pi_N(\sigma_i)$. Suppose that a result of decision making the alternative σ_i is selected and later it is excluded from S_a.

Let us designate the set of alternatives before S_{aN}, and after decision S_{aN-1}. The following versions of problem are possible:

1. The available resources R^{disp} have such an amount, that for $\forall j \in \overline{1,N}$ $R^{req}(\sigma_j) < R_N^{disp}$. The specified distribution of the required resources $R^{req}(\sigma_j)$ and expected new resources, obtained as a result of the problem P: $(\sigma_0 \to \sigma_j)$; $R^{exp}(\sigma_j)$ solution is assigned on S_{aN}.

Let us examine the distribution of the utilitarian preferences, which are formed in accordance witch supposition dependence on the relationship between resources of the three indicated types. Let at first the preferences are determined by the relationship between excess of the expected resources above the required resources, and this relationship be expressed in the form the relative resources

$$\bar{r}_j = \frac{\delta R^{exp}\left(\sigma_j\right)}{R^{req}\left(\sigma_j\right)}; \quad \delta R^{exp}\left(\sigma_j\right) = R^{exp}\left(\sigma_j\right) - R^{req}\left(\sigma_j\right). \tag{5.180}$$

It is assumed that after decision making: selection as the aim the alternative σ_i the available resources decrease for the value of $R^{req}(\sigma_i)$. In the set S_{aN} one of the alternatives is σ_i am „excluded" in the sense, that corresponding preference $\pi^+(\sigma_i)$ becomes zero.

The sign „+" speaks, that the preferences, are determined through the utility and correspond to the positive expectations:

$$\pi^+\left(\sigma_j\right) = \frac{f_j\left(R^{exp}\left(\sigma_j\right), R^{req}\left(\sigma_j\right)\right)}{\sum_{k=1}^{N-1} f_k\left(R^{exp}\left(\sigma_k\right), R^{req}\left(\sigma_k\right)\right)}. \tag{5.181}$$

If the condition,

$$R_{N-1}^{disp} = R_N^{disp} - R^{req}\left(\sigma_i\right) > R^{req}\left(\sigma_j\right); \text{ при } \forall j \neq i, \tag{5.182}$$

is satisfied then in S_{aN-1} all the remaining alternatives are saved, and standardization condition can be written down in the form

$$\sum_{j=1}^{N-1} \pi^+\left(\sigma_j\right) = 1\,. \tag{5.183}$$

If (5.182) is not fulfilled and L alternatives exists in S_{aN}, such that

$$R_{N-1}^{disp} \le R^{req}\left(\sigma_j\right),$$

then L alternatives are excluded from S_{aN} and preferences are determined by the formula:

$$\pi^+\left(\sigma_j\right) = \frac{f_j\left(R^{exp}\left(\sigma_j\right), R^{req}\left(\sigma_j\right)\right)}{\sum_{k=1}^{N-1-L} f_k\left(R^{exp}\left(\sigma_k\right), R^{req}\left(\sigma_k\right)\right)}; \quad j \in \overline{1, N-1-L}. \tag{5.184}$$

2. Distribution of preferences is determined by relationship between the available resources and required or relative expenditures

$$\bar{r}_j = \frac{R^{req}\left(\sigma_j\right)}{R^{disp}\left(\sigma_j\right)}.$$

For simplification we will assume that the available resources are universal and they are not the function of alternatives number. In this case with equal other conditions, the smaller expenditures are desirable. Preferences are determined on the set of „negative" possibilities:

$$\pi^{-}\left(\sigma_j\right) = \frac{f_j\left(R^{req}\left(\sigma_j\right), R^{disp}\right)}{\sum_{k=1}^{N-1-L} f_k\left(R^{req}\left(\sigma_k\right), R^{disp}\right)}. \tag{5.185}$$

We will examine examples below. Each time the initial entropy of preferences will be compared with the entropy of the new distribution of preferences after „selection" σ_i (or, in more, general case - the subset of the alternatives S'_{aN}) their difference will defined the subjective information, connected with decision making - *selection of aim* (*event A)*:

$$I(|A) = H_\pi(N) - H_\pi(N - 1 - L). \tag{5.186}$$

In the particular case $L = 0$:

$$I(|A) = H_\pi(N) - H_\pi(N - 1).$$

Let us determine in parallel the dispersion of preferences and its change as a result in the selection of the aim:

$$D_{\pi,N} = \sum_{j=1}^{N}\left(\pi\left(\sigma_j\right) - \frac{1}{N}\right)^2; \tag{5.187}$$

$$D_{\pi,N-1-L} = \sum_{j=1}^{N-1-L}\left(\pi\left(\sigma_j\right) - \frac{1}{N-1-L}\right)^2; \tag{5.188}$$

$$\delta D_\pi = D_{\pi,N} - D_{\pi,N-1-L}.$$

We can assume, that not the absolute, but relative value of the entropy is essential for the selection

$$\bar{H}_{\pi N} = \frac{H_{\pi N}}{H_{\pi\max}} = \frac{H_{\pi N}}{\ln N}.$$

$$0 \le \bar{H}_{\pi N} \le 1.$$

As the criterion, which determines the „moment" of selection, it is also possible to use the relative dispersion of preferences.

$$\bar{D}_\pi(N) = \frac{D_\pi(N)}{D_{\pi N\max}}.$$

It is easy to notice that $D_{\pi N\max} = \dfrac{N-1}{N}$. Thus

$$\bar{D}_\pi(N) = \frac{ND_\pi(N)}{N-1}. \tag{5.189}$$

When entropy is at maximum, dispersion is at minimum and vice versa. Maximum dispersion corresponds to minimum entropy, i.e., the following correspondence is carried out

$$H_\pi(N) = 0 \Leftrightarrow D_\pi(N) = \frac{N-1}{N}.$$

Let us examine the examples.

Example 1

S_{aN} contains 4 alternatives σ_1, σ_2, σ_3, σ_4, N = 4, the required resources of all alternatives are identical and are equal (in the arbitrary units) 8.

The available resources R^{disp} are universal and such, that after any selection they are sufficient for any subsequent selection, i.e., their remainder in the case of alternative choice with the maximum required resources exceeds any of the remained required resource. The expected (newly obtained) resources in all cases exceed the expenditures $R^{req}(\sigma_j)$ and are given in the tabele 1.

Distribution of preferences is positive and given by the formula

$$\pi^+(\sigma_j) = \frac{e^{\beta x_j}}{\sum_{k=1}^{N} e^{\beta x_k}}, \quad \text{where} \quad x_j = \frac{\bar{r}_j}{1-\bar{r}_j}.$$

Tabele 1

σ_i	σ_1	σ_2	σ_3	σ_4	
$R^{exp}(\sigma_j)$	2 + 8	4 + 8	3 + 8	1 + 8	
$R^{req}(\sigma_j)$	8	8	8	8	
$\bar{r}_j$	0,25	0,5	0,375	0,125	
x_j	0,333	1,0	0,6	0,14286	
$\pi^+(\sigma_j)$	0,1968	0,38345	0,2570	0,16273	Σ = 1,0

Initial entropy $H_\pi(N) \cong 1{,}3321$. Maximum entropy $H_{\pi\max}(N) = \ln N = \ln 4 == 1{,}38629$.
Standardized initial entropy

$$\bar{H}_{\pi}\left(N\right)=\frac{1,3321}{1,38629}=0,9609.$$

Dispersion of preferences $D_{\pi}(N)$ = 0,02839, and the maximum dispersion $D_{\pi\max}\left(N\right)=\frac{4-1}{N}=0,75,$ consequently

$$\bar{D}_{\pi}\left(N\right)=\frac{0,02839}{0,75}\cong 0,03785.$$

Let now the second alternative σ_2 as the aim is selected, as giving the greatest gain. Certainly, in accordance with the assumptions made, the critical entropy must exceed the obtained value $\bar{H}_{\pi}\left(N\right)$. The question of what are the critical values of entropy and (or) dispersion, represents itself the experimental problem. In the present chapter we only formulate assumptions and illustrate them by hypothetical calculations.

If the dispersion $D_{\pi}(N)$, as the entropy, plays role with the aim selection, then the corresponding condition must appear as follows

$$t^*_i\text{: } D_{\pi}(S_a, t^*_i) = D^* \text{ and } \left.\frac{dD_{\pi}}{dt}\right|_{t_i^*} > 0.$$

Let us continue the examination of an example and calculate entropy, using the relation $\bar{r}_j$, as x_j. It can be find:

$$H_{\pi}\left(N\right)=1,3766;\ \bar{H}_{\pi}\left(N\right)=0,99301;$$

We can see, that the sensitivity of entropy in the first case is greater. If the solution is accepted and the alternative σ_2 have been selected as the aim and if required resources did not change, then tabele 2 reflects the calculations of new preferences on S_{aN-1} and the new entropy.

Tabele 2

σ_j	σ_1	σ_2	σ_3
x_i	0,333	0,6	0,14286
$\pi^+(\sigma_j)$	0,31922	0,41693	0,26397

$$H_{\pi}(N-1) = H_{\pi}(3) = 1,08083;$$
$$H_{\pi\max}(N-1) = H_{\pi\max}(3) = \ln 3 = 1,0986;$$
$$\bar{H}_{\pi}\left(N-1\right)=\frac{1,08083}{1,0986}=0,98382.$$

The difference of absolute entropies is considered as the information, connected with decision making, in this case – the aim selection on S_{aN}.

$$I(|A) = H_{\pi}(N) - H_{\pi}(N-1) = 1,3321 - 1,08083 = 0,25127 > 0.$$

Thus, as a result of decision making the entropy of subject was decreased due to the import of information, the „pumping" occurred.

Example 2

Let it be now $\bar{r}_j = \frac{R^{req}(\sigma_i)}{R^{disp}}$, and the preferences model takes the form

$$\bar{\pi}(\sigma_j) = \frac{e^{-\beta x_j}}{\sum_{k=1}^{N} e^{-\beta x_k}}; \quad x_j = \frac{\bar{r}_j}{1-\bar{r}_j}.$$

Sign „-" indicates the circumstance, that the more $R^{req}(\sigma_j)$ less preferably the object of selection appears. Let us assume that available resources compose 8 units, and the distribution of required resources is shown in tabele 3, N = 4. Last alternative is most preferable. Let us assume, that additional conditions for decision making are satisfied and the alternative σ_4 is selected as the aim . Entropy determined on S_{aN} is

$$H_\pi(4) = 1{,}35273.$$

Tabele 3

σ_i	σ_1	σ_2	σ_3	σ_4	
$R^{req}(\sigma_j)$	2	4	3	1	
x_j	0,333	1,0	0,6	0,14286	
$e^{-\beta x_j}$	0,7168	0,36788	0,5488	0,8669	
$\bar{\pi}(\sigma_j)$	0,28667	0,14713	0,21948	0,34669	Σ = 1,0

Standardized entropy

$$\bar{H}_\pi(4) - \frac{1{,}35273}{1{,}38629} = 0{,}97579,$$

As a result of decision making, available resources change, decreasing in this case by one, i.e., $R^{disp}(3)$ = 8 – 1 = 7 units, and the number of topical alternatives is equal to 3. The result of calculation of preferences distribution and entropy is given in tabele 4.

Tabele 4

σ_i	σ_1	σ_2	σ_3	
$R^{req}(\sigma_j)$	2	4	3	
x_j	0,28857	0,57143	0,42857	
$e^{-\beta x_j}$	0,39997	1,3333	0,74999	
$\bar{\pi}(\sigma_j)$	0,47666	0,18744	0,33589	Σ = 1,0

New entropy

$$H_\pi(3) = 1{,}03348,$$

the standardized entropy

$$\bar{H}_\pi(3) - \frac{1{,}03348}{1{,}0986} = 0{,}94072,$$

„the information of solution”

$$I(|A) = H_\pi(N) - H_\pi(N-1) = 1{,}335273 - 1{,}03348 = 0{,}31925 > 0.$$

As in the preceding case, entropy decreased, the „pumping” of system occurred. The given examples bear illustrative nature. It is completely obvious that calculations of such type and estimations can be made for other models of the preferences distributions, being discussed above, in particular, for the biparametric distributions

$$\pi^{\pm}(\sigma_j) = \frac{x_j^{\alpha} e^{\pm\beta x_j}}{\sum_{k=1}^{N} x_j^{\alpha} e^{\pm\beta x_k}},$$

for the rating distributions, the distributions, which consider imperatives and others? As the parameter x_j the functions, containing simultaneously $R^{disp}(\sigma_j)$, $R^{req}(\sigma_j)$, $R^{exp}(\sigma_j)$ can be taken. The important expansion of theory appears usage along finite values of resources also the speeds of their alternation as well as calculation of the balance of different kinds of resource in dynamics.

Thus, for instance, for the available resources at each moment of time the balance condition occurs

$$R^{disp}(t) = R_0^{disp} - \sum_{k=1}^{P} \int_{t_k^*}^{t} V^{req}(\sigma_k)\,dt + \sum_{l=1}^{S} \int_{t_k^* + \tau_k}^{t} V^{exp}(\sigma_L)\,dt, \tag{5.190}$$

$$P < N,\ S < N,\ S \le P.$$

In this relationship the first sum reflects the expense of the resources, when several problems are solved simultaneously (t^*_k - the beginning moments of the corresponding investments), the second sum reflects an increase in the available resources due to the income, obtained in the process of the problems resolution. Here τ_k — delay of the moments, when return from the invested resources appears, V^{req} and V^{exp} –relay of resources „speeds”.

It is understood, that structures proposed here cannot be considered as the models of the economy, but only as the illustration of the analysis possibilities, which are opening in the case of their synthesis with the models of the preferences given by subjective analysis.

5.10. Ratings dynamics. Joint dynamics of the preferences of I and II type

The dynamics of the preferences of the II kind rating, as independent problem appears in connection with the analysis of the functioning of the subjects group. Above, in the chapter 4.4.1 the diverse variants of the group effectiveness functions are examined. Each of them independently or in the combination with other functions of effectiveness generates variational problem and, respectively - the specific canonical distribution of rating preferences. Here we will examine the diverse variants of the models of the rating preferences dynamics.

These models can prove to be the tool of investigation and prognostication of the processes, proceeding in the groups: structurization, differentiation, the appearance of inequality, conflicts, coalitions and so forth. In this case, having in mind the materials of the previous chapters, we assume that by this method it will be possible to consider exogenous and endogenous factors, influence of imperatives, past preferences.

In the sufficiently general case the model, which describes the joint dynamics of the preferences of the I and II type, can be represented in the form the system of the differential equations

$$\frac{d\xi}{dt} = F\left(\xi, \pi, U_{\xi}, U_{\xi}, \ldots\right); \tag{5.191}$$

$$\frac{d\pi}{dt} = G\left(\xi, \pi, U_{\pi}, U_{\pi}, \ldots\right), \tag{5.192}$$

where ξ and π are vectors or the matrixes of the corresponding dimensionality.

If the function of the utility (for the laconicism we do not include the „harmfulness" function L theory) U_{ξ}, U_{π} depends on the exogenous parameters σ, then to system (5.191), (5.192) the equation, which describes changes in these parameters, is added:

$$\frac{d\sigma}{dt} = g\left(\sigma, u\left(t\right), \pi,, \ldots\right), \tag{5.193}$$

where σ - vector of the exogenous parameters.

If β - vector of the endogenous parameters, then equations for these parameters in the sufficiently general case can be represented in the form:

$$\frac{d\beta}{dt} = f\left(\beta, \sigma, \ldots\right). \tag{5.194}$$

Finally, it can be assumed that the values, which characterize the influence of imperatives are the slowly changing factors, and submitted to the equations:

$$\frac{d\pi(I_\pi)}{dt} = h_1\left(\pi(I_\pi), \pi(I_\xi), \ldots\right); \tag{5.195}$$

$$\frac{d\pi(I_\xi)}{dt} = h_2\left(\pi(I_\pi), \pi(I_\xi), \ldots\right). \tag{5.196}$$

Here I_π is the imperatives, being concerned the preferences of the first type, while I_ξ is the imperatives, which have an effect on designation of ratings. After including in the right sides of the previous equations $\pi(I_\pi)$ and $\pi(I_\xi)$, we gives to emphasize that the presence of individual imperatives has an effect on group imperatives and, on the contrary, group imperatives must have an effect on the individual: man rarely is the „*white crow*".

The ellipses designate, that the right sides of the equations could contain the other variables and the constants. Thus, in the right side equation (5.193) in certain cases object preferences are contained in the manner as it was demonstrated with the modified Walras – Leontev problem. There are reasons to assume that in the equations (5.194) for the endogenous parameters both the exogenous parameters and the preferences of the I type can also be contained. So in the general case the equations above given comprise the system.

Naturally, following the path of generalizations, we can assume that instead of equations (5.191) - (5.196) in certain cases the equations with the delayed argument, or, even, integro-differential equations would be more adequate. In order to realize such models it is necessary to modify substantially the base settings of variational problems.

Let us examine the quotient versions of the system mentioned above. Suppose that preference functions of the I type are known and do not depend on ξ, then, if the right sides of (5.193) and (5.194) do not depend on ξ and π, we can examine the equation (5.191) separately. If, on the contrary, the preferences of the II type do not depend on ξ, and equations (5.193) and (5.194) do not contain ξ, then equations (5.192) can be solved independently.

To the functional of the form

$$\Phi_\xi^\pi(\sigma_k) = -\sum_{j=1}^{M} \xi_j(\sigma_k) \ln \xi_j(\sigma_k) + \beta_1 \sum_{j=1}^{M} \xi_j(\sigma_k) \pi_j(\sigma_k) + \gamma_1 \sum_{j=1}^{M} \xi_j(\sigma_k) \tag{5.197}$$

the canonical distribution corresponds, such that rating j depends on that how he „relates" to the alternative σ_k:

$$\xi_j(\sigma_k) = \frac{e^{\beta_1 \pi_j(\sigma_k)}}{\sum_{q=1}^{M} e^{\beta_1 \pi_q(\sigma_k)}}, \tag{5.198}$$

and to the functional

$$\Phi_{\xi}^{\pi}\left(\sigma_k\right)=-\sum_{j=1}^{M}\xi_j\left(\sigma_k\right)\ln\xi_j\left(\sigma_k\right)+\beta_1\sum_{j=1}^{M}\xi_j\left(\sigma_k\right)\ln\pi_j\left(\sigma_k\right)+\gamma_1\sum_{j=1}^{M}\xi_j\left(\sigma_k\right) \tag{5.199}$$

the distribution

$$\xi_j\left(\sigma_k\right)=\frac{\pi_j^{\beta_1}\left(\sigma_k\right)}{\sum_{q=1}^{M}\pi_q^{\beta_1}\left(\sigma_k\right)}. \tag{5.200}$$

Let us notice that $\sum_{q=1}^{M}\pi_q\left(\sigma_k\right)\neq 1$; therefore $\xi_j(\sigma_k)$ are not identical to $\pi_j(\sigma_k)$,even, if β_1 = 1. If, as it has been spoken the object preferences $\pi_j(\sigma_k)$ does not depend on ratings, then the latter can be determined by formulas (5.198) or (5.200).

However, if π (σ_k) depend on ξ (σ_k) (or ξ $(j, i|\ \sigma_k)$), for example, in accordance with formula (4.119) from p. 4.6, then it are necessary to use a system of type (5.191), (5.192). For the functions $\xi_j(\sigma_k)$ of form (5.200) we obtain the equations of the dynamics:

$$\frac{d\xi_j\left(\sigma_k\right)}{dt}=\beta_1\xi_j\left(\sigma_k\right)\left[\frac{1}{\pi_j\left(\sigma_k\right)}\frac{d\pi_j\left(\sigma_k\right)}{dt}-\sum_{q=1}^{M}\xi_q\left(\sigma_k\right)\frac{1}{\pi_q\left(\sigma_k\right)}\frac{d\pi_q\left(\sigma_k\right)}{dt}\right]. \tag{5.201}$$

For the function $\xi_j(\sigma_k)$, assigned by the relationship (5.198) we will find the differential equation:

$$\frac{d\xi_j\left(\sigma_k\right)}{dt}=\beta_1\xi_j\left(\sigma_k\right)\left[\frac{d\pi_j\left(\sigma_k\right)}{dt}-\sum_{q=1}^{M}\xi_q\left(\sigma_k\right)\frac{d\pi_q\left(\sigma_k\right)}{dt}\right]. \tag{5.202}$$

Let us examine the version, when the function preferences of the I kind is assigned by similar to (5.200) relationship:

$$\pi_j\left(\sigma_k\right)=\frac{\xi_j^{\beta_2}\left(\sigma_k\right)}{\sum_{p=1}^{N}\xi_j^{\beta_2}\left(\sigma_p\right)}. \tag{5.203}$$

Here the object preferences of alternatives σ_k of the subject j depend on his differential rating: in the group he is recognized an „expert" relative to this alternative; his rating is $\xi_j(\sigma_k)$ reflects the degree of recognition. The corresponding differential equation takes the form:

$$\frac{d\pi_j\left(\sigma_k\right)}{dt}=\beta_2\pi_j\left(\sigma_k\right)\left[\frac{1}{\xi_j\left(\sigma_k\right)}\frac{d\xi_j\left(\sigma_k\right)}{dt}-\sum_{p=1}^{M}\pi_j\left(\sigma_p\right)\frac{1}{\xi_j\left(\sigma_p\right)}\frac{d\xi_j\left(\sigma_p\right)}{dt}\right]. \tag{5.204}$$

The function of distribution (5.203) corresponds to variational problem with the functional

$$\Phi_{\pi_j}^{\xi}=-\sum_{k=1}^{N}\pi_j\left(\sigma_k\right)\ln\pi_j\left(\sigma_k\right)+\beta_2\sum_{k=1}^{N}\pi_j\left(\sigma_k\right)\ln\xi_j\left(\sigma_k\right)+\gamma_2\sum_{k=1}^{N}\pi_j\left(\sigma_k\right). \tag{5.205}$$

The „carrier" of both optimization problems is the subject „*j*".

If the function of effectiveness was selected in the form:

$$\varepsilon_{\pi_j}^{\xi}=\sum_{k=1}^{N}\pi_j\left(\sigma_k\right)\xi_j\left(\sigma_k\right), \tag{5.206}$$

which can be treated as the sum of the products of indices of „qualification" $\xi_j(\sigma_k)$ in the problem P: $\sigma_0 \to \sigma_k$ and the indices of the „desire" to solve this problem. The corresponding differential equation will take the form:

$$\frac{d\pi_j\left(\sigma_k\right)}{dt}=\beta_2\pi_j\left(\sigma_k\right)\left[\frac{d\xi_j\left(\sigma_k\right)}{dt}-\sum_{p=1}^{N}\pi_j\left(\sigma_p\right)\frac{d\xi_j\left(\sigma_p\right)}{dt}\right], \tag{5.207}$$

The system, comprised of equations (5.202) and (5.204) or of equations (5.202) and (5.207) is the uniform system of equations relative to the derivatives $\frac{d\xi_j\left(\sigma_k\right)}{dt}$ and $\frac{d\pi_j\left(\sigma_k\right)}{dt}$. It has the obvious trivial solution:

$$\xi_j(\sigma_k) = \text{const}; \ \pi_j(\sigma_k) = \text{const}.$$

Thus, the group can principally exist in the equilibrium conditions, when $\xi_j(\sigma_k)$ and $\pi_j(\sigma_k)$ are optimum in the sense of the functional given above. A question about existence of the nonequilibrium solution in such „locked" situation, when the preferences of the I and II type depend only on each other, remains open.

We have already examined the dynamics of the preferences of the I type $\pi_j(\sigma_k)$, when they do not depend on ratings. Dynamics of rating preferences $\xi_j(\sigma_k)$ (or modifications $\xi(j)$; $\xi(j\ i|\ \sigma_k)$) in the case of independence on the object preferences $\pi_j(\sigma_k)$ is similar to the dynamics of the latter. In order to leave the „vicious circle", when ratings are defined only through the object preferences, and object preferences only through the ratings, let us examine the „mixed" preferences, which depend both on the preferences of another type, and on the exogenous factors, for example, on resources.

Against the background of the large number of already introduced earlier canonical distributions, the introduction of the new additional distributions can seem by superfluous. However, it is worthwhile to repeat, that we do not consider theory the balanced and completed. This work reflects the process of search and tests of different assumptions and hypotheses and on this phase none of them must be rejected „a priori". Here we continue the discussion, begun in paragraph 4.12.

Let us examine the following functional:

$$\Phi_{\xi}^{\pi}\left(\sigma_k\right)=-\sum_{j=1}^{M}\xi_j\left(\sigma_k\right)\ln\xi_j\left(\sigma_k\right)+\beta_1\sum_{j=1}^{M}\xi_j\left(\sigma_k\right)\ln\pi_j\left(\sigma_k\right)+, \tag{5.208}$$

$$+\beta_1\sum_{j=1}^{M}\xi_j\left(\sigma_k\right)\overline{r}_j^{-1}\left(\sigma_2\right)+\gamma_1\sum_{j=1}^{M}\xi_j\left(\sigma_k\right);$$

$$\Phi_{\pi_j}^{\xi}=-\sum_{k=1}^{N}\pi_j\left(\sigma_k\right)\ln\pi_j\left(\sigma_k\right)+\beta_{2j}\sum_{k=1}^{N}\pi_j\left(\sigma_k\right)\ln\xi_j\left(\sigma_k\right)+, \tag{5.209}$$

$$+\beta_{2j}\sum_{j=1}^{N}\pi_j\left(\sigma_k\right)\overline{r}_j^{-1}\left(\sigma_k\right)+\gamma_2\sum_{j=1}^{N}\pi_j\left(\sigma_k\right).$$

The carrier - the „owner" of the second functional is the subject *j*. The second term reflects the circumstance that the choice of or another object preference $\pi_j(\sigma_k)$ depends for *j* on his „universally recognized" in the group rating $\xi_j(\sigma_k)$, which in turn, is connected with qualification with the respect to the problem P: $\sigma_0 \to \sigma_k$.

The third term determines dependence $\pi_j(\sigma_k)$ on the relative required resources

$$\overline{r}_j\left(\sigma_k\right)=R_j^{req}\left(\sigma_k\right),\ \left(R_j^{disp}\left(\sigma_k\right)\right)^{-1}.$$

First functional (5.208) appears to be the *„property"* of a certain *„hierarch"* which stands above the group (or being the member of the group), or *„ the collective reason"* - the carrier *„of public opinion"*. The parameters α, β, γ can be considered in this case as the characteristics „of the psyche" of virtual "collective reason", or, if rating $\xi_j(\sigma_k)$ distributes real hierarch, the characteristics of his psyche. It is natural to consider in the second functional, the parameters α_{2j}, β_{2j}, γ_{2j}, which are notice d by the index presence as the individual characteristics of subject *j*.

Let us notice that when the ratings of the group members from „point of view" of given subject *i*: $\xi(j, i|\ \sigma_k)$ are determined, it have to be supposed that the endogenous parameters are individualized α_i, β_i, γ_i, ... This must be taken into account during aggregation of such ratings. In this sense the second term of functional (5.208) reflects the fact that rating of subject *j* it can, or must, depends on his of larger or smaller desire to solve given problem. The third term in (5.208) considers the dependence of rating of subject *j* on the relation of the available and required resources and therefore it enters into functional with the positive sign. Thus, the presence of „consensus" in the group relative to ratings is assumed.

Canonical distributions take the form:

$$\xi_j\left(\sigma_k\right)=\frac{\pi_j^{\alpha_1}\left(\sigma_k\right)e^{\beta_1\bar{r}_j^{-1}(\sigma_k)}}{\sum_{q=1}^{N}\pi_q^{\alpha_1}\left(\sigma_k\right)e^{\beta_1\bar{r}_q^{-1}(\sigma_k)}};\tag{5.210}$$

$$\pi_j\left(\sigma_k\right)=\frac{\xi_j^{\alpha_2}\left(\sigma_k\right)e^{\beta_2\bar{r}_j^{-1}(\sigma_k)}}{\sum_{p=1}^{N}\xi_q^{\alpha_2}\left(\sigma_k\right)e^{\beta_2\bar{r}_p^{-1}(\sigma_k)}}.\tag{5.211}$$

Let us designate for the record reduction $\bar{r}_j\left(\sigma_k\right)=x_{jk}$. A change of preferences (5.210) and (5.211) is described by the differential equations:

$$\frac{d\xi_{jk}}{dt}=\xi_{jk}\left[\left(\alpha_1\frac{1}{\pi_{jk}}\frac{d\pi_{jk}}{dt}-\beta_1\frac{1}{x_{jk}^2}\frac{dx_{jk}}{dt}\right)-\sum_{q=1}^{M}\left(\alpha_1\frac{1}{\pi_{qk}}\frac{d\pi_{qk}}{dt}-\beta_1\frac{1}{x_{qk}^2}\frac{dx_{qk}}{dt}\right)\xi_{qk}\right];\tag{5.212}$$

$$\frac{d\pi_{jk}}{dt}=\pi_{jk}\left[\left(\alpha_2\frac{1}{\xi_{jk}}\frac{d\xi_{jk}}{dt}-\beta_2\frac{1}{x_{jk}^2}\frac{dx_{jk}}{dt}\right)-\sum_{p=1}^{M}\left(\alpha_2\frac{1}{\xi_{jp}}\frac{d\xi_{jp}}{dt}-\beta_2\frac{1}{x_{qk}^2}\frac{dx_{jp}}{dt}\right)\pi_{jp}\right].\tag{5.213}$$

System (5.212), (5.113) contains $2N\times M$ equations and in the represented form is not solvable relative to the derivatives $\frac{d\xi_{jk}}{dt}$ and $\frac{d\pi_{jk}}{dt}$. It is nonlinear and not uniform. Substituting, for example, in equations (5.212) the derivatives $\frac{d\pi_{jk}}{dt}$ by the right sides of equations (5.213), solving the obtained equations relative $\frac{d\xi_{jk}}{dt}$ and, then, returning the corresponded right sides in (5.213), we will obtain the system of equations in the Cauchy form. Since in equations (5.212), (5.213) the exogenous variables are contained, the equations, which describe the dynamics of these variables, must be added. The transformation of system (5.212) and (5.213) to Cauchy's form will bring to such nonlinear equations, whose right sides will have singularities, appearing with solution of the linear system of algebraic equations relative to the derivatives $\frac{d\xi_{jk}}{dt}$. It is possible to avoid these difficulties, if we accept the assumption that the production of the preferences of the I and II type occurs not only once, but in the determined time-sequential routine.

In this case we come to the scheme similar to that, which was examined in paragraph 5.5. After the solution of the converted system of equations the entropies of object (utilitarian) preferences are calculated for each subject j, and also rating group entropy for each alternative σ_k. Similarly, as this was done for the object preferences, in the ratings dynamics it is possible to take into account the influence of the past ratings and the rating imperatives which occurred at the previous moments of time.

An example of a priori ratings distributions (imperative) a priori requirement of the equality: $\xi(l)=\frac{1}{M}$. can serve. The calculation „of prehistory" is achieved via the postulation of the time chain of functional, which is equivalent to assumption about existence of the iterative process of the rating preferences optimization, when on each step at each sequential moment of time the subject solves optimization problem with the functional $\Phi_\xi(t)$, in the following moment $t + 1$ the problem with the functional $\Phi_\xi(t + 1)$ and so on. The connection between the sequential optimization problems realizes by the terms of the form

$$...+\beta\sum_{j=1}^{M}\xi_j(t+1)\xi_j(t)...\quad \text{or} \quad ...+\beta\sum_{j=1}^{M}\xi_j(t+1)\ln\xi_j(t)...$$

or generally

$$...+\beta\sum_{j=1}^{M}\xi_j(t+1)\varphi(\xi_j(t))...$$

In the second case occurs more „soft" (weak) of the previous preferences calculation in comparison with the first version. At conclusion of this paragraph let us examine a simple quantitative example of the rating preferences dynamics. Let us assume that ratings in the given moment are determined exclusive by individual available resources $R_j^{disp} = R_j^d$:

$$\xi(j)=\frac{e^{\beta R_j^d}}{\sum_{q=1}^{M}e^{\beta R_j^d}}. \tag{5.214}$$

Let there be group of three subjects: $M = 3$ and general set of alternatives S_a, which contains two alternatives $N = 2$. Both alternatives appear to be corporate and consolidation of resources is achieved. Let the „speed" of the available resources expense (equal to the required „speed")

$$u(t) = u(\sigma_1) + u(\sigma_2),$$

where $u(\sigma_1)$ - the required „speed" of the resources investment in the problem solution P_1: $\sigma_0 \to \sigma_1$; $u(\sigma_2)$ is in the problem solution P_2: $\sigma_0 \to \sigma_2$.

Both problems are solved simultaneously beginning with the moment $t = 0$. In the process of the solution of the problems P_1 and P_2 practically immediately appears the positive effect (income), the payment „speed" of which

$$b(t) = b(\sigma_1) + b(\sigma_2).$$

The inflow of the new resources, which increase those available, is described by the equations

$$\frac{db(\sigma_1)}{dt} = -\frac{1}{\tau_1} b(\sigma_1) + \frac{1}{\tau_1} f_1 u(\sigma_1), \tag{5.215}$$

$$\frac{db(\sigma_2)}{dt} = -\frac{1}{\tau_2} b(\sigma_2) + \frac{1}{\tau_2} f_2 u(\sigma_2). \tag{5.216}$$

Here τ_1 and τ_2 - the time constants of the appearance of resources „return"; f_1 and f_2 - technological coefficients, which determine the effectiveness of the resources investment in the solutions of the problems P_1 and P_2, correspondingly: $f_1 > 1$; $f_2 > 1$. If $\tau_1 = \tau_2 = 0$, the conversion of resources in the result (in the new resources) occurs instantly:

$$b(\sigma_i) = f_i u(\sigma_i).$$

Let us assume that the portions of resources, packed in P_1 and P_2 are constant, i.e., but not resources itself:

$$u(\sigma_1) = \mu u(t);\ u(\sigma_2) = (1 - \mu)u(t);\ \mu = \text{const}.$$

Let a change of the available resources of subjects be described by the equations

$$\frac{dR_j^d}{dt} = -\eta_j u(t) + \xi(j)\big(b(\sigma_1) + b(\sigma_2)\big), \tag{5.217}$$

where η_i is the standardized coefficients

$$\sum_{j=1}^{3} \eta_j = 1,$$

determining relative percentages of subjects in the process of the resources consolidation. We will examine two cases:

1. $$\eta_j = \frac{1}{M} = \frac{1}{3} = \text{const}.$$

2. $$\eta_j = \frac{R_j^d}{\sum_{q=1}^{3} R_j^q} \neq \text{const}.$$

In the second case η_j — the function of time is defined as the specific fraction of the available resources of the given subject in the general available resources of group. It is evident basing on (5.214) that $\xi(j)$ is also the function of time. The sense of last term in equations (5.217) lies in the fact, that newly obtained resources are divided proportional to the ratings of subjects in a given current time.

Ratings exponentially strengthen the influence of the available resources. The version of the profit distribution (income) considered here is very simplified and hypothetical. Completely legal appears the question: in what society this distribution is possible - in the society, which lives „according to the law" or „on the concepts". It is possible that this principle existed there in the prehistoric times.

The different scheme of the distribution of profit and distribution of the expenditures are known with solution of the corporate problems [113]. In this case the author pursues one purpose: to demonstrate based on maximally simple examples the influences on the economic situation development the calculation of ratings dynamics, for which there exist canonical distributions, received on the basis of the variation principle postulation. This problem is of paramount interest.
If M subjects participate in the solution of corporate problem and the total expected income $R^{exp} = b$, and the expenditures of each $R_j^{req} = u_j$, then profit is being made, if

$$b \geq \sum_{j=1}^{M} u_j \tag{5.218}$$

and it is equal

$$z = b - \sum_{j=1}^{M} u_j. \tag{5.219}$$

„Egalitarian" division of profit assumes division equally so, that the share of the subject j income

$$\text{"share } j\text{"} = u_j + \frac{z}{M}. \tag{5.220}$$

Aother model according to [113] – „proportional" division of the income: identical return per unit of individual expenditures:

$$\text{"share } j\text{"} = b\frac{u_j}{\Sigma u_i}. \tag{5.221}$$

Let us examine following [113] models of the expenditures distribution for manufacturing of the indivisible product with the general total „speed" of expenditures. Let b_j is income of the j-th subject. The solution of problem is effective, if

$$b = \sum b_j \geq u. \tag{5.222}$$

„Proportional" distribution of expenditures lies in the fact, that subject j „pays" the sum u_j proportional to his expected income b_i, i.e.

$$u_j = \frac{b_j}{\Sigma b_i};\ u_j \geq 0. \tag{5.223}$$

In this case no one pays more than his income.
„Egalitarian idea" consists in equalizing of the share of expenditures and equalizing of pure savings on the expenditures. The uniform distribution of expenditures assumes that

$$u_j = \frac{u}{M}.$$

The solution u_j with the equal profit is determined by the formula

$$u_j = b_j - \frac{\Sigma b_i - u}{M}. \tag{5.224}$$

As we can see, $b_i - u_j > 0$ are identical for $\forall i \in \overline{1,M}$.

Let us examine the following model: the available resources of subject j at the given moment $R_j^{disp} = Y_j$, total income per time unit $b = \Sigma b_j$, where b_j - individual „speeds" of obtaining income, expenditure per time unit

$$u = \Sigma u_j,$$

where u_j - individual „speeds" of expenditures. Let us assume that u_j are proportional to relative wealth:

$$u_j = u\frac{Y_j}{\sum_{i=1}^{M} Y_i}, \tag{5.225}$$

At the same time the income from „the solution of corporate problem" per unit of time is proportional to the integral rating of the subject

$$b_j = b\xi(j), \tag{5.226}$$

where the rating $\xi(j)$ is determined from the formula

$$\xi(j) = \frac{e^{\beta Y_j}}{\sum_{i=1}^{M} e^{\beta Y_i}}.$$

The scheme of the of income distribution can be named elite. Fig. 5.42 shows the simulation results for the problem solution of two corporate problems by group of three subjects with „egalitarian" scheme of expenditures division (equally) and „elite" schema of the income distribution, described by equations (5.227).

$$D(t,Y)=\begin{bmatrix} -\frac{1}{3}u(t)+(Y_3+Y_4)\sqrt{\dfrac{e^{\beta Y_0}}{e^{\beta Y_0}+e^{\beta Y_1}+e^{\beta Y_2}}} \\ -\frac{1}{3}u(t)+(Y_3+Y_4)\sqrt{\dfrac{e^{\beta Y_1}}{e^{\beta Y_0}+e^{\beta Y_1}+e^{\beta Y_2}}} \\ -\frac{1}{3}u(t)+(Y_3+Y_4)\sqrt{\dfrac{e^{\beta Y_2}}{e^{\beta Y_0}+e^{\beta Y_1}+e^{\beta Y_2}}} \\ -\delta_1 Y_3+\delta_1 f_1 \mu u(t) \\ -\delta_2 Y_4+\delta_2 f_2 (1-\mu) u(t) \end{bmatrix} \qquad Y_0 = \begin{pmatrix} 1,028 \\ 1,024 \\ 1,020 \\ 0 \\ 0 \end{pmatrix} \qquad (5.227)$$

β = 1; δ_1 = 0,03; f_1 = 1,1; δ_2 = 0,02; f_2 = 1,2; μ = 0,6; b = 0,02;
a = 0,002; $u(t)$ = a ln t + b; t_0 = 1; t_1 = 500; N = 5000.

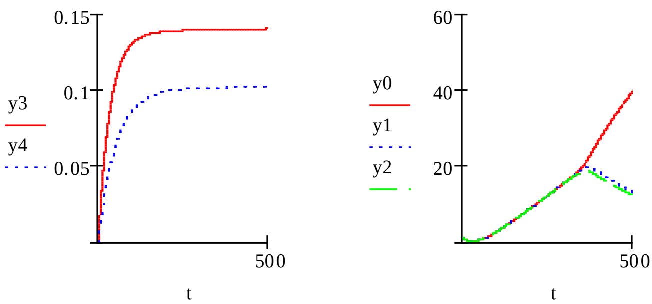


Fig. 5.42

$$\pi_i = \frac{e^{\beta Y_i}}{\sum_{q=0}^{2} e^{\beta Y_q}}, \ (j \in \overline{0,2}).$$

Two last equations describe the specific production process with the time constants $(\delta_1)^{-1}$, $(\delta_2)^{-1}$ and „technological" coefficients f_1 and f_2. We see, that initial conditions for

$$Y_j = R_j^{disp}$$

differ very little from each other. Through the certain time decisive „property" splitting and, respectively ratings splitting occurs. Along with this entropy of rating drops to zero (Fig. 5.43).

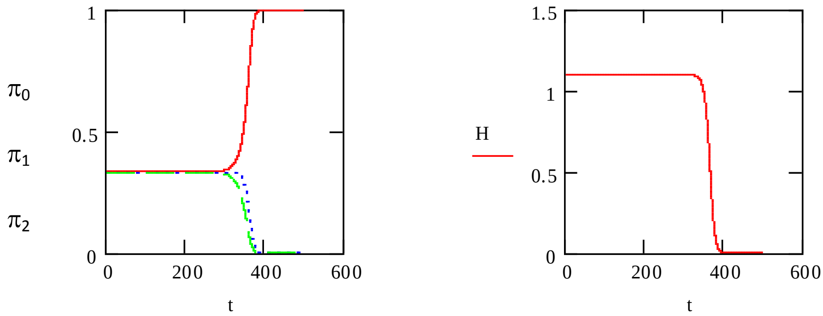


Fig. 5.43

$$H = -\sum_{j=0}^{2} \pi_j \ln \pi_j .$$

Fig. 5.44 and Fig. 5.45 show the result of solution of the same problem with the changed schemes of expenditures and incomes distribution. System of equations takes the form:

$$D(t,Y) = \begin{bmatrix} -\dfrac{Y_0}{Y_0+Y_1+Y_2}u(t)+(Y_3+Y_4)\sqrt{\dfrac{e^{\beta Y_0}}{e^{\beta Y_0}+e^{\beta Y_1}+e^{\beta Y_2}}} \\ -\dfrac{Y_1}{Y_0+Y_1+Y_2}u(t)+(Y_3+Y_4)\sqrt{\dfrac{e^{\beta Y_1}}{e^{\beta Y_0}+e^{\beta Y_1}+e^{\beta Y_2}}} \\ -\dfrac{Y_2}{Y_0+Y_1+Y_2}u(t)+(Y_3+Y_4)\sqrt{\dfrac{e^{\beta Y_2}}{e^{\beta Y_0}+e^{\beta Y_1}+e^{\beta Y_2}}} \\ -\delta_1 Y_3+\delta_1 f_1 \mu u(t) \\ -\delta_2 Y_4+\delta_2 f_2 (1-\mu) u(t) \end{bmatrix} \qquad Y_0 = \begin{pmatrix} 1{,}028 \\ 1{,}024 \\ 1{,}020 \\ 0 \\ 0 \end{pmatrix} \qquad (5.228)$$

β = 1; δ_1 = 0,03; f_1 = 1,1; δ_2 = 0,02; f_2 = 1,2; μ = 0,6; b = 0,02;
a = 0,002; $u(t)$ = $at + b$; t_0 = 1; t_1 = 500; N = 5000.

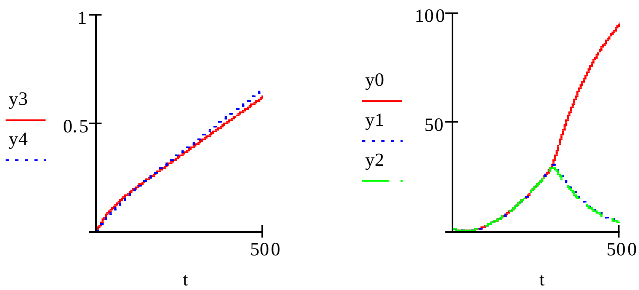


Fig. 5.44

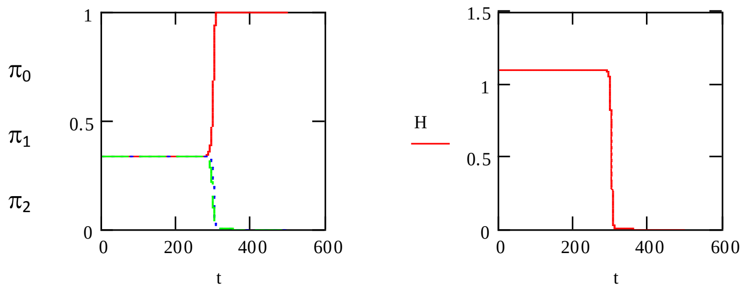


Fig. 5.45

Expenditures are divided proportionally to the available resources present, incomes are divided proportionally to square root of theirs rating (soft „elite" distribution). The corresponding system of equations (5.228) also contains two equations of „production". As in the preceding case, with the very small differences in the initial conditions occurs the sharp stratification of the available resources and its ratings (but considerably later, than in the first problem). System of equations (5.229) is characterized by the fact that in the right sides of the production equations the preferences of the I type of each of the alternatives are introduced as coefficients, which depend on the „speed" of obtaining income Y_3 and Y_4. The nature of the solution for Y_0, Y_1, Y_2 remains in this sense, that there is clearly expressed moment for „property" stratification of corporate problems solution participants.

$$D(t,Y)=\begin{bmatrix} -\dfrac{Y_0}{Y_0+Y_1+Y_2}u(t)+(Y_3+Y_4)\sqrt{\dfrac{e^{\beta Y_0}}{e^{\beta Y_0}+e^{\beta Y_1}+e^{\beta Y_2}}} \\ -\dfrac{Y_1}{Y_0+Y_1+Y_2}u(t)+(Y_3+Y_4)\sqrt{\dfrac{e^{\beta Y_1}}{e^{\beta Y_0}+e^{\beta Y_1}+e^{\beta Y_2}}} \\ -\dfrac{Y_2}{Y_0+Y_1+Y_2}u(t)+(Y_3+Y_4)\sqrt{\dfrac{e^{\beta Y_2}}{e^{\beta Y_0}+e^{\beta Y_1}+e^{\beta Y_2}}} \\ -\delta_1 Y_3+\delta_1 f_1\dfrac{e^{-\delta_1 Y_3}}{e^{-\delta_1 Y_3}+e^{-\delta_1 Y_4}}u(t) \\ -\delta_2 Y_4+\delta_2 f_2\dfrac{e^{-\delta_1 Y_4}}{e^{-\delta_1 Y_3}+e^{-\delta_1 Y_4}}u(t) \end{bmatrix} \qquad Y_0=\begin{pmatrix}1{,}028\\1{,}024\\1{,}020\\0\\0\end{pmatrix} \qquad (5.229)$$

δ_1 = 0,03; f_1 = 1,1; δ_2 = 0,02; f_2 = 1,2; b = 0,02;
a = 0,002; $u(t)$ = at + b.

Let us notice , that in all cases it was assumed that aggregation of ratings was already executed and, those in this case endogenous characteristics (α, β,...) of all subjects are identical. This is essential assumption, since in the case, if these parameters are different, this can be reflected in the behavior of the solution.

Fig. 5.46 and Fig. 5.47 show the result of solution process simulation of one corporate problem by five subjects. The principle of the division of expenditures proportional to the available resources and „elite" principle of the income division is system of equations (5.230) is accepted. Here also the „property" stratification is notice as well as stratification in corporate problem solution process.

$$D(t,Y)=\begin{bmatrix} -\dfrac{Y_0}{Y_0+Y_1+Y_2+Y_3+Y_4}u(t)+Y_5\dfrac{e^{\beta_0Y_0}}{e^{\beta_0Y_0}+e^{\beta_1Y_1}+e^{\beta_2Y_2}+e^{\beta_3Y_3}+e^{\beta_4Y_4}} \\ -\dfrac{Y_1}{Y_0+Y_1+Y_2+Y_3+Y_4}u(t)+Y_5\dfrac{e^{\beta_1Y_1}}{e^{\beta_0Y_0}+e^{\beta_1Y_1}+e^{\beta_2Y_2}+e^{\beta_3Y_3}+e^{\beta_4Y_4}} \\ -\dfrac{Y_2}{Y_0+Y_1+Y_2+Y_3+Y_4}u(t)+Y_5\dfrac{e^{\beta_2Y_2}}{e^{\beta_0Y_0}+e^{\beta_1Y_1}+e^{\beta_2Y_2}+e^{\beta_3Y_3}+e^{\beta_4Y_4}} \\ -\dfrac{Y_3}{Y_0+Y_1+Y_2+Y_3+Y_4}u(t)+Y_5\dfrac{e^{\beta_3Y_3}}{e^{\beta_0Y_0}+e^{\beta_1Y_1}+e^{\beta_2Y_2}+e^{\beta_3Y_3}+e^{\beta_4Y_4}} \\ -\dfrac{Y_4}{Y_0+Y_1+Y_2+Y_3+Y_4}u(t)+Y_5\dfrac{e^{\beta_4Y_4}}{e^{\beta_0Y_0}+e^{\beta_1Y_1}+e^{\beta_2Y_2}+e^{\beta_3Y_3}+e^{\beta_4Y_4}} \end{bmatrix} \qquad Y_0=\begin{pmatrix}1{,}14\\1{,}139999\\1{,}139995\\1{,}05\\1{,}01\\0\end{pmatrix} \tag{5.230}$$

$\delta_1 = 0{,}1$; $f_1 = 1{,}1$; $\delta_2 = 0{,}1$; $f_2 = 1{,}2$; $b = 0{,}2$; $a = 0$; $u(t) = at + b$;
$\beta_0 = 0{,}5$; $\beta_1 = 0{,}505$; $\beta_2 = 0{,}5055$; $\beta_3 = 0{,}505$; $\beta_4 = 0{,}5065$.

Ratings stratification leads to the zeroing of ratings entropy.

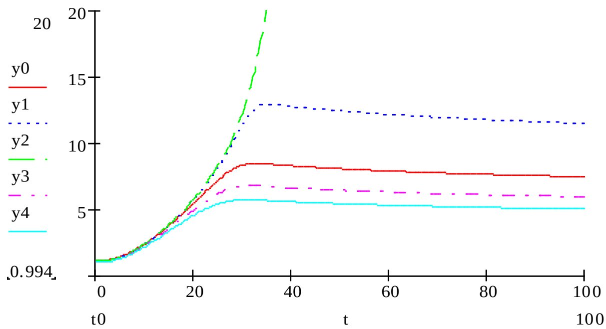


Fig. 5.46

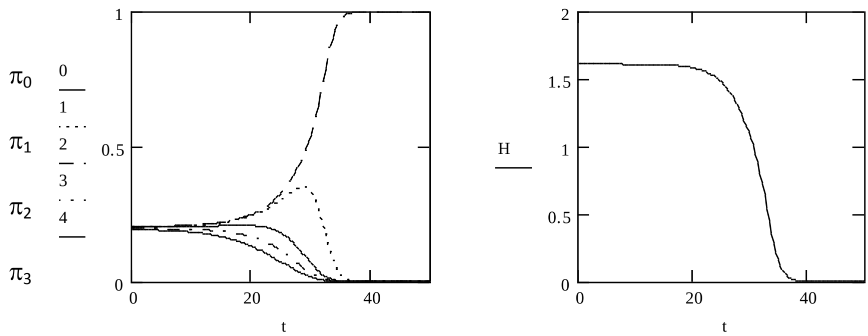


Fig. 5.47

Fig. 5.48 and Fig. 5 show the result of solution of the same problem with the insignificantly changed values of the parameters β_i ($i \in \overline{0,4}$). We see, that the stratification also occurs; however, the picture changed significantly. The process of stratification was extended for the more prolonged period. This can be considered as evidence of the fact, that the parameter β - is somehow connected with „the business activity" of subject during the solution of this corporate problem.

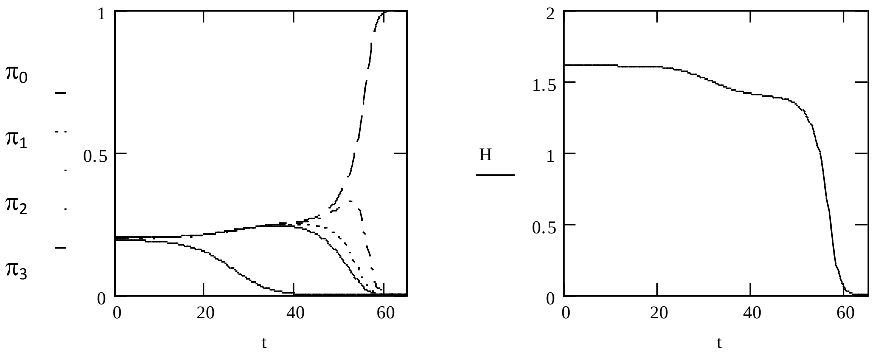


Fig. 5.48

5.11. On a change in the number of alternatives

Until now we assumed, that a number of alternatives *N* remains constant. However, the description of dynamics is complete, if we do not consider the possibility of changing the number of alternatives *N*. There are several reasons for a possible change of the alternatives number:

1. As a result of „decision making" one or several alternatives passes to the category of aim and, therefore, they are excluded from the set S_a.

2. Simultaneously with this the part of the available resources is withdrawn, directed to the realization of the selected aim.

In connection with this number of solvable problems is decreased and those alternatives, for which $R^{req}(\sigma_i) > R_1^{disp}$, where R_1^{disp} is new volume of the available resources are excluded from S_a. Scheme on Fig. 5.49, 5.50 illustrates these reasoning's. On the axis R^{req} the alternatives are designated by points, to which the given required resources correspond, the small crosses designate not realizable alternatives, for which $R^{req}(\sigma_i) \geq R^{disp}$.

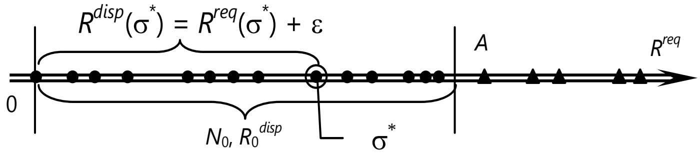


Fig. 5.49

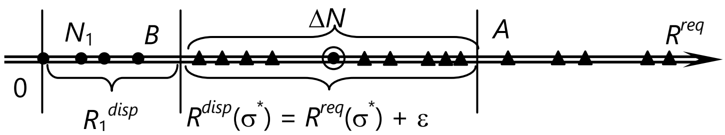


Fig. 5.50

Assume that as „aim" alternative σ^* is selected. For its realization the expenditures equal $R^{req}(\sigma^*)$ will be required (Fig. 5.49). Then, out of total available resources the value

$$R^{disp}(\sigma^*) = R^{req}(\sigma^*) + \varepsilon,$$

would have to exclude where ε„technological reserve" is. Laying the value $R^{disp}(\sigma^*)$ from point A in the opposite direction (Fig. 5.50), we will find the remainder of the available resources $R_1^{disp} = R_0^{disp} - (R^{req}(\sigma^*) + \varepsilon)$ — the section *OB*. Basing on these available resources it is possible to realize only those alternatives, which belong to section *OB* (in this case $N_1 = 3$).

The alternatives, concentrated within the limits of section *BA*, excluding alternative σ^*, pass into category of those not realized, σ^* passes in the category of the aim . Since σ^* belonged to section *BA*, its transformation into a „target" does not influence the number of alternatives in the section *OB*. Another situation is shown on Fig. 5.51. In this case the selected alternative does not belong to section *BA* and therefore the number of remained alternatives

$$N_1 = N_0 - \Delta N - 1,$$

as an alternative σ^* also leaves from S_a.

3. When the available resources are fixed the number of alternatives in S_a can change as a result of the required resources change, for example, if the „prices" increased or decreased. Fig. 5.52 demonstrates the modification of alternatives

distribution on the axis of required resources with the simultaneous proportional increase (a) or decrease (b) of „prices".

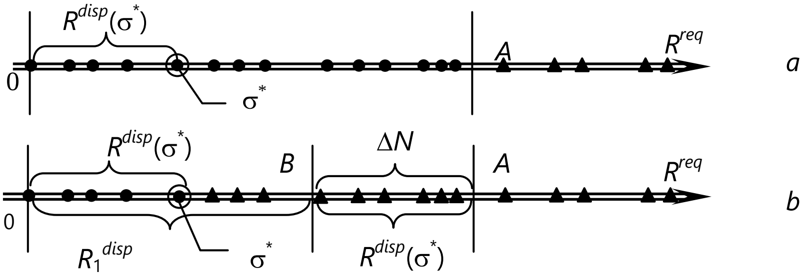


Fig. 5.51

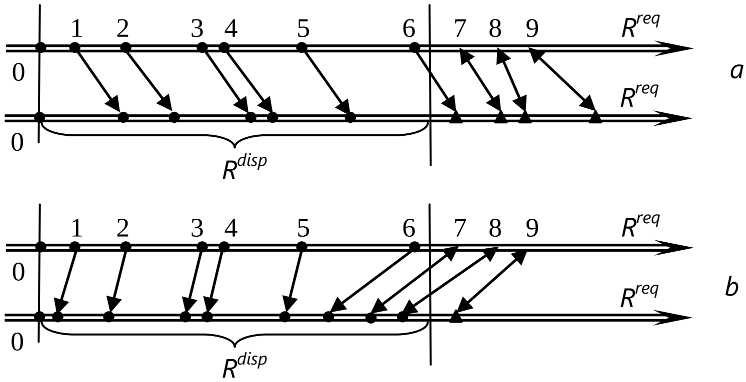


Fig. 5.52

In the case of an increase of the prices the number of alternatives in S_a decreases

$$N_0 = 6 \to N_1 = 5.$$

In the second case, with prices reduction the number of alternatives in S_a increases

$$N_0 = 6 \to N_1 = 8.$$

Let us break the section of the axis R^{req}, containing all N_0 alternatives for the sections ΔR and construct the histogram of the number N_0 distribution on classes (Fig. 5.53), and let $n(R)$ - the approximating continuous distribution.

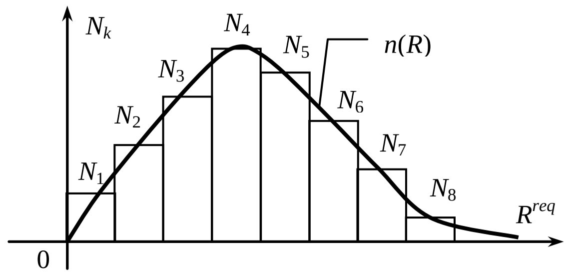


Fig. 5.53

Standardization conditions are determined as follows:

$$\int_0^{\infty} n(R)\,dR = N_0.$$

At each moment of the time, when $R^{disp} = R^d(t)$, and required resources are fixed the number $N(t)$ is:

$$N(t) = \int_0^{R^d(t)} n(R)\,dR. \qquad (5.227)$$

Let us find, differentiating:

$$\frac{dN(t)}{dt} = n\left(R^d\right)\frac{dR^d(t)}{dt}. \qquad (5.228)$$

If for $R^d(t)$ an exogenous model exists:

$$\frac{dR^d(t)}{dt} = g\left(R^d, t\ldots\right), \qquad (5.229)$$

that

$$\frac{dN(t)}{dt} = n\left(R^d\right)g\left(R^d, t\ldots\right), \qquad (5.230)$$

Let, for example $n(R) = N_0\mu e^{-\mu R}$ and the modification of this distribution occurs, as result of change in the value of required resources on the formula

$$R_1^{req} = mR_0^{req}.$$

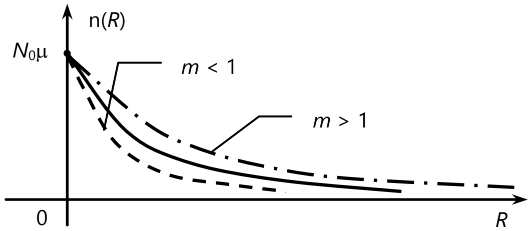


Fig. 5.54

If m > 1 is the „price" grows, with m < 1, the „price" diminishes proportionally.

Then

$$N(t,m) = \int_0^{R^d(t)} n(R,m)\,dR. \tag{5.231}$$

Let for example

$$n(R) = N_0 \frac{\beta}{m} e^{-\frac{\beta}{m}R}, \tag{5.232}$$

then

$$N(t,m) = \left[N_0 \frac{\beta}{m} \int_0^{R^d(t)} e^{-\frac{\beta}{m}R} dR \right] - N_0 \left[\left(e^{-\frac{\beta}{m}R^d(t)} - 1 \right) \right]. \tag{5.233}$$

Here [...] indicate taking the integer part of obtained number. Instead of relationship (5.230) it is possible to use the finite-difference approximation:

$$N(t) = N(t-1) + \left[\tau, n\left(R^d\right) g\left(R^d, t-1, \ldots\right) \right]. \tag{5.234}$$

The standardization of the preferences distribution is performed during each moment on current value $N(t)$. This means that in each new step t the variational problem and, the appropriate functional is renewed. It is possible to say, that the optimization „works" on one step forward.

4. Number of alternatives can change also as a result of the changes dependant on age, forgetting, and the displacement of some alternatives by others equivalent from an economic point of view, but being differed by the content.

Here approaches and models can be used analogous to models of scientific ideas propagations, advertizing campaign models, technologies „substitution" models [55].

5.12. „Manipulation of consciousness" - dynamic process of preferences modification

The problem of „consciousness manipulation" can be attributed to the division of dynamic, since the discussion deals with the process of the adjusted changes, proceeding in time. Control methods of complex objects, as well as theoretical and applied individual and group psychology development, penetration into recent quantitative methods and the appearance of the new technical capabilities: mass information media such as television, the Internet, mobile communication, gave birth to new ideas and new region of activity - control on mass consciousness in the interests of the „operators", which manage these medias and are ready to pay developers of theory and specific schemes.

The goals of the mentioned „operators" are not always noble and clean. In the most part they pursue the selfish interests of the specific group, class, state. It turned out that using technologies of „consciousness manipulation", it is possible to conquer without the „hot war", to subjugate and to seize entire states, moreover to make this so unnoticeable, that those subjugated continue to consider themselves free and independent. The Cold War gave enormous push to the development of theory and practice of consciousness manipulation. The impossibility to conquer in the „Hot War" led to the creation of new weapon – „the psychological weapon", from which there is no protection - neither bunkers, not steel doors, nor crossbar locks, nor armies equipped with the most perfected weapon do save one.

Human consciousness is defenseless because the technology of manipulation use exactly the well studied standard properties of human psyche and penetrate into soul as easily as radio waves through the concrete. Television – one of the tools of psychological war – is actually the „burglar", who achieves „breaking" into psyche for the purpose of encroachment on the most valuable that is in the human - on the personality, on soul.

„Operators - political technologists" penetrate unnoticeably the consciousness of each one and „pull the soul out in parts". This worst form of robbery is inevitable. The danger, which „psychological weapon" carries in itself for each one and for the world as a whole is enormous, the consequences of its application terrible, and commensurate with the consequences of thermonuclear war.

This question has long history. In the times of the Second World War within the staff of General Eisenhower, commander-in-chief of allied forces in Europe, the subdivision of psychological warfare was created. In the meaningful sense the methods and the approaches to manipulation of consciousness are well described in [75]. Our task in this paragraph is attempted to interpret *the problem of consciousness manipulation* in the terms of the developed version of subjective analysis. We will present this problem as problem of preferences modifications and sets of alternatives. This can be referred both to the preferences of the I type - object preferences $\pi(\sigma_k)$, and to the preferences of the II type is rating preferences $\xi(j)$, $\xi(j|\sigma_k)$...

If in the first case the discussion can deal with the change in the relation to the facts of material life, ideological, political, religious, cultural, and economic, then in the second case it is possible to speak about the redistribution of ratings, leaders, a change in the internal structure of group, change in ratings of the parties. It is going not about the spontaneous modification of preferences, but about the goal-directed controlled process, whose realization requires the expenditures of resources. „The consciousness manipulation" – what it consists of, from the point of view of subjective analysis.

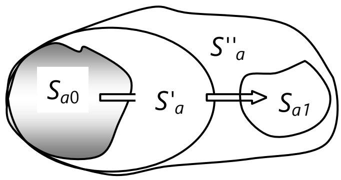


Fig. 5.55

Let S_{a0} — initial set of alternatives and S_{I0} — initial set of ethical imperatives, which do not contradict alternatives $\sigma_k \in S_{a0}$. Task of the „operator" consists in that, in modification of to modify the set S_a. Let the S_{a1} (Fig. 5.55) appears as destination set. Let us assume that on S_{a0} the initial entropy $H_{\pi 0}$ is small, and smaller than the threshold entropy $H_{\pi 0j}$, which separates the „reign of need" from the „reign of freedom". Then the sequence of the formal operations, necessary for the transfer j from S_{a0} into S_{a1} occurs as following:

1. It is necessary to „rock consciousness" first on S_{a0}, i.e., to increase entropy to the value $H'_{\pi j} > H_{\pi 0j}$, when alternatives $\sigma_k \in S_{a0}$ will appear approximately as equivalent and the solution selection on S_a will be hindered.

2. To enlarge the set S'_a by the addition of the new alternatives, not excepting the old alternatives. In this case the entropy increases due to an increase in the number of alternatives, if we preserve their approximate equivalence. The preferability of all alternatives, including old once will be additionally lowered.

If some new alternatives are prohibited by the reference system of ethics (S_I, $\pi(I)$), then it is necessary to modify the system of ethics or to completely substitute it by another ethics (for example, to replace Orthodoxy with the Uniate or „pure" Catholicism).

3. To widen alternatives set to S''_a of such, once that it would include desired „target" alternatives set S_{a1} without approaching a simultaneous increase of the

preferences $\sigma_k \in S_{a1}$. For example, it is possible to create many small parties with the tempting programs and the slogans in order to „pull" voters, to entice those being varying, deliberately knowing that these parties do not present danger for the leaders of voting race.

4. To modify the preferences distribution on S''_a so as they were „displaced" on the alternatives of S_{a1} set.

5. The new portion of controlling influences must reduce S''_a to the desired set S_{a1}. Let us see, what are the possibilities to influence on the preferences. In the sufficiently general case it is possible to assume that the preferences of subject j (for example, of the I type), are determined by the following factors: by the available resources R_j^{disp}, by the required resources $R_j^{req}(\sigma_k)$, by the expected resources (incomes) $R_j^{exp}(\sigma_k)$, by endogenous characteristics α, β,..., by ethical installations, by imperatives, by the mutual utilities of subjects in the group, by the presence of corporate problems and consolidated resources:

$$\pi_j(\sigma_k) = \pi_j(R_j^{disp}, R_j^{req}(\sigma_k), R_j^{exp}(\sigma_k), \alpha, \beta, ..., I_1, I_2, ..., I_L, ..., R_c^{disp},...).$$

Each of the enumerated factors exists as an object and simultaneously as the tool for the modification of preferences $\pi_j(\sigma_k)$. Thus, as it was already shown earlier, an increase of the available resources conducts to an increase in the entropy and vice versa, their decrease causes the H_π decrease. An increase of all $R_j^{req}(\sigma_k)$ simultaneously causes reduction in the entropy.

Let us give interpretation to the enumerated factors from the point of view of the mass consciousness:

1. Addition R_j^{disp} – the „bribery". This method is expensive, if we increase the resources of the large number of subjects, for example, of all members of the group, but it can be effective, if it is applied to the small number of decisive members of the group (let us say, that have the highest rating).

2. Differentiated change $R_j^{req}(\sigma_k)$. A simultaneous increase of the prices leads to the decrease in the entropy of each. Differentiated change: an increase in expenditures for undesirable alternatives and reduction in expenditures for alternatives, entering S_{a1} will lead to deformation of preferences distribution in favor of alternatives $\sigma_k \in S_{a1}$. This obviously is the complicatedly attained and expensive measure.

3. Influence on the expected results by using the false information, inaccurate estimations, that under certain conditions takes the form of fraud.

4. Influence on the endogenous parameters α_j, β_j, ... represents itself a kind of modification of the subject psyche. This is delicate and prolonged procedure.

5. Influence on the preferences through the ethical imperatives. The discussion deals with suppression of undesirable imperatives and start of new imperatives, i.e., about the directed modification of the ethics system.

Schematically described possibilities of influence on the consciousness of subject, to be more precise, on the system (S_a, $\pi(\sigma_k)$), either on the system (S_ξ, $\xi(j)$,..), or simultaneously on systems (S_a, S_ξ, $\pi(\sigma_k)$, $\xi(j)$,..) shown on Fig. 5.56.

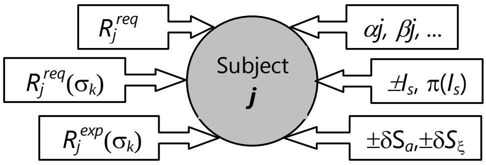


Fig. 5.56

The process of „consciousness manipulation" comes out in two forms:

- direct influence on the consciousness of each subject, entering the manipulated association;

- the use of technologies, based on the transmission of information from one subject to other $j \Leftrightarrow i$.

These processes are described by the equations, similar to the equations of the advertizing company, when there are several „competitive commodities". Commodities in this case are programs and the ideas of the parties or candidates, and the parties and candidates themselves (their ratings). As the starting point the models „Beast - victim" can be used when several forms of beasts and victims are present [55].

In general form assuming that M is total population of association, and M_i is number of followers of „ideology" i, we can assume that in the simplest case numbers M_i are subordinated to the equations

$$\frac{dM_i}{dt} = f_i\left(M_1, M_2, \ldots, M_n\right); \; \left(i \in \overline{1,n}\right). \tag{5.235}$$

This model appears as model „without memory". During the construction of probabilistic model we will arrive at the Markov scheme. More general and, evidently, more adequate to the processes, which take place in the human consciousness would be the integro-differential model

$$\frac{dM_i}{dt} = f_i\left(M_1, M_2, \ldots, M_n, t\right) + \sum_{j=1}^{n} \int_{t0_j}^{t} G_{ij}\left(t, s\right) g_j\left(M_1\left(s\right), M_2\left(s\right), \ldots, M_n\left(s\right), s\right) dS. \tag{5.236}$$

Here t_0 - the certain initial time, which, generally speaking, can be different for different groups of subjects. In the literature known to the author the models of such type are not encountered. To models of type (3.235) relate for example, model of Verhulst - Perl, model of Lotka - Volterra [55] and some others.

Let us examine model of following form

$$\frac{dM_i}{dt} = \mu\left(M - \sum M_j\right) - \varepsilon_i M_i + \sum{}' \varepsilon_j M_j + \sum_{j=1}^{n}{}' \alpha_{j \to i} M_i M_j - \sum_{j=1}^{n}{}' \alpha_{i \to j} M_i M_j . \tag{5.237}$$

Here $i, j \in \overline{1,n}$, dash in sum indicates the absence of term with $i = j$, M_i - subgroup population (electoral, for example), professing ideology i. Coefficients μ, ε_i, $\alpha_{j \to i}$ and $\alpha_{i \to j}$ represent by itself probabilities μ_i, ε_i — probability of autotrophic transitions into the state „i" or from the state „i"; $\alpha_{j \to i}$ — the probability of transition from „j" into „i" and, vice versa $\alpha_{i \to j}$ — transition from „i" into „j"; M – the total population of association. The first three terms describe autotrophic passages („sitting in front of the television set"), fourth and fifth terms describe the heterotrophic passages, caused by paired „encounters" and information exchange and reflect „second technology". It is obvious, that

$$\sum_{i=1}^{n} M_i \le M .$$

The source of models for reckoning of „consciousness manipulation" processes serves the theory of selection and competition [55], after the appropriate interpretation and adaptation to the task in question. Let us be turned to one more question: how „resource control" by consciousness can be achieved?

For this we will present the possible distribution of the I type preferences in the form:

$$\pi_j\left(\sigma_k\right) = \frac{e^{-\frac{R_j^{req}\left(\sigma_k\right)}{R_j^{disp} - R_j^{req}\left(\sigma_k\right)}} e^{-\frac{R_j^{req}\left(\sigma_k\right)}{R_j^{exp}\left(\sigma_k\right) - R_j^{req}\left(\sigma_k\right)}}}{\sum_{q=1}^{N} e^{-\frac{R_q^{req}\left(\sigma_k\right)}{R_q^{disp} - R_q^{req}\left(\sigma_k\right)}} e^{-\frac{R_q^{req}\left(\sigma_k\right)}{R_q^{\exp}\left(\sigma_k\right) - R_q^{req}\left(\sigma_k\right)}}}.$$

Let us designate

$$r_{ajk} = \frac{R_j^{req}\left(\sigma_k\right)}{R_j^{disp}}; \quad r_{ejk} = \frac{R_j^{req}\left(\sigma_k\right)}{R_j^{exp}\left(\sigma_k\right)}.$$

Here $R_j^{exp}(\sigma_k)$ — the expected effect (income) from the realization of alternative σ_k.

We have

$$\pi_j\left(\sigma_k\right) = \frac{e^{-\frac{r_{aj}}{1-r_{dj}}} e^{-\frac{r_{ejk}}{1-r_{ej}}}}{\sum_{q=1}^{N} e^{-\frac{r_{aj}}{1-r_{aj}}} e^{-\frac{r_{ejq}}{1-r_{ejq}}}}. \tag{5.238}$$

Simultaneous (for all alternatives) increase in the available resources leads to the fact, that $r_{aj} \to 0$ and distribution $\pi_j(\sigma_k)$ depends exclusively on r_{ej}, i.e. on the relationship between the required and expected („promised") resources. An increase (real and virtual) of the expected resources leads to the fact that $r_{ej} \to 0$ and distribution $\pi_j(\sigma_k)$ depends in the range only on the required and available resources. A selective (on σ_k) change of the expectations (expected resources) with the required and available resources kept constant leads to the repreferences distribution . Both the increase R_j^{disp} and an increase $R_j^{exp}(\sigma_k)$ (for all σ_k) causes an increase in the entropy.

Entropy grows, if simultaneously and evenly for all σ_k required resources decrease (for example, if simultaneous proportional reduction of the prices occurs). The most effective and cheap method to change the entropy H_π to the side of its increase, appears an increase in the expectations - the expected resources $R_j^{exp}(\sigma_k)$ ($k \in \overline{1,N}$).

The same method is applicable on the completing phase of the manipulation when the preferences of the undesirable alternatives should be decreased subject should be convinced in their final ineffectiveness. Recalling reasonings about the entropic „reign of freedom" and "the reign of need", we can graphically represent the process of consciousness manipulation in the form of scheme s on Fig. 5.57.

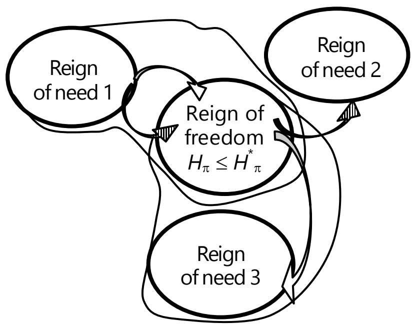


Fig. 5.57

Manipulation begins from certain "reign of need" (1), which is defined as the set of the subject is specific political views, belonging to the specific political party, group.

Then, with the aid of the methods and the technologies, mentioned above, occurs the „loosening" of his convictions to the state $H_{\pi j} \geq H^*_\pi$ and he is easily without the essential resistance transferred into the „reign of freedom", where flourishes the eclectism of views, persuasions, tastes, and, then; also easily he is transferred into the new „reign of need" (2), where his new convictions are fixed and attached.

The subject, after falling into the „reign of freedom" can prove to be the assault objective of different „operators" - the spreaders of different „epidemics". Since „political immunity" of subject in the „reign of the freedom" is substantially blown up, he becomes the victim of the most aggressive, insolent and unprincipled „operator". In this paragraph we gave only qualitative considerations about how it is possible to describe in the terms of subjective analysis the problem of „consciousness manipulation". The construction of mathematical models with the explicit use of canonical distributions of preferences of the I and II type is the matter of future.

5.13. Some additional tasks

In this paragraph we will discuss some additional dynamical problems as examples of subjective analysis application.

5.13.1. Beast – victim – hunter

Simplest ecological „beast - victim" model is a special case of the Lotka - Volterra model [55]:

$$\frac{dN_i}{dt} = N_i\left(\varepsilon_i - \sum_{j=1}^{K}\gamma_{ij}N_j\right),\ \ (i \in \overline{1,n}),$$

where N_i - number of populations. With n = 2, it is possible to use it for modeling of auto interferences, mutual interferences, competition and in some other cases. Coefficient ε_i characterizes the speed of the multiplication ($\varepsilon_i > 0$) or the speed of the natural mortality ($\varepsilon_I < 0$),coefficients γ_{ij} characterize the consequences of two individuals paired „encounters".

Terms $i = j$ characterize the decrease of the corresponding population as a result of the „closeness" effect. We will examine the case, when n = 2 and the model takes the form:

$$\frac{dN_1}{dt} = a_1N_1 - b_1N_1N_2 - f_1\left(N_1, N_2\right); \tag{5.239}$$

$$\frac{dN_2}{dt} = -a_2N_2 - b_2N_1N_2 - f_2\left(N_1, N_2\right).$$

Here $a_1 > 0$; $a_2 > 0$; $b_1 > 0$; $b_2 > 0$, N_1 — population of the „victims" is multiplying spontaneously proportional to their count N_1 and do not experiencing food deficiency. N_1 decreases as a result of the encounters of „victims" with the „beasts", with the speed proportional to the product of N_1 and N_2 .

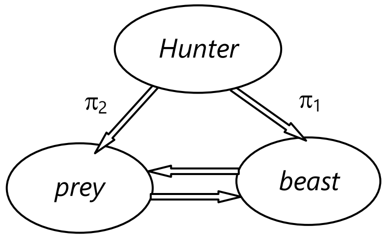


Fig. 5.58

"Beasts" die out proportional to their population N_2. Their population grows as a result of eating the „victims" at a rate proportional to N_1N_2. In the model the terms $f_1 > 0$ and $f_2 > 0$ are added, that describe the decrease of the populations both the „victims" and „beasts".

The „hunter" appears as the main „beast", while „victim" and „beast" in relation to the hunter are „victims". Let us assume that there is a specific „price", which hunter pays for prey, each yielded „victim" costs p_1, and each yielded „beast " is p_2. It is natural to assume that the „prices" p_1 and p_2 are connected with the number of populations and the number of hunters. Let us accept the following hypothetical scheme of price formation. Let the mean „price" is determined by the equality

$$(N_1 + N_2)\mu = p_1N_1 + p_2N_2$$

and, furthermore, the „prices" are related inversely proportional to numbers N_1 and N_2:

$$\frac{p_1}{p_2} = \frac{N_2}{N_1}.$$

It is easily find, that

$$p_1 = \mu\frac{N_1 + N_2}{2N_1}; \quad p_2 = \mu\frac{N_1 + N_2}{2N_2}$$

Let us assume also, that the mean „price" is as higher, as more hunters is: $\mu = \delta N_h$.

Then model will take the form:

$$p_1 = \delta\frac{N_h\left(N_1 + N_2\right)}{2N_1}; \quad p_2 = \delta\frac{N_h\left(N_1 + N_2\right)}{2N_2}. \tag{5.240}$$

Together with equations (5.239) we will examine the equation of the dynamics of the hunters numbers on the basis of the assumptions, that the more is rate of growth of this number, the more is the prey's output, and a natural loss of the hunters exist: if yield is absent, then exponential decay in the population of „hunters" is the main „beasts" occurs. Basing on this, the equation of the dynamics of number N_h can be written in the form:

$$\frac{dN_h}{dt} = -\gamma N_h + f_1 + f_2. \qquad (5.241)$$

Values f_1 and f_2 („kills" intensities of „victims" and „beasts") we will consider proportional to preferences of the hunter, in which there are two alternatives:

σ_1 – „victim"; the price per unit p_1; σ_2 – „beasts"; the unit price p_2. Let

$$f_1 = \lambda\pi(\sigma_1)N_1N_h;$$
$$f_2 = \lambda\pi(\sigma_2)N_2N_h.$$

In the relation $\pi(\sigma_1)$ and $\pi(\sigma_2)$ the following simplifying assumptions can be accepted:

a) „Hunter - amateur", hunts for the sport. He can prefer the game, hunting of which costs less:

$$\pi(\sigma_i) = \frac{e^{-\beta p_i}}{e^{-\beta p_1} + e^{-\beta p_2}}, \ \beta > 0.$$

b) „Hunter - professional", hunts for the profit from the sale of prey. He can prefer the more expensive game. Then

$$\pi(\sigma_i) = \frac{e^{\alpha p_i}}{e^{\alpha p_1} + e^{\alpha p_2}}, \ \alpha > 0.$$

c) „Hunter - amateur", burdened by the considerations of prestige. He selects the game, the hunting cost of which corresponds to his social status. Then we use the compromise model for $\pi(\sigma_i)$.

$$\pi(\sigma_i) = \frac{p_i^{\alpha} e^{-\beta p_i}}{p_i^{\alpha} e^{-\beta p_1} + p_i^{\alpha} e^{-\beta p_2}}, \ \alpha > 0, \ \beta > 0.$$

If there is only one hunter of only one type, then in accordance with Fig. 5.58 model of a change in the populations N_1, N_2, N_n takes the form:

$$\frac{dN_1}{dt} = a_1N_1 - b_1N_1N_2 - \lambda\pi(prey, p_1, p_2)N_1N_h; \qquad (5.242)$$

$$\frac{dN_2}{dt} = -a_2N_2 - b_2N_1N_2 - \lambda\pi(beast, p_1, p_2)N_2N_h;$$

$$\frac{dN_n}{dt} = -\gamma N_h + \lambda\left[\pi(prey, p_1, p_2)N_1 + \pi(beast, p_1, p_2)N_2\right]N_h.$$

If simultaneously in the hunting both the „amateur" and „professional" participates (not „firing back" on each other), then model contains two additional equations, describing the dynamics of „populations" of hunters of both types N_{ha}, N_{hp} , and equations for the populations N_1 and N_2 contain in the right sides terms, caused by the activity of both „amateurs", and „professionals":

$$\begin{aligned}
\frac{dN_1}{dt} &= a_1N_1 - b_1N_1N_2 - \lambda_1\pi_a\left(prey,p_1,p_2\right)N_1N_{ha} - \lambda_2\pi_p\left(prey,p_1,p_2\right)N_1N_{hp};\\
\frac{dN_2}{dt} &= -a_2N_2 - b_2N_1N_2 - \lambda_1\pi_a\left(beast,p_1,p_2\right)N_2N_{ha} - \lambda_2\pi_p\left(beast,p_1,p_2\right)N_1N_{hp};\\
\frac{dN_{ha}}{dt} &= -\gamma_aN_{ha} + \lambda_1\pi_a\left(prey,p_1,p_2\right)N_1N_{ha} - \lambda_1\pi_a\left(beast,p_1,p_2\right)N_2N_{ha};\\
\frac{dN_{hp}}{dt} &= -\gamma_pN_{ha} + \lambda_2\pi_p\left(prey,p_1,p_2\right)N_1N_{np} - \lambda_2\pi_p\left(beast,p_1,p_2\right)N_2N_{np}.
\end{aligned} \tag{5.243}$$

In these equations N_{ha} is number of „hunters - amateurs", N_{hp} is number of „hunters - professionals", a_1, a_2, b_1, b_2, γ_a, γ_p, λ_1, λ_2 is positive structural constants. On Fig. 5.59 is shown one of the versions of calculation of the dynamics of functions $N_1(t)$, $N_2(t)$, $N_{ha}(t)$, $N_{hp}(t)$ with the aid of equations (5.243). We see that the number of the „hunters" of both types diminishes. Fig. 5.60 shows the result of calculation with another combination of values of the structural parameters.
We see that in this case the game populations diminish, the number of „amateurs" grows, and the number of „professionals" diminishes. The case, when there are hunters of two types is reflected on Fig. 5.61.

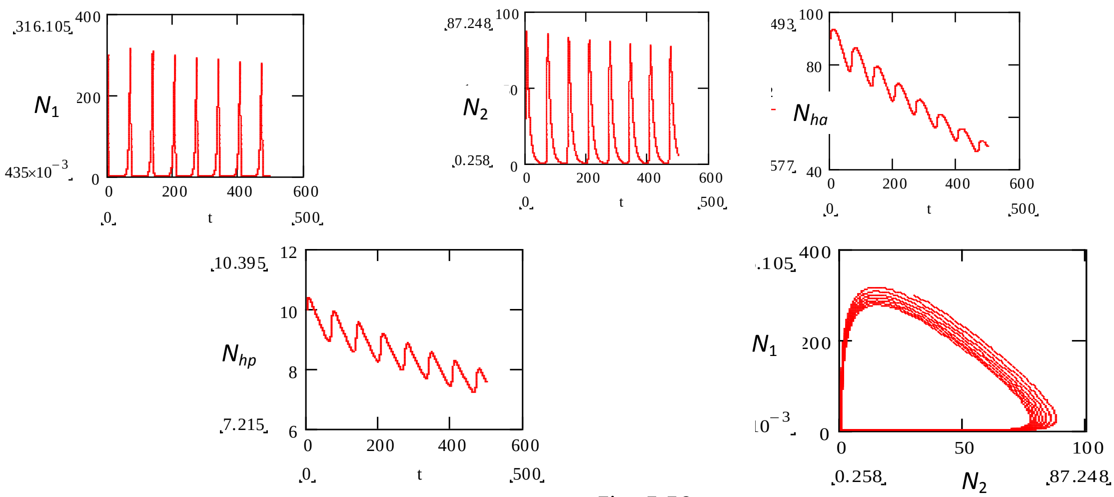


Fig. 5.59

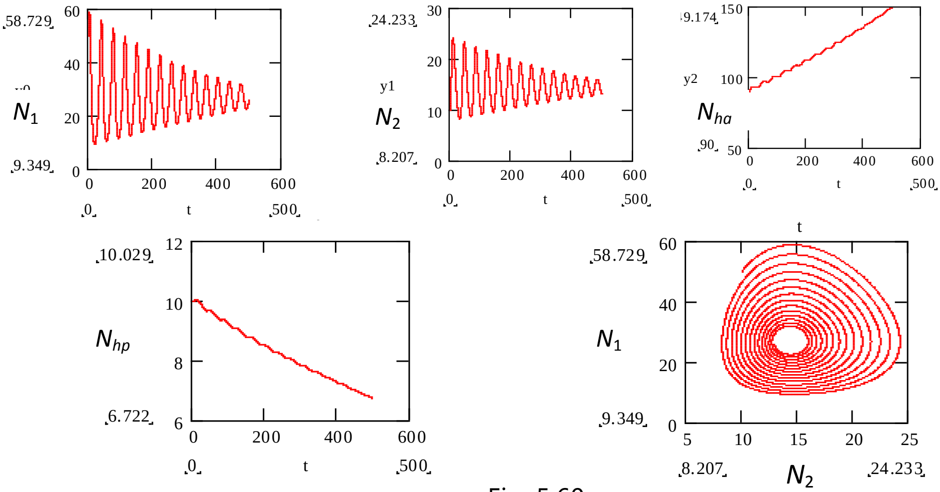


Fig. 5.60

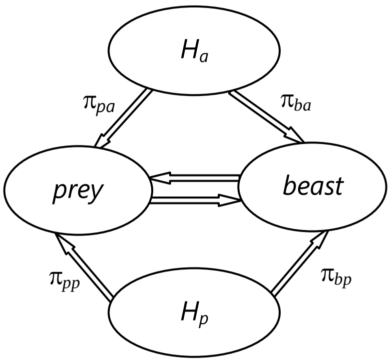


Fig. 5.61

5.13.2. On the models of advertising campaign

There is a large number of advertising campaign models. That is problem here to show, as it is possible to apply proposed version of subjective analysis to the analysis of the dynamic process of advertising campaign. Therefore let us examine the simplest model and we will see how it can be modified. Such model is one reduced to the Malthus equation:

$$\frac{dN}{dt} = \alpha N\left(N_0 - N\right), \tag{5.244}$$

where N — the number of informed customers, N_0 — number of potential customers, α - probability that as a result of the informed customer encounter with the potential customer, the latter will be informed. The following model answers the more general case:

$$\frac{dN}{dt} = \left(\alpha_1 + \alpha_2 N\right)\left(N_0 - N\right). \tag{5.245}$$

Here α_1 characterizes the intensity of the direct information of potential customers, α_2 - the effectiveness of advertizing campaign, caused by contact of informed customers with not informed potential customers.

Let us examine the hypothetical situation, when two commodities are on the market (two alternatives) σ_1 and σ_2 from two producers, and each of them conducts their advertizing campaign, moreover anti-advertisement is excluded. Let N_1 — number of those informed about the first alternative, N_2 — number of informed about the second alternative, N_{12} — number of those informed about both – about the first and the second alternatives.

The model, similar to model (5.245), in this case appears as follows:

$$\frac{dN_1}{dt} = \left(\alpha_0 + \alpha_1 N_1\right)\left(N_0 - N_1 - N_2 - N_{12}\right); \tag{5.246}$$

$$\frac{dN_2}{dt} = \left(\beta_0 + \beta_1 N_1\right)\left(N_0 - N_1 - N_2 - N_{12}\right);$$

$$\frac{dN_{12}}{dt} = \left(\gamma_0 + \gamma_1 N_1\right)\left(N_0 - N_1 - N_2 - N_{12}\right) + \gamma_2 N_1 N_2.$$

Here the parameters α_0, β_0, γ_0 characterize the „straight" advertisement effectiveness, moreover $\gamma_0 \neq 0$, if a certain part of the potential customers accepts advertisement of both producers, γ_2 characterize the intensity of an increase N_{12} as a result of the „encounters" of customers from subgroups N_1 and N_2.

The value of coefficients γ_0 and γ_1 can be selected from the following considerations. Company *A* (the first producer) and company *B* (the second producer) works with the contingent of potential customers N_0. The single customer information probabilities are α_0 and β_0, $P(A) = \alpha_0$, $P(B) = \beta_0$ respectively.

The probability that the customer will be informed about both alternatives is $P(A \vee B)$. If is assume that companies operate independently, we can consider, that

$$P(A \vee B) = P(A)P(B) \Rightarrow \gamma_0 = \alpha_0\beta_0,$$

Similarly, if customers behave independently having mutual information it is possible to assume $\gamma_1 = \alpha_1\beta_1$. In the general case number of informed customers will not coincide with the number of buyers, however, for the purpose of simplification we assume, that all informed customers appear as buyers. Their quantities M_1 and M_2 we will determine with the formulas

$$M_1 = N_1 + \pi(\sigma_1)N_{12}; \qquad (5.247)$$
$$M_2 = N_2 + \pi(\sigma_2)N_{12}.$$

From formulas (5.247) follows, that the number of buyers of the first commodity σ_1 is equal to the number of customers, informed only about σ_1 and the part of the group of N_{12} of customers, informed about both alternatives, which have the capability of selection and which achieve this selection in accordance with the distributions of preferences $\pi(\sigma_1)$, $\pi(\sigma_2)$.

In this case it is possible to express the function $\pi(\sigma_i)$ through the value of competitive capacity $C(\sigma_i) = C(p_i, q_i)$, where p_i — price of the commodity unit σ_1, and q_i — specific quantitative characteristic of its quality. Simplifying, we will assume that $C(\sigma_i) = C(p_i)$.Let

$$\pi(\sigma_i) = \frac{e^{-\mu p_i}}{e^{-\mu p_1} + e^{-\mu p_2}}; \qquad (5.248)$$

It is possible to propose the different versions of $\pi(\sigma_i)$ dependence on price. For example, in the case of two alternatives it is possible to consider, that

$$\pi(\sigma_1) = \frac{e^{-2\mu\frac{p_1-p_2}{p_1+p_2}}}{e^{-2\mu\frac{p_1-p_2}{p_1+p_2}} + e^{-2\mu\frac{p_2-p_1}{p_1+p_2}}}; \quad \pi(\sigma_2) = \frac{e^{-2\mu\frac{p_2-p_1}{p_1+p_2}}}{e^{-2\mu\frac{p_1-p_2}{p_1+p_2}} + e^{-2\mu\frac{p_2-p_1}{p_1+p_2}}}.$$

Hypothesis relative to the formation of price is further necessary. Here a maximally simple assumption is also accepted, that the price is proportional to a number of buyers, i.e., it is instantly formed. Let

$$p_1 = mM_1 + r_1;\ p_2 = mM_2 + r_2, \tag{5.249}$$

where r_i is additives to the price, with the aid of which producer governs both the advertizing campaign and volume of sales. If $r_i = 0$, then model proves to be locked and process occurs in the automatic regime. If additives r_1 and r_2 are constant, then

$$\frac{dp_1}{dt} = m\frac{dM_1}{dt};\quad \frac{dp_2}{dt} = m\frac{dM_2}{dt}.$$

Using formulas (5.247), after conversions we will obtain the equations:

$$\frac{d\pi_1}{dt} = -\frac{\mu m\pi_1\pi_2}{1+2\mu m\pi_1\pi_2}\left(A-B\right); \tag{5.250}$$

$$\frac{d\pi_2}{dt} = -\frac{\mu m\pi_1\pi_2}{1+2\mu m\pi_1\pi_2}\left(B-A\right).$$

Prices are determined from the formulas:

$$p_1 = m(N_1 + \pi_1 N_{12}) + r_1;$$
$$p_2 = m(N_2 + \pi_2 N_{12}) + r_2.$$

Finally, we can to assume, that producer makes expenditures for advertisement proportional to his income, what let us assume, that

$$\alpha_0 = \varepsilon D_1;\ \beta_0 = \varepsilon D_2, \tag{5.251}$$

where D_1 and D_2 is incomes, $D_1 = M_1p_1$; $D_2 = M_2p_2$, ε is coefficient, which determines the share of expenditures for advertisement and their effectiveness. Price dynamics in the given model is determined by equations:

$$\frac{dp_1}{dt} = \frac{m\left(A+\mu m\pi_1\pi_2\left(A+B\right)\right)}{1+2\mu m\pi_1\pi_2}; \tag{5.252}$$

$$\frac{dp_2}{dt} = \frac{m\left(D+\mu m\pi_1\pi_2\left(A+B\right)\right)}{1+2\mu m\pi_1\pi_2}.$$

Scheme on Fig. 5.62 schematically illustrates given model. Models with similar structure can be used for the rough simulation of the propagation process of scientific ideas, political ideologies, and replacement of old technologies by the new ones.

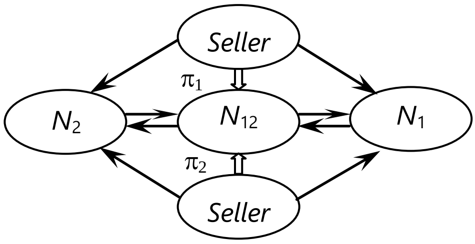


Fig. 5.62

In this case addition of relations to such models, relations which describe the influence of subjective factors - the distributions of the different types preferences, makes these models more meaningful. Fig. 5.63 shows dependences on time for groups population N_1, N_2, N_{12}, when N_0 = 1000 for the specific values of the structural parameters.

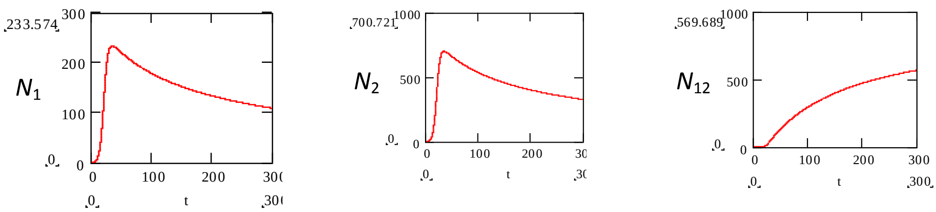


Fig. 5.63

The populations of informed clients (and buyers - by hypothesis) with the condition, that each of them will purchase commodities of one producer only (*A* or *B*) are presented on Fig. 5.64.

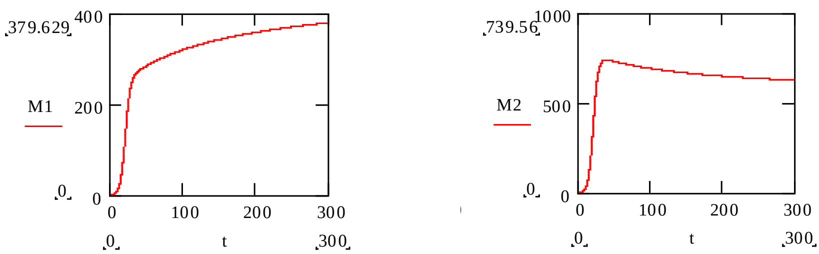


Fig. 5.64

We note that buyer's populations M_1 and M_2 in the initial period behave differently.

Since the price of commodity unit differs from number of population by a constant factor, then qualitatively prices change just as the population: the price of the first commodity p_1 constantly grows, the price of the second commodity p_2 at the beginning grows rapidly, it reaches maximum, and then, it is lowering constantly and finally it reaches equilibrium value, since in the course of time $N_1 \to 0$ $N_2 \to 0$ and all customers obtain information about both alternatives σ_1 and σ_2. Fig. 5.65 shows the data about the populations M_1, M_2, M_3, when it is supposed, that the subject informed about both commodities from the group N_{12} can purchase both of them. In these case alternatives set S_a contains the alternatives: σ_1, σ_2, $\sigma_1 \wedge \sigma_2 = \sigma_3$.

Last alternative σ_3 corresponds to the acquisition both the first commodity and second commodity by subject from the group N_{12}. The corresponding quantities of buyers M_1, M_2, M_3 are determined from the formulas:

$$M_1 = N_1 + \pi(\sigma_1)N_{12}; \tag{5.253}$$

$$M_2 = N_2 + \pi(\sigma_2)N_{12};$$

$$M_3 = \pi(\sigma_3)N_{12}.$$

In the detailed writing:

$$M_1 = N_1 + \frac{e^{-\mu p_1}}{e^{-\mu p_1} + e^{-\mu p_2} + e^{-\mu(p_1+p_2)}} N_{12}; \quad M_2 = N_2 + \frac{e^{-\mu p_2}}{e^{-\mu p_1} + e^{-\mu p_2} + e^{-\mu(p_1+p_2)}} N_{12};$$

$$M_3 = \frac{e^{-\mu(p_1+p_2)}}{e^{-\mu p_1} + e^{-\mu p_2} + e^{-\mu(p_1+p_2)}} N_{12}.$$

Here μ is structural endogenous parameter.

One assumes for simplification that it is identical for all subjects.

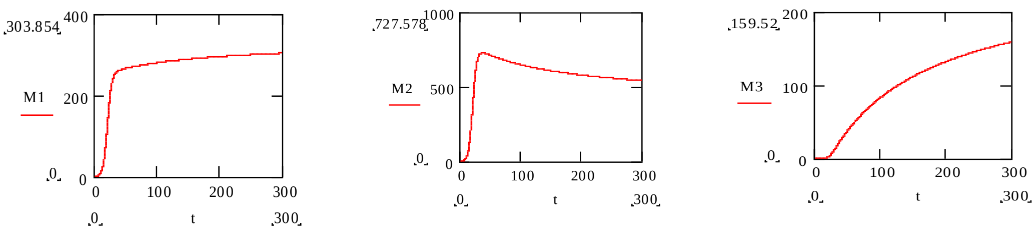


Fig. 5.65

In this case, it is possible to propose following model for the prices

$$p_1 = m(N_1 + M_3); \ \ p_2 = m(N_2 + M_3).$$

5.13.3. Auto-relaxation of the preferences distributions

Used previously everywhere in this book variational principle of entropy can be conditionally called „static". It makes possible to determine in each case the canonical preferences distribution „not attached" to any moment of time. In this case dynamic processes describe the dynamics of the preferences, which are remained optimum in each moment relatively to the „static" functional. In other words tuning on the optimality occurs instantly.

This circumstance, from the author's point of view, appears as drawback in the accepted concept. Can preferences experience the spontaneous changes, not caused by a change in the exogenous factors? Within the frameworks of the schemes used above there is no answer for this question.

The constructions in paragraph 5.5, based on use of functional, which contain preferences at different moments of time and allowing considering the „prehistory" of preferences, although provide an effective means of research of spontaneous or autorelaxation dynamics, nevertheless they appear palliatively.

The „experimental dynamic principle" possibly exists, in which as in the „static" principle the entropy plays important role, and dynamic equations appear as Euler-Lagrange equations for the specific functional. It is not excluded, that in the wording of this principle the Prigogine's theorem about the minimum speed of entropy change near the equilibrium state can play the important role.

The attempts of the author to form such principle were not crowned with success. In the present paragraph the heuristic simulation schemes of the „spontaneous" changes in the preferences are discussed, lying within the framework of static variation principle.

In this case we will satisfy two conditions: 1— at each moment of time the standardization condition is ensured, in this case the standardization can be standard (single), or non-single, when entropy reaching the absolutely maximum value $H_{\max,\max}$ (see Ch. 3.2); 2— arbitrary distribution at any moment of time coincides with the optimum, or asymptotically striving to optimum with $t \to \infty$.

Let us examine several scheme of the preferences auto-relaxation.

1. Let us assume that standardization is single: $\sum_i \pi_i = 1$ and let us examine the equation:

$$\frac{d\pi_i}{dt} = -k(t)\left(\pi_i - \frac{1}{N}\right), \ (i \in \overline{1,N}),\ k(t) > 0. \qquad (5.254)$$

As we see, standardization conditions are met, actually:

$$\sum_{i=1}^{N}\left(\pi_i-\frac{1}{N}\right)=0;\ \ \sum_{i=1}^{N}\frac{d\pi_i}{dt}=0.$$

Here in the course of time all π_i converge to $\frac{1}{N}$, and entropy is striving to $H_{\max}$ = ln$N$. The preferences canonical distribution can be taken as the initial conditions for π_i, optimum in the sense of a certain functional.

2. In the case, if standardization is non-single and corresponds to the conditions, when $H_{\max,\max}$ is reached, then for the equivalent alternatives we have to take as the limiting value $\pi_i=\frac{1}{e}$, $(\forall i\in\overline{1,N})$.

Then instead of equation (5.244) the following equation should be used:

$$\frac{d\pi_i}{dt}=-k(t)\left(\pi_i-\frac{1}{e}\right),\ \ i\in\overline{1,N}\ \ k(t)>0. \tag{5.255}$$

Generally it is possible to organize adaptation to any a priori assigned limiting constant distribution. For example to the canonical: $\pi_{i\text{opt}}$ with the condition, that it is correspondingly normalized

$$\sum_{i=1}^{N}\pi_{i\text{opt}}=1\ \text{ or }\ \sum_{i=1}^{N}\pi_{i\text{opt}}=\varphi,\ \ \varphi=\text{const}.$$

Equation (5.255) in this case takes the form:

$$\frac{d\pi_i}{dt}=-k_i(t)\left(\pi_i-\pi_{i\text{opt}}\right).$$

3. Let formation is non-single and appears variable:

$$\sum_{i=1}^{N}\pi_i=\varphi(t),$$

then $\pi_i^0=\frac{\varphi(t)}{N}$ is corresponds to the equivalence of all alternatives. Instead of equation (5.255) we must use the equation

$$\frac{d\pi_i}{dt}=\frac{1}{N}\frac{d\varphi(t)}{dt}-k(t)\left(\pi_i-\frac{1}{N}\varphi(t)\right). \tag{5.256}$$

4. Let us assume now, that the canonical distributions $\pi_{i\text{opt}}$ appear time functions as a result of, that exogenous situation changes (resources, utilities ,...) and perhaps, the endogenous parameters. Then relaxation is described by the equation:

$$\frac{d\pi_i}{dt}=\frac{1}{N}\dot{\varphi}(k)-k(t)\left(\pi_i-\pi_{i\text{opt}}\right),\ \ i\in\overline{1,N},\ k(t)>0,\ \sum_{i=1}^{N}\pi_{iopt}=\varphi(t). \tag{5.257}$$

Relaxation term ensures the adaptation of the current distribution π_i to the optimum canonical π_{iopt}, which corresponds to the given endogenous and exogenous situation. The success of adaptation depends on rate of function $\pi_{iopt}(t)$ change and coefficient $k(t)$, which, as structural parameters of canonical distributions (α, β,...) used earlier, can be attributed to endogenous parameters. $k(t)$ describes ability of subject the adapted to changing situation.

5. The following generalization of theory investigations the cause when we use for describing the adaptation process the differential equations of higher order, for example linear equation of second order.

With constant standardization the analog of (5.254) is the following equation:

$$\frac{d^2\pi_i}{dt^2}+a(t)\frac{d\pi_i}{dt}+b(t)\left(\pi_i-\frac{1}{N}\right)=0,\ i\in\overline{1,N},\ a>0,\ b>0. \tag{5.258}$$

Standardization conditions $\sum_{i=1}^{N}\pi_i=1$ are carried out at any moment of time. Coefficient $a(t)$ ensures damping, coefficient $\sqrt{b(t)}$ plays the role of fundamental frequency of vibrations. As the initial conditions is possible to take any values, which satisfy the conditions $\sum_{i=1}^{N}\pi_i(0)=1;\ \sum_{i=1}^{N}\dot{\pi}_i(0)=0.$ If standardization is variable: $\sum_{i=1}^{N}\pi_i=\varphi(t)$, then instead of (5.258) the following equation should be used

$$\frac{d^2\pi_i}{dt^2}+a(t)\frac{d\pi_i}{dt}+b(t)\left(\pi_i-\frac{\varphi(t)}{N}\right)=\frac{1}{N}\frac{d^2\varphi}{dt^2}+a(t)\frac{1}{N}\frac{d\varphi}{dt}. \tag{5.259}$$

It is necessary by selection of initial conditions and coefficients a and b, to ensure the conditions, which guarantee fulfilling of inequality $0\le\pi_i\le 1$ for $\forall i\in\overline{1,N}$.

At last, if adaptation is accomplishing to the given distribution $\pi_{iopt}(t)$ with a constant standardization on unit and constant a and b, we will use the equation:

$$\frac{d^2\pi_i}{dt^2}+a\frac{d\pi_i}{dt}+b\left(\pi_i-\pi_{iopt}(t)\right)=0. \tag{5.260}$$

The function $b\pi_{iopt}(t)$ can be considered as „perturbing" force, such, that $\sum_{i=1}^{N}\pi_{iopt}=1$. Coefficients a and b must be selected in a way that inequality $0\le\pi_i\le 1$ given above would be fulfilled. It is obvious that the resonance in certain cases can appear, if external conditions and $\pi_{iopt}(t)$ change periodically. This circumstance also impose limitations on the parameters *a* and *b*. Let us note one additional possibility.

We introduce new variables $\rho_i = \pi_i - \pi_{iopt}$, where in the beginning π_{iopt} we will consider constant and subordinating to the standardization $\sum_{i=1}^{N} \pi_{iopt} = 1$, then

$$\sum_{i=1}^{N} \rho_i = 0.$$

Let us examine linear system of equations

$$\frac{d\rho_1}{dt} = a_{11}\rho_1 + a_{12}\rho_2 + \ldots + a_{1N}\rho_N; \tag{5.261}$$

$$\frac{d\rho_2}{dt} = a_{21}\rho_1 + a_{22}\rho_2 + \ldots + a_{2N}\rho_N; \ldots; \quad \frac{d\rho_N}{dt} = a_{N1}\rho_1 + a_{N2}\rho_2 + \ldots + a_{NN}\rho_N.$$

If A is matrix of system, then for the asymptotic stability of zero solution ($\rho_i \to 0$ with $t \to \infty$) it is necessary and sufficient, so that all roots λ_k of the characteristic equation $\det(A - \lambda I) = 0$, where I — is the unit matrix, would have negative real parts, and for this, in turn, it is necessary and sufficient, that the principal minors of the matrix

$$\begin{bmatrix} a_1 & 1 & 0 & 0 & \ldots & \ldots & 0 \\ a_3 & a_2 & a_1 & 1 & 0 & \ldots & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 & 1 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix},$$

where a_i^- is coefficients of the characteristic equation

$$\lambda^N + a_1\lambda^{N-1} + \ldots + a_N = 0,$$

would be positive:

$$\Delta_1 = a_1 > 0; \; \Delta_2 = \begin{vmatrix} a_1 & 1 \\ a_3 & a_2 \end{vmatrix} > 0; \; \Delta_3 = \begin{vmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix} > 0; \; \ldots; \; \Delta_N = a_N\Delta_{N-1} > 0.$$

These are the Routh-Hurwitz conditions. Besides these conditions, in order to ensure the fulfillment of conditions for standardization, it is necessary to put additional conditions on the matrix coefficients. Since $\sum_{i=1}^{N} \frac{d\rho_i}{dt} = 0$, then summarizing, we obtain

$$0 = \left(\sum_{i=1}^{N} a_{i1}\right)\rho_1 + \left(\sum_{i=1}^{N} a_{i2}\right)\rho_2 + \ldots + \left(\sum_{i=1}^{N} a_{iN}\right)\rho_N.$$

This equality must be fulfilled for any distributions ρ_i (satisfying zero standardization). Consequently, each of the sums must be equal to zero:

$$\sum_{i=1}^{N} a_{ik} = 0; \;\; \forall k \in \overline{1,N}.$$

If canonical optimum preferences are the functions of time, then after obtaining solutions of the system (5.261) $\rho_i = \pi_i - \pi_{iopt}$ we will find the law of variation of the instantaneous preferences

$$\pi i(t) = \rho i(t) + \pi iopt(t), \quad \forall i \in \overline{1,N}.$$

As an example let us examine again the model economy of Walras - Leontev with two products (Y_2 and Y_3), two prices (Y_4 and Y_5), augmented by equations for the preferences ($\pi(\sigma_1) = Y_0$, $\pi(\sigma_2) = Y_1$), is equations (5.262).

In the equations for the preferences the relaxation members are added in the form of terms in the right sides:

$$\ldots + b(\pi_i - \pi_{i0}); \; \pi_{10} + \pi_{20} = 1.$$

Calculations were carried out for the following collection of the values of structural parameters:

$a_{11} = 0{,}7$; $a_{22} = 0{,}3$; $a_{12} = 0{,}3$; $a_{21} = 0{,}1$; $m_1 = 100$; $m_2 = 100$; $h_1 = 10$; $h_2 = 10$; $b = -0{,}1$;

$c_1 = 3$; $c_2 = 3$; $w_1 = 4$; $w_2 = 4$; $\beta = 0{,}03$; $q_1 = 1$; $q_2 = 2$; $a_1 = 1$; $a_2 = 1$; $\alpha = 0{,}01$.

$$D(t,Y)=\begin{bmatrix} Y_0Y_1\left[\frac{1}{h_1}\left(\alpha\frac{1}{Y_2}-\beta\right)\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)- \right. \\ \left. -\frac{1}{h_2}\left(\alpha\frac{1}{Y_3}-\beta\right)\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)\right]+b\left(Y_0-0{,}2\right) \\ Y_0Y_1\left[\frac{1}{h_1}\left(\alpha\frac{1}{Y_3}-\beta\right)\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)- \right. \\ \left. -\frac{1}{h_1}\left(\alpha\frac{1}{Y_2}-\beta\right)\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)\right]+b\left(Y_1-0{,}8\right) \\ \frac{1}{h_1}\left(a_{11}Y_4+a_{12}Y_5+c_1Y_0-Y_4\right)\left(\frac{4}{\pi}\operatorname{atan}\left(e^{4Y_2}\right)\right) \\ \frac{1}{h_2}\left(a_{21}Y_4+a_{22}Y_5+c_2Y_1-Y_5\right)\left(\frac{4}{\pi}\operatorname{atan}\left(e^{4Y_3}\right)\right) \\ -\frac{1}{m_1}\left(a_{11}Y_2+a_{21}Y_3-Y_2+a_1w_1+q_1\right)\left(\frac{4}{\pi}\operatorname{atan}\left(e^{4Y_4}\right)\right) \\ -\frac{1}{m_2}\left(a_{12}Y_2+a_{22}Y_3-Y_3+a_2w_2+q_2\right)\left(\frac{4}{\pi}\operatorname{atan}\left(e^{4Y_5}\right)\right) \end{bmatrix} \qquad Y_0=\begin{pmatrix} 0{,}5 \\ 0{,}5 \\ 60 \\ 60 \\ 30 \\ 30 \end{pmatrix} \qquad (5.262)$$

Coefficient $b < 0$.

This system without the relaxation terms was examined above, and the results of calculation were represented on Figs. 5.16-5.20 (for the same values of the structural parameters). Figs. 5.66-5.69 shows the analogous results obtained with the aid of system (5.262) with „attracting distributions" π_{10} = 0.2, π_{20} = 0,8.

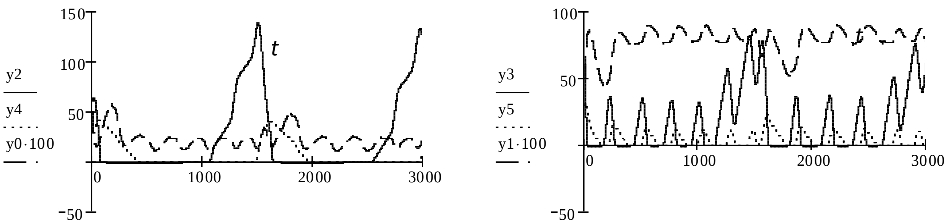


Fig. 5.66

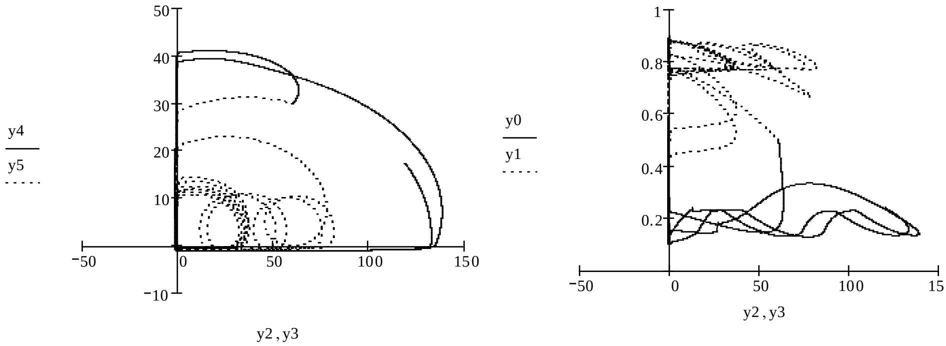
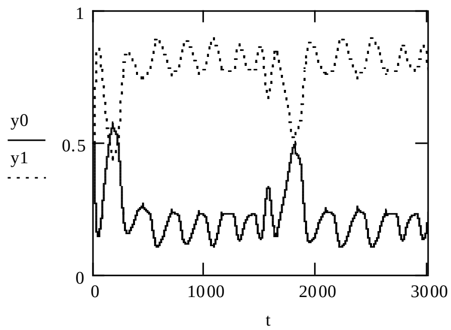


Fig. 5.67



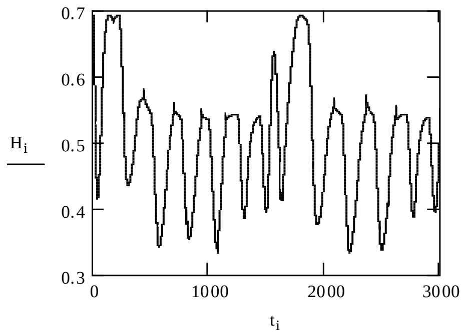


Fig. 5.68

Fig. 5.69

It seems evident, basing on 5.69 that the preferences in essence are concentrated near the „attracting" values, but oscillatory regime remains.

Entropy experiences the significant fluctuations (Fig. 5.70). Another case, when „attracting" appears the uniform distribution (π_{10} = 0,5; π_{20} = 0,5), with other conditions being equal is shown on Figs. 5.70-5.73.

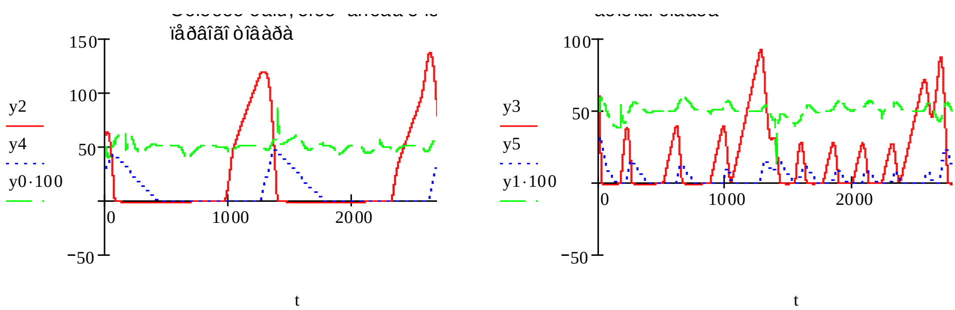


Fig. 5.70

We see from Fig. 5.71 that as a result the adaptations to „attracting" uniform distribution, current preferences oscillate in the vicinity of uniform values, and entropy (Fig. 5.73) remains high. This is also reflected on the „price- preference" phase portrait.

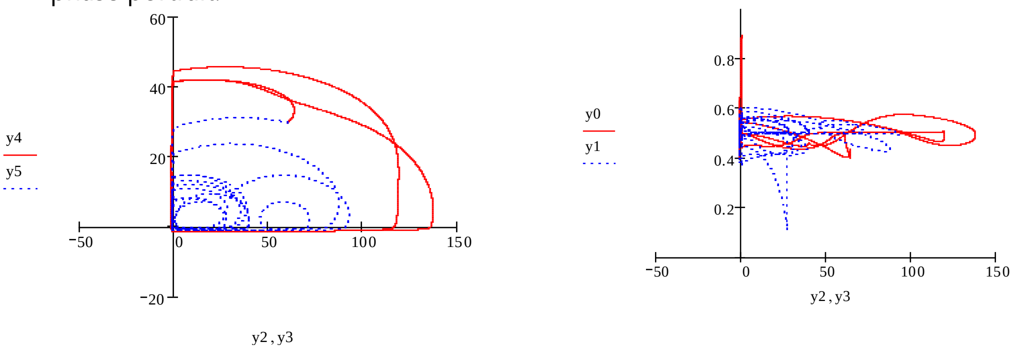


Fig. 5.71

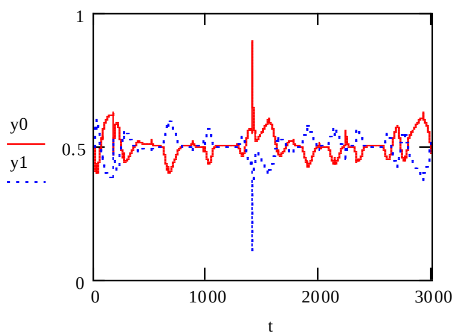


Fig. 5.72

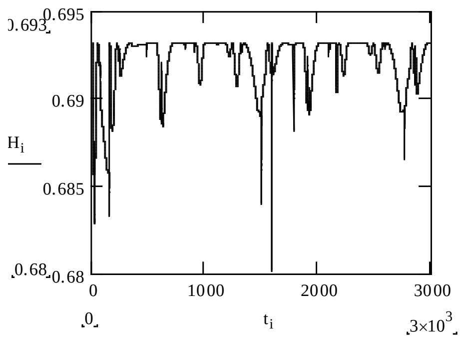


Fig. 5.73

5.13.4. Complete Walras - Leontev model, including preferences of consumer and producer

In paragraph 5.7 for simplification the truncated Walras - Leontev model was used, in which equations, which describe dynamics of production factors were omitted. In the qualitative sense the effects, described in 5.7, can be transferred to the complete Walras - Leontev model, and in which besides equations for the issues and prices of commodities the equations for prices of factors are contained.

In the present paragraph we examine this model with the following assumptions: two abstract subjects act: the „buyer", whose preferences $\pi_c(\sigma_i)$ depend on the prices of commodities (σ_i — alternative commodities) and the „producer", whose preferences $\pi_p(\sigma_i)$ depend on the expected profit. It is assumed, as it was above, that entire produced goods are realized. It is also assumed, that the structure of exogenous dynamics equations is such that it is possible to turn out without use of differential equations for preferences and to take then into account explicitly in right sides of equations for the prices of commodities and factors. Final demand for the commodity is considered to be proportional to the buyer's preference, which depends on the relationship of the prices:

$$c_i\pi_c(\sigma_i).$$

Here c_i is numerical coefficient.

The demand of factor depends on the producer preferences, which are, in turn, the functions of the expected profit.

$$r_i(b_{i1}\pi_p(\sigma_1) + b_{i2}\pi_p(\sigma_2));\ i \in \overline{1,2},$$

r_i is numerical coefficient.

The expected profit from the realization of the first commodity is equal q_1Y_2, from the realization of the second commodity is q_2Y_3.

$$D(t,Y) = \begin{bmatrix} \dfrac{1}{h_1}\left(a_{11}Y_2 + a_{12}Y_3 + c_1\dfrac{e^{-\beta_1 Y_0}}{e^{-\beta_1 Y_0} + e^{-\beta_1 Y_1}} - Y_2\right)\left[\dfrac{4}{\pi}\operatorname{atan}\left[e^{1\cdot Y_0 - 5}\right]\right] \\ \dfrac{1}{h_2}\left(a_{21}Y_2 + a_{22}Y_3 + c_2\dfrac{e^{-\beta_1 Y_1}}{e^{-\beta_1 Y_0} + e^{-\beta_1 Y_1}} - Y_3\right)\left[\dfrac{4}{\pi}\operatorname{atan}\left[e^{1\cdot Y_1 - 5}\right]\right] \\ -\dfrac{1}{m_1}\left(a_{11}Y_0 + a_{21}Y_1 - Y_0 + a_1w_1 + q_1 + b_{11}Y_4 + b_{21}Y_5\right)\left[\dfrac{4}{\pi}\operatorname{atan}\left[e^{1\cdot Y_2 - 5}\right]\right] \\ -\dfrac{1}{m_2}\left(a_{12}Y_0 + a_{22}Y_1 - Y_1 + a_2w_2 + q_2 + b_{12}Y_4 + b_{22}Y_5\right)\left[\dfrac{4}{\pi}\operatorname{atan}\left[e^{1\cdot Y_3 - 5}\right]\right] \\ \dfrac{1}{k_1}\left(b_{11}Y_0 + b_{12}Y_3 - r_1\dfrac{e^{\beta_2 q_1 Y_2}b_{11} + e^{\beta_2 q_2 Y_3}b_{12}}{e^{\beta_2 q_1 Y_2} + e^{\beta_2 q_2 Y_3}}\right)\left[\dfrac{4}{\pi}\operatorname{atan}\left[e^{1\cdot Y_4 - 5}\right]\right] \\ \dfrac{1}{k_2}\left(b_{21}Y_2 + b_{22}Y_3 - r_2\dfrac{e^{\beta_2 q_1 Y_2}b_{21} + e^{\beta_2 q_2 Y_3}b_{22}}{e^{\beta_2 q_1 Y_2} + e^{\beta_2 q_2 Y_3}}\right)\left[\dfrac{4}{\pi}\operatorname{atan}\left[e^{1\cdot Y_5 - 5}\right]\right] \end{bmatrix} \quad (5.263)$$

As earlier the first two equations describe the commodities price change (*Y0*, *Y1*). The following two equations determine the issues of commodities (Y_2, Y_3).

Last two are equations for the prices of commodities (Y_4, Y_5). The prices of commodities and prices of factors figure in the third and fourth equations.

The right sides of all equations are multiplied by the barrier functions, not „letting " the solution into the region of negative values. Figs. 5.74 - 5.77 shows the results of calculation for the following values of the structural parameters:

a_{11} = 0,7; a_{22} = 0,5; a_{12} = 0,3; a_{21} = 0,1; m_1 = 100; m_2 = 100; h_1 = 10; h_2 = 10; c_1 = 10;

c_2 = 10; w_1 = 4; w_2 = 4; β_1 = 0,02; β_2 = 0,05; q_1 = 0,1; q_2 = 0,1; a_1 = 1; a_2 = 1;

b_{11} = 0,05; b_{12} = 0,02; b_{21} = 0,03; b_{22} = 0,03; r_1 = 2; r_2 = 2; k_1 = 50; k_2 = 50 and the initial conditions:

$Y_0(0)$ = 30; $Y_1(0)$ = 30; $Y_2(0)$ = 60; $Y_3(0)$ = 60; $Y_4(0)$ = 30; $Y_5(0)$ = 30.

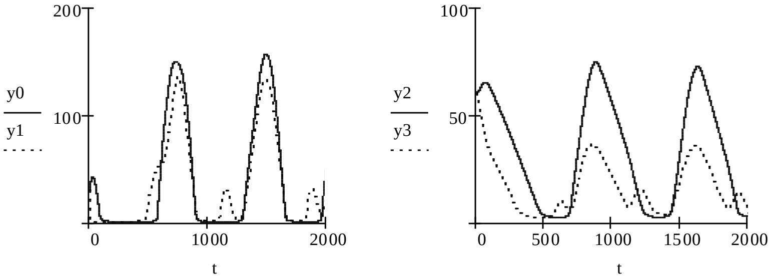


Fig. 5.74

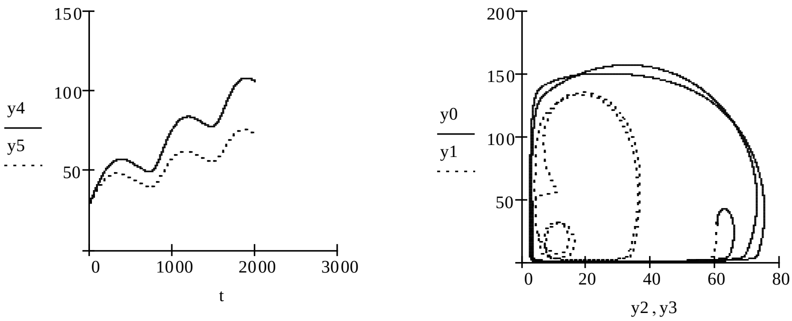


Fig. 5.75

Entropy of the buyer preferences

$$H_c(t) = \frac{\left(e^{-\beta_1 y_{0i}} \ln\left(\frac{e^{-\beta_1 y_{0i}}}{e^{-\beta_1 y_{0i}} + e^{-\beta_1 y_{1i}}}\right) + e^{-\beta_1 y_{1i}} \ln\left(\frac{e^{-\beta_1 y_{1i}}}{e^{-\beta_1 y_{0i}} + e^{-\beta_1 y_{1i}}}\right)\right)^{-1}}{e^{-\beta_1 y_{0i}} + e^{-\beta_1 y_{1i}}}$$

is shown on Fig. 5.76.

Entropy of the producer preferences

$$H_p(t) = \frac{\left(e^{\beta_2 q_1 y_{2i}} \ln\left(\frac{e^{\beta_2 q_1 y_{2i}}}{e^{\beta_2 q_1 y_{2i}} + e^{\beta_2 q_2 y_{3i}}}\right) + e^{\beta_2 q_1 y_{3i}} \ln\left(\frac{e^{\beta_2 q_1 y_{3i}}}{e^{\beta_2 q_1 y_{2i}} + e^{\beta_2 q_2 y_{3i}}}\right)\right)^{-1}}{e^{\beta_2 q_1 y_{2i}} + e^{\beta_2 q_1 y_{3i}}}$$

is shown on Fig. 5.77.

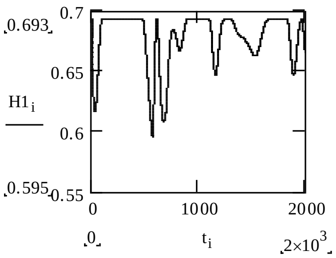


Fig. 5.76

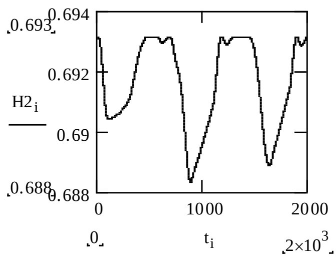


Fig. 5.77

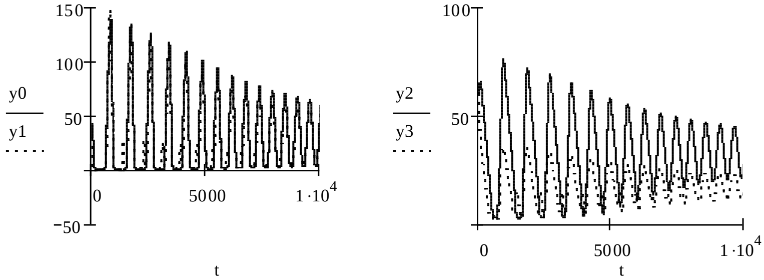


Fig. 5.78

The different version of calculations on the larger time interval (0, 10000) is shows on Fig. 5.78-5.80. The coefficients r_1 and r_2 are changed, so that average value of factors prices are stabilized on the specific level: was assumed r_1 = 59, r_2 = 59.

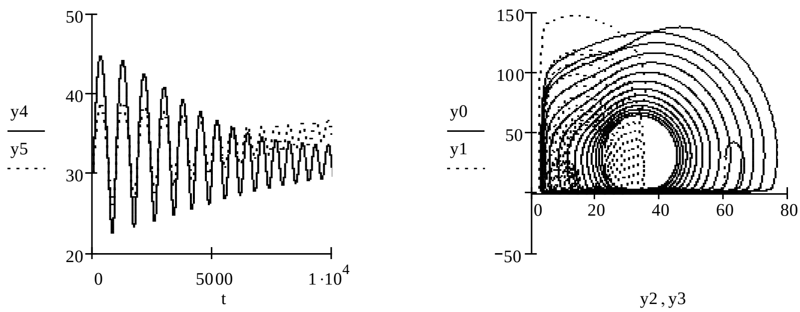


Fig. 5.79

Entropies $H_c(t)$ and $H_p(t)$ are given on Fig. 5.80.

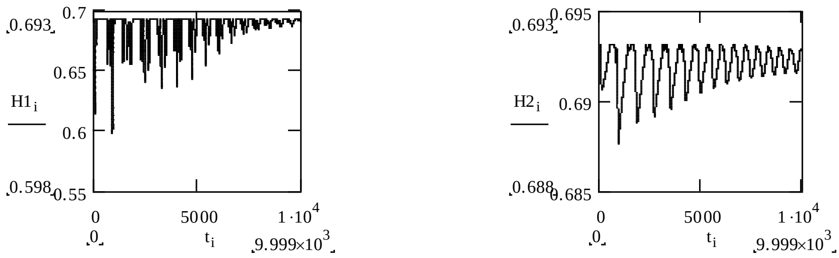


Fig. 5.80

The given model makes possible analyze the influence of the individual preferences of „customer" and „producer" in first approximation for on time dependence of releases, prices of goods and prices of factors.

5.13.5. Competition of ideas. One model of the sociodynamics

The simple model described here can be related to sociodynamics [211]. In this monograph it is also possible to find useful bibliography about sociodynamics. Let us specify immediately, that in the proposed model the certain palliative is reflected. Its equations are not derived from any variation principle, but the canonical preferences distribution of both I and II type participates in them. This deficiency is common for the majority of models in this chapter. Thus, we can see that equations for the exogenous variables and endogenous variables are formed on the basis of considerations not connected with any variation principle. In this specific inconsistency and retreat from the announced previously assertion of Leonard Euler is manifested.

However, let us note, that the dynamic equations for the population's numbers in the ecology and in some tasks of sociodynamics can be obtained at least in special cases on the basis of variation principle. Let us show, for example, how it is possible to obtain Lotka - Volterra equation.

The joint population number M_0, and the subpopulation numbers M_j, satisfy the condition $\sum_{j=1}^{M} M_j = M_0$. The indices of the structure $\nu_i = M_j M_0^{-1}$ satisfy the condition $\sum_{j=1}^{M} \nu_j = 1$. Let us examine the structural entropy

$$H_\nu = -\sum_{j=1}^{M} \nu_j \ln \nu_j$$

and let us try to find the differential equation, to which the indices ν_j are subordinated on the basis of the variation problem with the functional:

$$\Phi_\nu = -\sum_{j=1}^{M}\nu_j \ln\nu_j + \beta\sum_{j=1}^{M}\nu_j f_j + \gamma\sum_{j=1}^{M}\nu_j .$$

The corresponding canonical distribution takes the form:

$$\nu_j = \frac{e^{\beta f_j}}{\sum_{q=1}^{M} e^{\beta f_q}} .$$

Hence it follows that

$$\frac{d\nu_j}{dt} = \beta\left(\frac{\dot f_j}{f_j} - \sum_{q=1}^{M}\frac{\dot f_q}{f_q}\nu_q\right)\nu_j .$$

After designating $\varepsilon_j = \beta\dot f_j f_j^{-1}$, we obtain the equation

$$\frac{d\nu_j}{dt} = \beta\left(\varepsilon_j - \sum_{q=1}^{M}\varepsilon_q\nu_q\right)\nu_j ,$$

which is a special case of Lotka - Volterra equation? The sign, according to which occurs the separation of population on the subgroups, is contained in the structure of functions f_j. Further advance in this direction is interesting; however, in the given moment the author does not undertake to postulate the variation principle, which would cover both the structural characteristics of group (extensive variables) and preferences of type I and II (intensive variables). Let us examine the following task. Let the entire group (general population) consist of M_0 individuals.

There are two alternatives - ideas σ_1 and σ_2: (S_a: (σ_1, σ_2)) and two „operators" Σ_1 and Σ_2, each of them attempts to make the maximum number of group members as followers of idea σ_i. All members of group are neutral at the initial moment of ideas absence on the „market", in each following moment the general population is divided on three subgroups: M_1 is number of σ_1 followers; M_2 is number of σ_2 followers and $M_3 = M_0 - M_1 - M_2$ is number of neutrals.

Situation exactly copies the task of two commodities competition, if we examine „ideas" as the commodities. However, there are some essential differences, which make the model more rich.

As before, in the model of advertizing campaign in par. 5.13.2, this model considers the straight agitation (media outlets and any other means) and indirect (mediated) agitation, conducted by followers of one of the ideas in the contacts with the neutrals, and also transrecruiting, which occurs, when „encounter" the followers of different ideas. The following assumptions are taken:

- the effectiveness of straight agitation depends on the relationship of the I type preferences;

- the effectiveness of indirect agitation depends on the authority of this subgroup representative, who is evaluated as the relationship of appropriate ratings;

- the effectiveness of transrecruiting also depends on ratings relationship;

- only pairwise interactions of the group's members (straight calculation) are considered.

- The „crowd effect" is excluded.

The following assumption is important: the described mechanisms begin to „work" only, when the entropy of the corresponding distribution of preferences becomes less than the threshold value, which is the exogenous characteristic of the members of this group (subgroup) moreover for simplicity it is supposed, that these entropy thresholds are identical for all members of group.

The calculation of this circumstance will be carried out with the aid of introduction in the model the „barrier functions": $\varphi_1(H_\pi)$, $\varphi_2(H_\pi)$, $\psi(H_\xi)$. The relaxation model is used (par. 5.13.3), in order to describe asymptotic convergence of the instantaneous values of preferences to the canonical destinations.

The system of equations, which reflects the assumptions made above, appears as follows:

$$\frac{dM_1}{dt}=\left[\varphi_1\left(H_\pi\right)\alpha_{10}\frac{\pi_1\left(\sigma_1\right)}{\pi_1\left(\sigma_2\right)}+\psi\left(H_\xi\right)\beta_{10}\frac{\xi_1}{\xi_2}M_1\right]\times \tag{5.264}$$

$$\times\left(M_0-M_1-M_2\right)+\delta_0\psi\left(H_\xi\right)\left(\frac{\xi_1}{\xi_2}-\frac{\xi_2}{\xi_1}\right)M_1M_2\pm\mu_1M_1^2;$$

$$\frac{dM_1}{dt}=\left[\varphi_2\left(H_\pi\right)\alpha_{20}\frac{\pi_2\left(\sigma_2\right)}{\pi_2\left(\sigma_1\right)}+\psi\left(H_\xi\right)\beta_{20}\frac{\xi_2}{\xi_1}M_2\right]\times \tag{5.265}$$

$$\times\left(M_0-M_1-M_2\right)+\delta_0\psi\left(H_\xi\right)\left(\frac{\xi_2}{\xi_1}-\frac{\xi_1}{\xi_2}\right)M_1M_2\pm\mu_2M_2^2;$$

$$\left.\begin{aligned}\frac{d\xi_1}{dt}&=-\frac{1}{\tau_\xi}\xi_1+\frac{1}{\tau}\frac{e^{\Sigma M_1M_0^{-1}}}{e^{\Sigma M_1M_0^{-1}}+e^{\Sigma M_2M_0^{-1}}};\\ \frac{d\xi_2}{dt}&=-\frac{1}{\tau_\xi}\xi_2+\frac{1}{\tau_\xi}\frac{e^{\Sigma M_2M_0^{-1}}}{e^{\Sigma M_1M_0^{-1}}+e^{\Sigma M_2M_0^{-1}}};\end{aligned}\right\} \tag{5.266}$$

$$\left.\begin{aligned}
\frac{d\pi_1(\sigma_1)}{dt} &= -\frac{1}{\tau_\pi}\pi_1(\sigma_1) + \frac{1}{\tau_\pi}\frac{e^{\rho\hat{U}_1(\sigma_1)\xi_1}}{e^{\rho\hat{U}_1(\sigma_1)\xi_1} + e^{\rho\hat{U}_1(\sigma_2)\xi_2}};\\
\frac{d\pi_1(\sigma_2)}{dt} &= -\frac{1}{\tau_\pi}\pi_1(\sigma_1) + \frac{1}{\tau_\pi}\frac{e^{\rho\hat{U}_2(\sigma_1)\xi_1}}{e^{\rho\hat{U}_1(\sigma_1)\xi_1} + e^{\rho\hat{U}_1(\sigma_2)\xi_2}};\\
\frac{d\pi_2(\sigma_1)}{dt} &= -\frac{1}{\tau_\pi}\pi_2(\sigma_1) + \frac{1}{\tau_\pi}\frac{e^{\rho\hat{U}_2(\sigma_1)\xi_1}}{e^{\rho\hat{U}_2(\sigma_1)\xi_1} + e^{\rho\hat{U}_2(\sigma_2)\xi_2}};\\
\frac{d\pi_2(\sigma_2)}{dt} &= -\frac{1}{\tau_\pi}\pi_2(\sigma_2) + \frac{1}{\tau_\pi}\frac{e^{\rho\hat{U}_2(\sigma_2)\xi_2}}{e^{\rho\hat{U}_2(\sigma_1)\xi_1} + e^{\rho\hat{U}_2(\sigma_2)\xi_2}}.
\end{aligned}\right\} \tag{5.267}$$

First two equations (5.263), (5.264) describe the dynamics of the subpopulations numbers (coalitions) M_1 and M_2. Barrier functions $\varphi_1(H_\pi)$, $\varphi_2(H_\pi)$, $\psi(H_\xi)$ are assigned in the form:

$$\left.\begin{aligned}
\varphi_1(H_\pi) &= \left(\frac{2}{\pi}\operatorname{arctg}\left(H^*_{\pi_1} + H_{\pi_1}\right)\right)^{\frac{1}{p}} + 1;\\
\varphi_2(H_\pi) &= \left(\frac{2}{\pi}\operatorname{arctg}\left(H^*_{\pi_2} + H_{\pi_2}\right)\right)^{\frac{1}{p}} + 1;\\
\varphi_1(H_\xi) &= \left(\frac{2}{\pi}\operatorname{arctg}\left(H^* - H_\xi\right)\right)^{\frac{1}{p}} + 1.
\end{aligned}\right\} \tag{5.268}$$

here

$$H_{\pi_i} = -\left(\pi_i(\sigma_1)\ln\pi_i(\sigma_1) + \pi_i(\sigma_2)\ln\pi_i(\sigma_2)\right),$$

$$H_\xi = -\left(\xi_1\ln\xi_1 + \xi_2\ln\xi_2\right).$$

The parameter p is selected sufficiently large ($p \sim 200$—300).

Barrier functions ensure resetting to zero of the corresponding terms, until the instantaneous value of entropy is greater than threshold. This means that neither the straight nor indirect agitation causes the stratification of common population until entropies are high.

The threshold values of entropies in this model remain as the external parameters and must be determined preliminarily experimentally, or on the basis of comprehension and processing of retrospective data. Equations (5.265) describe the relaxation of instantaneous values of rating ξ_1 and ξ_2 ($\xi_1 + \xi_2 = 1$) to the canonical ratings, which are determined through the instantaneous coalitions numbers M_1 and M_2 and are calculated based on the formula

$$\xi_i = \frac{e^{\Sigma M_i M_0^{-1}}}{e^{\Sigma M_1 M_0^{-1}} + e^{\Sigma M_2 M_0^{-1}}}. \qquad (5.269)$$

Equations (5.266) analogously describe the relaxation of the I type object preferences to the canonical, which in this model are selected in the form:

$$\pi_j\left(\sigma_i\right) = \frac{e^{\rho \hat{U}_j(\sigma_i)\xi_i}}{e^{\rho \hat{U}_j(\sigma_1)\xi_1} + e^{\rho \hat{U}_j(\sigma_2)\xi_2}},$$

where $\hat{U}_j\left(\sigma_i\right)$ is the subject j estimation of the alternative σ_i utility of propagandized by „operator” Σ_1. It is also assumed, that the ratings of the compound coalitions influence the value of object preferences, the smaller utility assumed can be compensated by the higher rating of the corresponding coalition.

Let us note, that

$$\frac{\xi_1}{\xi_2} - \frac{\xi_2}{\xi_1} = \frac{1}{\xi_2} - \frac{1}{\xi_1} = \frac{2\xi_1 - 1}{\xi_1\left(1 - \xi_1\right)}.$$

This value can be both the positive and negative, which determines the „direction” of transrecruiting. The use of the proposed form of the right sides of the equations dependence on ξ_j is permissible, since with selected form (5.267) of ratings they do not become zero. Parameters τ_π and τ_ξ are the relaxation time constants of the I and II type preferences.

In equations (5.263) and (5.264) the term $\ldots\mu_i M_i^2$ is contained, which can give, depending on the sign, either cumulative effect or take into account the effect of „tightness” and, therefore, damping of the increase in given coalition. The „soft” barrier functions were used above (5.267). „Rigid” barrier ensures Heaviside's function, for example:

$$\psi_j = \psi_{0j}\frac{1}{2}\left(\frac{H_\xi^* - H_\xi}{\left|H_\xi^* - H_\xi\right|} + 1\right).$$

Above preference was returned by „soft” barrier functions, although in this case the problem of „boundary layer” influence appears on the solution behavior in the entire remaining region. Let us express additional considerations about the simulation of the cumulative effect within the framework of this model. It was already noted, that this effect can be taken into account with the aid of the term $\ldots\mu_i M_i^2$. In this case it is possible to assume that the factor μ_i is variable and changes the sign (and possibly the value), if either entropy H_π, or H_ξ, their monotonic function $f(H_\pi, H_\xi) > 0$ overcomes the certain barrier „from bottom to top”, i.e. the rule is carried out

$$\mu_i \le 0, \ \text{if} \ f\left(H_\pi, H_\xi\right) \le f\left(H_\pi^{(c)}, H_\xi^{(c)}\right);$$

$$\mu_i > 0, \ \text{if} \ f\left(H_\pi, H_\xi\right) > f\left(H_\pi^{(c)}, H_\xi^{(c)}\right),$$

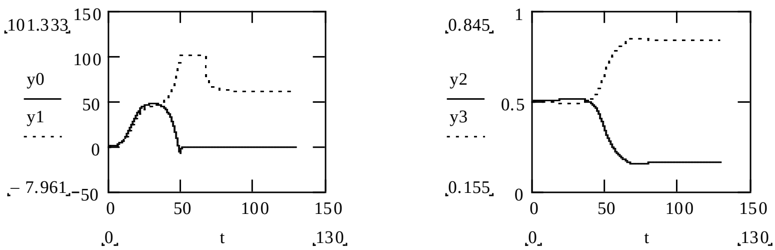
where $H_\pi^{(c)}$ and $H_\xi^{(c)}$ - the upper entropy thresholds, above which the cumulative effect appears, when the term $...\mu_i M_i^2$ contributes to rapid increase in the coalition numbers M_i. In a model example the following idea for the parameter μ_i is selected:

$$\mu_i = \mu_{0i} \frac{1}{2}\left(\frac{H_{\pi_i}^{(c)} - H_{\pi_i}(t)}{\left|H_{\pi_i}^{(c)} - H_{\pi_i}(t)\right|} + 1\right).$$

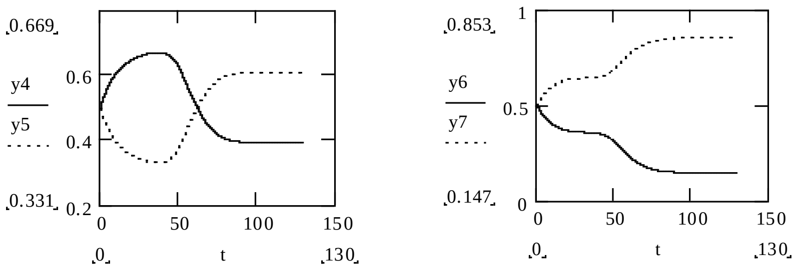
For the cumulative thresholds of the entropy $H_{\pi_i}^{(c)}$ the sufficiently high values are accepted, close to the maximum value ln 2: $H_{\pi_i}^{(c)} = 0{,}6925$. Under other circumstances it would be possible to name a similar threshold as „the panic entropy threshold".

Model contains the large number of structural parameters: α_i, β_i, δ, ε, μ_i, τ_ζ, ρ. Fig. 5.81 represents the results of simulation for one particular collection of the parameters values:

$\alpha_1 = 2$, $\beta_1 = 0{,}3$, $\alpha_2 = 1$, $\beta_2 = 0{,}5$, $\delta_0 = 0{,}5$, $c = 1$, $\tau_\pi = 0{,}1$, $\rho = 1$, $\varepsilon = 1$,
$\tau_\zeta = 0{,}1$, $M = 100$, $p = 300$, $H^*_{\pi 1} = 0{,}60$, $H^*_{\pi 2} = 0{,}63$, $H^*_\xi = 0{,}65$,
$\mu_1 = 0{,}01$, $\mu_2 = 0{,}01$, $\hat{U}_{11} = 2{,}38$, $\hat{U}_{12} = 1$, $\hat{U}_{22} = 2{,}3$, $u = 1$.



a *b*



c *d*

Fig. 5.81

On the figures the designations are used:

$$M_1 = Y_0;\ M_2 = Y_1;\ \xi_1 = Y_2;\ \xi_2 = Y_3;\ \pi_1(\sigma_1) = Y_4;\ \pi_1(\sigma_2) = Y_5;\ \pi_2(\sigma_1) = Y_6;$$
$$\pi_2(\sigma_1) = Y_7;\ \varphi_1(H_\pi) = Y_8;\ \varphi_2(H_\pi) = Y_9;\ \psi(H_\xi) = Y_{10}.$$

It follows from the graphs that at the specific moment of time the inversion of population's numbers of followers and their preferences occurs. Then spasmodic stratification occurs, moreover all socium members become followers of one of the „ideas" in particular (the second), a number of followers of another „idea" turns to zero.

It is seen, what significance has an assumption about entropy thresholds. In this case basic role plays the entropy threshold H_ξ^*. It follows, from graphs on Fig. 5.81, *a* number of followers of the second „idea" first reaches maximum (100%); however, then part of them is „disappointed" and passes into the „neutrals".

Fig. 5.82 represents the dynamics of the populations of ideas σ_1 and σ_2 followers that corresponds to following data:

$$\alpha_1 = 2,\ \beta_1 = 0{,}5,\ \delta_1 = 0{,}5,\ \mu_1 = -0{,}01,\ \mu_2 = 0{,}01,\ \alpha_2 = 1{,}6,\ \beta_2 = 0{,}9002,\ \delta_2 = 0{,}5,\ \varepsilon = 2,\ \tau_\pi = 0{,}1,\ \rho = 1,$$
$$\tau_\zeta = 0.1,\ M = 100,\ \rho = 300,\ H^*_{\pi 1} = 0{,}60,\ H^*_{\pi 2} = 0{,}63,\ H^*_\xi = 0{,}65,$$
$$\hat{U}_{11} = 2{,}38,\ \hat{U}_{12} = 1,\ \hat{U}_{21} = 1,\ \hat{U}_{22} = 2{,}3,\ n = 1,$$

The model of rigid entropy thresholds is used here, to what graph *f* corresponds. It follows from the figures *a*—*e*, that as in the first case the inversion of both numbers M_1 and M_2 and preferences, is observed. Figure *f* is the „phase portrait" of populations numbers M_1 and M_2.

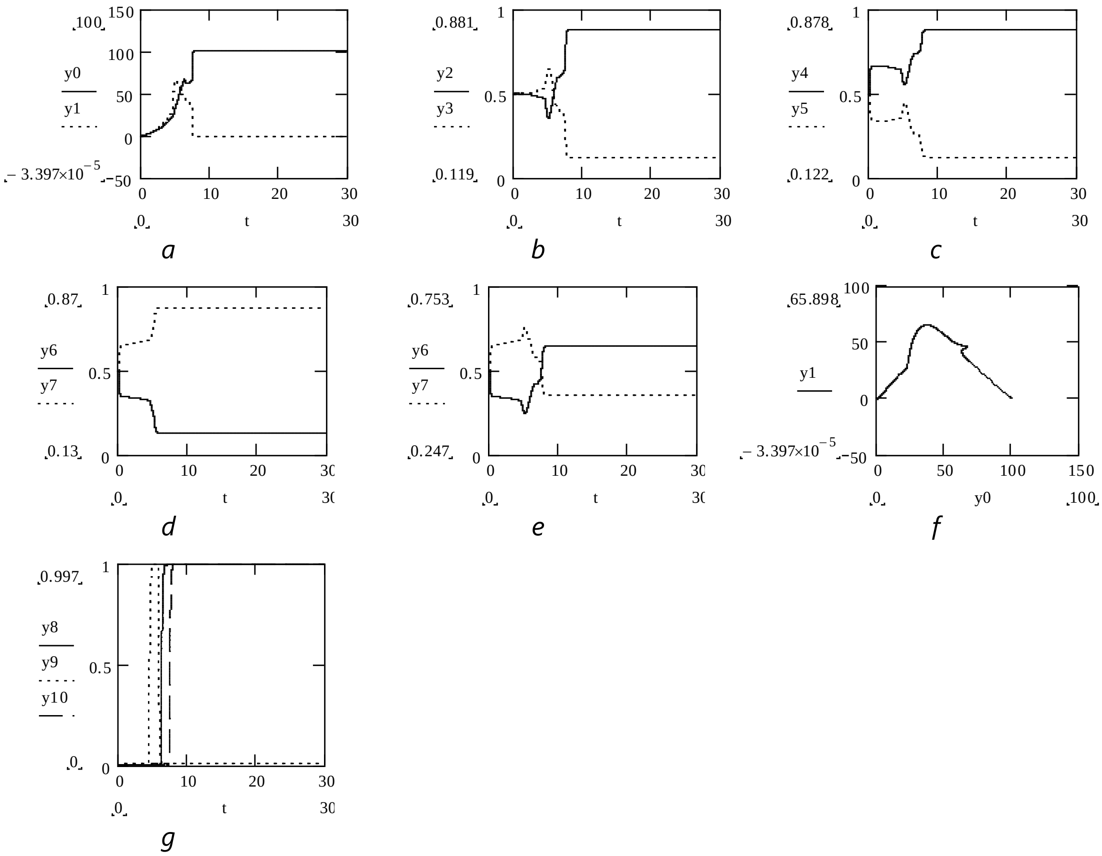


Fig. 5.82

The situation, represented on Fig. 5.83 radically differs from the previous, shown on Fig. 5.82, where after some fluctuations the first „idea" (σ_1) wins.

In this case the decisive victory gained the second „idea" (σ_2) - 100% of group subjects become its followers *a*. Initial data are characterized by the sign of coefficient μ_2 and the sign of the parameter β_2. The variation of the structural parameters shows the high sensitivity of the solution with respect to the parameters α_i and β_i.

In conclusion let us note, that in the given modeling solutions all structural parameters are considered constants, in particular the strategy of operators with respect to the choice of utility $\hat{U}_{ij}$ lies in the fact that they are assigned at very beginning of the process and then they do not change.

Obviously that some structural parameters and also utilities can come out as the controlling factors and in this sense the corresponding task of optimal control can be formulated.

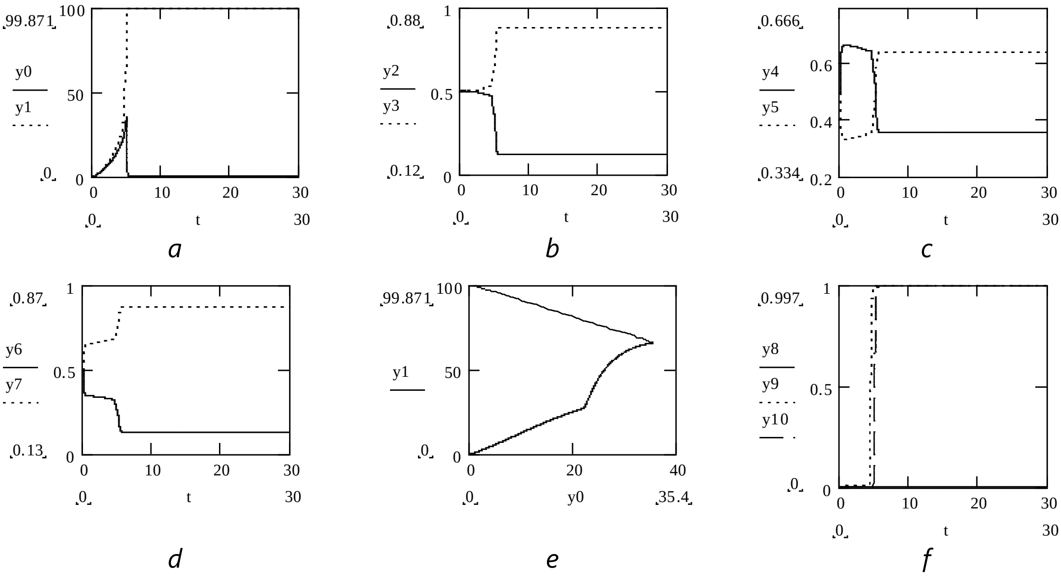


Fig. 5.83

The flow of the subjective information, caused by the competition of „ideas" is determined by formulas (for two alternatives σ_1, σ_2):

$$q_{\xi} = \frac{dH_{\xi}}{dt} = -\frac{d\xi_1}{dt}\ln\frac{\xi_1}{1-\xi_1};$$

$$q_{\pi_j} = \frac{dH_{\pi_j}}{dt} = -\frac{d\pi_j(\sigma_1)}{dt}\ln\frac{\pi_j(\sigma_1)}{1-\pi_j(\sigma_1)}.$$

It is possible to determine Using results of par. 4.14, the prices of the subjective information connected with processes of „ideas" competition.

6. SAFETY OF THE ACTIVE SYSTEMS

6.1. Approach to a study of the active systems safety

Concept „safety of active systems" differs essentially from the concept of technogenic, ecological, economic, and social and other systems safety. Besides the fact, that man present in all such systems, his role is usually interpreted from another point of view [32,54] comparison with the way which was chosen in the framework of the developed approach using subjective entropy as central element of us. The preference functions apparatus can be connected with some divisions of psychology and, in particular, with the psychometric methods.

Safety of active system - it is safety of the subject, who is the main body of active system. The active system could be very generally presented by the scheme, depicted on Fig. 6.1.

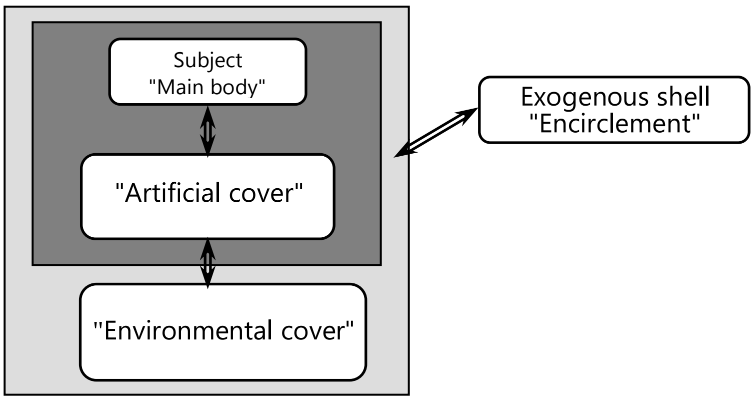


Fig. 6.1

Active system contains directly the parts enveloped by dotted line. The natural component of shell is not included in the system. Thus, active system is always open.

Subject interacts with the exogenous shell, which provides a change in the problem resources situation. Along with this the endogenous processes also occur. Exogenous and endogenous processes are connected with each other, and their connectedness makes it possible to speak about the „safety of active system" in the extended sense, after isolating, in particular, safety of the natural component of exogenous shell - environment (including other active systems) from the negative consequences of this active system functioning. We examine safety of active system in the sense of definition given above.

If subject can enter into with situation different danger level, then safety will be understood as his protection from entrance into these situations. The worst situation leads to the „loss" of subject. It is necessary to define concretely this term, in a way would give that possibility to describe security issue in the terms of subjective analysis. Not everything, that is being discussed, could be formalized, at least, in the present work.

We connected above the active and passive resources with the active system. By the „loss" of subject we will understand *the complete exhaustion of active resources* or the absolute impossibility of their use. Let us recall that any conversion or translation of passive resources is impossible without the attraction of the active resources. In the number of active resources of subject his available operating time is included. Expense, exhaustion of the available operating time occurs under the effect of interaction with the exogenous shell or spontaneously, as a result a natural change in the subject. Besides available active time the active resources include intellectual and physiological resources. Therefore by the „loss" it is possible to understand such situation, when subject physically exists, but his intellectual possibilities are completely exhausted.

Since the exogenous shell gives means and resources translation algorithms and is the object of application of active resources, it is possible to consider that the destruction of exogenous shell leads to the „loss" of active element and, therefore, entire system.

Below we will examine other concepts, connected with the safety of active systems and the series of particular models. Since safety of active system, from one side, is *objective* characteristic, and, from another side, with the subject selection of the safety maintenance strategy interpreted by him *on the subjective level*, then in the final result and the solutions taken by subject in the safety region with necessity bear *subjective* nature.

The examined models are directed to, reflection of subjectivism of the accepted solutions in the schemes of the safety analysis. This can be made, by usage an apparatus of the preference functions and an entropy approach.

Let us begin from the examination of the simplest schemes and examples. One of the possible schemes consists in the following. Let us assume that dangerous event A occurs against the background existence of several admissible strategies $\sigma_1, \sigma_2, \ldots, \sigma_N$: $\sigma_K \in S_a$ of averting it with the probabilities $P(A| \sigma_k)$ [70]. Probability $P(A| \sigma_k)$ is considered as objective characteristic.

The effectiveness function of the alternative strategy has in this case a nature of risk $\mathfrak{R}$. Let us assume that risk is composed of two components

$$\Re = \Re_1 + \Re_2,$$

where $\Re_1$ is „physical" risk, which is determined by the probability of event A, $\Re_2$ - „financial" risk determined by possible financial losses. Connected both directly with the event A and with the compelled investments into the safety of the part of the available resources. The „speed" of investment we will designate through V_s . This is the expenditure per time unit for the safety maintenance. Let us determine probability $P(A|\,\sigma_k)$ as probability that event A will occur in the moment t^* belonging to interval τ, which begins at in the moment t_0 with the condition, that strategy σ_k is realized:

$$P(A|\,\sigma_k) = P_A(t^* \in \tau,\, t_0|\,\sigma_k).$$

Let us connect the financial risk either with the probability of the specific losses or the mathematical expectation of the losses:

$$\text{a)} \qquad P(\Pi > \Pi_{кр},\, t \in \tau,\, t_0|\,\sigma_k); \tag{6,1}$$

$$\text{b)} \qquad E(\Pi,\, t \in \tau,\, t_0|\,\sigma_k), \tag{6.2}$$

where П — „loss", Π_{kr} — critical „losses".

Let us examine the case, when the flow of events A is subordinated to the Poisson law,

$$P_m(A) = \frac{(\lambda\tau)^m}{m!} e^{-\lambda\tau},$$

where λ is intensity of event flow, m is the number of events during τ.

Probability that in the interval τ not one event $P_0(A) = e^{-\lambda\tau}$ will occur, probability that $m \geq 1$, $P_{m>0}(A) = 1 - e^{-\lambda\tau}$. Let the losses with the single event A is equal C_A and not depend on the number of events m. Let us note that the last condition is not necessary.

The mathematical expectation of losses from A is equal

$$E(\Sigma C_A) = C_A e^{-\lambda\tau} \xi \sum_{m=1}^{\infty} \frac{\xi^{m-1}}{(m-1)!};\ \xi = \lambda\tau.$$

Substituting $m - 1 = k$, we obtain

$$E(\Sigma C_A) = C_A \lambda\tau,$$

since the series $\frac{\xi^k}{k!}$ absolutely converges and $\sum_{k=0}^{\infty} \frac{\xi^k}{k!} = e^{\xi}$. Let us assume that intensity λ depends on the intensity of investment into the safety Vs. One of the models of this dependence takes the form:

$$\lambda = \lambda_{\max} + (\lambda_{\min} - \lambda_{\max}) \frac{V_s}{V_s + 1};\ V_s \in [0, \infty) \tag{6.3}$$

and reflects the fact that there exists the minimum intensity λ_{min}, which cannot be reduced by any increase of the investments into the safety with given overall level of technological means perfection and personnel training (Fig. 6.2).

Another dependence model of intensity λ as the function of investments into the safety can be represented by logistic curve, for example, in the form

$$\lambda = \frac{\lambda_{min}}{1+\left(\frac{\lambda_{min}}{\lambda_{max}}-1\right)e^{-\alpha V_s^2}}, \tag{6.4}$$

where α - structural parameter, which determines the degree of sensitivity λ to a change in the value of investments. Dependence (6.4) is shown on Fig. 6.3.

When $Vs = 0$ on $Vs = \infty$ the derivative $\frac{d\lambda}{dV_s} = 0$.

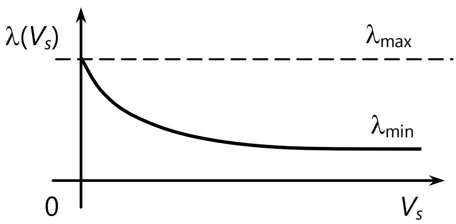


Fig. 6.2

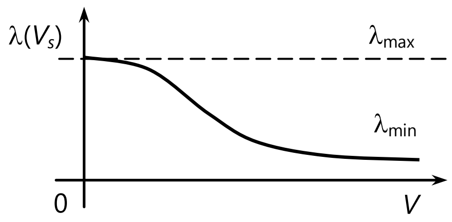


Fig. 6.3

It is completely obvious, that no matter how thoroughly pilots will be trained; nevertheless the probability of error different from zero exists. No matter how much investment was packed into an increase of reliability of the technology, nevertheless there is a nonzero probability of failure. It is proper to add to that, which was already abroad to the role of active element - the subject of system, the following reasons: no rules of the operator behavior in the normal and special situations how universal and detailed they would be, any actually appearing situation describe with the absolute accuracy.

In any case the „nonstandard" situation differs from the model, provided by the rules (in case of the civil aviation – „Flight Operations Manual"). Role of active element is subject in providing exactly the safety consists in reaction on the unregulated deviations from the conditions, provided by rules. Rules in this case play the role of sable imperatives.

Using probability $P(A|\sigma_k)$ let us introduce the value

$$\bar{P}(A\mid\sigma_k) = \frac{P(A\mid\sigma_k)}{1-P(A\mid\sigma_k)}.$$

If $P(A \mid \sigma_k) \in [0,1]$, that $\bar{P}(A \mid \sigma_k)$ is determined on the semi axis [0, $+\infty$). In this case

$$\bar{P}(m=0) = \frac{e^{-\lambda\tau}}{1-e^{-\lambda\tau}},\ \lambda\tau \in [0, +\infty),\ A\colon m = 0;$$

$$\bar{P}(m \geq 1) = \frac{1-e^{-\lambda\tau}}{1-\left(1-e^{-\lambda\tau}\right)} = \frac{1-e^{-\lambda\tau}}{e^{-\lambda\tau}},\ A\colon m \geq 1.$$

We note that

$$\bar{P}(m=0) = \frac{P(m=0)}{P(m \geq 1)};$$

$$\bar{P}(m \geq 1) = \frac{P(m \geq 1)}{P(m=0)} = \left(\bar{P}(m=0)\right)^{-1}.$$

Let us determine the mathematical expectation of financial losses by the relationship:

$$E(\Pi) = E(\Sigma C_A) + V_s\tau.$$

It is here assumed that the intensity of investments into the safety V_s remains constant during the time interval τ. Let us determine the relative losses

$$\bar{E}(\Pi) = \frac{E(\Sigma C_A) + V_s\tau}{V_d\tau},$$

where V_d is intensity of obtaining income from the active system functioning, which supposedly, as V_s, remains constant in the time distance τ. The relative mathematical expectation of profit Z can be expressed by the formula

$$\bar{E}(Z) = \frac{V_d\tau - E(\Sigma C_A) - V_s\tau}{V_d\tau} = 1 - \bar{E}(\Pi).$$

Total subjective risk is represented in the form:

$$\Re = \sum_{k=1}^{N} \pi(\sigma_k)\left(\alpha\Re_1(\sigma_k) + \beta\Re_2(\sigma_k)\right) = \sum_{k=1}^{N} \pi(\sigma_k)\left(\alpha\bar{P}(A \mid \sigma_k) + \beta\bar{E}(\Pi_k)\right), \quad (6.5)$$

where Π_k is losses with the use of strategy σ_k. It is assumed that values V_s and λ depend on the selection of strategy. On the selection of strategy can depend also C_A. Thus, we assume

$$\bar{E}(\Pi_k) = \frac{E(\Sigma C_{Ak}) + V_{sk}\tau}{V_{dk}\tau}.$$

The value V_{dk} depends on strategy σ_k, since with the appearance of events A changes profitabless level of the system. The value V_{sk} is considered in this example as the controlling parameter.

In this simplest case of strategy they are distinguished by the value of the controlling parameter V_{sk}. The more flexible form of risk takes the form:

$$\Re = \sum_{k=1}^{N} \pi(\sigma_k)\left(\beta_1\left(\alpha\Re_{1k} + \beta\Re_{2k}\right) + \beta_2 \ln\left(\alpha\Re_{1k} + \beta\Re_{2k}\right)\right).$$

One additional risk function takes the form:

$$\Re = \sum_{k=1}^{N} \pi(\sigma_k)\left(\alpha Q(A \mid \sigma_k) - \beta\Pi(\sigma_k)\right),$$

where $Q(A| \sigma_k) = 1 - P(A| \sigma_k)$ — the probability of reliable functioning. In the particular case $Q(A \mid \sigma_k) = 1 - e^{-\lambda_k \tau}$.

$$\bar{Q}(A \mid \sigma_k) = \frac{1 - P(A \mid \sigma_k)}{1 - \left(1 - P(A \mid \sigma_k)\right)} = \frac{1}{\bar{P}(A \mid \sigma_k)}.$$

As the illustration let us examine the functioning of aviation - transport system (ATS). There are two subjects. The first subject – it is ATS, which obtains profit, and at the same time, with the certain probability suffers the losses, connected with the aviation incidents, which inflict ATS heavy losses, including financial and sociopolitical. Company must carry out a selection of the parameter V_s and the price of tickets in such a way as to ensure the reasonable balance between the danger of catastrophe and the financial losses, caused by constant expenditures for safety. In the given above formulas $\pi(\sigma_k)$ are the preferences of the leader of ATS.

The second subject is the passenger, who in the presence on the transportation market of two or more ATS companies, also makes a selection in the conditions of two counterweights: he would want to fly safely and inexpensively. The cost of ticket can depend not only on the level of comfort, but also on the level of flight safety in given ATS. ATS, in which relatively frequently aviation incidents occur, is forced to reduce the cost of tickets in order to draw passengers.

Let us assume, that in given region two airlines act on the market, airlines which achieve voyages along the same routes. The incomes of airlines depend on prices on the airline tickets. The part of the incomes is directed to the maintenance of flight safety on the accept tabele level. Let the prices of airline tickets in both companies be identical. Let us designate the price of one ticket P_r and determine the intensity of investments by the formula

$$V_s(P_r) = P_r \frac{n}{t}\left(1 - kt - ke - kp\right),$$

where n is a quantity of tickets, sold in time τ, kt, ke, kp is coefficients of deductions for the payment of taxes, covering the operating costs, forming the profit, respectively. Total number of passengers is supposed to be n.

Let us assume, that

$$\lambda\left(P_r\right)=\lambda_{\max}+\left(\lambda_{\min}-\lambda_{\max}\right)\frac{V_s\left(P_r\right)}{V_s\left(P_r\right)+1}.$$

The physical risks of the 1st and 2nd airline are equal:

$$\Re_{11}^{(A)}\left(P_r\right)=\alpha_1\left(1-e^{-\lambda\left(P_r\right)\tau}\right);$$

$$\Re_{12}^{(A)}\left(P_r\right)=\alpha_2\left(1-e^{-\lambda\left(P_r\right)\tau}\right).$$

Financial risk - respectively:

$$\Re_{21}^{(A)}\left(P_r\right)=\beta_1\frac{P_r n_1+C_{A1}\lambda\left(P_r\right)\tau}{In_1};$$

$$\Re_{22}^{(A)}\left(P_r\right)=\beta_2\frac{P_r n_2+C_{A2}\lambda\left(P_r\right)\tau}{In_2}.$$

Here $\beta_i = 1 - \alpha_i$; $n_1 + n_2 = n$; C_{Ai} is single losses of airline in connection with the catastrophe (the separation of financial responsibility between ATS and insurance company is not examined), $I\ n_i$ is assumed incomes. We select the functions of the preference of the *i*-th ATS manager in the form:

$$\pi_i\left(P_r\right)=\frac{e^{-\mu\Re_i^{(A)}\left(P_r\right)}}{n\int\limits_0^\infty \pi_i\left(P_r\right)dP_r},$$

where i is the number of airline, and alternatives are determined by the parameters V_s and P_r. Total risk is calculated based on the formula:

$$\Re_i = \Re_{i1} + \Re_{i2}.$$

The idea of the risk in the form of sum of two terms reflects in this case the fact that ATS management allows not financial interest only, but also sociopolitical aspect, which is directly connected with the probability of the catastrophe $\overline{P}_1=\left(A\,|\,\sigma_k\right)$. As controlling influence can appear not the price of tickets only, but also the planned profit.

The second subject - passenger forms his preferences on the set S_a of two alternatives σ_1 and σ_2, where σ_1 is preference is given to the first airline, σ_2 is to the second airline. Physical risk for the passenger can also be connected with the probability of the event $A:\overline{P}_1=\left(A\,|\,\sigma_k\right)$, the financial losses of passenger is ticket price P_{rk}.

Thus, the risk of passenger, can be written down in the form

$$\Re^{(\text{п})}\left(\sigma_k\right)=\alpha P_{rk}+\beta\overline{P}\left(A\,|\,\sigma_k\right).$$

Values α and β are scale and weight factors. Let us present the preferences of passenger in the form:

$$\pi^{(\text{п})}\left(\sigma_k\right)=\frac{e^{-\left(\alpha P_{rk}+\beta \bar{R}\left(A|\sigma_k\right)\right)}}{\sum_{i=1}^{2} e^{-\left(\alpha P_{ri}+\beta \bar{R}\left(A|\sigma_i\right)\right)}}.$$

Fig. 6.4 shows the results of calculation for following initial data: $P_r \in [0, 10^3]$ (y.e); $n = 10^3$; $\tau = 9 \cdot 10^2$ eur, $kt = 0{,}25$; $ke = 0{,}5$; $kp = 0{,}15$; $\lambda_{\min} = 10^{-5}$; $\lambda_{\max} = 10^{-4}$; $C_{A1} = 2 \cdot 10^6$; $C_{A2} = 2 \cdot 10^6$; $In_1 = In_2 = 4 \cdot 10^2$; $\alpha_1 = 6 \cdot 10^{-1}$; $\alpha_2 = 4 \cdot 10^{-1}$.

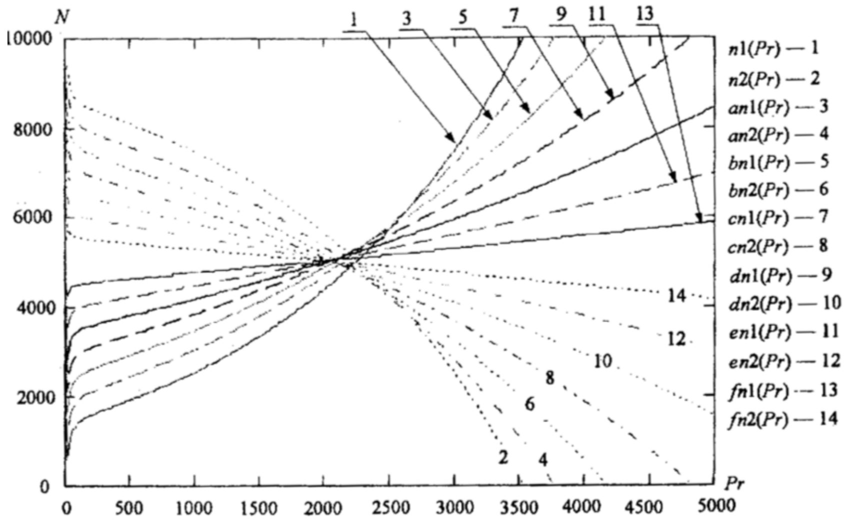


Fig. 6.4. Quantity of passengers of first and second ATS depending on P_r and ξ

It is seen that passenger's contingent is distributed between the airlines depending on the mean price (P_r) and respect to the prices of tickets ξ of first and second ATS, and also indices of the subjective evaluation of the level of safety of flights on the aircraft of the first of ATS - α_1 and by the second of ATS - α_2.

The numerical values of the parameters ξ and α_1, α_2 are presented in tabele 6.1.

Tabele 6.1

№	Quantity of the passengers of first and second ATS	ξ	Values of the coefficients α_1 and α_2
1.	$n_1(P_r)$	0,3	$\alpha_1 = 6 \cdot 10^{-1}$
2.	$n_2(P_r)$		$\alpha_2 = 4 \cdot 10^{-1}$
3.	$an_1(P_r)$	0,4	$\alpha_1 = 7{,}5 \cdot 10^{-1}$
4.	$an_2(P_r)$		$\alpha_2 = 5{,}5 \cdot 10^{-1}$
5.	$bn_1(P_r)$	0,5	$\alpha_1 = 8{,}8 \cdot 10^{-1}$
6.	$bn_2(P_r)$		$\alpha_2 = 6{,}8 \cdot 10^{-1}$

7.	$cn_1(P_r)$	0,6	$\alpha_1 = 9 \cdot 10^{-1}$
8.	$cn_2(P_r)$		$\alpha_2 = 7 \cdot 10^{-1}$
9.	$dn_1(P_r)$	0,7	$\alpha_1 = 9{,}1 \cdot 10^{-1}$
10.	$dn_2(P_r)$		$\alpha_2 = 7{,}1 \cdot 10^{-1}$
11.	$en_1(P_r)$	0,8	$\alpha_1 = 9{,}2 \cdot 10^{-1}$
12.	$en_2(P_r)$		$\alpha_2 = 7{,}2 \cdot 10^{-1}$
13.	$fn_1(P_r)$	0,9	$\alpha_1 = 9{,}3 \cdot 10^{-1}$
14.	$fn_2(P_r)$		$\alpha_2 = 7{,}3 \cdot 10^{-1}$

6.2. „Dangers” and „threats”

Let us examine the „dangers”, which accompany the functioning of active system.

„Entropy catastrophe” - this is such a situation, when the entropy H_π exceeds the critical level H^* on alternatives set S_a and subject cannot make a decision in the limit of the available time (time resources), in the more overall meaning, when required resources in order to overcome the barrier H^* exceed those available.

Buridan donkey fall as the first victim of the surplus of possibilities and high entropy. If in front of it there would be only one of haystack, and not absolutely equivalent two, it would remain among the living.

Its entropy with two equivalent alternatives

$$H_\pi = -2\pi\ln\pi = H_{\max} = 0{,}693$$

and, therefore, it did not have chances to rescue.

If the available resources are gradually drained, and subjective entropy remains high, then the „rescuing” condition is the inequality:

$$t^*_{\text{н}} < t^*_R, \tag{6.6}$$

where $t^*_{\text{н}}$ is moment of time, such that with $t \geq t^*_{\text{н}}$ occurs for the entropy the inequality $H_\pi \leq H^*_\pi$, t^*_R such, that with $t \geq t^*_R$ the inequality $R^{disp}(\sigma_k) \leq R^{req}(\sigma_k)$ is fulfilled for $\forall k \in \overline{1,N}$. Fig. 6.5. reflects these conditions.

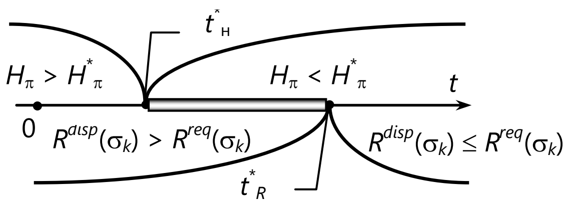


Fig. 6.5

If the inequality $t^*_{\text{н}} \geq t^*_R$, is fulfilled then „entropy catastrophe” occurs and unfavorable event A will most likely occur.

If it is previously known that inequality (6.6) doesn't occurs, then it is possible to pose the problem about an additional quantity of resources and about the methods of their investments into the system, which are necessary for the system „rescuing".

The negative consequences from the safety point of view can be produced by reciprocal effect of two and more subjects when solution in the extreme situation, combined with the selection of the alternative strategies should be made. „*The fetters of imperatives*" can be the reason for the entropy catastrophe, caused by the oppression of the system of imperatives, the prejudices, which enclose subject from the „external world", making him unable to adequately respond to the new „exogenous" challenges, reducing or altogether excluding his adaptability.

Catastrophe begins not inevitably as a result *of active resources exhaustion*, but also, for example, with the appearance of catastrophic changes in the exogenous shell, when active resources are not exhausted yet, but it is catastrophically missing passive resources and, in the consequence of this, all alternatives become unattainable. In this case we can indicate that the set S_a becomes empty: $S_a = \varnothing$, or contains only one (current) state σ_0, and subject falls into „*blind alley*".

When we speak about the exhaustion of active resources, it is corresponded with the previously accepted position (the use of each unit of passive resources, requires the expense of the specific portion of active resources), which is equivalent to the transformation of passive resources into „dead ballast".

Exhaustion of the available physiological resources is equivalent to the „physical death". Abrupt changes can bear stress nature and it is almost obvious that in this case the abrupt changes in the entropy occur. By examples serve:

- an abrupt change in the dimensionality of set S_a, i.e. an increase or decrease of alternatives number;
- the sharp price change, which draws an abrupt change in the required resources;
- an abrupt change in the quotation, when spasmodically change the available resources of the currency holders and so forth

„*The jump of entropy*" can serve as one of the characteristics of stress situation. Jump is characterized by depth and „direction".

Let us name „*entropy jump of the I type*" a sharp increase in the entropy:

$$\delta H_\pi = H_\pi(t + \varepsilon) - H_\pi(t) > 0, \tag{6.7}$$

„*entropy jump of the II type*" - the sharp decrease in the entropy

$$\delta H_\pi = H_\pi(t + \varepsilon) - H_\pi(t) < 0, \tag{6.8}$$

Entropy jump is connected with the instantaneous (proceeding in the very short period) „isolation" or the „absorption" of information.

In the case of the jump of the II type arises the question about the subject ability to receive and to master information in the short period ε. Most likely, in a view of the limitedness „capacity" of the information perception by subject the value of the negative entropy jump cannot be large, or time ε cannot be very small. This cannot be said about the jump of the I type. We can assume about the limited quantity of entropy jumps, which can withstand the psyche of the concrete subject $N^*_н$. Apparently, this number depends on an absolute value of jump, type of jump, time spaces between jumps. Dependence analogous to the fatigue curve $N^*_н = f(\delta H_\pi,\ldots)$ must exist.

Each subject, member of group, is also carrier of rating preferences $\xi(j, i| \sigma_k)$ or $\xi(j, i)$, and consequently, also of the rating entropy H_ξ referred to the subject *i*. The change of ratings, as it is known, is frequently accepted by subject very sharply, and an abrupt change in the distribution of ratings in the group appears as „*entropy ratings jump*". Here we will also introduce two types of the jumps: the entropy rating jump of the I type - the entropy H_ξ sharply grows

$$\delta H_\xi = H_\xi(t + \varepsilon) - H_\xi(t) > 0, \tag{6.9}$$

This is the „*jump to the equality*", to the ratings leveling and, correspondingly, in a certain sense, ranks. If public shakings, „*revolution*", lead to an equality increase , conduct to an increase of the rating entropy, on the contrary, of different kind „*counterrevolutionary*" events (military coups, Fascist and monarchist putsches) correspond to the decrease of rating entropy - to decrease of subjective equality and to the jump of the II type.

The structurization of group, the complication of structure is not compulsorily equivalent to the decrease of entropy H_ξ. Thus, for instance, if in the group of 4 subjects is an absolute rating equality (quantity of rating levels $k = 1$), then in a view of standardization $\Sigma\xi(j) = 1$ all ratings are equal $\xi(j) = 0{,}25$ and $H_\xi = 1{,}3863$. Let now in the group $k = 2$, i.e. there are two levels of ratings. This means that there are two classes of rating equivalence.

Version 1.

$$\xi_1 = \xi_2 = 0{,}1;\ \xi_2 = \xi_4 = 0{,}4;$$
$$H_\xi^{(1)} = 1{,}1935.$$

Version 2.

$$\xi_1 = \xi_2 = \xi_3 = 0{,}1;\ \xi_4 = 0{,}7;$$
$$H_\xi^{(2)} = 1{,}0573.$$

If in the group there are three levels of ratings, then there are 3 classes of equivalence, for example:

Version 3.

$$\xi_1 = 0{,}1;\ \xi_2 = 0{,}1;\ \xi_3 = 0{,}2;\ \xi_4 = 0{,}6;$$
$$H_\xi^{(3)} = 1{,}0889.$$

Version 4.

$$\xi_1 = 0{,}1;\ \xi_2 = 0{,}1;\ \xi_3 = 0{,}22;\ \xi_4 = 0{,}58;$$
$$H_\xi^{(4)} = 1{,}1935.$$

As we see, the complication of structure - addition of the additional rating level in the given example does not lead to the decrease of the rating entropy: the values $H_\xi^{(3)}$ and $H_\xi^{(4)}$ lie between $H_\xi^{(1)}$ and $H_\xi^{(2)}$. Thus, the value of entropy depends not only on a quantity of ratings levels, but also on the distribution of the group members along the levels. We already spoke, about the connection of ratings and ranks, and also about the fact, that the rank of subject is tightly connected with his imperious authorities, which, in turn, can be connected with the authorities to manage resources. We see also from the given example that in the case of group the rating entropy, being referenced to the given subject, is not the comprehensive characteristic, which reflects the degree of structural uncertainty in the group. The structured group is the hierarchical system, where its rating entropy is determined against the level of each rank and this entropy depends on the rank of this subject.

The concept *„entropy catastrophe"* for the group is considerably more complex and is less obvious, than the same concept for the individual. It should be determined within the framework of the hierarchical active systems theories, which is only planned here.

Being applied to the active system with one (unitary) subject, „danger" is reduced to the following:

1. Spontaneous destruction of „nucleus" - the „loss" of subject (endogenous catastrophe).

2. Destruction of nucleus under the action of exogenous factors, conflict between the exogenous shell and the subject (endoexogenous catastrophe).

3. Spontaneous destruction of artificial component of exogenous shell without the loss of subject (exogenous catastrophe).

4. Destruction of exogenous shell of the subject as a result of the erroneous or incompetent actions of the latter (exo- endogenous catastrophe).

Let us determine a difference between „*danger*" and „*threat*" concepts in application to the active system. We will consider that *danger* is the objective possibility of negative action, events, which are formalized in the terms of the functions of utility, harmfulness, or resources, as unfavorable for the subject changes in the latter.

Threat is the subjective possibility of negative action or event, expressed in the formalism of the preference functions through a change of the preferences of another subject in the group, or out of this group. In other words, introducing such definitions, we exclude expression „threat of flood", „the threat of earthquake".

More suitable expressions are: „the danger of flood" and „the danger of earthquake". The expression completely correspond to the sense „threat of attack", the „threat of an increase in the taxes"... These actions can be reflected as changes in the preferences of a certain other subject, who has available resources and authorities, in order to lead „threat" at the point of the performance. It is possible to say: „in connection with *the threat* of attack the specific *dangers* appear".

6.3. Flight safety and the subjective analysis

The attraction of subjective analysis for the study of the flight safety problems, is promising, since makes it possible to look against the problem from the new point of view [70,71].

The calculation of human factor in the problems of flight safety is the necessary component of a study, and in the case of concrete aviation events - by the component part of the investigation. Statistics speaks, that this factor in the causal chain occupies „honorable" first place. Experts attribute „human factor" to be a reason from 60 to 80% of special flight situations, incident or accidence. If it is prospect of the subjective analysis usage in form, as it is represented in this work, we have in mind the following: it is possible with the confidence to assert that each time with the appearance of special situation several alternative strategies of the behavior of crew or entire personnel exist, which ensures the execution of flight, directed on the countering the negative circumstances.

The exception, of course, composes the cases when already nothing depends on crew. In the majority of the cases, however, the crews take measures to the rescuing of aircraft and the satisfactory completion of flight. Let us designate the alternative strategies through σ_k, and entire set of admissible strategies through S_a. On the set S_a are formed the preferences $\pi(\sigma_k)$. The distribution of preferences changes in the course of the flight situation development . Set S_a enters into the formal description of flight situation and it is also the dynamic object: the composition of alternatives changes in the course of time. As a rule, there is a sharp scarcity of time on decision making: the „best" alternative selection. We took the word „best" in the quotation marks in order to emphasize the fact that „best" strategy from the crew point of view at the moment of the appearance of special situation, can prove to be not best in actuality.

Let us give several examples, based on which it is evident, as the alternative strategies appear.

If the engine failure occurs during the take-off run before the take-off, then there exists two possibilities: to continue take-off (strategy σ_1) and to end take-off (strategy σ_2) each time the selection of strategy occurs via the comparison of the speed at the moment of failure with the so-called critical speed.

Rule is such: if speed $V(t_{\text{stop}}) \geq V_{\text{kr}}$, take-off should be continued, but if $V(t_{\text{stop}}) < V_{\text{kr}}$ take-off should be ceased. Flight situation however, can be complicated by other circumstances and then this simple rule in the pure form cannot be used. Strictly speaking, two alternatives exist with the condition $V(t_{\text{stop}}) < V_{\text{kr}}$:

$$S_a|\, V_{\text{stop}} < V_{\text{kr}}: (\sigma_1, \sigma_2); \;\; S_a|\, V_{\text{stop}} \geq V_{\text{kr}}: (\sigma_1).$$

Let us imagine that the engine failure is complicated by fire. Then there are available resources of time t^{disp}, connected with the rate of fire development. The required time t^{req}, in the case of the continued take-off, on the maneuver of return, landing, and evacuation - can prove to be more than t^{disp}.

At the same time the curtailment of take-off (with the condition $V_{\text{stop}} \geq V_{\text{kr}}$) is combined with the danger of rolling outside the limits of runway, and as consequence, with the emergency or the catastrophe. Under such conditions, there are two alternatives for a while. The available time for decision making is very small. On S_a appears the distribution of preferences, which evolves rapidly under the action of the development of the exogenous situation: $\pi(\sigma_1)$, $\pi(\sigma_2)$. The problem consists in this, to model the distribution of preferences, to trace its modification in the time, and also to determine the conditions of decision making.

Simplified motion of aircraft during the take-off is described by the equations:

$$m\frac{dV}{dt} = P(V,t) - X(V) - N(V)f; \quad 0 = Y(V) - G + N,$$

where V is velocity of the center of masses, m is mass, $P(V, t)$ is the summary thrust of engines, which depends on velocity V and time t (explicit dependence on time t appears in the case of the rejected take-off - thrust reversal, the explicit dependence X on t is explained by a possible change in the aerodynamic configuration with the rejected take-off , for example, the release of speed brakes), G is weight of aircraft, $X(V)$, $Y(V)$ is drag and lift, N is the total normal reaction of runway, f is coefficient of friction. Covered path is determined by the equation

$$\frac{dx}{dt} = V.$$

Converting the previous equations, we obtain the relationships:

$$\frac{dV}{dx} = \frac{1}{mV}P(V,t) - X(V) - \big(G - Y(V)\big)f; \quad \frac{dt}{dx} = \frac{1}{V}.$$

If we select the speed V as the independent variable (taking into account that throughout the beginning of braking it varies monotonically), then we will obtain the relationships

$$\frac{dx}{dV} = \frac{mV}{P(V,t) - X(V,t) - \big(G - Y(V)\big)f};$$

$$\frac{dt}{dV}=\frac{m}{P(V,t)-X(V,t)-(G-Y(V))f}.$$

Direct calculation of the covered path and time spent allows via the comparison of these values with the available values L^{disp} and t^{disp} to determine the remainders of the space and time resources, which, in turn, influence the distribution of preferences.

The second example of the presence of alternative situations gives landing approach due to the complicated conditions, when with a descent two alternatives appear: σ_1 is the continuation of approach and landing and σ_2 is withdrawal on the second circle. The latter can be caused by unforeseen interferences on the strip, large inadmissible deviations from the heading-slope line, change in the meteorological conditions, by other circumstances. The problem about the maximum deviations with the aircraft landing approach was examined in the works [18,19,and 65].

Regions of permissible lateral deviations: lateral deviation from the course plane Z and the course angle ψ are shown on Fig. 6.6. Each point on the border of regions is available by the solution of maximum high speed problem on the conditional extremum with the fulfillment of different, including not differentiated limitations on the phase and controlling variables. Maneuvering with the withdrawal on the second circle was studied in the work [65].

The problem of the substantiation of the certification requirements to the aircraft was solved, in the correspondence to the II category ICAO (the International Civil Aviation Organization). Statistical flight simulation was used.

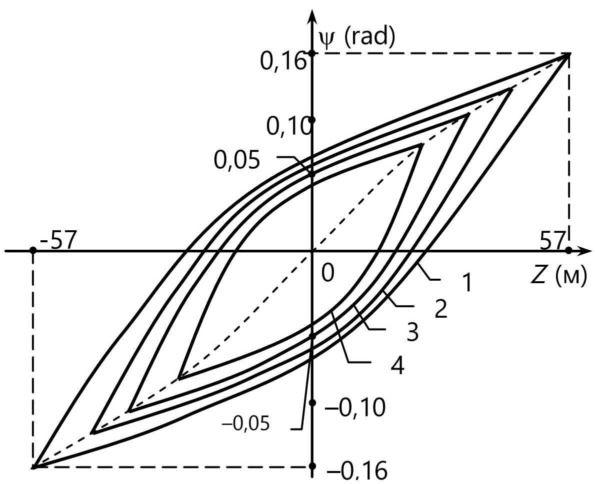


Fig. 6.6

1 — distance 1000 m from the runway head; 2 — distance 800 m from the runway head;3 — distance 600 m from the runway head; 4 — distance 500 m from the runway head;

Fig. 6.7 shows as height H_{ky} of the beginning of withdrawal on the second circle (aircraft Il-86 with the two slotted flap) depends on the value of the first type error α and on the most strongly influencing parameter τ - the delays of reaction on decision making „we depart".

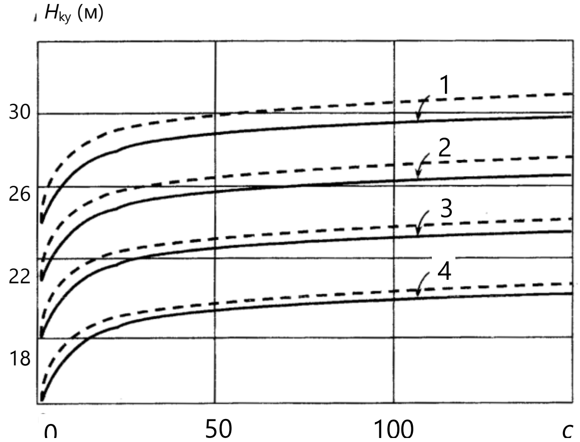


Fig. 6.7

Value α is connected with the mean statistical value of the number of landings, against which is one withdrawal on the second circle and the previously selected probability of the appearance of an improbable event $P_{\text{MBC}} = (10^{-5}...10^{-7})$, by the formula

$$\alpha = nP_{\text{MBC}}.$$

We see that H_{Hy} substantially depends on the indicated parameters and, for example, when $\alpha = 10^{-3}$ and τ = 3,16 s (according to requirements ICAO) $H_{\text{Hy}} \approx 30$ m.

In the work [19] was used the sufficiently detailed mathematical model of the aircraft dynamics. In particular, the dynamics of engine revolutions after „setting the throttles" on the take-off conditions, the time, expended on taking steering wheel on itself, and other components was considered, that was necessary in view of the small available reserves of time.

Let us bring a maximally simplified model, which makes possible to demonstrate the approach of taking subjective factor into consideration. We will assume that the motion of aircraft occurs in the vertical plane, yawing motion and wind are absent. In the trajectory coordinate system the motion of the center of the masses of aircraft is described by the equations:

$$m\frac{dV}{dt} = P(V,t) - G\sin\theta - X(V,t);$$

$$mV\frac{d\theta}{dt} = Y(V,t) - G\cos\theta,$$

motion around the center of masses is expressed by equation

$$I_z\frac{d\omega_z}{dt} = M_z(\alpha, \omega_z, V, t)..$$

The dependences P, X, Y, M_z on time are explicitly connected with of the controlling influences. θ - flight path angle, the angle of attack α and the pitch angle υ are linked by the condition:

$$\alpha = \upsilon - \theta.$$

Angular velocity ω_z is determined by the equation:

$$\frac{d\upsilon}{dt} = \omega_z.$$

Let us add the equation, which describes the measurement of the height H of the masses center above the earth:

$$\frac{dH}{dt} = V\sin\theta.$$

Let us assume that there are limitations on the speed V and on the flight path angle θ:

$$V \in [V^*_1, V^*_2];\ \theta = [\theta^*_1, \theta^*_2],$$

and also the minimum height of the lower point of the aircraft above the earth with the withdrawal on the second circle

$$H > H_{min}.$$

Solving equations of motion and then, calculating „disagreements", we can control the value of „reserves" (or the remained „resources" of these values with respect to the maximum value):

$$V^{(1)} = V - V^*_1;\ V^{(2)} = V - V^*_2;$$
$$\theta^{(1)} = \theta - \theta^*_1;\ \theta^{(2)} = \theta - \theta^*_2,$$

and also

$$H^{(1)} = H - H^*.$$

Since the height of the beginning of withdrawal $H_{\text{ну}}$ (Fig. 6.7) is known, it is possible at each moment of time to determine stored up altitude $H - H_{\text{ну}}$, and, therefore, knowing speed, stored up time to the moment of decision making about „withdrawal" on the second circle.

It is proposed, the following scheme of the subjective factor calculation. As earlier, let us determine, the relation of the resources

$$\bar{r}_k = \frac{R^{req}(\sigma_k)}{R^{disp}(\sigma_k)}, \tag{6.10}$$

and also the adduced relation $\bar{r}_k' = \bar{r}_k\left(1-\bar{r}_k\right)^{-1}$. Let us select operating time as resources. If the operating time t_{op} coincides with the astronomical time t, then

$$\frac{dt_{\text{op}}}{dt} = 1 \Rightarrow t_{\text{op}} = t + c. \tag{6.11}$$

Let us designate required operating time through $t^{req}(\sigma_k)$, available operating time — through $t^{disp}(\sigma_k)$ and assume that condition (6.11) is satisfied. If $t^{req}(\sigma_k)$ is the required time for the realization of strategy σ_k , then total required time must include the time, necessary for decision making $t^{dec}(S_a)$:

$$\tilde{t}^{req} = t^{req}\left(\sigma_k\right) + t^{dec}\left(S_a\right).$$

The designation $t^{dec}(S_a)$ speaks, that the solution starts on the set S_a of the alternative strategies. Let us assume that each of strategies is combined with the additional „troubles", smallest of which, for example, in the case of withdrawal on the second circle - the additional fuel consumption. „Troubles" can be, of course, more significant (crash of Greek aircraft in the region of Athens (2005) with the two-fold withdrawal on the second circle). It is possible to imagine such situation, when the available time $t^{disp}(\sigma_k)$ is determined by the development of the specific unfavorable process on board (fire, the gradual creeping failure). Situation with the failure of the undercarriage retractable system can serve as an example. There exist two possibilities: landing from the first approach on the fuselage belly (σ_1) and dangers combined with this, withdrawal on the second circle (σ_2) with a certain nontrivial probability of work fitness restoration of the undercarriage retractable system, but with the additional complications, connected with the go around, possibly, with the maneuvering with the complicated conditions. It is assumed that the pilot can evaluate the comparative danger of the consequences of one or another selection on S_a.

Let us define this estimation as *the subjective probability* $P(\sigma_k)$ of event. Let

$$\bar{r}_k'(\sigma_k) = \bar{\tau}(\sigma_k) = \frac{t^{req}\left(\sigma_k\right) + t^{dec}}{t^{disp}\left(\sigma_k\right) + t^{dec} + t^{req}\left(\sigma_k\right)}. \tag{6.12}$$

Let us select the canonical distribution of preferences in the form:

$$\pi^{-}\left(\sigma_k\right) = \frac{P^{-\alpha}\left(\sigma_k\right)e^{-\beta\bar{\tau}\left(\sigma_k\right)}}{\sum_{q=1}^{2} P^{-\alpha}\left(\sigma_q\right)e^{-\beta\bar{\tau}\left(\sigma_q\right)}}. \tag{6.13}$$

This distribution possesses the following properties: with $\overline{t}_k = \frac{t^{req}\left(\sigma_k\right)}{t^{disp}\left(\sigma_k\right)} \to 0$ preferences are determined only by subjective probabilities $P(\sigma_k)$, with $\overline{t}_k \to 1$ the corresponding preference becomes zero. Distribution (6.13) appears as solution of optimization problem with the functional:

$$\Phi_{\pi^-} = -\sum_{k=1}^{N} \pi^-\left(\sigma_k\right) \ln \pi^-\left(\sigma_k\right) - \beta \sum_{k=1}^{N} \pi^-\left(\sigma_k\right) \overline{\tau}\left(\sigma_k\right) - \tag{6.14}$$
$$-\alpha \sum_{k=1} \pi^-\left(\sigma_k\right) \ln P\left(\sigma_k\right) + \gamma \sum_{k=1}^{N} \pi^-\left(\sigma_k\right).$$

Special feature of this functional is structure of the effectiveness function

$$\varepsilon_{\pi^-} = -\sum_{k=1}^{N} \left(\alpha \ln P\left(\sigma_k\right) + \beta \overline{\tau}\left(\sigma_k\right)\right) \pi^-\left(\sigma_k\right), \tag{6.15}$$

in which the logarithm of subjective probability is included.

As earlier, realizing simulation on the basis of the Walras - Leontev economic model, we will consider that the parameters α and β variables determining endogenous dynamics, certain property of the subject psyche. The complexity of decision making in this model is determined by the value of subjective entropy - function characterizing degree of indetermination and, therefore, uncertainty. In order to make a decision the subject must overcome „*entropy barrier*". In the example in question the solution must be accepted to the moment t^*. It is necessary to select such a model for the functions $\alpha(t)$ and $\beta(t)$, that it would reflect internal, endogenous factors uncertainty, „fluctuations", which appear in proportion to approach for moment t^*. This reasoning Fig. 6.8 demonstrates.

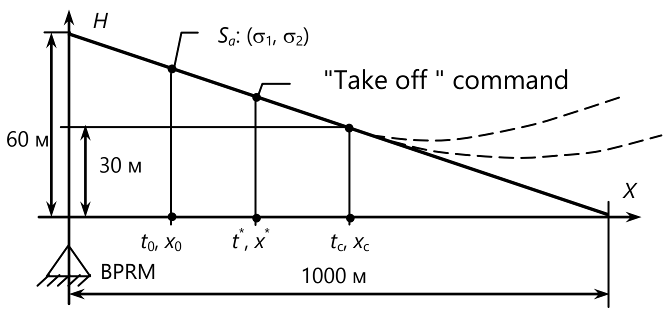


Fig. 6.8

Here BPRM inner homing beacon, above which the height of glide path equals 60 m. In the moment t_0 alternative situation appears, t^* is the moment of making a decision about withdrawal on the second circle, t_c is the moment of the withdrawal beginning. According to ICAO requirements $\tau = t_c - t^* = 3{,}16$ s, which corresponds to the distance of ~ 220 m with the approach speed 70 m/s. Thus, at point t_0 the set of alternatives and selection problem appear

$$S_a: (\sigma_1, \sigma_2)$$

and as well as the distribution of the preferences $\pi(\sigma_1)$ and $\pi(\sigma_2)$, which vary with time at the point of two reasons: the force of exogenous changes (first of all - the decrease of the available time before the moment of the solution adoption) and by the endogenous processes, caused by the uncertainty of situation and respectively, by uncertainty and „the fluctuations" of subject. If till the moment t^* the entropy threshold H^* will be overcome, the solution starts, in consequence of which set S_a changes:

$$\begin{array}{c} S_a: (\sigma_1, \sigma_2) \\ t < t^* \\ \pi(\sigma_1) + \pi(\sigma_2) = 1 \end{array} \begin{array}{c} \nearrow \\ \searrow \end{array} \begin{array}{c} S_{a1}: (\sigma_2);\ \pi(\sigma_2) = 1;\ \pi(\sigma_1) = 0 \\ S_{a2}: (\sigma_1);\ \pi(\sigma_1) = 1;\ \pi(\sigma_2) = 0 \\ t \geq t^* \end{array}$$

If the entropy threshold H^* is overcome later than the moment t^*, then begins the complication of flight situation and the reaction time for the solution „we depart" must be reduced. With the larger required time on decision making $t^* — t_0$ the exception of alternative σ_2 („withdrawal")occurs as a result of exogenous alternations (decrease or complete disappearance of the available time on decision making and controlling actions).

The results of the preference dynamics simulation are presented below. At the point of basis the model of preferences (6.13)is accepted. The structural parameters α and β are considered as the variables, presenting endogenous dynamics. For the simulation of endogenous dynamics, the modified „Lorenz attractor" is used. Modification consists in the introduction of damping and „disturbing" factor, into attractor's equations.

Of course the application of this attractor as the model of mental processes is sequential hypothesis. In actuality there are no any foundations to assume that Lorenz's attractor relation to the processes, proceeding in the consciousness of man. In the given case if plays the role „of black box", similar to that role, which the regression models play, as well as the models of autoregression, moving average and others similar for description of statistical experimental data.

We do not have for a while statistical data is the usual sense, but we have the ideas about the manifestations of the psyches based on the experience, which we would want to reflect with the aid of the models of the type of Lorenz's attractor.

A certain justification, lies in the fact that this model is sufficiently rich: before the modified version it includes 8 structural parameters, by change in which it is possible to obtain different effects, and to give to them the plausible interpretation, which does not contradict experience and „the common sense”. In this case the discussion does not deal with proposing of any method of the identification of the parameters. For this it would be necessary to conduct a series of psychological experiments using, for example, a complex aviation trainer.

The modified Lorenz attractor is described by system of equations (5.106). The components of attractor are redesignated:

$$Q_0 = x;\ \ Q_1 = y;\ Q_2 = z.$$

As earlier, in chapter 5 the equations of attractor are added with quadratic terms, that ensure damping of nonlinear vibrations, and the term *f* of sinks, which excludes the uniform solution and plays the role of additional disturbance. Instead of the sinusoidal disturbance could be used random process.

For evaluating the degree of the uncertainty of preferences the standardized entropy is used:

$$\bar{H}_\pi = \frac{H_\pi}{\ln N}. \tag{6.16}$$

In this example $N = 2$ lnN = 0,693.

It is assumed that the time of decision making is negligibly small in comparison with the values of the available and required times. Glide-slope descent occurs with the following parameters: the weight of the aircraft G = 106 N, the speed V = 70 m/s, the wing area S = 100 m^2, air density ρ = 1,25 kg / m^3, wing aspect ratio λ = 7, specific resistance with the zero lift C_{xo} = 0,02, the required thrust P = 9,4×104 N, the required lift coefficient of 3,259. Let us identify the parameters α and β with variable z and y, i.e., let us assume $\alpha = z$, $\beta = y$.

The parameter x we will interpreted as the concealed endogenous variable. Fig. 6.9 demonstrates change in the variables of attractor and two-dimensional projections of three-dimensional phase portrait for the values of the structural parameters

a = *8;* b = *8;* c = *20;* d = *0,43;* f = *0,8;* h = *0,065;* k = *0,1;* m = *0,065;* n = *0,065.*

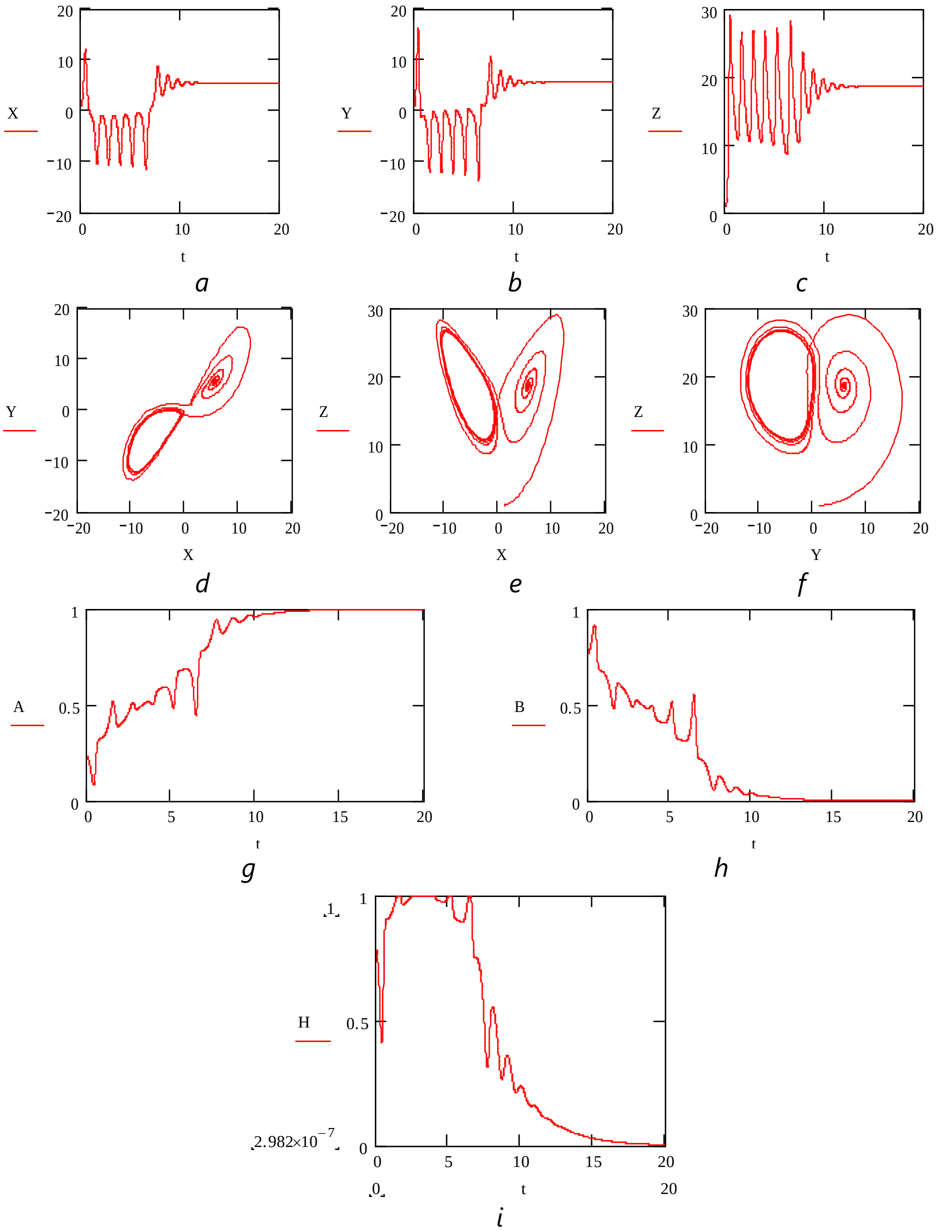


Fig. 6.9

Subjective probabilities are accepted as far as equal $p(\sigma_1) = 0,53$, $r(\sigma_2) = 0,6$. It was accepted that $\bar{\tau}(\sigma_k)$ vary with time about the linear law

$$\bar{\tau}_1 = 5{,}5 + 0{,}01t; \;\; \bar{\tau}_2 = 5{,}4 + 0{,}04t,$$

which corresponds to the gradual decrease of the difference between the required and the available time?

Preferences were calculated based on the formulas:

$$\pi_1^-\left(\sigma_1\right) = \frac{\left(p_1\right)^{-0{,}01z} e^{-y\tau_1}}{\left(p_1\right)^{-0{,}01z} e^{-y\tau_1} + \left(p_2\right)^{-0{,}01z} e^{-y\tau_2}};$$

$$\pi_2^-\left(\sigma_2\right) = \frac{\left(p_2\right)^{-0{,}01z} e^{-y\tau_2}}{\left(p_1\right)^{-0{,}01z} e^{-y\tau_1} + \left(p_2\right)^{-0{,}01z} e^{-y\tau_2}}.$$

Relative entropy was calculated with the formula:

$$\bar{H}_\pi = -\frac{\pi^-\left(\sigma_1\right)\ln\pi^-\left(\sigma_1\right) + \pi^-\left(\sigma_2\right)\ln\pi^-\left(\sigma_2\right)}{\ln 2}.$$

On the figures the designations are used:

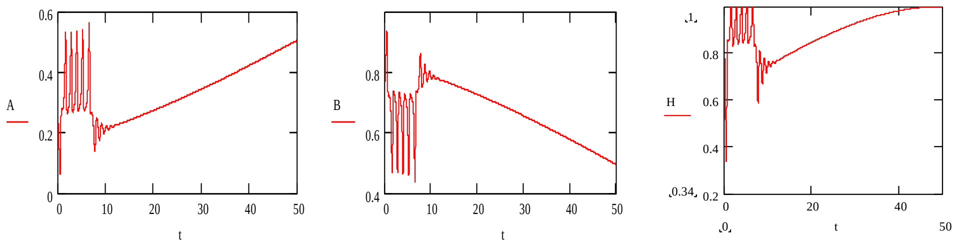
$$\pi^-(\sigma_1) = A; \; \pi^-(\sigma_2) = B.$$

Graphs Fig. 6.9 shows the motion of changing the preferences $\pi(\sigma_1)$ and $\pi(\sigma_2)$. We see that the period of sharp oscillations of preferences in the course of first 10 s. occurs.

In this case the entropy remains high and also experiences fluctuations. If we accept the critical value of the relative entropy $H^* = 0{,}7$ then in this special case „solution" can be accepted approximately in 10 s., and, if σ_1 it corresponds to version „withdrawal", then will be accepted the solution about the withdrawal down the second circle, since after the end of the period „of uncertainty" $\pi^-(\sigma_1)$ sharply it grows to approach 1.

Situation however, is critical, since the total time of a decrease from BPRM (close-in homing beacon) to the end of landing strip is about 14 s. Thus, if alternative situation arose after passage BPRM, then the time to decision making about withdrawal on the second circle can occur insufficient taking into account the selected conditions and parameters. In the second case, Fig. 6.10, *a*—*e* shows alternation of the preferences and relative entropy with the same values of the determining parameters, but with the slower change $\bar{\tau}\left(\sigma_k\right)$:

$$\bar{\tau}_1 = 5{,}5 + 0{,}001t; \;\; \bar{\tau}_2 = 5{,}4 + 0{,}004t.$$



a *b* *c*

Fig. 6.10

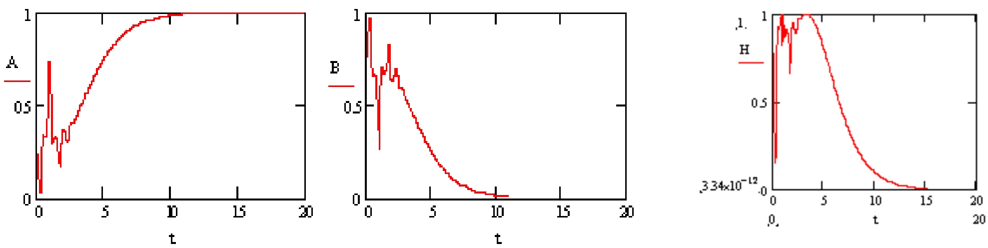
We see that the period „of oscillations", as it is above, continue about 10 s, but relative entropy further remains high and the solution cannot be accepted in the limits of the available time.

The following case (Fig. 6.11, *a*-*c*) corresponds to the following set of the determining parameters:

$$a = 10;\ b = 10;\ c = 35;\ d = 1;\ f = 0;\ h = 0{,}065;\ k = 0{,}1;$$
$$m = 0{,}065;\ n = 0{,}065;\ p(\sigma_1) = 0{,}53;\ p(\sigma_2) = 0{,}6.$$

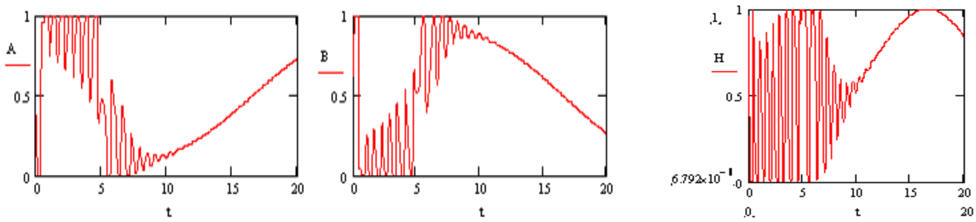
The period of uncertainty composes for about 3 s., after this the entropy $\bar{H}_\pi$ decreases rapidly and there is reason to believe that the solution about „the withdrawal" can be accepted in the limits of 5 s.

Fig. 6.12 (*a*-*c*) shows one additional case, when the internal parameters of attractor take the values: a = 10; b = 10, c = 35, d = 1, damping remains high: h = n = m = 0,05.



a *b* *c*

Fig. 6.11

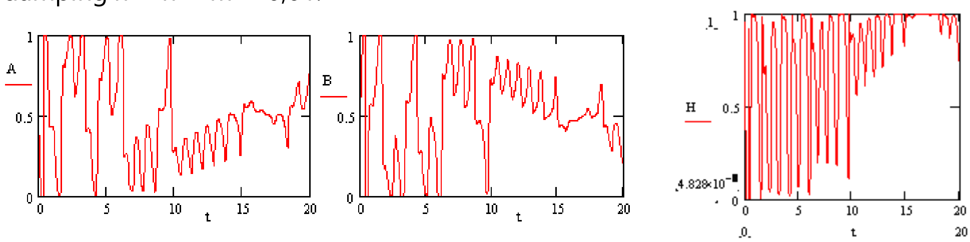


a *b* *c*

Fig. 6.12

We see that the period „oscillating uncertainty" is about 10 s. In this case on the average the inversion of preferences occurs, although as earlier $r(\sigma_1)$ = 0,53; $r(\sigma_2)$ = 0,6; $\overline{\tau}_1(\sigma_1) = 5{,}5 + 0{,}01t$; $\overline{\tau}_2(\sigma_2) = 5 + 0{,}04t$. By this the entropy $\overline{H}_\pi$ firstly decreases, and then it begins to grow. This example demonstrates such a possibility, when the solution "we land", accepted after 10 s. of descent, subsequently proves to be „erroneous" (if it world make on 20th second, then it would be opposite).

Fig. 6.13 (*a*-*c*) presents analogous process, but during the considerably smaller damping $h = n = m = 0{,}01$.



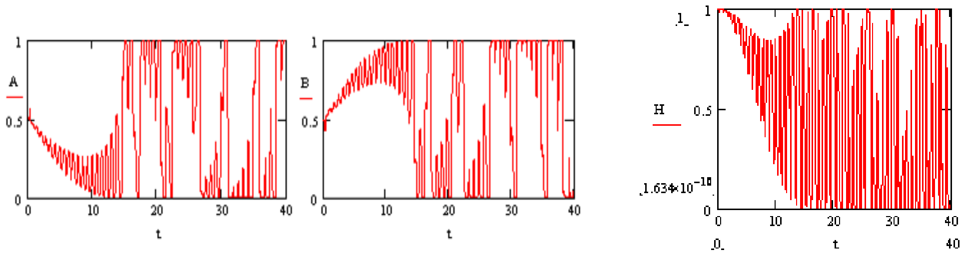
a *b* *c*

Fig. 6.13

As we see, the fluctuations of preferences continue in the larger interval of time (to 15-18 s.), and entropy on the average remains high and „solution" will not be accepted as minimum in the limits, of 20 s. But this means that the alternative σ_1 - „withdrawal" drops off by itself since „withdrawal" will become physically impossible.

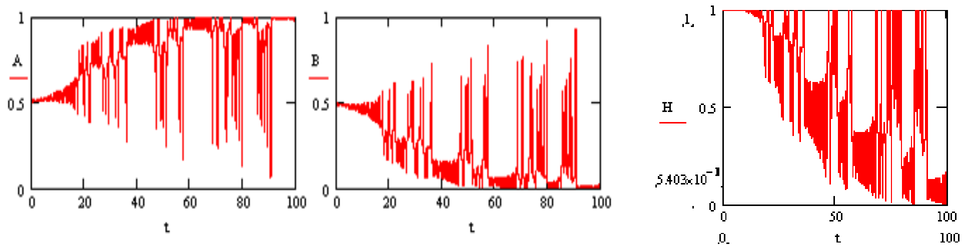
On the following two series of figures the versions, uncertainty oscillating increasing in the course of time are represented (Fig. 6.14 and Fig. 6.15). In order to ensure the positivity of the index of exponential curve the positive addition to *y* is introduced, furthermore, it is assumed that $\overline{\tau}(\sigma_k)$ changes according to the law

$$\overline{\tau}(\sigma_k) = 5 + \mu_k e^{\nu_k t};\ \ \nu_k > 0,\ \mu_k > 0.$$



a b c

Fig. 6.14



a *b* *c*

Fig. 6.15

Fig. 6.16 presents solution, which corresponds to the following set of the structural parameters is represented,

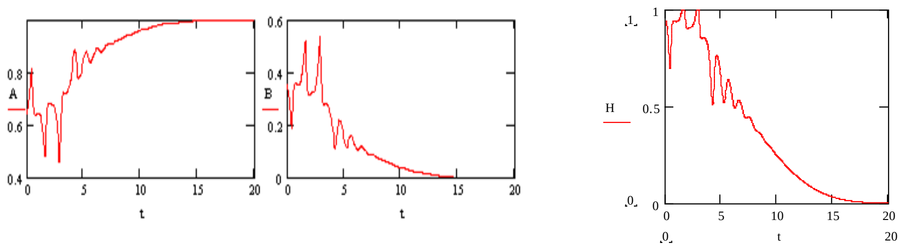
$$a = 8;\ b = 8;\ c = 20;\ d = 0{,}43;\ f = 0{,}8;\ h = 0{,}067;\ k = 0{,}1;$$

$$m = 0{,}067;\ n = 0{,}067;\ p(\sigma_1) = 0{,}53;\ p(\sigma_2) = 0{,}1.$$

$$\overline{\tau}(\sigma_1) = 5 + 0{,}05e^{0{,}05t};\quad \overline{\tau}(\sigma_2) = 5 + 0{,}10e^{0{,}10t},$$

i.e. different growth rates $\overline{\tau}(\sigma_1)$ and $\overline{\tau}(\sigma_2)$. are assigned.

It is possible to find such cases, when bifurcations in the limits of the assigned interval of time are absent select such a values of the parameters.



a *b* *c*

Fig. 6.16

One additional example illustrates the connection of the problems of flight safety with the subjective analysis. Fig. 6.17, *a* schematically shows the special situation, which can arise in flight as a result „closing" of some reasons for destination airport (point *c*). It is assumed that at this moment there are three possible solutions - three alternate (alternative σ_1, σ_2, σ_3), distances to which depend on the moment of the appearance of special situation t^*. With each alternative is connected the certain risk, which supposedly depends on the position of aircraft against the moment t^*, determined as far as two parameters: R and φ.

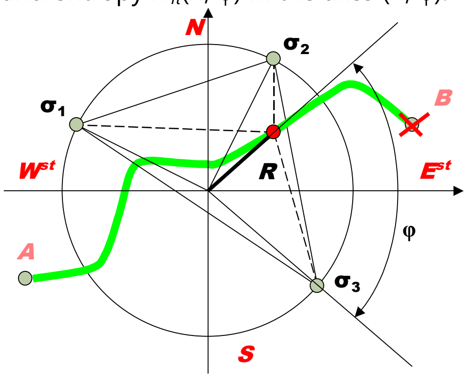
Let us assume that preferences are expressed by the formula

$$\pi\left(\sigma_k \mid R,\phi\right) = \frac{e^{-a_k(R,\phi)}}{\sum_{j=1}^{3} e^{-a_j(R,\phi)}},$$

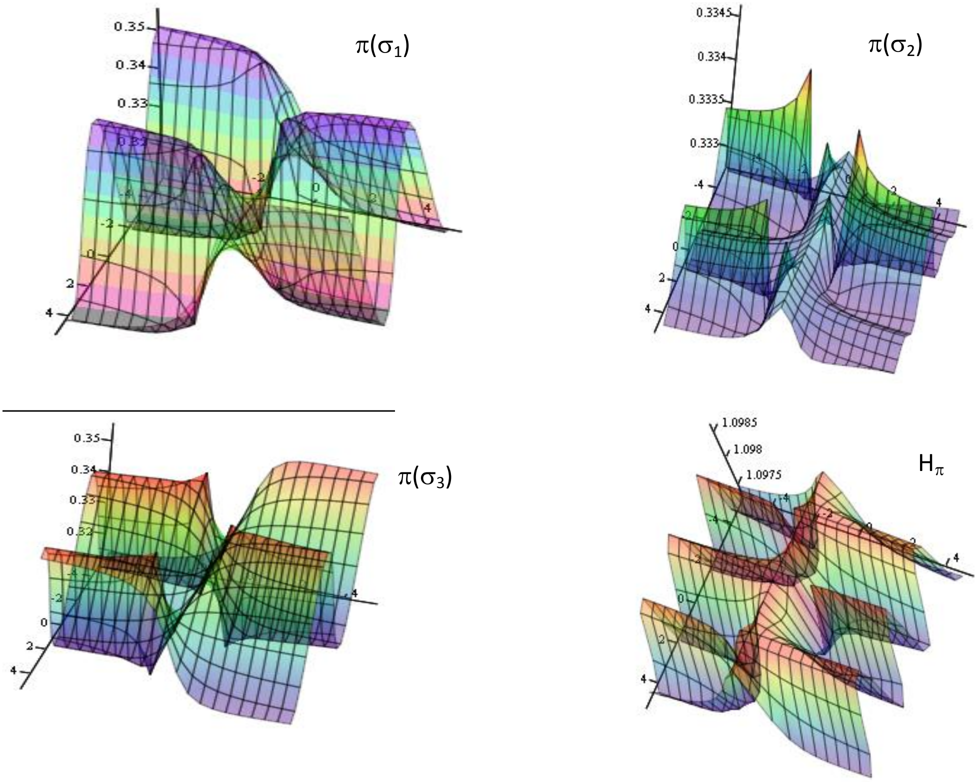
where $a_k(R, \varphi) = dL_k(R, \varphi)$, d is coefficient, which reflects the connection „of risk" with the distance $L_k(R, \varphi)$ to alternate airport. Subjective entropy

$$H_\pi = -\sum_{k=1}^{3} \pi\left(\sigma_k \mid R,\phi\right) \ln \pi\left(\sigma_k \mid R,\phi\right)$$

and, therefore, also it depends on two parameters: R and φ. Fig. 6.17, *b* shows the preferences $\pi(\sigma_k \mid R, \varphi)$ and entropy $H_\pi(R, \varphi)$ in the axes (R, φ).



a

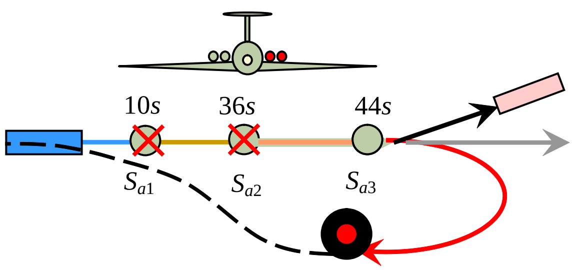


b

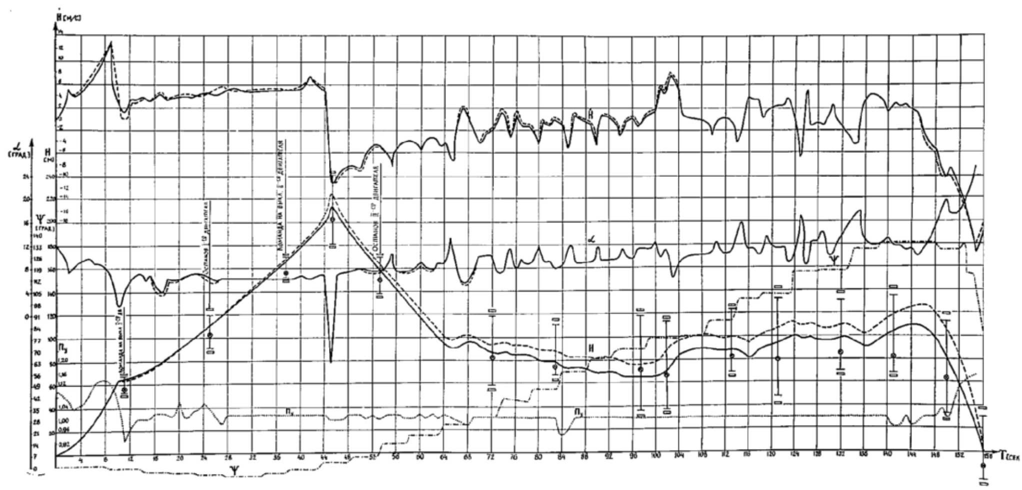
Fig. 6.17

It is evident that the distribution of these values significantly depends on "geographical" position of the point of the special situation appearance. For the certain combinations of R and φ values the entropy $H_\pi(R, \varphi)$ is great if means that decision making on the version selection is hindered. Obviously, in these cases the additional information will be drawn.

At conclusion of this division let us describe the real flight situation, based on example by which it is possible to trace the moment of decision making and in which there were alternative strategies (Fig. 6.18, *a*).



a



b

Fig. 6.18

The signaling about the fire of engine operates with the take-off of the long-range liner aircraft 7 seconds after rotation, pilot switches an engine off and take-off on three engines continues, signaling about the fire of the second engine and the fire in the pods operates after 40 more seconds. Pilot turns off the second engine in complete agreement with the requirements „flight operations manual" in the point of the case of the engine fire. Aircraft with the complete fuel stock and about 100 people aboard cannot accomplish further climb, but it cannot fly horizontally. In spite of the use of all discharges of fire extinguishing signaling about the fire does not go out.

Theoretically there are two possibilities: 1 landing „before themselves" before the straight course on the unequipped areas (ploughed field) with the great probability of catastrophe, since the matter occurs at night, 2 – turn and landing on the airport of departure with the reverse heading also with the great probability of catastrophe, in view of the transient nature of the fire of engine.

Specifically, pilot makes this decision, but as is explained this alternative it is not realized physically - height is not sufficient (potential energy) for fulfilling of maneuver and landing. Nevertheless, in the consciousness of pilot (on the subjective level) these two alternatives exist and are received as realizable and the second - as more preferable.

Fig. 6.18, *b* shows the results of evaluating the parameters of flight with the use of mathematical simulation for restoring the missing parameters, since because of the strong fire on the place of aircraft crash the records of emergency chart recorder MSRP -12 were preserved only, which was available on the tail section and it suffered less.

On the graph of bank angle the moment of decision making about return to the airport of departure, a pilot "placed" the sharp bank (to 27°) on the starboard, is outlined well. As a result a deficiency in the altitude margin increased even more greatly: because of slip appearance, flight altitude additionally decreased. Pilot, increasing angle of attack, brought aircraft to the stall. As a result of investigation it turned out that actually the false response of the temperature sensors occurred, which signal about the fire.

6.4. Probabilistic flight models. Combining of the probabilistic and subjective analysis

Aviation- transport system relates to the class of active systems. In connection with this the problem of using of subjective analysis categories and methods for studying the safety of the functioning of this system appears. The single flight vehicle, which accomplishes flight in the nonautomatic regime, is a special case. In the practice of the analysis of flights, during the investigation of aviation incidents is used the term „*flight situation*". In order to use quantitative methods in the problems of flight safety, it is necessary to pass from the intuitive concept „flight situation" to that formalized. The concepts are distinguished „state", „events" and "situation". „State" determines position and configuration of system irrespectively to the moment of time, „event" is defined as a „state" at the certain moment of time, „situation" is connected with the problem, which must be solved. Taking into account introduced earlier concepts and terminology, it is possible to indicate that in connection with active systems adequate concept is the concept of problem resource situation $c(t)$.

This concept includes indication of

- the state $\sigma_0$of system in the given moment;
- the available resources in the generalized understanding (finances, time, energy resources, the permitted range of change of controlling influences). The description of resources can be the part of the description of state σ_0;
- set of alternatives S_a.

In particular flight situation can be defined as the following collection:

$$c(t)\text{: } (x_0,\ t_0,\ \omega(t_f \in [t_0,\ t_0 + \tau]);\ R^{disp}(t_0),\ R^{req}(t_0),\dots)$$

or in the designations of Chapter 1

$$c(t)\text{: } (P\text{: } \sigma_0(t_0) \to \sigma_j(t_f \in [t_0,\ t_0 + \tau]) \in S_f \subset S_a,\ R_{disp}(t_0),\ R_{req}(t_0),\dots),$$

where x_0 is vector of the state parameters at the initial moment t_0, τ is available time, such, that falling of vector x in terminal set ω must occur not later than through τ: $t_f \in [t_0, t_0 + \tau]$, P is the designation of the problem, which consists besides the transfer of system besides the initial state beside one of the states of subset $S_f \subset S_a$ is not later than through τ.

Formalization of the concept of flight situation is given in [65]. In the process of flight one flight situation changes another. In this sense it is possible to speak about *the situation dynamics*. Let us present flight as the motion of the representative „point" $c(t)$ in the space of flight situations C.

Schematically situation we represent as the combination of the cone of the attainability Ω, that emanates based on the initial state x_0, either from $\sigma_0(t_0)$, the permissible terminal set ω, or $S_f \subset S_a$, and set Ω_f or $S_{attf} \in S_a$.

The problem, which determines the content of situation, is solved, if $\Omega_f \cap \omega \neq \varnothing$, or $S_f \cap S_{attf} \neq \varnothing$ (Fig. 6.19).

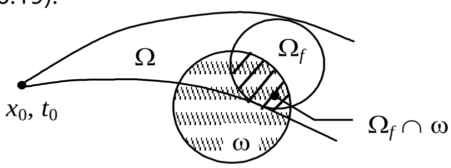


Fig. 6.19

In the general case the motion is considered as random process. The model of disconfigures Markov processes is adequate to the prevailing ideas. This model possesses relative simplicity and, at the same time, contains the necessary arsenal of means for the reflection of the basic technical, natural and subjective factors, which associate appearance and development of flight situations. Let us examine the brief description of model, following [10]. Let $E \subset C$ is subset of the flight situations: if $C(t) \in E$, then event E occurs. On the set C probability measure $P(E)$ is assigned such, that

$$0 \leq \mathrm{P}(\mathrm{E} \subset \mathrm{C}) \leq 1; \ \mathrm{P}(\mathrm{C}) = 1; \ \mathrm{P}(\varnothing) = 0 \tag{6.17}$$

and the transition probability

$$P(E(t); \xi(t_1)) = P(c(t) \in E | \ c(t_1) = \xi(t_1)); \ t > t_1. \tag{6.18}$$

It is assumed that the passage of one situation to another occurs as a result an instantaneous change in the parameters of those determining situation. The ordinariness of the flow of the events occurs, when at the point of the small interval of time Δt can be carried out only one „jump" and a change only in one essential parameter. Spasmodic passages are caused as far as the errors of crew or personnel of air traffic control system, as far as a rapid change in the external conditions.

Although the real changes occur in finite time, we examine the case, when this time is small in comparison with the duration of flight, either the duration of the stages of flight or the duration of maneuver. Transition probabilities satisfy Kolmogorov- Chapman equation:

$$P\left(E(t);\xi(t_1)\right)=\int_{(C)}P\left(E(t);\eta(s)\right)P\left(d\eta(s);\xi(t_1)\right)\quad \text{if}\ \ \forall t\geq t_1;\ s\in(t_1,t). \quad (6.19)$$

Let $q\ (\xi\ (t_1))\tau\Delta$ is probability that in the interval $[t_1, t_1+\Delta t]$ occurs abrupt change in one of the determining parameters. Then $q(\xi\ (t_1))$ is the density of probability distribution of transition. Density $q(\xi(t_1))$ is determined by investigating the reliability of functional systems, probabilities of errors, committed by personnel, the probabilities of occurrence of the extreme atmospheric conditions and other contributing the safety factors.

Probability $Q(E,\ \xi(t_1))$ that as a result of „jump" the system will prove to be in one of the situations, which belong to subset $E\subset C$, is determined by investigating the statistical dynamics of system after „jump", tracing of controllability set and (or) attainability set relative to the terminal set ω. „Jump" can be realized as change in the actions of crew at the moment, when the recognition of special situation occurs, the solution starts and crew begins to undertake actions on its correction.

Thus, for instance, as the landing approach the moment of detecting the significant deviations from the course plane it can be considered the moment of „jump", since the accomplishment of intensive maneuver for the correction of position begins after this. Functions $q(\xi(t_1))$ and $Q(E,\ \xi(t_1))$ satisfy the conditions:

$$q(\xi(t_1))\geq 0;\ \xi(t_1)\in C; \quad (6.20)$$

$$0\leq Q(E,\ \xi(t_1))\leq 1;\ E\subset C;$$
$$Q(\varnothing,\ \xi(t))=0;\ t\geq 0;$$
$$\int_{(C)}Q\left(dE,\xi(t)\right)=1\quad \text{for}\ \ \xi(t)\in C.$$

Transition probability satisfies the direct and inverse equations of Feller [10]:

$$\frac{\partial P\left(E(t),\xi(t_1)\right)}{\partial t}=-\int_{(E)}q\left(\eta(t)\right)P\left(d\eta(t),\xi(t_1)\right)+\int_{(E)}Q\left(E,\eta(t)\right)q\left(\eta(t)\right)P\left(d\eta(t),\xi(t_1)\right); \quad (6.21)$$

$$\frac{\partial P\left(E(t),\xi(t_1)\right)}{\partial t_1}=q\left(\xi(t_1)\right)\left[P\left(E(t),\xi(t_1)\right)-\int_{(C)}P\left(E(t),\eta(t_1)\right)Q\left(d\eta,\xi(t_1)\right)\ \right]. \quad (6.22)$$

In these equations $P(E(t),\ \xi(t_1))$ is the transition probability, which can be considered as one of the criteria of safety.

In the aviation it is accepted to divide all flight situations on five categories: normal, complicated, dangerous, emergency and catastrophic. Let us assign to them the designations respectively E_0 , E_1 , E_2 , E_3, *and* E_4 . The separation of situations sets in the subsets and the determination of the corresponding gradation of the types of flight situations is to a considerable extent the result of the agreement between the specialists.

Probabilistic analysis makes it possible to refine this gradation and to give the quantitative attributes of the reference of situations to one or other type or another on the basis of the calculation of transition probabilities.

One of the problems of standardization consists in the determination of boundaries $\Gamma^{(i)}$ ($i \in \overline{1,4}$) the subsets of flight situations. The most important characteristic are the transition probabilities beside the catastrophic situation, i.e. $P(E_4(T), \xi(t_1))$, where $\xi(t_1) \in E_0$. Let P_{ij} is probability of transition of system from E_i into E_j (more precise - from a certain “initial” state $c_0 \in E_i$ into any state from E_j).

The schemes (Fig. 6.20) represented below make it possible to determine clearly the role of various transition probabilities P_{ij} of passage of the less complex situation to more complex, determine in the level of vulnerability, *hazard* of system. Probabilities $\overline{P}_{ij} = 1 - P_{ij}$ characterize the vulnerability of system from worsening in the situation, increase in the danger.

The probabilities P_{ji} of passages from the worse situation to the better characterize the property of system to restore safer states - to „survive” with the appearance of contingency situations and therefore it is possible to consider them the primary characteristics *of the vitality* of system, almost more important than first type probabilities.

The transition probabilities are normalized for each initial state, i.e.,

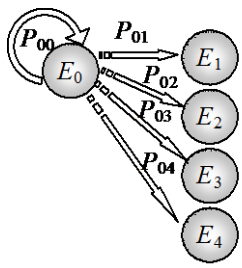
$$\sum_{j=0}^{4} P_{ij} = 1. \tag{6.23}$$

The given in Fig. 6.20 inequalities, which contain the probability P^*_{ij} on the right side , assume that these probabilities are normalized in some manner. Usually only the probabilities, which correspond to the first scheme (Fig. 6.20, *a*) are normalized. Remaining probabilities are not normalized .

More complete volume of the requirements, presented on the system from its safety point of view, consists in normalization of all matrix elements of the transition probabilities

$$P = \begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} & P_{04} \\ P_{10} & P_{11} & P_{12} & P_{13} & P_{14} \\ P_{20} & P_{21} & P_{22} & P_{23} & P_{24} \\ P_{30} & P_{31} & P_{32} & P_{33} & P_{34} \\ P_{40} & P_{41} & P_{42} & P_{43} & P_{44}. \end{bmatrix} \qquad (6.24)$$

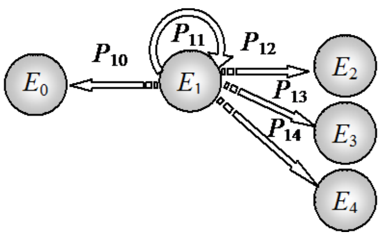
a



$P_{01} < P^*_{01}$
$P_{02} < P^*_{02}$
$P_{03} < P^*_{03}$
$P_{04} < P^*_{04}$

$P_{00} > P^*_{00}$

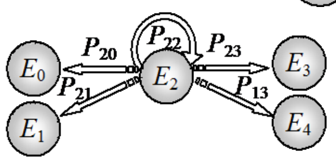
b



$P_{12} < P^*_{12}$
$P_{13} < P^*_{13}$
$P_{14} < P^*_{14}$

$P_{10} > P^*_{10}$
$P_{11} > P^*_{11}$

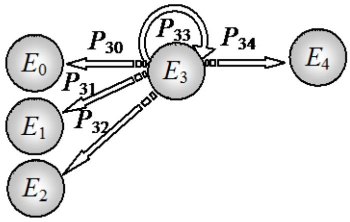
c



$P_{23} < P^*_{23}$
$P_{24} < P^*_{24}$

$P_{20} > P^*_{20}$
$P_{21} > P^*_{21}$
$P_{22} > P^*_{22}$

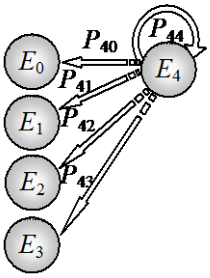
d



$P_{34} < P^*_{34}$

$P_{30} > P^*_{30}$
$P_{31} > P^*_{31}$
$P_{32} > P^*_{32}$
$P_{33} > P^*_{33}$

e



$P_{40} = P_{41} = P_{42} = P_{43} = 0$
$P_{44} = 1$

Fig. 6.20

The values of probabilities in the last row of matrix P correspond to the definition of the catastrophic situation as such, from which „there is no exit".

In particular, E_0 contains only „one situation" is normal. Transition probabilities of the less complex situation into more should be considered to be the criteria *of danger*, while transition probabilities of the more complex situation into less complex - as a criteria *of vitality. The ability of system „to resist" passage from given situation to the less favorable (more dangerous), is naturally to name as reliability*.

In this connection let us define the catastrophic situation as such, from which there is no exit. Thus, if trajectory in the space of flight situations falls into the subset E_4, then it remains there „forever". *The ability of system „to return" from the more complex and more dangerous situation into the less complex and the dangerous situation is called vitality*.

Taking into account the aforesaid the property of vitality is connected with the subsets E_1, E_2, E_3. Let us examine *direct* and *reverse* problems of safety in the terms of probabilistic analysis.

Direct problem - certification problem. It is considered, that the types of flight situations and their gradation are determined, the boundary $G^{(i)}$ of subsets $E_i \subset C$, are described quantitative and values of the transition probabilities, in particular, passage in the catastrophic situation E_4 have been standardized . „A type aircraft certification" consists in that, to prove by tests and calculations that the transition probabilities for this type of aircraft do not exceed the standardized values.

The more complete characteristic of safety includes all transition probabilities as *„straight"* so that and *„reverse"* passages. *Inverse problem – standardization problem* . Problem of rate setting to lies in the fact that, in the first place, appoint the values of the normalized probabilistic indices, including of the transition probabilities and, in the second place, to construct in the situation space of boundary $G^{(i)}$, that separate subsets E_i.

The problems of certification and rate setting so in a general manner appear and in this form they are applicable not only to the security issue of flights, but also to the safety of other complex systems.

The interpretation of the given formulations for the application to the active systems requires the certain refinements, in connection with the determination of active system safety as safety of the subject of this system (in chapter 6.1). In this connection, the safety must consider the possibility of putting to the subject of any loss including death. In connection with the safety problem of civil aviation it is necessary to enlarge the concept of subject, after including in this concept in s the pilots (crew), who make decisions with the appearance of special situation and accomplish direct control, also the passengers, who also make the decisions, connected with their own safety selecting one or another airline, if there is a possibility of selection.

The prescription of the normalized levels of probabilistic criterion is the original problem. Its solution is based on retrospective data about the aviation incidents, already achieved statistical levels, taking into account the technical and financial possibilities of increase and maintaining safety level.

Let us assume that „flight situation" as the object of quantitative analysis can be parametrized. The discussion deals on the parametrization of the sets, entering the formal description of flight situation. Then the study of situation dynamics comes down to the dynamics, generally speaking, of the stochastic object, described by the set of the numerical parameters. An example of the parametric description of terminal sets and sets, which assign operational limitations, are the distance of visibility with the landing approach before the steering regime, permissible range of centering, and the permissible value of crosswind at the landing.

The region of the allowed values of the introduced parameter can be quantized. In this case each component will have the finite (or denumerable) number of values. Assume that process $Z(t)$; $t \geq 0$ is Markov chain, where $Z(t)$ is the vector-parameter, containing the parameters, which determine situation, including that of characterizing the state of air vessel (attitude, engine operating mode, the position of the control elements and the like).

In connection with the introduction of the flight situation quantitative image each flight can be represented as the random walk of „representative point" $c(t)$ in the space of flight situations C (Fig. 6.21). If „point" $c(t)$ falls in the subset of the catastrophic situations C_4, it remains there forever. In other cases the return into the subset of the normal situations C_0 is possible.

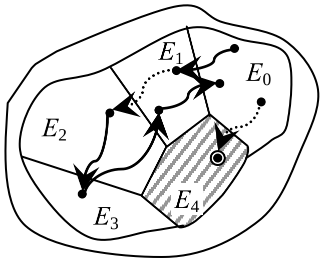


Fig. 6.21

In the models of Feller and Kolmogorov the random process of point walk appears explosive, passage from situation to situation has a nature of jump. Feller's equations are substituted by the equations of Kolmogorov [10,15]:

$$\frac{\partial P_{i,j}(t,t_1)}{\partial t} = -q_j(t)P_{i,j}(t,t_1) + \sum_{k=0}^{N} q_k(t)Q_{k,j}(t)P_{j,k}(t,t_1); \tag{6.25}$$

$$\frac{dP_{i,j}(t,t_1)}{\partial t_1} = q_i(t_1)\left[P_{i,j}(t,t_1) - \sum_{k=0}^{N} Q_{i,k}(t_1)P_{k,j}(t,t_1)\right]. \tag{6.26}$$

Here $q_i(t)$ is the density of probability distribution of random changes, if system is presents in the situation with the number i at the moment t. $Q_{i,\ j}(t)$ is the conditional probability that system, which is present in the situation i at the moment t as a result of „jump" will complete passage into the situation j.

Transition probabilities $P_{i,j}(t,\ t_1)$ satisfy the conditions:

$$0 \le P_{i,j}(t,\ t_1) \le 1, \text{ for } \forall(i, j, t, t_1);$$

$$\sum_{j=1}^{N(\infty)} P_{i,j}(t,t_1) \le 1, \text{ for } \forall(i, j, t, t_1),$$

and also to Kolmogorov- Chapman equation:

$$P_{i,j}(t,t_1) = \sum_{K=0}^{N} P_{i,k}(t,s)P_{k,j}(s,t_1)$$

with the initial conditions:

$$P_{i,j}(t_1,t_1) = \delta_{i,j} = \begin{cases} 1, i = j; \\ 0, i \ne j. \end{cases}$$

Let us examine an example of the analytical solution of Kolmogorov's equations under the simplifying assumptions:

- probability $q_1(t)\Delta t$ of the reasons for withdrawal from the normally stationary situation $q_1(t) = q_1$;

- there is no cause of referrals from this particular situation ($i \in$ 1, 2, 3, 4) in any other $q_i = 0$;
- matrix elements Q: Q_{ij} = const;
- flight situations in the limits of one subset E_i are not distinguished and are defined as situations of the type i.

Relatively to the matrix Q let us assume additionally that all passages lead to the worsening situation, the jump, unfolding in a catastrophic situation, does not derive system from this situation. Standardization conditions are satisfied

$$\sum_{j=0}^{4} Q_{i,j} = 1 \qquad \text{for} \qquad \forall i \in \overline{0,3}, \qquad\qquad Q_{44} = 1.$$

When above assumptions is hold the analytical solution of equations (6.25), (6.26) takes the form:

$$P_{00}\left(t,t_1\right) = e^{-q_0\left(t-t_1\right)}; \quad P_{04}\left(t,t_1\right) = Q_{04}\left(1 - e^{-q_1\left(t-t_1\right)}\right); \ Q_{04} = 1.$$

$$P_{0j} = 0 \quad \text{for} \quad j = 1,2,3; \ P_{i,j} = 1 \quad \text{for} \quad i = 1, 2, 3, 4.$$

All the remaining matrix P elements are equal to zero. If $t \to \infty$, $P_{00} \to 0$, $P_{04} \to Q_{04} = 1$. With the integration of equations the equality $P = I$ is accepted as the initial conditions, where I — unit matrix. It was accepted that $Q_{04} = 1$.

This corresponds to the fact, that any initiating jump in the normal state transfers system into the catastrophic situation. If we waive this condition and assume, $Q_{00} \neq 0$, which corresponds to the case, when the event, proceeding in the normal situation, can with the probability $Q_{00} > 0$ not lead to a change in the situation and with the probability $Q_{04} = 1 - Q_{00}$ lead system in the catastrophic situation, transition probabilities are determined by the formulas:

$$P_{00}\left(t,t_1\right) = e^{-q_0\left(1-Q_{00}\right)\left(t-t_1\right)};$$

$$P_{04}\left(t,t_1\right) = \frac{Q_{04}}{1-Q_{00}}\left(1 - e^{-q_0\left(1-Q_{00}\right)\left(t-t_1\right)}\right) = 1 - e^{-q_0\left(1-Q_{00}\right)\left(t-t_1\right)}.$$

Finally, if all probabilities Q_{1j} in the first row of matrix Q are not equal to zero, then transition probabilities are given by the formulas:

$$P_{0j}\left(t,t_1\right) = \frac{Q_{0j}}{1-Q_{00}}\left(1 - e^{-q_0\left(1-Q_{00}\right)\left(t-t_1\right)}\right).$$

$$P_{04}\left(t,t_1\right) = \frac{Q_{04}}{1-Q_{00}}\left(1 - e^{-q_0\left(1-Q_{00}\right)\left(t-t_1\right)}\right) = 1 - e^{-q_0\left(1-Q_{00}\right)\left(t-t_1\right)}.$$

The model of situation dynamics, represented by equations (6.25), (6.26) is purely probabilistic.

The influence of subjective factors is in no way reflected in it. Construction *of the combined „probabilistic- subjective"* model of situation dynamics can be carried out as follows. Air transport system or its part, in particular, system „aircraft- pilot" within the framework of the terminology taken in this book is *the active system*, which in each moment is located in the certain problem - resource situation.

The subject of system - its active element makes decisions on the set S_a of alternatives.

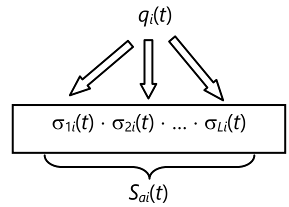


Fig. 6.22

Let $\sigma_{mi}(t)$ is alternative or strategy, the solution *of i-th* type flight situation, and $S_{ai}(t)$ — set of realizable strategies, which is examined by subject at the given moment t. The nature of the alternative strategies and their number depend on the appearing flight situation:

$$\sigma_{mi}(t) \in S_{ai}(t),\ (m \in \overline{1, L}).$$

Each time, when the initiating event with the probability $q_i(t)\Delta t$ occurs, that leads to a change in the situation, pilot analyzes certain number of the alternative strategies of unfavorable development countering, and the one of many strategies shall be selected. The events, which consist in the selection of that or another strategy from $S_{ai}(t)$ are inconsistent and compose complete group.

Therefore conditional probability $Q_{i,j}(t)$ can be written in the form:

$$Q_{i,j}(t) = \sum_{m=1}^{L} P\left(\sigma_{m,i}(t) \mid i\right) Q_{i,j}\left(t \mid \sigma_{m,i}(t)\right), \tag{6.27}$$

where $P(\sigma_{m,i}| i)$ — probability that as a result of appearance of the initiating event with an intensity of $q_i(t)$ the pilot will select strategy $\sigma_{m,i}(t) \in S_{ai}(t)$, $Q_{i,j}(t| \sigma_{m,i}(t))$ — the probability of system transition into the state j from the state i with the use $\sigma_{m,i}(t)$.

Probabilities $P(\sigma_{m,i}(t)| i)$ are normalized by the condition:

$$\sum_{m=1}^{L} P\left(\sigma_{m,i}(t) \middle| i\right) = 1, \tag{6.28}$$

These probabilities are determined not only on objective factors, orders of normative documents, but also as subjective evaluation of flight situation and effectiveness of particular strategies $\sigma_{m,i}(t)$.

None of the appearing flight situations is described with the complete certainty not in one of the statutory documents. In each partial case there are deviations from the model scheme and the role of pilot (subject of active system) consists not only in, carrying out the order of action, but also reacting on the deviations from the previously foreseen schemas.

The solutions about the of strategy $\sigma_{m,i}(t)$ schemes are taken on the basis of the analysis of resources (required and available), of knowledge and habits, obtained with the instruction, and also such factors as personal psychophysical data: the level of intellect, training, the level of common culture. The one of the methods to reflect these factors at the formal level and to ensure the possibility of the quantitative analysis of their influence appears with the introduction in the review of subjective preferences, as well as the combination of probabilistic and subjective analysis.

If one assumes that the probabilities of alternatives choice $P(\sigma_{m,i}(t)|\, i)$ are directly proportional to the correspondent preferences, which are normalized to the value of one like probabilities are, then it is possible to make the nonformal substitution

$$P(\sigma_{m,i}(t)|\, i) \rightarrow \pi(\sigma_{m,i}(t)|\, i)$$

and use the relationship instead of relationship (6.27)

$$Q_{i,j}(t) = \sum_{m=1}^{L} \pi\left(\sigma_{m,i}(t) \mid t\right) Q_{i,j}\left(t \mid \sigma_{m,i}(t)\right). \tag{6.29}$$

In this case in the equations of Kolmogorov (6.25), (6.26) the canonical functions of the preferences are included, which are, in their turn, the result of solving one of the variational problems, described above. Thus, we obtain the combined *probabilistic - subjective model of dynamic situation*.

Thus, for instance, if the formula of preference takes the form

$$\pi\left(\sigma_{m,i}(t) \mid i\right) = C_i e^{-\beta_i \hat{Q}_{i,j}\left(\sigma_{m,i}(t) | i\right)}, \tag{6.30}$$

where C_1 is normalizing coefficient, and $\hat{Q}_{i,j}$ is the subjective evaluation of the probability of transition $i \rightarrow j$, then it should be assumed, that such estimation is accessible to subject (in this case - to pilot) in one or another form. It, can be based on the experience for example, on a priori expert's estimations, information, which is contained in the technical manuals.

In the more general case, the function of preference is determined through a priori „utility" $U_{i,j}(\sigma_{m,i}(t)|\ i)$, or „harmfulness" $L_{i,j}(\sigma_{m,i}(t)|\ i)$. In this case certainly, the available and required resources are considered in the structure of these functions.

The modified equations of Kolmogorov take the form:

$$\frac{\partial P_{i,j}(t,t_1)}{\partial t} = -q_j(t) P_{i,j}(t,t_1) + \tag{6.31}$$

$$+\sum_{k=0}^{N(\infty)} q_k(t) \sum_{m=1}^{L_k} \pi\big(\sigma_{m,k}(t) \mid t\big) Q_{k,j}\big(t \mid \sigma_{m,k}(t_1)\big) P_{j,k}(t,t_1);$$

$$\frac{\partial P_{i,j}(t,t_1)}{\partial t_1} = -q_j(t_1)\big[P_{i,j}(t,t_1) - \tag{6.32}$$

$$-\sum_{k=0}^{N(\infty)} \sum_{m=1}^{L_i} \pi\big(\sigma_{m,i}(t_1) \mid i\big) Q_{i,k}\big(t_1 \mid \sigma_{m,i}(t\ )\big) P_{k,j}(t,t_1)\big].$$

As an example we will examine the simplified graph, depicted on Fig. 6.23, when there are only two subsets E_1 is „normal situations" and E_2 is „unfavorable situations".

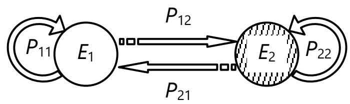


Fig. 6.23

Let at the initial moment the system be located in the situation, which belongs to the subset of „normal" situations E_1.

The initiating event occurs with the probability $q_1\Delta t$, as a result of which system either remains in set E_1, or passes into E_2. Corresponding transition probabilities are P_{11} and P_{12}. The probabilities P_{22} and P_{21} characterize the retention of situation in E_2 and reverse passage in E_1.

Below the results of simulation are represented on the assumption, that the subset E_2 is a subset of catastrophic situations (subset „without the output"), and when the initiating event appears with the probability $q_1\Delta t$ there are two alternative strategies: σ_1, σ_2.

Let us assume that, the initiating events, which cause passages can appear only in the normal situations. Let the conditional probabilities $Q_{11}(\sigma_1) = Q_{111}$ и $Q_{11}(\sigma_2) = Q_{112}$ (probability to preserve initial position $c(t) \in E_1$) change in time according to formula

$$Q_{11i} = \frac{\alpha_i}{\alpha_i + \beta_i t}, \quad \alpha_i > 0,\ \beta_i > 0, \tag{6.33}$$

i.e. in the course of time they decrease. α_i and β_i depend on that which strategy of allowed transition countering will be chosen. In accordance with (6.29) the conditional probability $Q_{11}(t)$ is assigned by the formula

$$Q_{11}(t) = \pi(\sigma_1)Q_{11,1}(t) + \pi(\sigma_2)Q_{11,2}(t). \tag{6.34}$$

In connection with the assumptions made

$$Q_{22} = 1;\ Q_{21} = 0;\ q_2 = 0;\ P_{21} = 0.$$

Calculations are performed for the case, when $\alpha_1 = 5\cdot10^3$; $\alpha_2 = 5\cdot10^2$. The probability $Q_{12}(t)$ is calculated from the formula $Q_{12}(t) = 1 - Q_{11}(t)$, and preferences can be formed with the formulas:

$$\pi(\sigma_1) = \frac{e^{\beta R_1(t)}}{e^{\beta R_1(t)} + e^{\beta R_2(t)}};\ \pi(\sigma_2) = 1 - \pi(\sigma_1), \tag{6.35}$$

Where

$$R_1(t) = \frac{R_d - \delta_1 - V_1 t}{\delta_1 + V_1 t};\ R_2(t) = \frac{R_d - \delta_2 - V_2 t}{\delta_2 + V_2 t};$$

R_d is available resources; V_1 and V_2 are „speeds" of the expense of resources with the use, respectively strategies σ_1 and σ_2, δ_1 and δ_2 are the initial expenditures, connected with the selection of one or another strategy (δ_1 = 0,35; δ_2 = 0,35; V_1 = 0,0002; V_2 = 0,00005 was accepted).

The algorithm of the strategy selection is such: subjective entropy is defined as

$$\bar{H}_\pi = -\frac{1}{\ln 2}\left(\pi(\sigma_1)\ln\pi(\sigma_1) + \pi(\sigma_2)\ln\pi(\sigma_2)\right). \tag{6.36}$$

The critical value of the entropy is established

$$\bar{H}^* = 0,5.$$

At the moment, the condition $\bar{H}_\pi \le \bar{H}^*$ is satisfied first time subject makes decision, the selects one of strategies. The dependence $\pi(\sigma_1)$ and $\pi(\sigma_2)$ on time is shown on Fig. 6.24, entropy monotonically decreases. The shown above inequality is fulfilled with $t \ge t^* = 2213$.

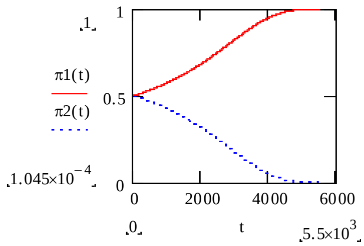


Fig. 6.24

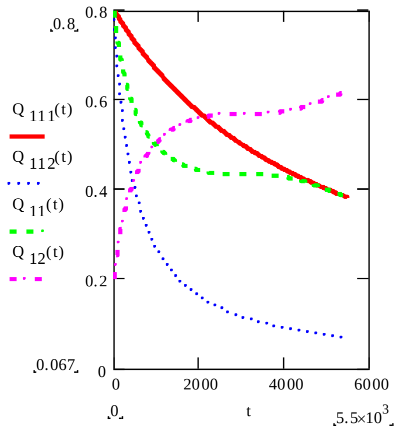


Fig. 6.25

Fig. 6.25 shows the probabilities $Q_{11,1}(t)$; $Q_{11,2}(t)$; $Q_{11}(t)$; $Q_{12}(t)$, while Fig. 6.26 and Fig. 6.27 shows the change of probability $Q_{11}(t)$ it at the moment t^* the one of strategies is selected. In this case the preferences are redistributed in such a way, that with the selection σ_1 $\pi(\sigma_1) = 1$; $\pi(\sigma_2) = 0$, and vice versa, with the selection σ_2 $\pi(\sigma_1) = 0$; $\pi(\sigma_2) = 1$. The function $Q_{11}(t)$ at the moment t^* undergoes jump, and its further motion depends on the selected strategy.

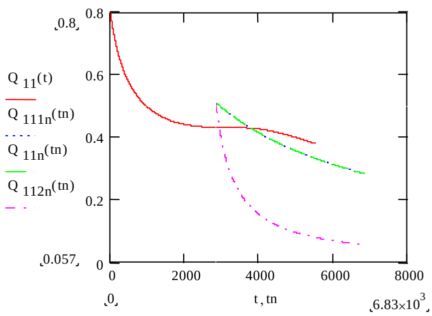


Fig. 6.26

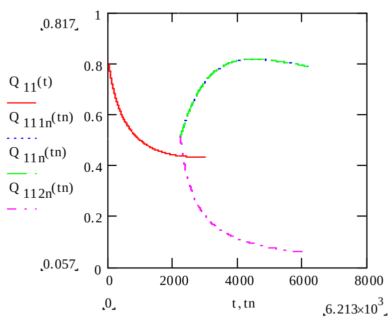


Fig. 6.27

It is evident from Fig. 6.28, that, since the moment t^* the dependences of transition probabilities on time change. Fig. 6.27 corresponds to the case, when for the transition probabilities $Q_{11,l}$ one assumes a somewhat outstanding law of their dependence on time. In this case also occurs the jump of the probability Q_{11} at the moment of time t^*. Further motion of dependence $Q_{11}(t)$ substantially depends on that, which alternative is selected: σ_1 or σ_2. In the given case the alternative σ_1 proves to be best.

One additional much simpler model, is based on the assumption about the special nature of the flows of events (for example, Poisson).

Adapting probabilistic model to the gradation of situations accepted in the aviation, examine the graph, depicted on Fig. 6.29.

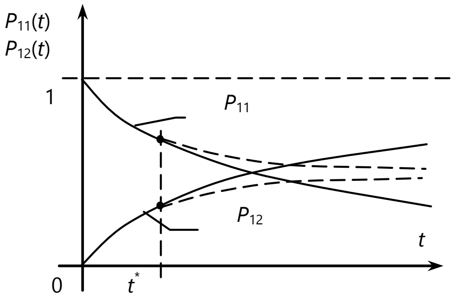


Fig. 6.28

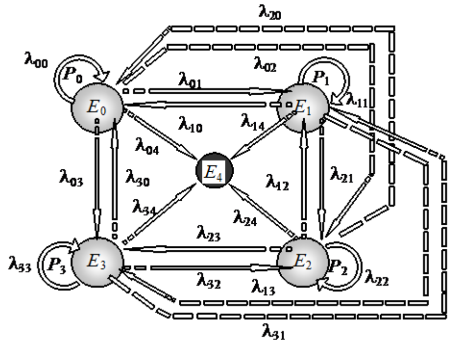


Fig. 6.29

Here are λ_{ij} the intensity of passages, $p_i\left(i \in \overline{0,4}\right)$ - the probability of stay in the appropriate subsets E_i. By hypothesis the subset E_4 is a subset „without output" - the subset of catastrophic situations. System is nontransitive. Passages from subset to subset bear spasmodic nature. System of equations for p_i, that corresponds to graph, takes the form:

$$\frac{dP_0}{dt} = -\left(\lambda_{01} + \lambda_{02} + \lambda_{03} + \lambda_{04}\right)P_0 + \lambda_{10}P_1 + \lambda_{20}P_2 + \lambda_{30}P_3; \tag{6.37}$$

$$\frac{dP_1}{dt} = -\left(\lambda_{10} + \lambda_{12} + \lambda_{13} + \lambda_{14}\right)P_1 + \lambda_{01}P_0 + \lambda_{21}P_2 + \lambda_{31}P_3;$$

$$\frac{dP_2}{dt} = -\left(\lambda_{20} + \lambda_{21} + \lambda_{23} + \lambda_{24}\right)P_2 + \lambda_{02}P_0 + \lambda_{12}P_1 + \lambda_{32}P_3;$$

$$\frac{dP_3}{dt} = -\left(\lambda_{30} + \lambda_{31} + \lambda_{32} + \lambda_{34}\right)P_3 + \lambda_{03}P_0 + \lambda_{13}P_1 + \lambda_{23}P_2;$$

$$\frac{dP_4}{dt} = \lambda_{04}P_0 + +\lambda_{14}P_1 + \lambda_{24}P_2 + \lambda_{34}P_3.$$

Suppose that in each situation (excluding the situations, which belong to the subset E_4) there are a certain number of strategies, and pilot selects the best one from his point of view:

$$\sigma_k^j\left(k \in \overline{1,N_j}\right)..$$

The relationship occurs:

$$\lambda_{iq}\Delta t = \sum_{k=1}^{N_j} P\left(\sigma_k^j\right)P\left(j \to q \mid \sigma_k^j\right)\Delta t, \tag{6.38}$$

where $P(\sigma_k^j)$ is the probability of the selection of strategy σ_k^j, $P\left(j \to q \mid \sigma_k^j\right)$ is the probability of transition $j \to q$, if is selected strategy σ_k^j.

Standardization condition is satisfied:

$$\sum_{q=0}^{4} \sum_{k=1}^{N_j} P\left(\sigma_k^j\right) P\left(j ® q \mid \sigma_k^j\right) = 1. \tag{6.39}$$

Relationship (6.39) corresponds to the assumption that the collection of strategies forms complete group each time. Also, as this was done earlier, suppose that probabilities $P(\sigma_k^j)$ are proportional to the value of the corresponding preferences $\pi(\sigma_k^j)$ and we will replace in the equations given above the probabilities $P(\sigma_k^j)$ by $\pi(\sigma_k^j)$.

Such a replacement bears in a certain sense of a priory supposition type and lead to the combined probabilistic subjective model.

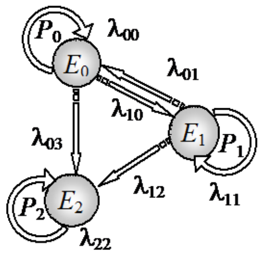


Fig. 6.30

In the general case the model equations take the form:

$$\frac{dP_i}{dt} = -\sum_{j=0}^{N} \sum_{k=1}^{M_i} \pi\left(\sigma_k^j\right) P\left(i \to j \mid \sigma_k^j\right) P_i\left(t\right) + \sum_{q=0}^{N} \sum_{k=1}^{M_q} \pi\left(\sigma_k^q\right) P\left(q \to i \mid \sigma_k^q\right) P_q\left(t\right). \tag{6.40}$$

Assume that in each situation, including the normal situation, pilot selects and realizes the certain strategy. As has already been told above, the use of subjective preferences in the quantitative models makes possible to consider subjective probabilities and influence of a priori imperatives, as which can come out the different kind of rules, standards, including ethical standards. Let us examine an example of simulation with the use of model (6.40) for the simplified graph (Fig. 6.30).

It is assumed that there are only three subsets of the situations: E_0, E_1, E_2, moreover E_2 is subset „without output", i.e. the subset of catastrophic situations. Furthermore, assume that the alternative situation appears only if system falls in the subset E_1 („dangerous situations"), and each time only two alternative strategies: $S_A \to (\sigma_1, \sigma_2)$ presents. Let us designate $\pi(I_i)$ ratings of imperative $I_i \in S_i$ certain quantitative measure of the imperative significance.

Let the system of imperatives with respect to the set S_a be bijective (the one-to-one correspondence between S_a and S_i: $S_a \leftrightarrow S_I$, Fig. 6.31). Let us accept the following model of preferences:

$$\pi(\sigma_i) = \frac{\pi(J_i)^{\alpha} e^{-\beta\bar{r}(\sigma_i)}}{\sum_{j=1}^{N} \pi(J_j)^{\alpha} e^{-\beta\bar{r}(\sigma_j)}}, \tag{6.41}$$

where $\bar{r}(\sigma_i) = \dfrac{\bar{t}}{1-\bar{t}}$, $\bar{t} = \dfrac{t^{req}}{r^{disp}}$,

and let

$$\lambda_{1,0} = \pi(\sigma_1)P(1 \to 0 \mid \sigma_1) + \pi(\sigma_2)P(1 \to 0 \mid \sigma_2);$$
$$\lambda_{1,2} = \pi(\sigma_1)P(1 \to 2 \mid \sigma_1) + \pi(\sigma_2)P(1 \to 2 \mid \sigma_2).$$

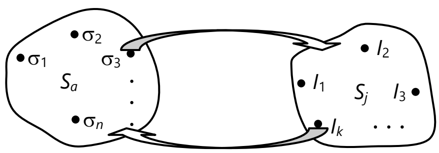


Fig. 6.31

Taking into account all made assumptions we can for the diagram on Fig. 6.30 write down the following model of a change in the probabilities $P_i(t)$, ($i \in \overline{0,2}$):

$$\frac{dP_0}{dt} = -(\lambda_{01} + \lambda_{02})P_0 + \left(\pi(\sigma_1)P(1 \to 0 \mid \sigma_1) + \pi(\sigma_2)P(1 \to 0 \mid \sigma_2)\right)P_1; \tag{6.42}$$

$$\frac{dP_1}{dt} = \lambda_{01}P_0 - \left(\pi(\sigma_1)P(1 \to 0 \mid \sigma_1) + \pi(\sigma_2)P(1 \to 0 \mid \sigma_2)\right)P_1 - $$
$$-\left(\pi(\sigma_1)P(1 \to 2 \mid \sigma_1) + \pi(\sigma_2)P(1 \to 2 \mid \sigma_2)\right)P_1;$$

$$\frac{dP_2}{dt} = \lambda_{02}P_0 + \left(\pi(\sigma_1)P(1 \to 2 \mid \sigma_1) + \pi(\sigma_2)P(1 \to 2 \mid \sigma_2)\right)P_1.$$

Initial conditions: $t = 0$; $P_0 = 1$; $P_1 = P_2 = 0$.

Let us designate:

$$\eta = \frac{\pi(I_2)}{\pi(I_1)}; \quad (\pi(I_1) + \pi(I_2) = 1), \quad Z = \bar{r}(\sigma_2) - \bar{r}(\sigma_1).$$

Then $\pi\left(\sigma_1\right)=\dfrac{1}{1+\eta^{\alpha}e^{-\beta Z(t)}}$; $\pi\left(\sigma_2\right)=\dfrac{1}{1+\eta^{\alpha}e^{-\beta Z(t)}}$.

Assuming, that in the course of time the given resources $\bar{r}\left(\sigma_i\right)$ diminish according to the law

$$\bar{r}\left(\sigma_i\right)=a_i-b_it;\ a_i > 0;\ b_i > 0,$$

then find, that

$$Z = c - bt.$$

It is of interest to explain, what subjective information is „separated" with a change in the relation of imperatives ratings. Entropy of the preferences

$$H_{\pi}=-\left(\pi_1\ln\pi_1+\pi_2\ln\pi_2\right); \qquad (\pi_i = \pi(\sigma_i)).$$

The information flow, connected with a change of η, is determined by the relationship:

$$I_{\pi\eta}=\frac{dH_{\pi}}{d\eta}=\frac{\alpha}{\eta}\pi_1\pi_2\ln\frac{\pi_1}{\pi_2}.$$

The equations of model (6.42) in the new designations take the form:

$$\frac{dP_0}{dt}=-\left(\lambda_{01}+\lambda_{02}\right)P_0+\frac{P_{101}+\eta^{\alpha}e^{-\beta Z}P_{102}}{1+\eta^{\alpha}e^{\beta Z}}P_1;$$

$$\frac{dP_1}{dt}=\lambda_{01}P_0-\frac{P_{101}+P_{121}+\eta^{\alpha}e^{-\beta Z}\left(P_{102}+P_{122}\right)}{1+\eta^{\alpha}e^{\beta Z}}P_1;$$

$$\frac{dP_2}{dt}=\lambda_{02}P_0+\frac{P_{121}+\eta^{\alpha}e^{-\beta Z}P_{122}}{1+\eta^{\alpha}e^{-\beta Z}}P_1.$$

Fig. 6.32 shows the results of calculations for following initial data :

λ_{01} = 0,001; λ_{02} = 0,000001; $P(1 \to 0|\sigma_1) = P_{101}$ = 0,001;
$P(1 \to 2|\sigma_1) = P_{121}$ = 0,0004; $P(1 \to 2|\sigma_2) = P_{122}$ = 0,00001.

The versions were calculated for η = 0,05; η = 0,5; η = 5; η = 50, Z_0 = 0; Z_0 = 0,5; and α = 0,05; β = 2; b = 0,0003. One additional example shows, as relationship of imperatives ratings can influence the „period of decision making" t^*.

Preferences, as in previous case, are calculated from the formulas

$$\pi\left(\sigma_1\right)=\frac{1}{1+\eta^{\alpha}e^{-\beta Z(t)}};\ \ \pi\left(\sigma_2\right)=1-\pi\left(\sigma_1\right).$$

It is assumed that $Z(t)$ changes linearly on time, α = 0.2; β = 0,8; $\bar{r}\left(\sigma_1\right)=a_1+b_1t$; $\bar{r}\left(\sigma_2\right)=a_2-b_2t$; $Z\left(t\right)=\bar{r}\left(\sigma_2\right)-\bar{r}\left(\sigma_1\right)$. Fig. 6.33 shows the results of the entropy calculations for the values a_1 = 1,2; a_2 = 2; b_1 = 0,0003; b_2 = 0,00003 and different values of η: 4,0; 1,5; 0.667; 0,25.

Critical value H^* = 0,58 is taken. A series of critical time t^* values makes possible to construct the dependence of critical value t^* from the η relation (Fig. 6.34).

Each time $t^*(\eta_i)$ is determined by condition $H_\pi(\eta_i) = H^*$.

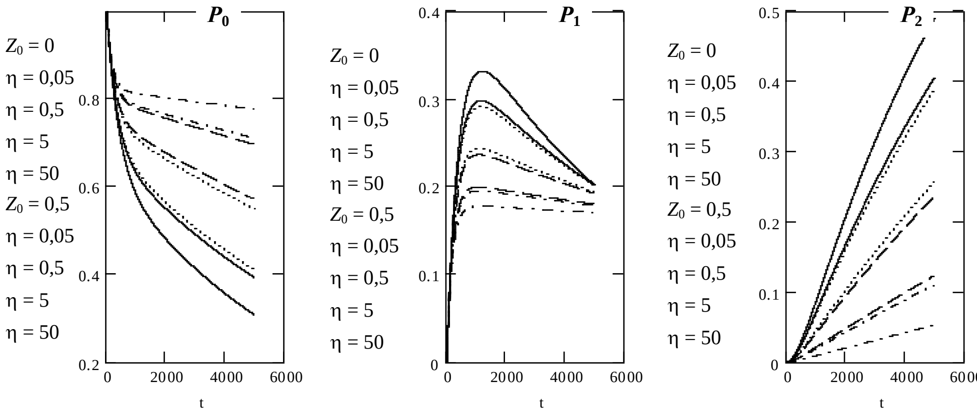


Fig. 6.32

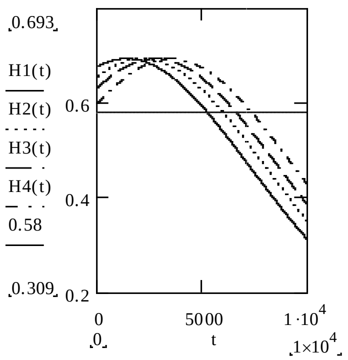


Fig. 6.33

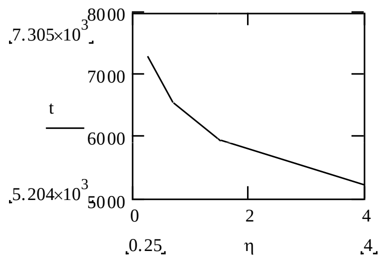


Fig. 6.34

The given theoretical diagrams contain significant arbitrariness in the structural coefficients of models, the selection of the attractor type, assigned numerical values of some probabilities. The structure of the functions of preference, as it is evident from the previous chapters, is not uniquely determined, there is an uncertainty in the selection of the endogenous parameters and, especially, in interpretation of their sense.

As a justification for the selection of models, as earlier, it is possible to refer to that circumstance that, in the absence a sufficient information about „physical" content of a certain process, the simulation is accomplished via selection of model from a defined class; moreover the model must have the parametric or structural arbitrariness, whose liquidation then serves the task of parametric or structural identification.

The canonical distributions of preferences, which contain the indeterminate endogenous parameters are selected as the models of the functions of preference. Analogously the selection of Lorenz's attractor is to a considerable extent arbitrary, however the equations of attractor contain the large number of indeterminate parameters, which should be estimated on the basis of experimental information processing. The matter is reduced to the solution of the certain problem of identification.

One of the possibilities consists in the determination according to the data from onboard system for technical parameters and voice signals registration of the actual decision making moments t^*_k to the subsequent estimation of the endogenous parameters taking these data into account.

7. CONFLICTS, COGNITIVE DISSONANCE, SUBJECTIVE FREEDOM

7.1. Conflicts from the point of view of the subjective analysis

In the „psychological dictionary" s given following determination of conflict: *„Conflict – it is the collision of the oppositely directed purposes, interests, positions or views of opponents or subjects of interaction"*. The authors give the specific classification of conflicts. In particular „interpersonal", „intra-personal", „between-group" conflicts are selected. On the basis of second type conflicts usually lie the „ambivalence of desires", or the tendencies of subject, and also the so-called „cognitive dissonance", introduced into the theoretical psychology by Festinger defined as *„the negative motive state, which appears in the situation, when subject simultaneously has available psychologically contradictory „knowledge"... about one „object"*.

An example of such knowledge is simultaneous knowledge about the „evidences" and the „contra-evidences" of medicine, i.e., about the possible benefit and the possible harm. Above we already introduced the „functions of harm" together with the „functions of utility" ($L(\sigma_i)$, $U(\sigma_i)$).

The conflicts are distinguished as „constructive" and „destructive". In connection with the conflicts „problematic situations" are examined, which tightly agree with the concepts and the terminology, used in the present work.

According to Bulding [143] „conflict - it is the contest, in which the sides are attempted to reach the incompatible positions". In one of the monographs, dedicated to the theory of conflicts, to the development of mathematical models and methods of the study of conflicts [52], the conflict is represented as interaction of complex systems.

Conflict assumes fight, moreover the two stages are defined: the first - preparatory, characterized by origin and increase of contradictions and the second - resolution of conflict – the fight, the completion of fight, the „victory" of one of the sides or „agreement" (saddle point according to Paretto).

The „human factor" role decreases according to [52] with an increase in the technological extent of outfitting and apparently, the decrease of the conflicts sharpness are assumed. It is difficult to agree with this, since the weight of solutions taken by the subject, their price and consequences (technogenic, ecological, social) repeatedly grows. We can observe this on the global level.

An exponential increase of the technology in all areas of life and human activity correlates with an increase in the frequency, bitterness and destructiveness of conflicts.

The extreme case of conflict is the war, when resolution of conflict assumes suppression by power or destruction of one of the sides and when the interlacing of the intra-personal, interpersonal and group conflicts occurs, which correlate, they strengthen each other to such an extent, that all other process control capabilities drop off, leaving only one possibility – the war path. Here we encounter the problem of interaction, cumulative strengthening, catalyzation of conflicts, which, as it is presented, also lies in the area of subjective analysis.

The huge amount of works is devoted to the theory of conflicts in the contemporary scientific literature. Even the quick survey of them and the systematization attempt would be the independent study, which requires the large efforts of many participants and the significant time.

In the present work we do not pretend to development and generalization of the conflicts theory, including their psychological basis. In this case the task lies in the fact to explain, how it is possible to use developed methods of subjective analysis, entropy-information technology and functions of preference in the theory of conflicts, in attempt to assume approach to the quantitative description of conflict.

Nevertheless let us give some sources about the conflicts theory, both the monographs and the textbooks, which will allow the reader to be introduced to the contemporary state of the matters in this area.

Very frequently is cited the monograph of L. Kozer [79] (2000), and also the monograph of T. L. Saati [143], the majority of works on the theory of games, since the game is most frequently formulated as the solution of situation conflict. To this direction can be attributed the book of N. N. Moiseeva [109] and J. B. Germeyera [37, 38, 39], V. F Krapivina [78], the works of A. P. Nazaretyana [126, 127]. The textbooks include works of I. V. Vashchenko [34], O. I. Bondarchuk [23], A. A. Girnik [43], V. O. Gneusheva [44], S. Grushevskoy [45], N. V. Grishina [46], A. D. Dmitriev [59], N. I. Leonov [98], N. D. Luktyanenko [97] and others.

This enumeration can be continued. The selection of the enumerated sources can seem as random and, of course, it is by far incomplete. Let us switch over to the description of conflicts in the terms of subjective analysis.

In the present work the main station for experiments are the distributions of preferences, therefore it is natural to attempt to give the interpretation of some positions of the conflicts theory, to explain, in particular, what role plays the subjective entropy and moments of the distributions of the preferences: dispersion, covariances.

We will distinguish the following forms of conflicts:

– *self conflict* - conflict of subject with itself, internal conflict, for example, the conflict of object (utilitarian) preferences with the ethical preferences, formed on the basis of ethical imperatives;

– *the intersubjective conflict of object preferences*;

– *the intersubjective conflict of rating preferences*;

– *conflict between the subject and the group*, including, in the hierarchical system, between the subjects and the groups on the different hierarchical levels.

We concretely define some forms of conflicts beginning starting from the selfconflict. In the Festinger book [150] we find the determination of the internal conflict, which is separated from the „cognitive dissonance": *"it is also necessary to discuss the difference between the conflict and the dissonance, since the dynamics of these processes are different. Human is located in the situation of conflict before he must take a decision. After decision is made, there is no longer the conflict: human made his selection. It in other words, he resolved the given conflict. From this point on, he is located within the framework of the selected modus operandi"*.

And further: *"When... they speak about the conflict between the opinions and the values (or ethical imperatives and utility), it is frequently sufficiently difficult to understand what it is intended specifically"*. In the Festinger book [150] the conflict is understood as the „collision of the factors, which act on the individual, in the situation of vagueness (or uncertainty)". As far as cognitive dissonance is concerned, to it the interval of time after decision making is removed. Referring to Adams, Festinger writes: *„By itself decision making - this is altogether only half of problem. Dissatisfaction and remaining tension of the deflected alternative continue to have their effect, if further process does not occur"*. It is assumed, that this treatment narrows both the conflict acceptance and the fact that is called „cognitive dissonance".

Apparently, the clear semantic and temporary boundary between that and by other does not exist. Conflict is represented by more general common category. It can be, it would be more adequate *„intra-personal conflict"* identify with *„cognitive dissonance"*. Even more attractive is the following concept, which orders relations between the *„conflict"* and *„dissonance"* categories.

Conflict is a form of the dissonance realization in the specific action form or, on the contrary, the inaction. In any case, conflict is the external manifestation of dissonance, which can be identified and fixed. Through the analysis of conflict, apparently, it is possible to determine the characteristics of dissonance. One additional feature, which makes possible to separate the conflict from dissonance, is the fact that the carrier of dissonance is always the individual, whereas the conflict between the individuals is possible.

Furthermore, as we will see further, the conflict is possible due to the conditions of consonance.

The attempt undertaken here is the determined means to formalize the concept of conflict, is directed not so much on the development of the theory of conflicts as such, as on that, in order to this concept comprehension process based on the of subjective analysis positions, to verify the accepted approaches fitness for work, theoretical scheme s of the one more following important subject of a study, to refine some concepts and interrelations between them.

From the quotations given above it is possible to draw some conclusions, namely:

1. The conflict, like the dissonance, can be treated as the state, however, in actuality they come out as the phenomena, which develop in time, and therefore adequate description and understanding of these categories is combined with the treatment of their dynamics. Thus, both conflict and dissonance - they are processes, tightly interconnected and, most likely, dissonance and consonance are attributes - components of conflict.

Strictly speaking, the conflict - is sufficiently general universal category - entire life consists of the sequence of the large and small conflicts, which change each other. It can be, this is simply expressed in other terms the thesis about the „unity and the struggle of opposites", which composes the essence of life from the materialist dialectics point of view. It seems that the category of „conflict" is more convenient for formalized and is more adequate to the method of subjective analysis.

2. After decision is made, the human *„is located within the framework of selected modus operandi*". This thesis exactly coincides with the assertion, that decision making each time is combined with the passage from the „reign of freedom" into the „reign of necessity". These „reigns" most likely have the boundary washed away, since in the general case a particular decision making does not leads to the complete degeneration of problematic set S_a , if the discussion deals with the object preferences, and to the degeneration of set S_ξ, if the discussion deals with the rating preferences. By decision making on the rating set S_ξ , the redistribution *of ranks* in the group is understood, i.e. *the organizational solution.*

The example, which illustrates the expressed above thesis about the passage into the „reign of necessity", can serve the situation, when someone, making decision, packs large resources in the selected direction and, after connecting them with the defined problem and, correspondingly, - the target, no longer it can be deflected from the selected course and it becomes as the „slave" of its own solution. There is *„a tendency of individual to remain true to its solution*".

We have already said, that supposedly for each individual there is a lower entropy barrier H_π^*, reaching whose „on top" offers the possibility of decision making and the upper entropy barrier H_{π}^{**}, exceeding whose is in principle impossible, since the corresponding distribution of the preferences is „nontransferable". Psychological collapse begins. These limits are strictly individualized for each subject.

The entropy layer, which lies between $H_{\pi j}^{*}$ and $H_{\pi j}^{**}$, can be conditionally called the *„entropy reign of freedom"*. Inside this layer several levels - the thresholds can be still located, where in the process of analysis the passages from one variational problem to another occur. Since, as it was already noted above, the forming of different types of preferences occurs each time with the use of one or other specific entropy or another (H_{π}^{+}, H_{π}^{-}, H_{υ}^{+}, H_{υ}^{-}, ...), and the presence of the corresponding „layer of freedom" should be allowed for each of them, then now we can speak about the multidimensional space of subjective entropies and the about presence in this multidimensional space *„the freedom hyper-layer*" isolated by the upper and lower thresholds.

This layer has the moving boundaries, whose dynamics is determined by the endogenous and exogenous factors. Obvious also that all self-conflicts are conceived and developed in this layer. According to the views of the Festinger, the fall outside the limits of the „layer of freedom" indicates resolution of the conflict and simultaneously the onset of the dissonance, which subsequently leads to the new conflicts. With obtaining of the new resources, or, more precise, with a change in the resource situation, the subject returns to the *„reign of freedom*", where again the new conflicts bloom with bright color. It would be tempting postulate this schematics (or concept): *„The reign of freedom is simultaneously the reign of conflicts, the reign of the necessity for - is a reign of dissonances"*.

However, more natural is the assumption about the fact that also in the „reign of freedom" the dissonances exist, especially, near its boundaries. The example, which already was given, with the acquisition of the medicine, when description presents simultaneously the positive and negative information and against the background of dissonance appearing in connection with this situation, the solution about the medicine acquisition or nonacquisition starts, it clearly shows that the dissonance appears before decision making, and not only after it. At the moment of decision making the dissonance changes the nature: before solution it directs to the future and is related with the forecast; after the solution it directs into the past and manifests as dissatisfaction, doubting the correctness of the accepted solution.

Since each time the solution limits the directions of further activity of subject - it pushes slightly it into the „reign of necessity", this "*retrospective*" dissonance creates negative psychological experiences. Let us bring some reasoning from the Fastinger's book [150], which corresponds to the assumptions made above about the role of subjective entropy as the characteristics of the mental processes, connected with the decision making, with outbreak and development of conflicts.

Festinger refers on Martin, who examines three categories, which relate to the decision-making process - selection types:

1. *„The preference. This process of the selection realization is characterized by the* marked preference of one of the alternatives. Although the importance of the solution is high, *the need for selection usually does not lead to the appearance of strong internal conflict"*.

From our point of view, this is actual thus, since in this case the subjective entropy is small and the intersection of the lower threshold H_π* (solution threshold) does not require the great mental stress and significant volumes of additional information. When selection is already made *„there exists tendency to justify the selected alternative by the specific reason, which frequently were carried out with the „pressure"*, *what increased the degree of satisfactoriness with the selection made.*

This process of the selection justification after its accomplishment has as a goal the faster mental satisfaction of the subject himself, than the search for the logical foundations of its accomplishment". From the position of the entropy analysis this can indicate, that, if selection is made on the sufficiently high threshold value of the entropy H_π^* and, correspondingly, in „near-stress" mental condition, then short while is required for relaxing the stress, connected with decision making, and further decrease of entropy. And further

2. „C*onflict. This type of decision making is characterized by formidable difficulties with the realization of selection, which is caused by very small differences in attractiveness level of accessible alternatives, which is so low that the solution comes slowly and with great difficulty. The decision-making process can be accompanied by the expression of doubts about the correctness of the selection made and connected with the sensation of discomfort in contrast to calmness and feeling of satisfaction, characteristic for the type described above. Sometimes can even appear the tendency to regret about the selection made"*.

The bright description of the fluctuations of Caesar is given in Plutarch: „when it approached the river named Rubikon he enveloped in deep meditation with thought about the begun minute and he reeled under the sublimity of his daring... Leaving the vehicle, he again kept silent for a long time, considering his concept from the all sides, assuming the one, or another solution... Finally, as if having rejected reflections and being courageously fixed towards the future, it pronounced words common for people, who enter into the courageous enterprise, outcome of which is doubtful".

In the language of entropy analysis the „conflict" here corresponds to the state of subject (or process) with the very high value of the subjective entropy, the aim is in its maximum value

$$H_\pi \to H_{\pi\max} \ln N.$$

The upper threshold $H_\pi^{**} \leq H_{\pi\max}$, apparently, never reaches the theoretical maximum of ln$N$, but approximation to H_π^{**} causes the reaction, directed to the accomplishment of the actions, which decrease the entropy: retrieval for additional information, the rejection of alternatives (decrease of the set S_a dimensionality,...). The sense of the quotations given above also exactly will be coordinated with the assumptions made earlier about the role of subjective entropy (measure of the uncertainty of preferences) in the decision-making process.

In chapter 5 are shown the particular results of simulation based on the example to the modified Walras – Leontief dynamic model with the excitation of the Lorenz attractor, which supposedly simulates psychological reaction to the entropy change. Evidently, as the section of uncertainty appears - repeated bifurcations, gradually damped as a result of the damping terms presence in the attractor equations.

Third type of the selection:

3. *„Indifference. Such process of the realization of selection is characterized by the absence of clearly expressed preference of one of the alternatives, and also of high indifference with respect to the generally given alternatives. In this case the solution has very low significance for the subject"... „In case of indifference the time interval, necessary for decision making, also must be relatively small"... „However, the interval of time, necessary for decision making in the case of conflict, must be sufficiently large"...*

In support of these considerations in [150] the following experimental tabele is given.

Tabele 7.1

MEAN TIME, SPENT ON THE DECISION MAKING (seconds)

The proposed task (N= 2)	Type of situation with decision making		
	Preference	Conflict	Indifference
Selection from the two hypothetical situations	23,3	51,0	37,2
Selection from the two proposed smells	4,1	14,1	6,2

In our case the „preference" is the category, the tool of analysis, utilized in each of three cases and it is, therefore, understood in a broader sense.

Let us note that in the two cases „conflict" and „indifference", the entropy is great; however, the degree of mental stress is different and depends, apparently, on additional circumstances.

In the chapter 6, dedicated to the safety of active systems, it was noted, that the possibility of decision making depends not only on *the value of entropy, but* *also on the relationship of the available time (time resources) and the time, necessary for the achievement of the entropy lower threshold H_π^*.*

The situation of indifference can be characterized by the fact that all alternatives examined by subject have relatively small „cost" $R^{req}(\sigma_i)$ to comparison with the available resources. Into the number of these resources it also enters the available operational time. It is necessary to attempt to formalize the difference between the „conflict" and the „indifference" in that sense, as this is understood in the [150]. Let us introduce the index

$$\mu = \frac{\max\limits_{i \in \overline{1,N}} R^{req}\left(\sigma_i\right)}{R^{disp}}, \quad \mu \in \left[0, 1\right].$$

If it is small, then the portion of required resources is small in comparison with the available resources and even when $H\pi$ = Hmax, the subject should make decision relatively easily and should be expected small dissonant aftereffect, furthermore, this solution can be easily changed, since there is a sufficient resource margin for this. On the contrary, if μ is large ($\mu \to 1$), the solution will be additionally complicated, and a change in the solution can prove to be generally impossible, since for this may miss the available resources (after the withdrawal of the part of the resources necessary for the solution of the selected problem).

As *the model of the criterion,* which signals about the possibility of decision making, it is possible to propose the criterion

$$K_\pi^{(1)} = \mu H_\pi \tag{7.1}$$

For this criterion the thresholds K_π^* and K_π^{**} should also be established. Another possible approach lies in the fact that consider the thresholds H_π^* and H_π^{**} as functions of μ, in the particular case, as functions of relative required time

$$\overline{\tau} = \frac{\max\limits_{i \in \overline{1,N}} t^{req}\left(\sigma_i\right)}{t^{disp}} \tag{7.2}$$

If one assumes that $H_\pi^* = \phi\left(\overline{\tau}\right)$ and, then one should consider that $\phi\left(\overline{\tau}\right)$ - the increasing function $\overline{\tau}$, moreover

$$\lim_{\overline{\tau} \to 1} H_\pi^*\left(\overline{\tau}\right) = H_{\pi max} = \ln N \tag{7.3}$$

a $\psi\left(\overline{\tau}\right)$ is decreasing function.

This means that with the increase of time deficit ($\overline{\tau} \to 1$) subject will make decisions with the higher degree of uncertainty - with the much higher threshold H_π^* and, on the contrary, the upper threshold H_π^{**} must be reduced, i.e. with the smaller uncertainty the subject will approach the relaxation of problematic-resource situation.

Thus, with the increase of time deficit both thresholds displace towards each other, the „layer of freedom" narrows. With the condition $H_\pi^{**} \leq H_\pi^{*}$ the solution will be taken „immediately". Returning to model (7.1), let us try on the experimental data basis to estimate the value of the lower threshold of the entropy H_π^{*}, allowing, that after decision making (selection of one from the two alternatives) the retrospective entropy in the specific short time preserves its value, equal approximately to the threshold value H_π^{*}.

In the publication [150] the two additional tabele s are given, which serve as the indirect evidence of the realism of model presentations presented here.

Tabele 7.2

AVERAGED EXPONENTS OF CONFIDENCE IN THE ACCEPTED SOLUTION (measured in the [0, 1] scale)

The proposed task (N = 2)	Type of situation with decision making		
	Preference	Conflict	Indifference
Selection from the two hypothetical situations	0,925	0,600	0,500
Selection from the two proposed smells	0,975	0,700	0,475

From this tabele follows, that the maximum uncertainty appears after the solution, taken due to the conditions *of indifference,* and the minimum uncertainty, when the preference of the selected alternative is expressed definitely in full.

Tabele 7.3

THE PORTION OF THE SOLUTIONS, WHICH SUBJECTS WERE UNABLE TO CHANGE (%)

The proposed task (N = 2)	Type of situation with decision making		
	Preference	Conflict	Indifference
Selection from the two hypothetical situations	90,3	75,8	40,6
Selection from the two proposed smells	84,2	50,0	10,0

Continuing tabele 2, let us assign „*the degree of uncertainty*" to the new name in the spirit of the subjective analysis: „the subjective entropy". Then „*the degree of confidence*" (as in the tabele 7.3) can be described by the value

$$\overset{\vee}{K}{}_\pi^{(2)*} = 1 - \overline{H}_\pi^{*} \tag{7.4}$$

Where

$$\bar{H}_\pi^* = \frac{H_\pi^*}{\ln N};\ H_{\pi\max} = \ln N = 0{,}693\ (N = 2).$$

Identifying data of tabele 7.2 with the values of the parameter K_π, let us find the appropriate values of the entropy H_π:

$$H_\pi^* = \left(1 - K_\pi^{(2)*}\right)\ln N = 0{,}963\left(1 - K_\pi^{(2)*}\right) \tag{7.5}$$

The values H_π^* with $N = 2$ are presented in tabele. 7.4.

Tabele 7.4

VALUES OF SUBJECTIVE ENTROPY AFTER DECISION MAKING („residual" entropy)

The proposed task (N = 2)	Type of situation with decision making		
	Preference	Conflict	Indifference
Selection from the two hypothetical situations	0,052	0,277	0,346
Selection from the two smells proposed	0,017	0,208	0,364

Entropy is minimum, when preferences are clearly expressed, entropy is considerably greater, when the accepted solution was preceded by „conflict", and entropy is even greater, if the solution was started in the conditions of indifference.

This tabele tells, that more realistic the condition of decision making is, for example, the condition

$$K_\pi^{(1)} \le K_\pi^*,$$

including besides entropy the factor of urgency („emergency") μ.

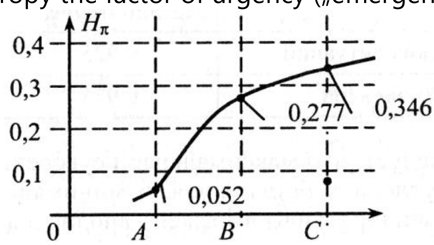


Fig. 7.1

A – „preference", *B* – „the conflict", *C* – „indifference"

Scheme on Fig. 7.1 is illustrative, since the immeasurable characteristics are noted on the X-axis.

7.2. Model characteristics of the internal conflict

7.2.1. Utilitarian internal conflicts

Internal conflict either self conflict appears and develops on the background cognitive dissonance or consonance. After usage of introduced earlier preferences functions $\pi^+(\sigma_i)$, $\pi^-(\sigma_i)$, $\upsilon^+(\sigma_i)$, $\upsilon^-(\sigma_i)$, we will examine the possible models. Let us recall their sense:

$\pi^+(\sigma_i)$ is to accept σ_i on the basis of positive analysis;

$\pi^-(\sigma_i)$ am to accept σ_i on the basis of negative analysis;

$\upsilon^+(\sigma_i)$ is to reject σ_i on the basis of positive analysis;

$\upsilon^-(\sigma_i)$ am to reject σ_i on the basis of negative analysis.

We will indicate that, if distributions π^+ and π^- coincide, the complete consonance on the set S_a occurs. Let us determine the degree of consonance or dissonance by the Pearson correlation coefficient of the subjective preferences:

$$\rho\left(\pi^+,\pi^-\right)=\frac{\sum_{i=1}^{N}\left(\pi^+\left(\sigma_i\right)-\frac{1}{N}\right)\left(\pi^-\left(\sigma_i\right)-\frac{1}{N}\right)}{\sqrt{\sum_{i=1}^{N}\left(\pi^+\left(\sigma_i\right)-\frac{1}{N}\right)^2\sum_{i=1}^{N}\left(\pi^-\left(\sigma_i\right)-\frac{1}{N}\right)^2}} \qquad (7.6)$$

Complete consonance corresponds to the value of $\pi(\pi^+, \pi^-) = 1$, dissonance - to the value of $\rho(\pi^+, \pi^-) = -1$, in the remaining cases $-1 < \rho(\pi^+, \pi^-) < 1$.

Generally we will consider by consonance, the case when $\rho(\pi^+, \pi^-) > 0$, by dissonance – the case, when $\rho(\pi^+, \pi^-) < 0$.

The presence of consonance or dissonance does not guarantee the presence of conflict yet, since the latter is understood as such situation, when it is difficult to the subject to make a decision and to make a selection. Internal conflict due to the conditions of consonance appears, if both subjective entropies

$$H_\pi^+=-\sum_{i=1}^{N}\pi\left(\sigma_i^+\right)\ln\pi\left(\sigma_i^+\right);\; H_\pi^-=-\sum_{i=1}^{N}\pi\left(\sigma_i^-\right)\ln\pi\left(\sigma_i^-\right)$$

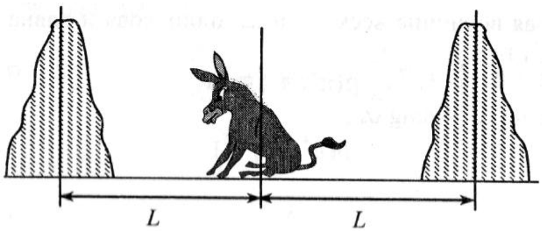
are located in the following „ freedom layers"

$$H_\pi^+\in\left(H_\pi^{**+},H_\pi^{*+}\right);\; H_\pi^-\in\left(H_\pi^{**-},H_\pi^{*-}\right).$$

Let, for example, with $N = 2$, $\pi^+(\sigma_1) = 0{,}55$; $\pi^+(\sigma_2) = 0{,}45$, $\pi^-(\sigma_1) = 0{,}52$, $\pi^-(\sigma_2) = 0{,}48$. The consonance occurs between the distributions π^+ and π^-, but internal conflict lies in the fact that both entropies H_π^+ and H_π^- are high and it is difficult to accept the solution.

In this case, the greater is the sharpness of conflict, than the nearer is the entropy to a maximally possible value of ln 2.

Consonant conflict can be conditionally named „Buridan's conflict", and the standing, in which the subject is in this case, – „Buridan's stress", having in mind the known philosophical parable about the donkey, died from starvation, which cannot make a selection between two identical stacks of hay.



Animal perished from the surplus of possibilities - from the surplus „entropy freedom" and, obviously, before death it was in a state of extreme stress ($H_\pi = \ln 2$). In connection with this parable we can raise the very nontrivial and important in the theoretical plan question about the conflicts stability with respect to small changes in the exogenous situation. Solution of this question lies in the dynamic theory of conflicts.

Now suppose that when $N = 2$, $\pi^+(\sigma_1) = 0{,}55$; $\pi^+(\sigma_2) = 0{,}45$, $\pi^-(\sigma_1) = 0{,}48$ and $\pi^-(\sigma_2) = 0{,}52$. In this case $\rho(\pi^+, \pi^-) = -1$, $H_\pi^+ = 0{,}6881$, $H_\pi^- = 0{,}692$.

Dissonance occurs, but in view of great significances of both entropies the conflict does not appears sharp. The solution, however, will not be accepted, since both entropies are above the „thresholds", if we assume: $H_\pi^{*+} = 0{,}683$, $H_\pi^{*-} = 0{,}683$.

The layer of freedom is determined by the inequalities:

$$0{,}683 \le H^+_\pi \le 0{,}693;\ 0{,}683 \le H^-_\pi \le 0{,}693.$$

In following case one of the entropies remains high, while the second has a value smaller, than the threshold value.

Let with $\pi^+(\sigma_1) = 0{,}55$; $\pi^+(\sigma_2) = 0{,}45$, $\pi^-(\sigma_1) = 0{,}25$ and $\pi^-(\sigma_2) = 0{,}75$. It is obvious that $\rho(\pi^+, \pi^-) = -1$. For the entropy we find $H_\pi^+ = 0{,}6881$, $H_\pi^- = 0{,}5624$. We see that $H_\pi^+ \in [0{,}683, 0{,}693]$, but $H_\pi^- < H_\pi^+ = 0{,}683$.

There is no conflict, and the solution will be accepted on the basis of „negative" analysis (about the π^-distribution).

Let us examine the case, when due to the conditions of dissonance both entropies are small and are approximately identical with respect to the value: $\pi^+(\sigma_1) = 0{,}75$; $\pi^+(\sigma_2) = 0{,}25$, $\pi^-(\sigma_1) = 0{,}23$ and $\pi^+(\sigma_2) = 0{,}77$. In this case $\rho(\pi^+, \pi^-) = -1$, $H_\pi^+ = 0{,}5624$, $H_\pi^+ = 0{,}5392$. Thus, in this case the degree of the confidence I as the result both of positive and negative analysis is high and approximately identical I. We have here a case of the clearly expressed dissonant conflict.

As we see, in the case of dissonance with the comparison of distributions π^+ and π^- the high value of at least one of the entropies: H_π^+ or H_π^- testifies about the impossibility of dissonant conflict.

With $N > 2$ let us assume $\pi_i^+ = \frac{1}{N} + \alpha_i^+$; $\pi_i^- = \frac{1}{N} + \alpha_i^-$, where α_i^+ and α_i^- — have low values (in comparison with unity), for which are satisfied the following conditions:

$$\sum_{i=1}^{N} \alpha_i^+ = 0; \ \sum_{i=1}^{N} \alpha_i^- = 0.$$

Let us find that

$$\rho\left(\pi^+, \pi^-\right) = \frac{\sum_{i=1}^{N} \alpha_i^+ \alpha_i^-}{\sqrt{\sum_{i=1}^{N} \alpha_i^{+2} \sum_{i=1}^{N} \alpha_i^{-2}}}.$$

If the absolute value of all α_i^+ and α_i^- is identical and equal α, then, if each time ($\forall i$) $\alpha_i^+ = -\alpha_i^-$, then

$$\rho(\pi+, \pi-) = -1,$$

but if each time sing α_i^+ = sing α_i^-,

$$\rho(\pi+, \pi-) = 1.$$

Entropy

$$H_\pi^+ = -\sum_{i=1}^{N} \left(\frac{1}{N} + \alpha_i^+\right) \ln\left(\frac{1}{N} + \alpha_i^+\right) = -\sum_{i=1}^{N} \left(\frac{1}{N} + \alpha_i^+\right) \ln\left(\frac{1}{N} + \frac{\alpha_i^+}{\frac{1}{N} + \alpha_i^+} + \ldots\right) \approx \ln N.$$

It is accounted here, that $\sum_{i=1}^{N} \alpha_i^+ = 0$. Consequently, if α_i^+ (and α_i^-) is so small, that it is possible to limit for two members in the Taylor's decomposition, then the entropies H_π^+ and H_π^- take maximum values. Suppose now the distribution of preferences $\pi^+(\sigma_i)$ and $\upsilon^+(\sigma_i)$ is compared.

We will say, that there is a complete dissonance, if the correlation coefficient

$$\rho\left(\pi^+, \upsilon^+\right) = \frac{\sum_{i=1}^{N} \left(\pi^+\left(\sigma_i\right) - \frac{1}{N}\right)\left(\upsilon^+\left(\sigma_i\right) - \frac{1}{N}\right)}{\sqrt{\sum_{i=1}^{N} \left(\pi^+\left(\sigma_i\right) - \frac{1}{N}\right)^2 \sum_{i=1}^{N} \left(\upsilon^+\left(\sigma_i\right) - \frac{1}{N}\right)^2}} \tag{7.7}$$

is equal to +1 and complete consonance, if it is equal -1, in the remaining cases

$$-1 < \rho(\pi^+, \upsilon^+) < 1.$$

The condition $\rho(\pi^+, \upsilon^+) \approx 1$ is already the base for the internal conflict, since even with the condition $H_\pi^+ < H_\pi^{+*}$ and $H_\upsilon^+ < H_\upsilon^{+*}$ although the selection of determined $\sigma_i \in S_a$ can be made, nevertheless a question remains „to accept or not to accept" the corresponding solution. Actually, in this case $\pi^+(\sigma_i) \approx \upsilon^+(\sigma_i)$.

If additionally are satisfied the conditions

$$H_\pi^{+*} < H_\pi^+ < H_\pi^{+**};$$
$$H_\upsilon^{+*} < H_\upsilon^+ < H_\upsilon^{+**},$$

The conflict is complicated in connection with the fact that there is uncertainty with the selection $\sigma_i \in S_a$. In the case of the consonance, when $p(\pi^+, \upsilon^+) \approx -1$, question with the choice of alternative $\sigma_i \in S_a$ does not arise, if the entropy H_π^+ is small. The conflict appears, when H_π^+ belongs „to the layer of freedom" (H_π^{+*}, H_π^{+**}). Similar reasoning can be conducted in the case of the distributions $\pi^-(\sigma_i)$ and $\upsilon^-(\sigma_i)$ comparison.

Another model of dissonant conflict appears, when to subject are known not only the utility of alternatives $U(\sigma_i)$, but also the probability of resolution of the corresponding problems $p(P\colon \sigma_0 \to \sigma_i) = p_i$, moreover $\sum_{i=1}^{N} p_i = 1$. In any case, p_i can be considered as the estimations of real probabilities, accessible to subject.

Let us assume, that the distribution of preferences is formed as the solution of variational problem with the functional

$$\Phi_\pi = -\sum_{i=1}^{N} \pi(\sigma_i) \ln \pi(\sigma_i) - \alpha \sum_{i=1}^{N} \pi(\sigma_i) \ln p_i + \sum_{i=1}^{N} \pi(\sigma_i) U(\sigma_i) + \gamma \sum_{i=1}^{N} \pi(\sigma_i) \qquad (7.8)$$

We exclude here the division of preferences on the types (π^+, π^-, υ^+, υ^-). The corresponding canonical distribution takes the form:

$$\pi(\sigma_i) = \frac{p_i^\alpha e^{\beta U(\sigma_i)}}{\sum_{j=1}^{N} p_j^\alpha e^{\beta U(\sigma_j)}} \qquad (7.9)$$

The entropy H_π approaches the maximum value, if for $\forall i$ $p_i \to \frac{1}{N}$ and for $\forall i, j$ $U(\sigma_i)$ — $U(\sigma_j) \to 0$. The degree of dissonance is characterized by correlation coefficient

$$\rho(\pi, p) = \frac{\sum_{i=1}^{N} \left(p_i - \frac{1}{N}\right)\left(\pi_i - \frac{1}{N}\right)}{\sqrt{\sum_{i=1}^{N} \left(p_i - \frac{1}{N}\right)^2 \sum_{i=1}^{N} \left(\pi_i - \frac{1}{N}\right)^2}} \qquad (7.10)$$

In the particular case, when α = 1, $U_i - U_j$ = 0 V/, $\forall i, j \in \overline{1,N}$, $\rho(\pi, p)$ = 1, dissonance does not appears, conflict, however, can occur, if $H_\pi > H^*_\pi$.

Let further the subject forms the preferences exclusively as utilitarian:

$$\pi(\sigma_i) = \frac{e^{\beta U(\sigma_i)}}{\sum_{j=1}^{N} e^{\beta U(\sigma_j)}},$$

to it is also known the probabilities (or their estimation) of the successful permission of the problems P: $\sigma_0 \to \sigma_i$. In this case the dissonance can occur, when $\rho(\pi, p) \to -1$, since the most preferable alternative can have the smallest probability of realization. If additionally $H_\pi > H^*_\pi$, conflict proves to be deeper, since the selection $\sigma_i \in S_a$ is hindered.

Thus, consonant conflict corresponds to the high (close to the maximum) value of entropy and to the correlation coefficient, close to the +1, if examined conflict occurs between „similar" distributions (π^+ and π^-, or υ^+ and υ^-) and close to the κ –1, if examined conflict occurs between the opposite distributions (π^+ and υ^+, π^- and υ^-, π^+ and υ^-, π^- and υ^+).

The possible types of internal conflicts depending on the distributions of preferences, and also on the value of the correlation coefficients and entropies participating in them are given in Tabele 7.5.

Tabel 7.5

Correlation coefficient	Values of the entropy	Type of the conflict
$\rho(\pi^+, \pi^-) \to +1$	$H_\pi^+ > H_\pi^{*+}$; $H_\pi^- > H_\pi^{*-}$	Consonant
$\rho(\pi^+, \pi^-) \to -1$	H_π^+ and H_π^- are small	Dissonant
$\rho(\upsilon^+, \upsilon^-) \to +1$	$H_\upsilon^+ > H_\upsilon^{*+}$; $H_\upsilon^- > H_\upsilon^{*-}$	Consonant
$\rho(\upsilon^+, \upsilon^-) \to -1$	H_υ^+ and H_υ^- are small	Dissonant
$\rho(\pi^+, \upsilon^+) \to -1$	$H_\pi^+ > H_\pi^{*+}$; $H_\upsilon^+ > H_\pi^{*+}$	Consonant
$\rho(\pi^+, \upsilon^+) \to +1$	H_π^+ and H_υ^+ are small	Dissonant
$\rho(\pi^-, \upsilon^-) \to -1$	$H_\pi^- > H_\pi^{*-}$; $H_\upsilon^- > H_\upsilon^{*-}$	Consonant
$\rho(\pi^-, \upsilon^-) \to +1$	H_π^- and H_υ^- are small	Dissonant
$\rho(\pi^+, \upsilon^-) \to -1$	$H_\pi^+ > H_\pi^{*+}$; $H_\upsilon^- > H_\upsilon^{*-}$	Consonant
$\rho(\pi^+, \upsilon^-) \to +1$	$H\pi^+$ and H_υ^- are small	Dissonant
$\rho(\pi^-, \upsilon^+) \to -1$	$H_\pi^- > H_\pi^{*-}$; $H_\upsilon^+ > H_\upsilon^{*+}$	Consonant
$\rho(\pi^-, \upsilon^+) \to +1$	H_π^- and H_υ^+ are small	Dissonant

With the development of conflict situation in the „binary" time the comparisons can change each other, and the conflict of one type is substituted by conflict of another type until the solution is accepted – the target is selected on the set S_a.

In the following example let us attempt to show, how a change in the resource situation affects the indices of conflict and, therefore, its „sharpness". Let us make assumptions about the nature of the positive preferences π_i^+ and negative preferences π_i^- dependence on the resources. Let the positive preferences be determined by the relationship between the expected resources R_i^{exp} and the required resources R_i^{req}, namely, let us assume that the higher is the preference of alternative σ_I, the greater is the relation

$$\frac{R_i^{\exp} - R_i^{req}}{R_i^{req}} = \overline{r}_i^{\,e} - 1 \,,$$

where $\overline{r}_i^{\,e} = R_i^{\exp}\left(R_i^{req}\right)^{-1}$. It is assumed also that $R_i^{\exp} \geq R_i^{req}$. Value $x_i = \overline{r}_i^{\,e} - 1 \in \left[0, +\infty\right)$. In other words, the higher the expected excess of income above the invested resources, the better.

Let us assume that the negative sensations appear in connection with the expenditures and they are more acute, the nearer the required resources to those available resources. This circumstance can be reflected quantitatively, if we select as the index the value

$$y_i = \frac{\overline{r}_i^{\,r}}{1 - \overline{r}_i^{\,r}} \,,$$

where $\overline{r}_i^{\,r} = R_i^{req}\left(R^{disp}\right)^{-1}$. Since for all $\sigma_i \in S_a$ the condition: $R_i^{req}\left(R^{disp}\right)^{-1} \in \left[0,1\right]$ must be satisfied, then $y_i \in [0, +\infty)$.

Let us examine two distributions:

$$\pi_i^+ = \pi_i^+\left(\sigma_i\right) = \frac{e^{\beta\left(\overline{r}_i^{\,e} - 1\right)}}{\sum\limits_{j=1}^{N} e^{\beta\left(\overline{r}_j^{\,e} - 1\right)}} \,;\; \pi_i^- = \pi_i^-\left(\sigma_i\right) = \frac{e^{-\alpha \overline{r}_i^{\,r}\left(1 - \overline{r}_i^{\,r}\right)^{-1}}}{\sum\limits_{j=1}^{N} e^{-\alpha \overline{r}_i^{\,r}\left(1 - \overline{r}_i^{\,r}\right)^{-1}}} \,.$$

Let there be two alternatives σ_1 and σ_2 and resource situation is assigned by the tabele

	σ_1	σ_2	
R_i^{exp}	1,5	2,7	β = I
R_i^{req}	1,0	2,0	α= 1
R^{disp}	–3		

The distributions π_i^+ and π_i^-, that correspond to these data, are given in the tabele

	σ_1	σ_2
π_i^+	0,53742	0,46257
π_i^-	0,81757	0,182439

The consonance occurs between π_i^+ and π_i^-, therefore $\rho(\pi^+, \pi^-) \to +1$.

H_π^+ = 0,69034, H_π^- = 0,47506.

The analysis of positive circumstances in this case gives the high value of entropy H_π^+, close to the maximum $H^+_{\pi\max}$, whereas the analysis of negative circumstances gives the low value of entropy H_π^-, it is for sure lower than the threshold H_π^{*-}. This means that there is, apparently, no conflict and the solution will be taken on the basis of the negative analysis.

Let us assume, that the available resources increased. In the previous case available resources are universal (for example, money) and equal to 3 units. Let it be now R^{disp} = 6 units. The entropy H_π^+ will not change, since the distribution π_i^+ does not depend on the available resources and, as earlier, H_π^+ = 0,69034.

The distribution π_i^- as a result of an increase in the available resources is „equalized":

	σ_1	σ_2
π_i^+	0,53742	0,46257
π_i^-	0,57444	0,42556

The entropy H_π^- = 0,68203, it means that it grows and, most likely, exceeds the threshold H_π^{*-}. Therefore consonant conflict occurs. One can say, that in the case of consonant conflict the comparison of the two distributions is not essential.

In this case for resolution of conflict it is possible to attempt to select another „pair" - another distribution (υ_υ^+ or υ_υ), that is to change the type of possible conflict. If in this case one of the entropy proves to be below the threshold value, the solution can be accepted and conflict will be resolved.

Assume that a simultaneous proportional increase in the required resources occurred now (for example, the prices). Let us suppose, the prices on all goods increased 1,2 times. Initial tabele appears as follows:

	σ_1	σ_2
R_i^{exp}	1,5	2,7
R_i^{req}	1,2	2,4
R^{disp}	3	

The distributions π_i^+ and $\pi_i^-$are shown in the tabele

	σ_1	σ_2
π_i^+	0,53120	0,46878
π_i^-	0,96555	0,03444

As above $\rho(\pi^+, \pi^-) \to +1$, that is, there is a consonance. Entropies are equal

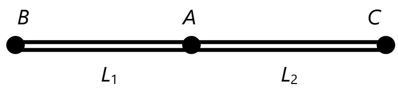
$$H_\pi^+ = 0{,}69110, \qquad H_\pi^- = 0{,}14986.$$

Since one of the entropies (H_π^-) is small, there is no conflict, and the solution can be accepted on the basis of negative analysis.

It is possible to say, that in the case of consonance with the high entropies of all positively correlating distributions, one of them is sufficient to speak about the presence of internal conflict, connected with the impossibility of the solution acceptance with the high entropies. In the case of dissonance the correlation plays the decisive role. Correlation coefficient forms as the „third axis" and together with the entropies forms the space, whose each point characterizes the state of conflict situation.

As can be seen from Tabl.7.5, in the case of the dissonance the entropies are small and, nevertheless, the solution cannot be accepted because of the presence of the corresponding correlation of the competing distributions. Let us note in conclusion that, fortunately „Buridan's donkey" did not die.

If we examine „problematic-resource" situation, in which it proved to be, in the dynamics, then we will come to the conclusion rapidly that initial position (point *A)* is unstable



$L_1 + L_2 = L$ = const

Let it be at the initial moment t = of 0 distances L_1 and L_2 identical: = L_2, $L_1 + L_2 =$ $= L$ = const and let the preferences of alternatives σ_1 and σ_2 be expressed by the formulas

$$\pi(B) = \frac{e^{-\beta L_1}}{e^{-\beta L_1} + e^{-\beta L_2}};\ \pi(C) = \frac{e^{-\beta L_2}}{e^{-\beta L_1} + e^{-\beta L_2}};\ \boldsymbol{\beta > 0}.$$

It is evident that $\pi(B) + \pi(C) = 1$. Let us take the following model of change L_1 and L_2 ($K > 0$):

$$\frac{dL_1}{dt} = -K\left(\pi(B) - \pi(C)\right);\ \frac{dL_2}{dt} = -K\left(\pi(C) - \pi(B)\right).$$

Taking into account standardization condition for $\pi(B)$ and $\pi(C)$, and also the condition $\frac{dL_1}{dt} = -\frac{dL_2}{dt}$, let us reduce the previous system to one equation:

$$\frac{dL_1}{dt} = -K\left(\frac{2}{1 + e^{-\beta L} e^{2\beta L_1}}\right).$$

Let there be at the initial moment the small deviation from the position of equilibrium $L_1 = L_2$:

$$L_1 = \frac{1}{2}L - \varepsilon;\ \ 0 < \varepsilon \ll 1$$

toward the position *B*. Value e satisfies the equation

$$\frac{d\varepsilon}{dt} = K\beta\varepsilon.$$

Hence it follows, that with $K > 0$, $\beta > 0$, position *A* is exponentially unstabele : $\frac{d\varepsilon}{dt} > 0$.

Derivative reverses the sign, if β reverses the sign.

This is evident from the original equation. With any $\beta > 0$

$$\left.\frac{dL_1}{dt}\right|_{L_1=0} = K\left(1 - \frac{2}{1+e^{-\beta L}}\right) < 0,$$

on the contrary, with any $\beta < 0$

$$\left.\frac{dL_1}{dt}\right|_{L_1=0} > 0$$

that is the position *A* is unstabele . In the given model the distances *A* and *B* or *C* are assumed as the „required resources" and in this case the instability of equilibrium position occurs. The „death of hunger" possibility, however, remains, if in the position *A* „indecision" will be manifested. It is possible to attempt to model this circumstance, after adding into the preferences „remnant" additive component $\pi'(t)$:

$$\pi(B) = \pi_0(B) + \pi'(t);\ \ \pi(C) = \pi_0(C) - \pi'(t).$$

Standardization conditions are not disrupted. As $\pi'(t)$ it is possible to take any random process with the zero average, or component of Lorenz attractor. Simulation shows, that with the presence of remnant component a stay of subject in the environment of point A is delayed, and the time of arrival to the point *B* (where it is possible to „quench the hunger") may become unacceptably high. Let in the simplest case $\pi'(t)$ be given in the form of periodic sinusoidal oscillations $\pi'(t) = \mu\sin\omega t$ ($\mu > 0$). In this case the equation for $L_1(t)$ takes the form:

$$\frac{dL_1}{dt} = K\left(1 - 2\mu\sin\omega t - \frac{2}{1+e^{-\beta L}e^{2\beta L_1}}\right).$$

Assuming again $L_1(t=0) = \frac{1}{2}L + \varepsilon$, we reveal that during the specific combinations of the parameters ε, β, μ the time of arrival at point *B* substantially depends on the frequency of remnant vibrations ω, moreover with some values of ω it is minimal, and in other cases – it is considerably longer, which corresponds to the absence of remnant component ($\mu = 0$).

Fig. 7.1 shows the dependence $L_1(t)$ for „decisive donkey", not experiencing mental fluctuations with the initial condition $L_1(0) = L - 0{,}05$, where $L = 5$, $\beta = 0{,}3$, $k = 0{,}1$.

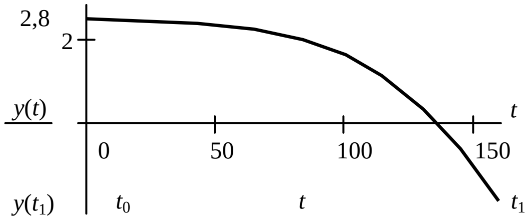


Fig. 7.2

„Goal" is achieved approximately in 130 s. Fig. 7.3 shown the solution of equation for *L1* (*t*), when the endogenous parameter p is variable and is defined as one of the components of Lorenz attractor, for which the damping terms - $\beta = \lambda(Q_2 - 18{,}877)$ are added. Here number 18,877 is this steady state value of variable Q_2 after „damping" of the perturbed process in the attractor when the structural parameters are assigned as follows:

a = 8; b = 8; c = 20; h = 0,01; т = 0,03; п = 0,015; k = 0,3; s = 5; β = 0,1; λ = 0,3.

Thus the initial conditions for variables Q_i are assigned

$$Q(0) = \begin{pmatrix} 0{,}0001 \\ 0 \\ 0 \\ 2{,}45 \end{pmatrix}.$$

As we see, there is an initial deviation from the equilibrium position to the left on value of 0,05. System of equations takes the form:

$$D(t,Q) = \begin{bmatrix} aQ_1 - bQ_0 - hQ_0^2 \\ -Q_1 - Q_0Q_2 + cQ_0 - mQ_1^2 \\ Q_0Q_1 - dQ_2 - nQ_2^2 \\ k\left(1 - \dfrac{2}{1 + e^{-\lambda(Q_2 - 18{,}877)s} e^{2\lambda(Q_2 - 18{,}877)Q_3}}\right) \end{bmatrix}.$$

It is evident from the Fig. 7.3 that in this case the solution $L_1(t)$ is stabilized near the equilibrium position even after „damping" of the large excited motions of attractor. The remaining small fluctuations Q_2 suffices in order to make position $h_1(t) \cong 2{,}5$ steady. This effect resembles the case, when the oscillations of the suspension point of simple pendulum make upper position of equilibrium steady.

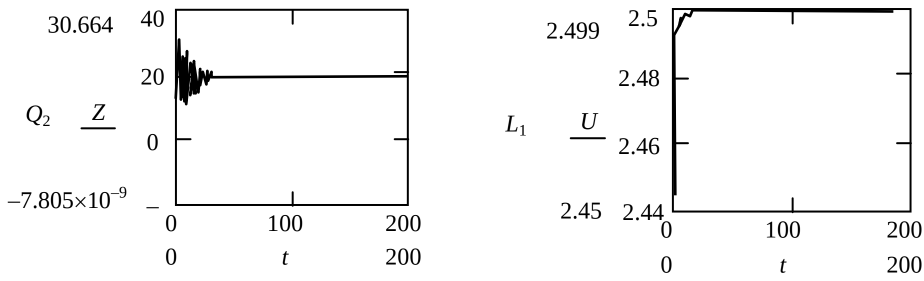


Fig. 7.3

7.2.2. Internal ethical conflicts

Conflicts examined earlier relate to the type *of utilitarian conflicts*, since the distributions of preferences depend on the utilitarian characteristics: utility, harmfulness (U, L), resources (R_i^{exp}, R_i^{req}, R^{disp}, ...).

The internal conflicts, associated with the rivalry of systems of ethics, the distributions of ethical preferences with the utilitarian, are *the ethical conflicts*. Ethical conflicts can be „competing" and „antagonistic". We have already mentioned about the difference between the antagonistic and competing relations from the theory of binary relations point of view. Let us add that „resolution of antagonistic conflict is combined with the „loss" (not necessarily physical) of one of the competing parties. Thus, the resolution (liquidation) of the attack of the antagonistic relations between the buyer and the salesman is possible only, if one of them disappears. If the „buyer" will cease to be „buyer", then at the same time status of „salesman" will change: it will cease to be the „salesman".

As a result of revolution the capitalist class was destroyed and, as a result, the proletariat ceased to be the proletariat. Competition does not imply (although it does not exclude) the „loss" (including of physical) one of the competing parties. For battle of the ethical systems the „battlefield" is necessary each time. Such field is object alternatives set S_a, or rating alternatives set S_x, and also the „flock" - *the „carriers"* of both utilitarian and ethical preferences. The following assertion is fundamental: *any ethics (ethical system) „will sleep" until it would not be „presented" with certain alternatives sets (S_a, S_x,...), of utilitarian nature*.

Ethics does not exist out of the consciousness of its „carriers" are ethical subjects, the following fundamental fact: *ethics "dwells" in the groups of subjects* (M > 1). *Ethics becomes meaningless in relation with one isolated subject.*

They can ask: „and what about Robinson Crusoe? Was his behavior ethical?" Answer - was positive.

However, he was not „isolated", first of all, from his creator Daniel Defoe, or from the readers, to whom the novel was addressed. When to the ethics the „battlefield" is presented (S_a, S_ξ, ...), it is „spilled" and it begins to influence the decision-making processes, thus manifests itself as one of the fundamental „ordinary" ethics:

- injective,
- surjective,
- everywhere-defined,
- functional,

or as combined – bijective. In addition to what was already been said about the ethical imperatives, is necessary to add that they are conveniently divided on *"forbidding", "prescribing", "recommending"* (similarly, as the signs in the rules of street traffic are subdivided), for example, "forbidding" imperatives: do not kill, do not steal, do not wish of the neighbor wife… „Prescribing" imperatives: help the poor, protect the native land, believe… One might say that ethics - are the traffic rules on the „road of life", that the formal rules and regulations, laws of states, juridical codes are translated into reality through the ethics and by means of the ethics.

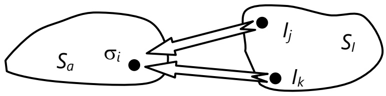
It can be imagined as „ the bridge from the past to the future, thrown through the present". Purely utilitarian preferences and decisions are based ultimately on a certain system of ethics. Thus neoliberalism, Friedmann model recognizes the ethics, which consists of one postulate: good is (ethically), that ensures maximum profit. If there are two ethical systems, which claim to one and the same „living space" – the consciousness of the subject, then one of the ethical conflicts forms appears. In this case each of the ethics has their „test" collection of the alternatives, with respect to which it gives categorical recommendations. Let these be sets $S_{a1}^{(0)}$, $S_{a2}^{(0)}$. If there are common elements - alternatives, that is the non-empty intersection

$$S_{a1}^{(0)} \cap S_{a2}^{(0)} \neq \varnothing,$$

on this intersection the collision of ethics occurs, the fight of ideologies, religions, philosophical systems. In order to judge the essence of fight, it is necessary to explicitly describe set of alternatives (behavioral, economic, and social). In this work however, as has been mentioned above, the task consists in, that, being distracted from the essence of particular alternatives and problems, to develop on the conceptual level the approach to the formalized description of the processes of subjective analysis, including quantitative description.

The system of ethics (system of imperatives) *is not contradictory relative to set* S_a, if on this set does not appear competitions or antagonism between the imperatives, included in this system.

It is easy to visualize the examples, when with respect to the specific alternative $\sigma_i \in S_a$ the conflict appears between the imperatives (being contained in this system: $I_j, I_k \in S_I$). This can occur, when imperatives I_j and I_k „attend" simultaneously the one and the same alternative $\sigma_i \in S_a$



Either consonance or dissonance is possible between I_j and I_k.

Let $\rho\Delta$ ($\sigma_i \in S_a$) is the relation of the binary dissonance of ethical imperatives S_I relative to alternative σ_I, $\rho_c(\sigma_i \in S_a)$- the relation of binary consonance relative to $\sigma_i \in S_a$. Record:

$$I_j \, \rho_D \, (\sigma_i \in S_a) \, I_k$$

it means that the imperatives I_j and I_k are dissonant relative to $\sigma_i \in S_a$, and the record $I_j\rho_c(\sigma_i \in S_a)I_k$ means that the imperatives I_j and I_k are consonant relatively $\sigma_i \in S_a$. It is obvious that the relation ρ_c — is reflexive and transitive:

$$I_j \, \rho_c \, I_k \Leftrightarrow I_k \, \rho_c \, I_j; \quad (I_j \, \rho_c \, I_k \, ; I_k \, \rho_c \, I_m) \Rightarrow I_j \, \rho_c \, I_m.$$

Relation p_D is reflexive, but it can be negatively transitive:

$$I_j \, \rho_c \, I_k \Leftrightarrow I_k \, \rho_c \, I_j; \; \left(I_k \bar{\rho}_D I_j \, ; I_k \rho_D I_m\right) \Rightarrow I_j \rho_D I_m \, .$$

Let $\rho_0(\sigma_i \in S_a)$ is a binary relation on S_I, when either both imperatives I_j and I_k or one of them are indifferent with respect to the alternative σ_i. Based on the above considerations, we will clarify the concept of a *consistent system of ethics relatively* S_a. One can say that this is a system where there are no dissonant pairs (I_j, I_k) for $\forall\sigma_i \in S_a$ and $\forall j,k \in \overline{1,M}$ (I_j, $I_k \in S_I$). The system of ethics *is indifferent to this set S_a*, if it has „nothing to say" on at least one alternative from S_a.

The action of ethical imperatives in the „reign of freedom" leads to the redistribution of preferences and, in the final analysis, has an effect on the „selection" - decision making. After decision making - passage into the „reign of necessity" the imperatives continue to influence the mental condition: subject experiences either satisfaction or regret in connection with the accepted solution, that in common parlance is called „remorse".

Thus, it is possible to say that the ethics from the active phase in the „reign of freedom" passes into the passive phase in the „reign of necessity". The passive phase is not „harmless": here are prepared „explosions" - in the terminology of nonlinear systems - bifurcations.

Transitions from the „reign of freedom" into the „reign of necessity" are carried out mainly on the basis of utilitarian interests with the corrective influence of ethics and involve an increase in the prevalence and the consonance of similar distributions (π^+, π^- or υ^+, υ^-). Inverse passages - into the „reign the freedom" are connected with intensification of dissonance homologous distributions of their mutual gain - accumulation.

Thus, it is possible to make the assumption that the motion „upward" on the scale of subjective entropy is connected with the growth of dissonances of the indicated type.

In this case the dissonances are strengthened with the approximation „from below" to an absolute entropy threshold H_π^{**}. On the contrary, „downward" motion on the entropy scale is connected with the decrease of such dissonances and the growth of the corresponding consonances. Lower boundary, when $H_\pi = 0$ meets the complete absence of dissonances. In this case S_a degenerates into the singular set, which contains one alternative.

What is the role of ethics in the „wandering" of active system in the entropies space? It is possible to expect that the most active corrective role the ethical imperatives play in the region of „reigns" boundary: $H_\pi = H_\pi^*$. Further, it is natural to assume, that passage „upward" and passage „downward" occurs on the different levels of the entropy $H_\pi^*\downarrow$ and $H_\pi^*\uparrow$. Their mutual arrangement is not obvious. Thus, the „boundary" $H_\pi^*\uparrow$ most likely is washed away, since the discussion deals with increase of utilitarian possibilities, the gradual expansion of accessible alternatives set S_a (excluding the cases of sudden spasmodic appearance of „wealth").

Spasmodic passages are also possible both „upward" and „downward". In this case it can seem, that

$$H_\pi^*\uparrow > H_\pi^*\downarrow.$$

It would be careless to call such passages as „revolutions".

However, passage „upward" - from the „necessity" to the „freedom" in certain cases is received as „revolution", since it is connected with the high level of dissonance between the groups of subjects and the high level of consonance inside the groups. After „upward" passage the consonances inside the groups diminish, and dissonances increase, and vice versa, discords between the groups diminish and consonances between groups increase.

In other cases and in other conditions the „downward" passage - from the „freedom" to the „necessity" is received as „*revolution*".

The corresponding examples are well known. This heuristic description is speculative, but it is based at the analysis of the interrelation of categories - examined within the framework of subjective analysis.

The role of ethical systems with respect to the appearance and development of problematic-resource situations and, in particular, conflict situations in the groups is the object of subjective analysis.

7.3. Model characteristics of the intersubject conflict

In this division we examine conflicts between two subjects, bearing in mind that the transfer of the corresponding results to group of 3 and more subjects does not present fundamental difficulties.

The conflict of the I type individual preferences distributions depends on the nature of alternatives. Let two subjects have distributions of preferences $\pi_1(\sigma_i)$ and $\pi_2(\sigma_k)$, $\sigma_i \in S_{a1}$, $\sigma_i \in S_{a2}$. Let us define these distributions so, that on $S_a = S_{a1} \cup S_{a2}$ the standardization conditions would be satisfied for them. Correlation coefficient is determined by the formula:

$$\rho\left(\pi_1,\pi_2\right)=\frac{\sum_{i=1}^{N}\left(\pi_1\left(\sigma_i\right)-N^{-1}\right)\left(\pi_2\left(\sigma_i\right)-N^{-1}\right)}{\sqrt{\sum_{i=1}^{N}\left(\pi_1\left(\sigma_i\right)-N^{-1}\right)^2\sum_{i=1}^{N}\left(\pi_2\left(\sigma_i\right)-N^{-1}\right)^2}} \tag{7.11}$$

Case I.

All alternatives are „one-place", that is each alternative σ_i can be realized only by one subject, then, if $\rho(\pi_1, \pi_2) \to 1$, then on S_a conflict situation occurs, moreover the smaller entropies $H_{\pi 1}$ and $H_{\pi 2}$, the sharper the conflict. If they approach the appropriate thresholds $H^*_{\pi 1}$ and $H^*_{\pi 2}$ simultaneously in proportion to the decrease of entropies, then these thresholds themselves can be reduced, since decision making is hindered by the presence of conflict. The aggravation of conflict will occur and the „pulling" of thresholds into the region of low entropies. This corresponds to the higher certainty of the individual contradicting each other desires of subjects. In case in question the conflict is absent, if $\rho(\pi_1, \pi_2) \to -1$.

If in the conflict situation ($\rho(\pi_1, \pi_2) \to 1$) one of the entropies more rapidly approaches its threshold, and the second remains high, then conflict weakens and one of the subjects „makes decision" earlier than other. In this case the conflict from passive preparatory phase will pass to the active phase, related with the expense of resources of one of the subjects, a significant change in the distribution of its preferences and entropy. Appearance and development of conflict situation is connected with production or absorption of subjective information. Quantitative description is based on the use of canonical distributions of preferences and the study of their dynamics.

The development of conflict is characterized by a change in the preferences of each of the subjects, and, correspondingly, by change in both entropies and correlation coefficient. The dynamics of preferences is examined in chapter 5, where, in particular, it is shown that as a result of the decision making the problematic-resource situation changes „abruptly".

Here interests us in essence the „passive" phase of the conflict development, that is the events, proceeding in the „reign of freedom". It is at the same time obvious, that the conflict can deform the boundaries of this „reign", changing the entropies threshold values.

Case II.

All alternatives are corporate. In this case by interpersonal conflict we will understand the divergence in the preferences of different alternatives $\sigma_i \in S_a$, which are also characterized quantitatively by the value of correlation coefficient $\rho(\pi_1, \pi_2)$. The divergence of preferences is large, when $\rho(\pi_1, \pi_2) \to -1$, and this can be treated as conflict situation. The divergence of preferences is small, if $\rho(\pi_1, \pi_2) \to 1$. In this case the conflict is absent.

In the study of the dynamics of conflict situations some models from the chapter 5 can be used. The discussion deals with the equations, which describe a change in the preferences (both the I and II kind) taking into account the dynamics of exogenous and endogenous factors. It is additionally necessary to observe the dynamics of correlation coefficients. For the correlation coefficient (7.11) we obtain the following differential equation:

$$\begin{aligned} \frac{d\rho}{dt} &= \frac{1}{S}\sum_{i=1}^{N}\left(\left(\pi_{2i} - N^{-1}\right)\dot{\pi}_{1i} + \left(\pi_{1i} - N^{-1}\right)\dot{\pi}_{2i}\right) - \\ &- \frac{\rho}{S^2}\sum_{i=1}^{N}\left(\left(\pi_{1i} - N^{-1}\right)\dot{\pi}_{1i} + \left(\pi_{2i} - N^{-1}\right)\dot{\pi}_{2i}\right), \end{aligned} \tag{7.12}$$

where $\rho = \rho(\pi_1, \pi_2)$; $S = \sqrt{\sum_{i=1}^{N}\left(\pi_{1i} - N^{-1}\right)^2 \sum_{i=1}^{N}\left(\pi_{2i} - N^{-1}\right)^2}$.

In the case, if the number of alternatives $N = 2$, correlation coefficient takes only two values: +1 and -1, moreover $\rho(\pi_1, \pi_2) = -1$, when simultaneously $\pi_{11} > 0,5$ and $\pi_{22} > 0,5$ and $\rho(\pi_1, \pi_2) = 1$, when either $\pi_{11} > 0,5$ and $\pi_{22} < 0,5$ or $\pi_{11} < 0,5$ and $\pi_{22} > 0,5$.

A change in the entropy is determined, as earlier in chapter 5, by the relationships:

$$\frac{dH_{\pi_1}}{dt} = \sum_{i=1}^{N}\left(\ln \pi_{1i} + 1\right)\dot{\pi}_{1i} \tag{7.13}$$

$$\frac{dH_{\pi_1}}{dt} = \sum_{i=1}^{N} \left(\ln \pi_{2i} + 1 \right) \dot{\pi}_{2i} \tag{7.14}$$

It is simple to construct the scheme s by analogy, when in the conflict participate the canonical distributions of other types (υ^+, υ^-, ...).

It is interesting to trace the development of conflict in the whole range of entropy and correlations. An important question is, what and how to resolve conflict and what happens in the interval of time following this event. Such analysis can be carried out by modeling the dynamics of preferences, just as it was done in Chapter 5 with respect to other tasks.

In particular, if there are only two subjects and only two alternatives, then as a result of the antagonistic conflict resolution (the binary relation between the subjects *A* and *B* appears as the „antagonism") occurs the complete „destruction" of the system: if „buyer" disappears, then simultaneously „salesman" disappears. They do not exist without each other. Consequently, entire active system disappears. This does not necessarily mean physical death of *A* and *B*. At the site of previous active system can arise another active system. As a result of competitor conflict resolution the subjects *A* and *B* do not disappear, the active system does not disappear also.

It appears, that under certain conditions *the antagonism may proceed in the competition and vice versa*. Due to the conditions of competition the presence of the third subject *C*, who plays the passive role (Fig. 7.4), is implied.

Fig. 7.4

In the scheme *a,* Fig. 7.4, between *A* and *B* antagonism occurs, in the scheme *b*, Fig. 7.4 between *A* and *B* occurs competition, while between *A* and *C* and *B* and *C* an antagonism exists.

In order to visualize as it is possible to express this in the terms of the preferences of the I type, let us examine this model situation: salesman *A* can sell one and the same goods on the price p_1 or on the price p_2 and $p_2 > p_1$. Buyer *B* can purchase goods on the price p_1 or p_2 respectively. If π_{Ai} — preferences of salesman, and π_{Bi} — preferences of buyer, then

$$\pi_{A2} > \pi_{A1}, \quad \text{and} \quad \pi_{B2} < \pi_{B1}.$$

This means that

$$\rho(A, B) = -1$$

and conflict between *A* and *B* is antagonistic. In the case *b* distribution of preferences *A* and *B* is such, that

$$\pi_{A2} > \pi_{A1,} \quad \text{and} \quad \pi_{B2} > \pi_{B1}.$$

Consequently,

$$\rho(A, B) = +1.$$

Conflict exists, since there is the third subject *C* (kind of „arbitrator"), but this conflict is competitive. In this second case it is possible to visualize analogy with two subjects and with two chairs: α and β. Each alternative is „one-place". Even if *A* and *B* both preferred chair α, their preferences are in consonantal respect, but there is a conflict. If at least one of the subjects no matters, on what chair to sit, then its entropy $H_\pi - H_{\pi \max}$ = ln2 and conflict cannot arise.

Conflict of ratings unfolds over the set S_ξ of subjects in the group. In Chapter 4 were examined different types of the rating preferences distributions: integral ratings $\xi(j)$, conditional ratings $\xi(j|\ i)$, differential ratings $\xi(j|\ i, \sigma_k)$ and others. Let us examine some forms of rating conflicts. The binary conflict of the conditional rating distributions of two subjects *i* and *k* is defined as divergence with respect to the estimation of its associates - rating distributions $\xi(j|i)$ and $\xi(j|k)$. On the presence of rating conflict signals the correlation coefficient of the conditional ratings distributions:

$$\rho_\xi\left(\xi_i, \xi_{j|i}\right) = \frac{\sum_{j=1}^{M}\left(\xi\left(j|i\right) - M^{-1}\right)\left(\xi\left(j|k\right) - M^{-1}\right)}{\sqrt{\sum_{i=1}^{M}\left(\xi\left(j|i\right) - M^{-1}\right)^2 \sum_{i=1}^{M}\left(\xi\left(j|k\right) - M^{-1}\right)^2}} \tag{7.15}$$

Under conditions of the „mutual responsibility", as it was shown in Chapter 4, all ratings were identical, $\rho_\xi(i, k) = +1$ for $\forall j, k \in \overline{1, M}$. Matrix $\hat{\rho}_\xi = \left\|\rho_\xi\left(i, k\right)\right\|$ is symmetrical, moreover $\rho_\xi(i, i) = 1$ for $\forall i$. With the presence of complete consensus, that is the agreement of the conditional distribution of all subjects: $\xi(j|\ i) = \xi(j|\ k)$ for $\forall j, k \in \overline{1, M}$ all $\rho_\xi(i, k) = 1$ and $\det \hat{\rho} = 0$.

As a measure of discrepancy between the preferences for the entire group can take the value

$$1 - \det \hat{\rho} \geq 0.$$

Assume that, for example, in the group of 3 subjects the distribution of centered ratings preferences is assigned by the tabele :

j	$\xi(j \mid 1) - M^{-1}$	$\xi(j \mid 2) - M^{-1}$	$\xi(j \mid 3) - M^{-1}$
1	0,5	–0,2	–0,3
2	–0,25	0,6	–0,3
3	–0,25	–0,4	0,4

In this case the matrix $\hat{\rho}_\xi$ has the form:

$$\hat{\rho}_\xi = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -0,3397 & -0,4901 \\ -0,3397 & 1 & 0,0951 \\ -0,4901 & 0,0951 & 1 \end{bmatrix}.$$

Then

$$\det \hat{\rho} = 1 + 2\rho_{12}\rho_{13}\rho_{23} - \rho_{12}^2 - \rho_{13}^2 - \rho_{23}^2 \cong 0,6670$$

and

$$1 - \det \hat{\rho} = 0,333 .$$

For the binary conflict, when $p\xi(i, k) < 0$, additional information give the individual entropies $H_{\xi i}$ and $H_{\xi k}$

$$H_{\xi i} = -\sum_{j=1}^{M} \xi\left(j \middle| i\right) \ln \xi\left(j \middle| i\right);$$

$$H_{\xi k} = -\sum_{j=1}^{M} \xi\left(j \middle| k\right) \ln \xi\left(j \middle| k\right).$$

The „sharpness" of conflict is the greater, the smaller are these entropies. If a study relates to the „pre-election stage", which within the framework of used here terminology should be considered the „reign of freedom", then the maximum „sharpness" of ratings conflict is reached at the moment of achieving the threshold values $H^*_{\xi i}$ and $H^*_{\xi k}$, this is, figuratively, in the moment of „filling the bulletins" by subjects „*i*" and „*k*". Conflict is absent, or almost absent, if at least one of the subjects „anyway who is chosen" and his entropy is high: $H_{\xi i} \to \ln M$.

Another type of ratings conflict is based on a comparison of the individual conditional distribution $\xi(j \mid i)$ with integral ratings $\xi(j)$, as defined in Chapter 4 (formula (4.129)).

According to the accepted concept for any distribution of preferences should be given the „carrier", in this case the „carrier" for distribution of integral ratings is the „operator" - someone which determines these ratings basing on the conditional processing of individual ratings and then uses them in their „own interests", or - it is the „virtual subject" that represents the entire group, as a unified social body that we called above the „collective wisdom", the „public opinion". Correlation coefficient between the distributions $\xi(j)$ and $\xi(j \mid i)$

$$\rho_\xi\left(\xi_i, \xi_{j|i}\right) = \frac{\sum_{j=1}^{M}\left(\xi\left(j\right) - M^{-1}\right)\left(\xi\left(j\middle|k\right) - M^{-1}\right)}{\sqrt{\sum_{i=1}^{M}\left(\xi\left(j\right) - M^{-1}\right)^2 \sum_{i=1}^{M}\left(\xi\left(j\middle|k\right) - M^{-1}\right)^2}} \tag{7.16}$$

is characterized by the degree of the divergence of the subject individual rating preferences with the „public opinion" including evidence of a conflict between the subject „i" and the group, if

$$\rho(\xi j, \xi j \mid i) \to 1.$$

We do not examine here a question, what follows from such conflict: will subject „*i*" manifest activity and attempt to change „public opinion" and will be there someone, representing public opinion who will attempt to act on „*i*" for the purpose to change its individual preferences.

As in the case of object preferences, it is possible to visualize antagonistic and nonantagonistic, for example, competitor conflicts on the set S_ξ. Integral preferences $\xi(j)$, determined by formula (4.129), always exist. With respect to the ratings $\xi(j)$ the group, in which these ratings exist, is „locked". Let us examine group of three subjects: $M = 3$.

The conditions must be satisfied according to formula (4.129):

$$\begin{aligned} \xi\left(1\right) &= \xi\left(1\right)\xi\left(1\middle|1\right) + \xi\left(2\right)\xi\left(1\middle|2\right) + \xi\left(3\right)\xi\left(1\middle|3\right) \\ \xi\left(2\right) &= \xi\left(1\right)\xi\left(2\middle|1\right) + \xi\left(2\right)\xi\left(2\middle|2\right) + \xi\left(3\right)\xi\left(2\middle|3\right) \\ \xi\left(3\right) &= \xi\left(1\right)\xi\left(3\middle|1\right) + \xi\left(2\right)\xi\left(3\middle|2\right) + \xi\left(3\right)\xi\left(3\middle|3\right) \end{aligned} \tag{7.17}$$

Furthermore, the conditions for the standardization must be satisfied

$$\sum_{j=1}^{M}\xi\left(j\right) = 1\,;\ \sum_{j=1}^{M}\xi\left(j\middle|i\right) = 1\,;\ \forall i \in \overline{1,M}\,.$$

Three equations given above (7.17), and three equalities, which follow from standardization conditions, compose the system of 6 equations relative to 9 values $\xi(j \mid i)$ (taking into account the fact that in the general case $\xi(j| i) \neq \xi(i \mid j)$), therefore, these conditions do not determine unambiguously conditional preferences $\xi(j \mid i)$. On the contrary, it is possible to show that, if we consider condition (7.17) as equations for the integral ratings $\xi(i)$, then taking into account standardization conditions the determinant of this uniform system is identically equal to zero.

Consequently, the integral ratings, determined by formula (4.129), always exist, which was discussed in Chapter 4:

$$\det(Z - 1) = 0,$$

where Z is a matrix of conditional ratings.

One of the integral ratings can be selected arbitrarily. Let this be ξ(1) = ξ∗(1). In our example, when M = 3, we can take any two equations of system (7.17) and conditions for standardization.

As a result we will obtain the system of three equations, for example:

$$\begin{gathered}\xi^*(1)+\xi(2)+\xi(3)=1 \\ \xi(2|1)\xi^*(1)+\left(\xi(2|2)-1\right)\xi(2)+\xi(2|3)\xi(3)=0 \\ \xi^*(1)\xi(3|1)+\xi(3|2)\xi(2)+\left(\xi(3|3)-1\right)\xi(3)=0\end{gathered} \tag{7.18}$$

The matrix determinant of this system

$$\det Z^* = \begin{vmatrix} 1 & 1 & 1 \\ \xi(2|1) & \xi(2|2)-1 & \xi(2|3) \\ \xi(3|1) & \xi(3|2) & \xi(3|3)-1 \end{vmatrix} > 0 \tag{7.19}$$

because all ratings are positive. Thus, system (7.18) has unique solution.

Let us determine the matrix

$$W = \left[\begin{array}{c:c} 1 & \hat{\rho}^0(k) \\ \hdashline \hat{\rho}(k) & \hat{\rho}(i,k) \end{array}\right] \tag{7.20}$$

where $\hat{\rho}(k)$ is an vector, the column of the correlation coefficients of the individual conditional preferences of subject „k" with the „public opinion", and $\hat{\rho}(i,k)$ is a matrix of coefficients of correlation of different subjects preferences between themselves. Then, by analogy, with the statistics it is possible to introduce the coefficient of subjective multiple correlation

$$R = \sqrt{1-\frac{\det W}{\det\ \hat{\rho}(i,k)}} \tag{7.21}$$

As is known, R lies within the limits

$$0 \le R \le 1$$

and characterizes the „degree of agreement" relative to distribution of ratings in the group. There is no agreement, if $R \to 0$, if $R \to 1$, the high level of agreement occurs (consensus, convergence).

The next type of conflict is related to potential conflicts of preference *ratings* of the subject with the distribution of ranks in the group (in society), or a mismatch of

„*public opinion*", that is integral ratings to the distribution *of rank*s (read „posts", „imperious authorities").

Let us designate, as earlier, the subject rank in group $\eta_k \left(k \in \overline{1,L}\right)$, $L \leq M$, where L — the number of classes of rank equivalence. The correlation coefficient of ranks distribution in the group and of individual rank preferences of subject „*i*" is determined by the formula:

$$\rho_\xi\left(\eta,\xi_i\right) = \frac{\sum_{j=1}^{M}\left(\eta_i - M^{-1}\right)\left(\xi\left(j\middle|i\right) - M^{-1}\right)}{\sqrt{\sum_{j=1}^{N}\left(\eta_j - M^{-1}\right)^2 \sum_{j=1}^{M}\left(\xi\left(j\middle|i\right) - M^{-1}\right)^2}} = \tag{7.22}$$

$$= \frac{\sum_{k=1}^{L}\left(\eta_k - M^{-1}\right)\sum_{j=M_{k-1}+1}^{M_k}\left(\xi\left(j\middle|i\right) - M^{-1}\right)}{\sqrt{\sum_{j=1}^{N}\left(\eta_j - M^{-1}\right)^2 M_k \sum_{j=1}^{M}\left(\xi\left(j\middle|i\right) - M^{-1}\right)^2}}$$

Analogously is calculated the correlation coefficient of the ranks distribution and distribution of integral ratings, this is the „public opinion":

$$\rho_\xi\left(\eta,\xi\right) = \frac{\sum_{k=1}^{L}\left(\eta_k - M^{-1}\right)\sum_{j=M_{k-1}+1}^{M_k}\left(\xi\left(j\right) - M^{-1}\right)}{\sqrt{\sum_{j=1}^{N}\left(\eta_k - M^{-1}\right)^2 M_k \sum_{j=1}^{M}\left(\xi\left(j\right) - M^{-1}\right)^2}} \tag{7.23}$$

The last two forms of correlation coefficient signal about the possible presence of the divergence between the ratings preferences of the group members and the established objective rank hierarchy and, correspondingly, the distribution of imperious authorities. It is possible to express the assumption, that the integrating correlations play the decisive role in the „reign of freedom" („before elections"), while mixed rank-ratings correlations determine the level of conflict after the distribution of ranks („after elections"). Earlier, in chapter 4 the relevant entropies were examined, which in this case are involved in the characterization of the situation in the group.

The analysis of possible conflicts can be extended to the ethical systems. Let us explain, how it is possible to present the conflict between two ethical systems. Let us recall, that the ethics in order to appear, to enter the action, must have the „battlefield". In all cases without the exception each ethical system has the certain

„standard" (or canonized) collection of the alternatives $S_a^{(0)}$, relative to which it „commands", or „advises" relative to the choice of preferences.

The ranking of ethical imperatives we will conditionally assume „a priori". Assume that this ranking is reflected in the form of distribution $\pi(I_k)$, $I_k \in S_I$.. Distribution $\pi(I_k)$ can be ordinal or cardinal.

If there are two systems of ethical imperatives with a priori distributions $\pi_1(I_k)$ and $\pi_2(I_k)$ on one and the same canonical set $S_a^{(0)}$, then the degree of correspondence or divergence of ethics can be determined by correlation coefficient $\rho(\pi_1(I_k)$ and $\pi_2(I_k))$, and the sharpness of ethical conflict, if any, by the values of the entropies

$$H_{\rho 1} = -\sum_{k=1}^{S} \pi_1(I_k) \ln \pi_1(I_k)\,;\; H_{\rho 2} = -\sum_{k=1}^{S} \pi_2(I_k) \ln \pi_2(I_k)\,.$$

Under $S_a^{(0)}$ we mean again the union $S_{a1}^{(0)} \cup S_{a2}^{(0)}$ with distributions supplemented by zeros on alternatives „not employed".

The more general and more realistic determination of ethical conflict consists in the following: let as the „battlefield" for two ethical systems are proposed alternatives set S_a and $\pi(\sigma_k)$, where $\sigma_k \in S_a$, are utilitarian preferences. Let us „place" the first ethical system on the distribution $\pi(\sigma_k)$, and let as a result the distribution $\pi(\sigma_k| S_I)$ appear different from initial $\pi(\sigma_k)$. Similarly, looking on the set S_a through the „prism" of the second ethical system, we will obtain another distribution $\pi(\sigma_k| S_I)$. The information, connected with the „action" of ethics on the utilitarian preferences, is determined by the formula

$$I(S_a | S_{Ii}) = H_\pi(S_a | S_{Ii}) - H_\pi(S_a) \tag{7.24}$$

The sense of designations is clear here. Difference between the conditional entropy

$$I(S_a | S_{Ii}, S_{Ik}) = H_\pi(S_a | S_{Ii}) - H_\pi(S_a | S_{Ij}) \tag{7.25}$$

is subjective and due to the replacement of one ethical system to another one and on the same „battlefield" - a set of meaningful alternatives S_a. Adopted designations (for example, $\pi(\sigma_k | S_I)$) they imply that each alternative $\sigma_k \in S_a$ is „organized into a trust" by all imperatives $I_j \in S_I$ (Fig. 7.3), as a result of which the corrected, considering ethical factors, preference $\pi(\sigma_k| S_I)$ is selected.

Degree of the divergence of distributions $\pi(\sigma_k| S_{Ij})$ and $\pi(\sigma_k| S_{Ik})$is reflected by the value of correlation coefficient

$$\rho\left(S_{li}, S_{lj} \middle| S_a\right) = \frac{\sum_{j=1}^{N}\left(\pi\left(\sigma_k \middle| S_{li}\right) - N^{-1}\right)\left(\pi\left(\sigma_k \middle| S_{lj}\right) - N^{-1}\right)}{\sqrt{\sum_{k=1}^{N}\left(\pi\left(\sigma_k \middle| S_{li}\right) - N^{-1}\right)^2 \sum_{k=1}^{N}\left(\pi\left(\sigma_k \middle| S_{lj}\right) - N^{-1}\right)^2}} \tag{2.26}$$

If $\rho(S_{li}, S_{lk} | S_a) \to -1$, then it is possible to indicate, that on the set S_a two ethical systems are in the state of conflict.

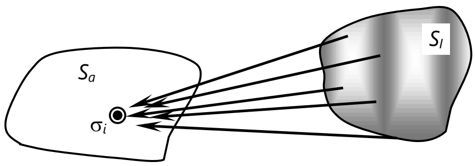


Fig. 7.5

Is it possible to count that one and the same subject is the carrier of two "ethics".

Most likely, answer is negative. Two subjects can have one and the same set of object alternatives, but be the carriers of different ethics.

In this case correlation coefficient $\rho(S_{li}, S_{lj}| S_a)$ signals about *the interpersonal conflict*. Let us assume now that $\pi(\sigma_k)$, $\sigma_k \in S_a$ — the distribution of utilitarian preferences, and $\pi(\sigma_k | S_l)$ — the distribution of the preferences of the same subject on the same set *Sa*, corrected as a result of ethical imperatives calculation from the set S_l. Let us determine correlation coefficient

$$\rho\left(\pi\left(\sigma_k\right), \pi\left(\sigma_k \middle| S_l\right)\right) = \frac{\sum_{j=1}^{N}\left(\pi\left(\sigma_k\right) - N^{-1}\right)\left(\pi\left(\sigma_k \middle| S_l\right) - N^{-1}\right)}{\sqrt{\sum_{j=1}^{N}\left(\pi\left(\sigma_k\right) - N^{-1}\right)^2 \sum_{j=1}^{N}\left(\pi\left(\sigma_k \middle| S_l\right) - N^{-1}\right)^2}} \tag{7.27}$$

In this case we can speak about the intra-personal conflict between the purely utilitarian preferences and the preferences, which consider ethical standards. Conflict occurs, if

$$\rho(\pi(\sigma_k), \pi(\sigma_k | S_l)) \to -1.$$

Similar considerations apply to other kinds of subject preferences (π^+, π^-, υ^+, υ^-) and ratings preferences ($\xi(j)$, $\xi(j | i)$, ...). Suppose, for example, $\xi(j | i)$ — the distribution of rating preferences on S_a, based on utilitarian considerations (benefit of subject *j* for the subject *i*,...), and $\xi(j| i, S_l)$ — the distribution of rating preferences on the same set S_ξ, corrected accounting ethical imperatives from the set S_l. By analogy with previous the value

$$I(S_\xi / S_l) = H_\xi(S_\xi / S_l) - H_\xi(S_\xi) \tag{7.28}$$

there is the information, related with the action of imperatives from S_I on the ratings ideas of subject „*i*". The correlation coefficient signals on the internal conflict

$$\rho(\xi(j \mid i), \xi(j \mid i, S_I).$$

If it approaches к –1, this testifies about the presence of „*conflict*", if it is close to к +1, this speaks, that the initial ratings distribution $\xi(j \mid i)$ is in „agreement with given ethics". Let now the utilitarian ratings preferences of two subjects coincide:

$$\xi(j \mid i) = \xi(j \mid k) \ (\forall j \in \overline{1,M}).$$

Then, conditionally speaking, one of them assumes Christianity (ethics „*X*"), and another - Islam (ethics „*M*"). Distribution $\xi(j| i, S_{IX})$ differs from distribution $\xi(j| k, S_{IM})$. The divergence between them is characterized by correlation coefficient

$$\rho(\xi(j| i, S_{IX}) \quad \xi(j| k, S_{IM})),$$

and the relative information, connected with a change in the „faith"

$$I(S_\xi| S_{IX}, S_{IM}) = H_\xi(S_\xi| S_{IX}) - H_\xi(S_\xi| S_{IM}). \tag{7.29}$$

In connection with all aforesaid above in this paragraph, let us make the following observation. Let the P_a is set of sets of all possible problem -resource situations, (S_I, $\pi(I)$) is a certain ethical system. The P_a can be divided on the subsets P_a^+ and P_a^-, such, that in P_a^+ such sets of the utilitarian distributions $\pi(\sigma_k)$ on S_a are contained, which do not enter into the conflict with given system of ethics, but in the P_a^- includes those subsets of the distributions, which are in conflict with given system of ethics.

The development of conflicts in entire range of entropies changes and correlations is of interest to simulate. The question appears essential, which we practically did not touched here: how is the conflict of each type resolved and what does occur after its solution?

The problem of conflicts control is practically important. It departs to the region the of games theories [39,109,110,125]. However, within the framework of subjective analysis given version, this problem acquires unique painting and promises the possibility of obtaining the new and interesting results. Obvious, in particular, there is a question which the exogenous effect on active system is required for the decreasing of entropy to the lower boundary of the „reign of the freedom" H_π^*, if at the given moment $H_\pi > H_\pi^*$. Necessary information is

$$I^* = H_\pi - H_\pi^*. \tag{7.30}$$

Required information is maximum, when system at first is found on the upper boundary of the „reign of freedom"

$$I^*_{\max} = \ln N - H_\pi^*. \tag{7.31}$$

Since $I^*_{\max} = f(R^{disp})$, it is possible to determine the necessary additional required resources ΔR^{disp}.

Another question: what exogenous actions are necessary for weakening or liquidation of the conflict?

a — internal;

b — interpersonal;

c — intragroup.

7.4. Entropy map

7.4.1. On the quantization of the preferences

The ordinal distributions of preferences appear as a special case of cardinal distributions, since in the ordinal distributions the alternatives can be numbered, and then „weighed" with a weight $S_N = 0{,}5N(N + 1)$, i.e. it can be assume that

$$\pi\left(\sigma_i\right) = \frac{i}{S_N} = \frac{2i}{N\left(N+1\right)}.$$

Assumption about the quantization of preferences is not connected with the „quantum psychology" theories, similar to this, which is presented in the works [135,148] and it is not so arrogant. Here are not used ideas about the Aristotelian „essences", Platotelian „deep realities", metaphysical „specters", and manifestations of quantum-mechanical effects on the psyche level.

Elsewhere, where we introduce the „virtual subject" or „collective reason" concepts, we also do not connect them with the above designated concepts. As the basis of assumption about preferences quantity simple and, from our point of view, appropriate „to the common sense" considerations are placed:

1. Subject can simultaneously analyze the limited number of discrete alternatives.

2. There are ranges of the indistinguishability of „cost" and, therefore, the preferability of alternatives; moreover these ranges are individualized, they are variable, depend on type and the „sharpness" of problematic- resource situation, on presence or absence of conflict situation.

3. Distinguishability makes subjective sense here, so, if the real „costs" of alternatives are $ 5,01 and $ 4,99, then subjectively they are frequently imagined as essentially different.

4. The greater the absolute value of the available resources R^{disp} in the comparison with the required resources, the greater the „step" - the indistinguishability range. The more precise estimation of the ranges of the preferences indistinguishability can be obtained by using elasticity of preferences (see Section 5.8).

Thus, for instance, on the basis of formula (5,169) we find that

$$\left|\Delta\pi_i\right| = \beta\left(1-\pi_i\right)R_i^r\left|\Delta R_i^r\right|. \qquad (7.32)$$

If $\left|\Delta R_i^r\right|$ is the range of the „indistinguishability" of the required resources level for the given subject, who possesses the resources R_i^{disp}, then the corresponding range of the „indistinguishability" of preferences is $\left|\Delta\pi_i\right|$. Analogous dependences can be obtained, also, in other cases. The alternatives, which fall into one and the same range of „indistinguishability", are received as equivalent (on the required resources) and can be related to one class of equivalence.

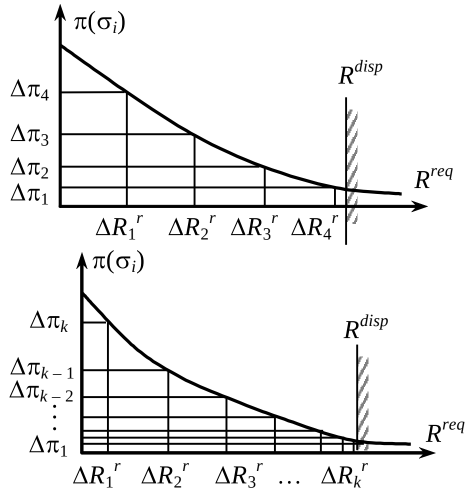
The quantized preferences introduce the intermediate class of distributions, being located between the ordinal and cardinal distributions. They can be named „*quantum distributions*". A question about, how to select the range of „indistinguishability" on the scale of required resources; have to be made additional assumptions. It is evident from Fig. 7.6, *a* that when the range of the indistinguishability of alternative with respect to the required resources is identical at any point of R^{req} axis, then for more distant from zero sections of R^{req} axis the one and the same range ΔR^{req} matches the smaller uncertainty range $\Delta\pi_i$. As the different version let us examine the assumption, that ΔR^{req} decreases in proportion to the approximation R^{req} and R^{disp} (Fig. 7.6, *b*).

Certainly, in this case in any manner must be established the boundaries of the equivalence ranges on the axis of required resources. It would be possible to make, for example, if „cost" of the most expensive alternative is considered as the upper boundary of range of indistinguishability. All alternatives, which fall in this range, relate to the first class of equivalence, lower boundary of this cashbox is upper boundary of the following class, and so on.

Fig. 7.6, *a* shows, as the ranges of equivalence on the axis of preferences change on the assumption that the ranges of resource indistinguishability are identical and they do not depend on the relationship between the available and required resources. For first type canonical distribution, the wider the ranges on the $\pi(\sigma_i)$ axis, are the smaller required resources.
Difference in the value of the equivalence ranges for $\pi(\sigma_i)$ grows, if one assumes, that the range of indistinguishability decreases, when $R^{req}(\sigma_i) \rightarrow R^{disp}$ *in accordance with assumption 4* (Fig. 7.6, b).

a

b



$$\pi(\sigma_i)=\frac{e^{-\beta R^{req}(\sigma_i)}}{\sum_{j=1}^{N} e^{-\beta R^{req}(\sigma_j)}};$$

$$\Delta R_i^r=\text{const};$$

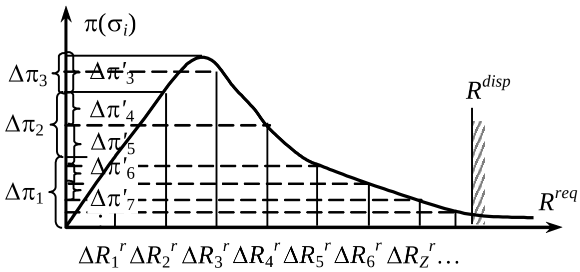
$$\Delta\pi_1<\Delta\pi_2<\Delta\pi_3<\Delta\pi_4.$$

$$\Delta R_1^r>\Delta R_2^r>\Delta R_3^r>\ldots>\Delta R_k^r;$$

$$\Delta\pi_1<\Delta\pi_2<\Delta\pi_3<\ldots<\Delta\pi_k.$$

Fig. 7.6

For second type canonical distribution the dependence between ΔR_i^r and $\Delta\pi_i$ is characterized by the decrease of ranges $\Delta\pi_i$ in the region of the distribution peak Fig. 7.7).



$$\pi(\sigma_i)=\frac{R_{(\sigma_i)}^{req^{\alpha}}\, e^{-\beta R_{(\sigma_i)}^{req}}}{\sum_{j=1}^{N} R_{(\sigma_j)}^{req^{\alpha}}\, e^{-\beta R_{(\sigma_j)}^{req}}}$$

Fig. 7.7

The quantization of preferences leads to the quantization of subjective entropies and subjective correlations and in this sense it is connected with the problem of decision making, and also with the conflicts analysis.

7.4.2. What is an entropy map?

Let us name the graph „entropy map", on axes of which are placed the entropies „combined" distributions of preference: π_i^+, π_i^-, υ_i^+, υ_i^-.

As earlier, the following designations are used: H_π^* — entropy barrier, which separates the „reign of freedom" from the „reign of need", i.e. the threshold, which must be overcome „from top to bottom" for the finding the possibility of decision making. The „reign of freedom" can be treated as the „mixing layer" or the „discussion layer".

H_π^{**} — the upper „priestess" threshold of entropy.

In the situation, when $H_\pi^{**} \le H_\pi \le H_{\pi\max}$, subject by hypothesis is situated in the stress situation - in the „Buridan stress" state.

Let us examine Fig. 7.8. It schematically reflects the conditions of the internal conflict appearance, when there are two alternatives σ_1 and σ_2 (M = 1, N = 2).

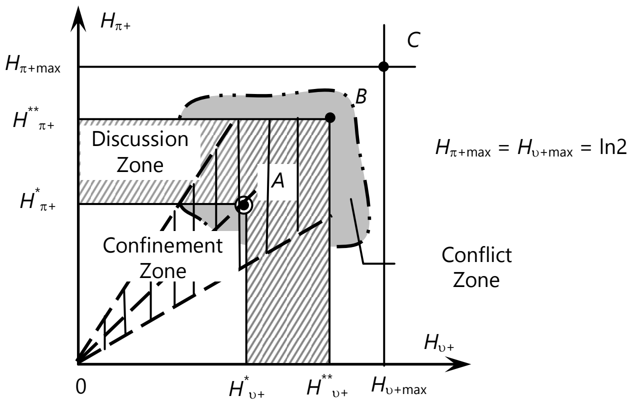


Fig. 7.8 Consonant internal conflict.

Sense of the alternatives: answer on the certain question A: σ_1 — „yes", σ_2 — „no". Distributions π_i^+ (to accept on the basis of positive information, for example, about the usefulness U_i) and υ^+ (to reject A on the basis of positive information about the usefulness U_i) are compared.

The necessary conditions must be fulfilled for decision making:

$$H_{\pi+} < H^*_{\pi+};\ H_{\upsilon+} < H^*_{\upsilon+}. \tag{7.33}$$

Both boundaries, however, are not reached simultaneously, therefore the weaker conditions of decision making

Another

$$H_{\pi+} < H^*_{\pi+};\ H_{\upsilon+} \in [H^*_{\upsilon+}, H^{**}_{\upsilon+}], \tag{7.34}$$

$$or \qquad H_{\upsilon+} < H^*_{\upsilon+};\ H_{\pi+} \in [H^*_{\pi+}, H^{**}_{\pi+}].$$

In the first and second cases the result will be different: σ_1 is in the first case and σ_2 are in the second. In spite of fulfilling of absolute inequalities the solution can be not accepted, if $H_{\pi+} \approx H_{\upsilon+}$. The zone *of consonant conflict* covers the neighborhood of points *A* and *B* and can stretch to the point O in the form the neighborhood „highway" *OA*, where the condition of the approximate equality of „combined" entropies $H_{\pi+}$ and $H_{\upsilon+}$ is satisfied. Besides the values of entropies the presence of conflict is determined by the value of the correlation coefficient $r_{\pi+\upsilon+}$.

If $r_{\pi+\upsilon+} = -1$ consonant conflict is absent. This assertion confirms Fig. 7.9. Here the entropies $H_{\pi+}$ and $H_{\upsilon+}$ can be identical and satisfy the inequalities given above, nevertheless conflict is absent, since $r_{\pi+\upsilon+} = -1$.

The situation, shown on Fig. 7.10 corresponds to consonant conflict, since the foundations for accepting *A* and rejecting *A* are identical. We will conditionally name this type of conflict consonant, since it appears on the basis of the study of one and the same information apropos a „*A* question" - usefulness based on different „points of knowledge": to accept *A* or to reject *A*.

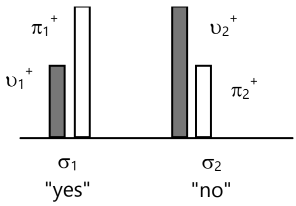


Fig. 7.9

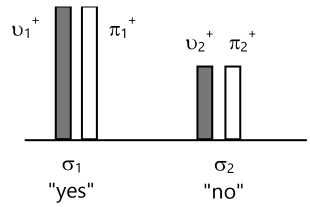


Fig. 7.10

Examine the case *of dissonant conflict*. Let be also solved a question about the selection of two alternatives σ_1 and σ_2 on the basis of the parallel study of positive information (usefulness) and negative information (harmfulness) about each of the alternatives, as a result of which two distributions of preferences π_i^+ and π_i^- appear. Fig. 7.11 shows the „Entropy map"

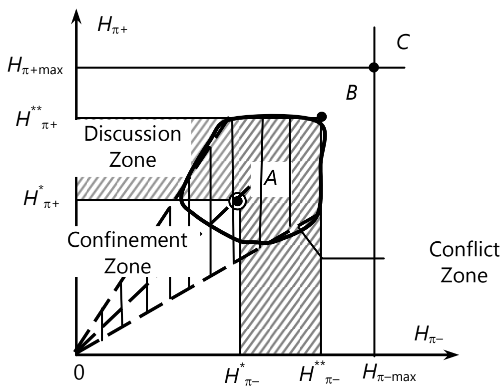


Fig. 7.11

The essence of dissonant conflict explains Fig. 7.12 and Fig. 7.13. Fig. 7.12 expose the case, when distributions π_i^+ and π_i^- coincide (or they are close), respectively the correlation coefficient $r_{\pi+\pi-}$ = +1. (This occurs, if $\pi_1^+ > 0{,}5$ and $\pi_1^- > 0{,}5$). Here conflict is absent, since both distributions testify in favor of the one alternative selection. Fig. 7.11 shows the case of dissonant conflict. Correlation coefficient $r_{\pi+\pi-} = -1$.

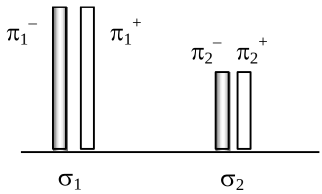


Fig. 7.12

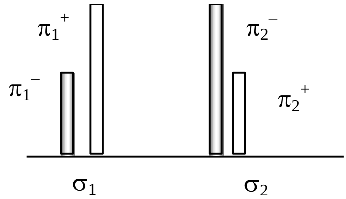


Fig. 7.13

The shaded region on Fig. 7.11 is the zone of dissonant conflict. Apparently the „residual" conflict remains also after adoption, and the zone of conflict, includes points *O*, *A*, *B*. If $r_{\pi+\pi-} = -1$, that is greater sharpness of conflict is the smaller entropies $H_{\pi+}$ and $H_{\pi-}$ are. It means that till decision making the greatest sharpness of conflict occurs at point *A*, but after decision making it have place at point *O* (if as before $r_{\pi+\pi-} = -1$).

The entropies, which form two-dimensional „entropy map" can be found in the interdependence, since in this case one and the same subject is their carrier. Consequently, the configuration of „reigns" can be more complex, what is schematically reflected by Fig. 7.14.

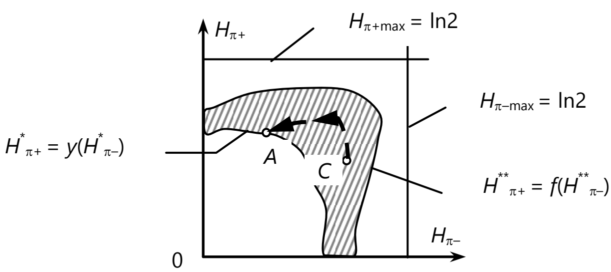


Fig. 7.14

As an example the map in the $H_{\pi+}$, $H_{\pi-}$ coordinates is shown.

Construction and analysis of „entropy maps" can prove, to be useful tool for a study not conflicts only, but also generally decision-making processes. It is possible to visualize that the „discussion" begins at point „*C*" (Fig. 7.14). Line *CA* reflects the process of wandering in the „reign of freedom". At the certain moment of time the point A on the border between the „reign of freedom" and the „reign of need" is reached, from where the passage is possible as a result of decision making. The presence of conflict can deform two-dimensional picture. In this sense it is expedient to examine three-dimensional „entropy - correlation map", where on the third axis the correlation coefficient is placed, for example, $r_{\pi+\pi-}$. The three-dimensional picture (in the case N = 2) is concluded in the parallel with the sides of ln2, ln2, [- 1, +1] (Fig. 7.15).

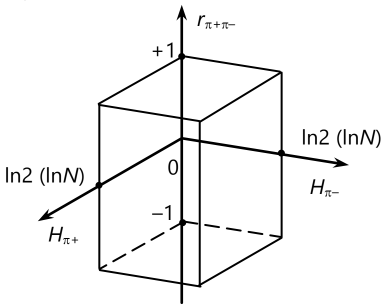


Fig. 7.15

The boundaries of „reigns" are not most likely constant, but they do change under the effect of the endogenous and exogenous processes. We cannot propose in the given moment any models relative to the changeability of these boundaries. It is possible, however, to express some general assumptions:

1. With aggravation of situation, an increase in the scarcity of resources, first of all – of time, the „reign of need" is enlarged and, on the contrary, the „reign of freedom" becomes smaller. The „poor" cannot be free. „The total quantity of freedom" in socium is distributed approximately proportional „to wealth" and is determined first of all as a quantity of solving problems, i.e. with the dimensionality of the set S_a.

2. In the calm situation, the presence of the large available resources, including the heavy time stocks and, correspondingly, the possibilities for the more weighed solutions the „reign of need" becomes smaller, and the „reign of freedom" is enlarged.

These assertions, apparently, bear relative nature. What obviously occurs to the „reigns", it strongly depends on the mental type of subject, on his ability to risk, from the presence of the restrictive or encouraging ethical factors, the relations between the subjects and between the subject and the group. From the concept of „freedom" for one subject, we should switch over to the „freedom" of group concept, „freedom", connected with the interrelations of subject with other members of group. Let us examine the interpersonal conflict, when there are two subjects and two alternatives:

$$M = 2,\ S_a: (\sigma_1, \sigma_2).$$

Let $\pi_1(\sigma_i)$ and $\pi_2(\sigma_i)$ are the preferences of the first and second subject respectively. Name conditionally alternatives "one-place", if „state" σ_i can be achieved (occupied) only by one subject (two cannot sit on one chair), and let us name alternatives „potentially corporate", if both subjects can reach each of the states σ_i not only without interfering, but perhaps helping each other, uniting their resources.

Figs. 7.16 - 7.18 represent possible situations. Fig. 7.16 reflects the consonant conflict of two subjects, who attempt to engage one and the same position (to sit down on one and the same chair). If each of the alternatives is „one-place", then consonant conflict occurs. In this case there can be $H_{\pi 1} \approx H_{\pi 2}$, and $r_{\pi 1,\pi 2} = +1$. It is obvious that the worse the conflict is, the both entropies are smaller. It is also obvious, that if the entropy at least of one of two subjects is big

$$H_{\pi i} > H^*_{\pi i},$$

than conflict cannot arise, since to this subject „all the same on which chair to sit". Thus, the necessary condition of the appearance of conflict is low (close to zero) value of both entropies.

In the situation, shown in the diagram on Fig. 7.17, in the case of „one-place" alternatives the conflict is absent, since $r_{\pi 1,\pi 2} = -1$ independently of the individual entropies value.

Let now the alternatives σ_1 and σ_2 be „potentially corporate". The conditions of conflict here are different (Fig. 7.16).

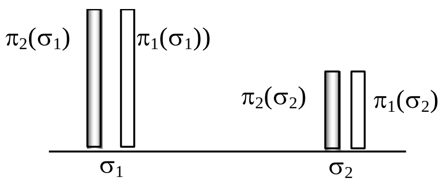


Fig. 7.16

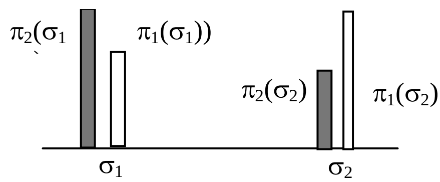


Fig. 7.17

Here both subjects give preferences to the first alternative and, therefore, are they ready to solve together the appropriate problem perhaps uniting (consolidating) their resources. Correlation coefficient

$$r_{\pi 1,\pi 2} = +1$$

and conflict is absent.

The situation, represented on Fig. 7.18 reflects the conflict situation, result of which is the failure of the joint solution of corporate problem and of the consolidation of resources in this direction.

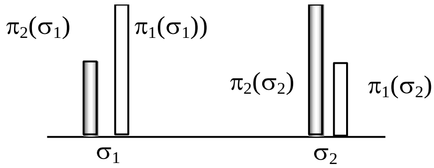


Fig.7.18

The correlation coefficient $r_{\pi1,\pi2}$ = –1, entropies can be equal. But as it way above, the sufficiently low value of the entropies is the condition of the conflict appearance

$$H_{\pi1} < H^{*}_{\pi1};\ H_{\pi2} < H^{*}_{\pi2}.$$

Let us recall that the threshold values of entropies are individualized and during the study of conflicts in the group M > 2 this circumstance must be considered. Note that in connection with interpersonal conflicts it is also possible to construct two-dimensional and three-dimensional entropy and entropy-correlation maps. Considerations the mobility of the boundaries of „inequalities" should be supplemented with observation the fact that the boundaries of passage from the „reign of freedom" to the „reign of need" and the reverse passage may not coincide. Two versions are possible:

1. Boundary of „downward" passage (Fig. 7.19, *a*) is located above boundary of the reverse passage

$$H_\pi^{*}\downarrow > H_\pi^{*}\uparrow$$

2. Boundary of „downward" passage (Fig. 7.19, *b*) is located below boundary of the reverse passage

$$H_\pi^{*}\downarrow < H_\pi^{*}\uparrow.$$

a *b*

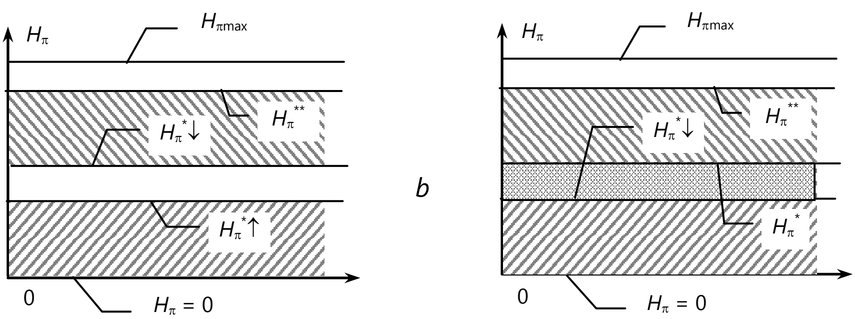


Fig. 7.19

Partial discussion of passages was given earlier.

In the conclusion let us note, that the threshold values are not only individualized, but, most likely, are distinguished for the entropies of different distributions: π^+, π^-, υ^+, υ^-.

7.5. Category „freedom" from the point of view of subjective analysis. Brief historical excursus

Since above repeatedly the term "freedom" is used as one of the categories of the developed version of subjective analysis, and "the reign of freedom" borrowed from the work [133], is connected for the sake of the subjective entropy, it is necessary to attempt to find the connection of the concept "freedom", utilized here, for the sake of the treatments of this category in different philosophical systems.

Let us conduct the brief historical excursus, subordinated only to one task - of finding any correlation of the general philosophical treatments of „freedom" with that sense, which is packed beside this concept in this work. In this case we in no account intend to develop or to improve the appropriate division of philosophy.

Propriety of this excursus to us is obvious, since it is clear that the study of many alternatives, which contain is more than one alternative, and the possibility of selection is combined for the sake of a priori assumption about the presence of this possibility.

We will follow in essence of monograph [177].

Category "freedom" was born together with the philosophy. Problem as a whole was realized in Ancient Greek philosophy; moreover the main attributes of the freedom were immediately picked out:

1. Responsibility, voluntariness, the freedom of will.
2. Freedom of choice and solution between the different possibilities.
3. Independence of action.

Further history of philosophy - these are interpretation and the development of these concepts, complication, unbinding the kinds, caused as far as semantic uncertainty, accumulated contradictions and appearance of new concepts. All this up to the last time occurred and occurs against the verbal level.

The study of problem as a whole attests to the fact that at the present times the treatment of category „freedom" not only did not reach perfection, but it became more tangled and indeterminate. The categorical divergence of series of philosophical directions and schools for this question testifies about this.

Let us examine in the historical context the basic stages of the development of category „freedom".

In ancient Greece they knew the concepts „free selection" and „will", synonym of which was autonomy - the internal independence from the internal and external factors (Antisthenes, Diogenes).

Socrates separates the social and political and individual- subjective aspects of freedom. One of the first models of totalitarian state is contained in the work of Plato „state".

He asserts that in the democratic state the freedom treats as „greatest wealth", which frequently leads to the oblivion of other basic values: excessive freedom not in that another is converted, as soon as beside *„exorbitant confinement both for the man and for the state*".

Greeks distinguished three types of the political system: monarchy, elite aristocracy and democracy. Plato considered that the elite aristocracy easily can become oligarchy. Aristotle in the work "Politics" separates freedom individual and social political. „Democracy it is the organization, in which to the greatest degree are realized the bases of equality". By the basis of democracy is establishment the protection of freedom. Aristotle considered that democracy undergoes the deformation, when it is governed exclusively by the interests of majority, but not by the good of all.

Freedom cannot enter into contradiction with law and order, is necessary the superiority of right. Political activity must be based beyond the morals and beyond *the presence of free time*. In the „Three ethics" Aristotle analyzes individual- moral freedom, the „free selection" - concept close to the Christian „liberum arbitrum" and referring straight to the „will freedom" concept that exists in the later theories.

The „will freedom" correlates with the responsibility both for the choice of the alternative and for the realization methods of the corresponding problems. Another antique concept of freedom is the part of the philosophy of Stoicism, representatives of which were Seneca, Epictetus and Mark Aurelius. Stoics assumed that there is „ecumenical freedom", the material carrier of which is a human. Most valuable in the human is the „free will".

Freedom is not the same, that permissiveness, willfulness. *„Reasonable freedom" requires* the knowledge of the laws of nature and laws of public life, feeling of responsibility. *This is sufficiently close to the concept „freedom as the recognized indispensability"*.

Epictetus distinguished *„internal freedom*" and *„external freedom*". This idea has historical continuation up to our days. Thanks to the freedom of will the human is capable to achieve a selection between the good and the evil. To a question „how to reach the internal freedom?" Epictetus answered: *to be subdued with yours own fate and do not to desire anything*. Because of the internal freedom human does not appears a slave to nothing and to anybody.

The things, which do not depend on the human, are ethically neutral things.

The Cartesian philosophical system, to which R. Descartes, Kartezius, E. Kant is referred also, treats the „freedom" category. One of the basic theses: „Cogito ergo sum" (I think it means I exist) moves starting point from the of external world into the internal world. This is to a certain extent consonant with initial idea accepted in our concept, that any preference (as experience), any alternative as subjective concept has its personal carrier and it, therefore, belongs to internal world.

Here also as in the works of antique philosophers the freedom of will is the fact, which indicates the sublimity of human and his similarity to God. The opponents of the will freedom theory refer to the fact of fluctuations, doubts, which human experiences, preparing to make a decision. Kartezius asserted that, on the contrary, the ability to doubt confirms existence of freedom. In our work in the simulation tasks in chapter 5 we made the attempt to model fluctuations, using attractors.

Emmanuel Kant in the „criticism of clean reason" attempted to coordinate achievements in the field of natural sciences and new image of material world with the acknowledgment of the exclusiveness of human and his autonomy in this world.

From the Kant's point of view the exclusiveness of human is concluded in his freedom. He separated in the human existence unique duality - dichotomy: the totality of external manifestations - *Homo phenomenon* and phenomena of psyche - spiritual - *Homo noumenon*.

If *Homo phenomenon* is an object of sensual, empirical perception, then *Homo nouomenon* - is the „personality", which possesses freedom and independence from the surrounding world, free in its activity. *Indispensability* and *freedom* coexist in the human. Kant's views are based on the antagonism and the transcendent idealism: phenomena, the phenomenon of the external world - figment, the arbitrary creations of the human imagination: true nature of things is unrecognizable. He separated two types of the knowledge: „*clean reason*" and „*practical reason*".

The first is connected with a priori ideas; the second is subordinated to moral acts. „Freedom" is not subjected to empirical verification and it is possible to speak about its existence in the same way as about the transcendental idea. Does any reality answer to this idea? This question is connected with previous statement.

Kant called transcendental idea „heuristic figment". The general human tendency – according to Kant, is a tendency to the realization of moral goods. In connection with this there is an internal motive: make good. This is one of *the categorical imperatives* of Kant. Postulates *of the practical reason*:

- the freedom of will;
- the immortality of soul;
- existence of the God

are not theoretical dogmas, but there is a unique hypothesis of knowledge.

From the Kant's point of view the freedom has two aspects: *negative and positive*. The first is understood as independence from laws and forces of material nature. The selection and the solutions *of free will* are not caused by nature, but they are subordinated by the postulates of moral right and exceed the scope of empirical experience. *The positive* aspect of freedom consists in its independence: *it is a law in self for itself*. The real freedom of human is the autonomy of his will. Freedom is the fundamental value, the authority of intellect. The autonomy of will - this is the only principle of all laws of morals.

The Kantian concept of freedom is not a concept of anarchism. „People are the competent members of the kingdom of morality, possible because of „freedom"". This thesis agrees with the term „reign of freedom". Freedom of will is nothing more but the definition of the will through the laws of morals.

Kant examined the problem of the social and political freedom: human - entity reasonable and free; therefore is the „goal" and it never must treat as instrument for the realization the of the „strangers" goals. To the basic criteria of state he refers: the respect of the human independence, the acknowledgment of law, concordance with the standards of morals. This concept is not conformed to the concepts of utilitarianism and political makiavelism.

The criteria of state are evinced more accurately by the following assertions:

1. Each member of association must be free.
2. All citizens must be equal as participants in the public processes.
3. Each citizen has right to the independence.

Based on the positions of contemporary experience and understanding these postulates cannot be accepted as a system. It is clear that they contradict each other and present very rigid and practically never realizable ideals. Kant's observation is curious about the fact that „human is the entity, which requires master", which is absolutely correct, but it is in contradiction with the requirement of autonomy. As a whole the concept of Kant's „freedom" is contradictory and can be qualified as anthropological dualism.

The problem of „freedom" is the central concept of the classical liberalism, creators of which are Hobbes, Locke, Hume, Mill, Rousseau, Montesquieu. In the Hobbes interpretation „freedom" in the Ancient World was understood not as the freedom of personality, but as the freedom of state. „The freedom of personality" is the later product of Christian ideology and morals.

Hobbes, as Kant, distinguished *the negative* and *positive* aspects of freedom, postulated the opposition between *the freedom* and *the coercion*, but not between *the freedom* and *the indispensability*. This was step in the side of the future understanding of freedom in the materialism as gotten to know indispensability. The roots of this interpretation are located in the antique and medieval philosophical systems.

Negative freedom in the Hobbes sense is understood as the absence of obstacles to the individual actions. The *positive* freedom possesses that human, which can make what he wants. The concept of freedom in the Hobbes system was naturalistic, defining human as „the living body, which has sensations, reasonable".

Among the philosophers *of the classical liberalism* most consonant to the constructions, carried out in the present work appears the concept of „freedom", proposed by English philosopher - empiricist Locke. He, like Hobbes allowed for the co-existence *of negative* and *positive* freedom, however, with the deviations in the interpretations. Let us pause in greater details on the Locke [177] views.

„... For the creators of the liberal- individualistic theory of conditional ism" the thought is characteristic, that... on the dawn "... humanity occurred the pre-public, or „natural" period.

Natural „freedom" is an absence of dependence on any public authority and insubordination in the bases of existence of any human of another will or authority, except *„the natural law"*. In connection with this „the contemporary human is born with the right of complete freedom and unlimited use of all laws and privileges *of natural right* in the same degree as all other people on the earth".

The type of freedom, besides the freedom of the natural state, is social and political freedom. „The freedom of human in the association is reduced to nonsubordination of any other legislative authority, except that, which is established as a result of public agreement". Is distinctly visible incompleteness, internal discrepancy in the concept of freedom and, as a whole, utopianism from the point of view of its actual realization? The same relates also to the previously examined concepts. Further however, Locke recognizes that the establishment *„of state"* lies in the fact that each citizen rejects the part of his natural rights and transfers them to „elites" of leaders.

Locke separated in the power structures *„authority legislative"* and *„authority executive"*. *In the terms of subjective analysis the conscious refusal of citizen from the part of the rights can be treated as the exception of the part of the alternatives from individual sets S_{aj}, either their transfer in the category of the corporate alternatives, on which decisions are taken collectively or by leadership elite, as the transfer of the part of resources under the jurisdiction of „authority".*

Locke was not the worshipper of anarchism, he considered only that *„the purpose of law is not the abolition or limitation, but retention and an increase in the freedom* - where is no *law*, there is no *freedom*. A deficiency in the law opens road to the lawlessness, the coercion, and the abuses". Locke interpretation of freedom is ambivalent. *„The classical understanding of freedom is reduced to the assertion, that it is ability of selection of one from set of alternatives"*. This very tightly correlates with understanding of freedom in our version of subjective analysis.

Locke, however, experienced fluctuations relative to the classical interpretation of the freedom of will. *„Freedom does not consist in the selection of the specific possibility"*. Freedom can be understood as ability, internal force to govern own desires. In the idea of „freedom" Locke saw the element of psychological determinism.

„From the one side he recognized existence of freedom as the ability to act, or to abstain from the action, from another side asserted that since human was forced to make the specific selection, he thus no longer is free. *He cannot avoid the acts of the manifestation of will; He cannot avoid desires, since they are the consequence of natural needs. The opposition of freedom is internal coercion."* The freedom being understood positively is the ability to manage self emotions and desires.

To be free - this means to decide freely, in accordance with own conscience, to govern own actions with the observance of laws. This does not indicate failure from its responsibilities. Locke went beyond the framework *of the negative-juridical* understanding of freedom and recognized *positive - moral* freedom.

Rousseau and Montesquieu represented „continental liberalism". The problem of freedom was also the main issue in their philosophical systems. Rousseau, for example, distinguished three types of the freedom: natural, public and moral.

To the pantheistical flow in the philosophy relate Spinoza, Fichte, Shelling, Hegel and partially Schopenhauer. Spinoza confessed naturalistic pantheism, which recognizes existence of the „God" and of „Nature"; however, asserts their indissoluble unity in a determined sense of identity. Their functions are divided, they form, speaking in modern language the two-step hierarchical system, beyond upper level of which the „God" is located. He is the creator of „Nature" - *natura naturans*.

On the lower level stands „Nature" - the object of creation - *natura naturata*. From this follows the interpretation of freedom category as the freedom of God and freedom of human. The freedom of human does not have the external reason for its existence, i.e. it is the original attribute of man and, therefore, the absolute.

From the classical philosophy Spinoza borrowed the identification of personal freedom with the auto - definition of individual activity and connected *the categories of „freedom" and „indispensability"*. In this case he separated two varieties *of the indispensability*: *„free indispensability"* and *„forced indispensability"*.

The freedom of human is „free indispensability", whereas coercion is „forced indispensability". As we see in this model „*indispensability*" stands above and generates „*freedom*". Spinoza distances himself from the classical understanding of „freedom" as *the freedom of choice of one of many alternatives*. Spheres of knowledge and desires - this is one and the same. „Will and reason are identical".

It is a mental phenomenon being rested on the desires. „The will of freedom" Spinoza considered figment, imaginary thing.

As it is possible to note, there are certain close interpretations in pantheism and classical liberalism, and also discrepancy and incompleteness of both concepts.

Correlation with the concepts, used in the present work, consists in particular in the fact that the appearance of the desires is actually necessary, but desires can become preferences, if they are compared with „external" circumstances, which become alternatives, being compared, in turn with the desires. The logical component, which connects „external possibilities" with the desires – they are the „preferences".

It is possible to attempt to coordinate the model of subjective analysis with one of the theses of classical liberalism about the fact, that the „freedom" includes in itself also freedom *„not to make any selection"*.

For this it is sufficient each time in alternatives set S_a to provide one more additional special „alternative" σ_e: „not to make any selection among the meaningful utilitarian alternatives". Alternative σ_e would correspond to the absence of any desires. To singular distribution

$$\left.\begin{aligned}&\pi(\sigma_i) = 0; \forall i \in \overline{1,N}\\&\pi(\sigma_e) = 1\end{aligned}\right\}\dim S_a = N + 1$$

corresponds the entropy $H_\pi^{(e)} = 0$, i.e. complete certainty occurs. Alternative σ_e would be possible to define by the conditions

$$\sigma_i \wedge \sigma_e = \sigma_e; \;\; \sigma_i \vee \sigma_e = \sigma_i.$$

Under this hypothesis the internal contradiction is also contained: not to desire desires are also unique „desire" is the certain act of will. This is actually the inherent right of human and the element of his internal freedom.

It is necessary to note that the „desires" are formed not only by internal factors as the result of natural needs, but also, to a considerable extent, by the external circumstances. Where is the boundary between „internal freedom" and „external freedom", or in the Spinoza terminology - between the „freedom of God" and the „freedom of human".

It is possible, in our opinion, to draw newer to understanding of this issue being based on the developed approach to the subjective analysis. Commanding are in this respect the views of another representative of pantheism Fichte, which are reduced, in particular, to the fact that „the freedom - is the capability of control the future".

Hence it follows that „freedom" is the dynamic concept, which can be adequate defined only in the context of development as a process. Assertion, that human is his own „creator" - this understanding of „autocreation" although is painted with the pantheistical perception of reality, correlates well with the point of view that human as *the basic element of each active system himself* forms his desires and problems and every goal from outside converts to his own problem. The important position of the Fichte system is also the place, allocating by him to the public component of the human consciousness, understanding the impossibility of individual isolation from the society.

We already said that such fundamental notions as ethics, freedom, and justice do not have a sense out of the social organism. Hegel was under the strong influence of Spinoza, perceived and developed, tightly interconnected, concepts *of freedom* and *indispensability*. At least, the terminologically operable in the present work concepts of the *„reign of freedom"* and the *„reign of the need"* are directly connected with the appropriate concepts.

In the Hegelian concept of „freedom" the „absolute freedom" and the „freedom of human" are distinguished, what is close to the ideas of Fichte. One considers, that in Hegel's system it acknowledges:

the freedom, as the possibility of selection from many alternatives, achieved by the mental authority, which stands higher than the intellect is knowledge as well as. the freedom as the definition of himself, achieved by reasonable way. It connects the first with the spirit of the absolute. difference and contrast of the „individual will" and the „general will " occurs. Last concept is solidary with the concept of „virtual subject".

The Hegelian concept of freedom is rational, it connects the realization of freedom with the functioning of the intellect: *„The consciousness, which thinks... is self-knowledge of freedom"*, *„ignorant is not free"*. Understanding, reason open doors into the „reign of freedom". Hegel also uses this term. We give to it the sense, designated above in chapter 5. In the historical plan the next step in the study of „freedom" category was made by Marxism. The idea of „freedom" was one of the central notions of Marx's concept, in which three aspects are distinguished: phenomenological, ontological - anthropological and public - philosophical.

The freedom of socium he placed above individual freedom, assuming that the realization of the latter is possible through the public freedom. Like Hegel he presented „freedom" as dynamic category. The realization of freedom today, must not prevent, decrease the freedom of the future generations.

He connected the realization of the freedom of entire association and each individual with the Communist arrangement of society.

Marx understood „freedom" as the natural attribute of human.

Following the concept of Hegel, Marx and Engels asserted that *„the freedom is the realized indispensability"*. We saw that understanding freedom and indispensability as the tightly interconnected categories occurred also in the earlier, pre-Hegelian philosophical systems.

In the last assertion essential for us is the fact that the dynamic nature is reflected in it: knowledge is a process and, consequently *freedom*, in this case, follows *indispensability*.

The philosophical systems of the past century include „philosophy of life" (Nietzsche, Bergson, Dewey, Peirce), phenomenology Sellars, Heidegger, Ingarden). These directions are one way or another connected with the attraction of God to the development of „freedom" category. This relates also to existentialism (Kierkegaard, Sartre, Marcel, Jaspers) and to neothomism (Maritain, Velte).

Most important from the point of view of problem stated in the beginning of paragraph: to look through the prism of different philosophical concepts of freedom against the ideas about the freedom operable in the present work and its connections with the entropy of preferences, appears the concept of neoliberalism, which present first of all Hayek, Friedman, Berlin, Novak.

In the Hayek work „Constitution of freedom" [170] basic conditions of Anglo-American type utilitarian liberalism, which differs from the so-called „continental liberalism" are given:

1. Admission of the need for changes in the social and political life.
2. Negation of the excessive control of state.
3. Admission of the spiritual proximity of conservatism and nationalism.
4. Admission of fact that the supporters of conservatism attempt to impose their model of civilization.

The egocentrism of liberal model is expressed in the concept, according to which individual activity is the consequence of external needs and tendencies and, as a result, leads to the competition. This interpretation excludes the idea of the „public goods", and leads to the extreme individualism. Hayek sees in the human not the social being, but egocentric individual, he separates personal freedom from the social and political freedom. Speaking about "*the internal freedom*" and "*the external freedom*", Hayek; however, assumed that there is only one form of freedom, which is manifested in two ways.

Following tradition, it separated *negative freedom* and *positive freedom*. He identified the first with the absence of coercion from the side of others, and positive freedom traditionally connected with the possibility of selection among several alternatives. However, Hayek considered *negative freedom* as the most fundamental. Relating critically to the idea of internal freedom in this form, as it was understood by stoics also in the Christian tradition, he connected „*negative freedom*" with the absence of external obstacles or coercion in the realization of individual goals.

It is possible to say that so understood „freedom" relates to the stage, following after decision making, i.e. in the conditions, when the entropy $H_\pi < H_\pi^*$ (smaller than the threshold H_π^* is lower boundary of the „reign of freedom" or the „layer of discussion"), and at the same time, realizes until making decision and consists in the possibility to form sets of alternatives on the basis of the individual desires and to distribute their preferences on these sets.

„Negative freedom" is connected with the personal responsibility: the „greater" this freedom is the greater personal „*responsibility*". „*The responsibility*" is in turn connected directly with the „*indispensability*". Coercion is always considered the evil, which cannot be justified. It is difficult to agree with these theses. History and daily private life refute it. In this connection neoliberalism considers that state law limits „freedom", but it is not coercion. Justifying the value of neoliberalism, Hayek even alternative of the hunger of worker and his family does not consider coercion that contradicts his assertion about existence of moral coercion.

Neoliberalism has well-defined understanding of democracy, where the main role is removed to individual freedom. If individual freedom is not provided for, democracy „*degenerates into the demagogy*".

Discussion, however, of this question is not our purpose. The „Constitution of freedom" examines a question about the relationship between the „freedom" and the admission of ethical standards.

This question also lies within the limits of interests and possibilities of subjective analysis. Speaking about the moral bases, Hayek asserted that they are not the creation of human intellect, moral values are the result of agreement and undergo social-historical evolution. As we see, here again the „dynamism" of basic categories presents.

Assuming that the normative ethics does not disrupt the freedom of human, we allow, however, the possibility *of conflict* between the „freedom" and a strict faithfulness to the postulates of morals. This point of view is important, since it justifies the examination of category „freedom" in the chapter, dedicated to conflicts, and reflects the close connection of „conflict" category with the „freedom" category.

One of the main representatives of neoliberalism Friedman, Nobel prize winner, in the works „Capitalism and freedom" and „Freedom of choice" proclaims as the starting idea *the utilitarianism*, based on the empiricism, positivism and pragmatism.

The widely understood *utility* and *extreme individualism* is principal criterion. Friedman connects „*freedom*" with the social relations, actually the discussion deals about *the political freedom*, which is defined as the absence of the coercion of given subject from the side of others, moreover by the condition of the freedom of subject is freedom economic.

The latter, is determined by the presence of resources (on which depends, by the way, both the „size" of alternatives set S_a and the level of subjective entropy).

In the works of Berlin „Two concepts of freedom" and „Four essays about the freedom" the basic concepts of the freedom are compared, which were discussed above:

Kant - *freedom* - subordination to the standards of morals;

Mill - *negative freedom* - the possibility of the realization of own desires;

Montesquieu - *freedom* - the possibility to do what should be done;

Rousseau - *individual freedom* must be subordinated *to the general freedom*.

It is assumed in all these postulates that *the freedom* is incompatible with the possibility to create evil. Berlin in contrast to the mentioned philosophers assumes that the real *freedom* must not exclude alternatives, *both the good and the evil*.

The given brief survey of the basic philosophical treatments of „freedom" category is far from complete. Nevertheless, it makes it possible to conclude that understanding „freedom" term and, in particular, use of concepts *„reign of freedom"* and *„reign of need" basically do not contradict the well known beliefs*. In certain cases there is „single-valued" correspondence. Thus, the definition of the Lewicki's freedom directly falls on the scheme, accepted in this work: *„preference"* is a consequence *of „desire"*, the divergence of preferences assumes „selection", and the selection of goal assumes action on reaching it.

From the other side, is the incompleteness of the division of philosophy, concerning the development of the concept of freedom is obvious. The author does not undertake to defend here either one or another concept or to create the new model of „freedom". It seems, however, that view on the described interpretations of this category through the prism of subjective analysis, attempts to formalize the use of utilitarian and ethical distributions of preferences, possibilities of the analysis of dynamics can prove to be useful both for the general philosophical analysis and for the development of the methods of subjective analysis.

In connection with the given survey it is possible to place number of questions. It seems, that with the attempt to classify different „freedoms" each time the questions must be answered:

Freedom whom (what)?

Freedom from whom (from what)?

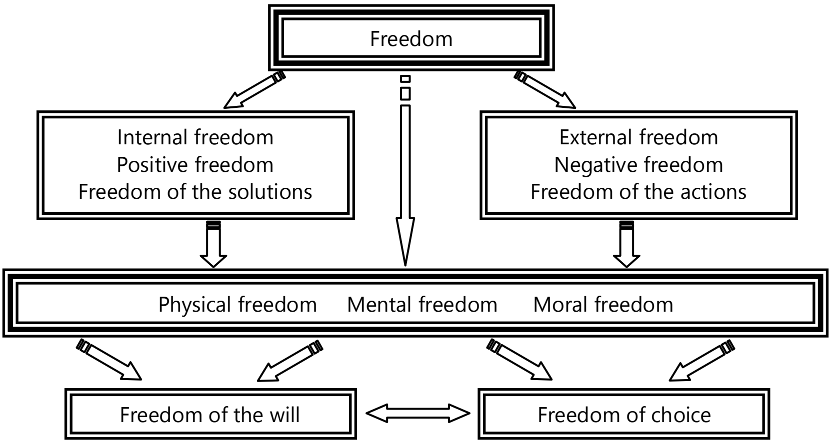
Freedom for which, with what purpose?

Freedom - by what means?

Figuratively speaking, it is necessary to „decline" word „freedom". Which connection exists between the „freedom" and „law" concepts, where is the „boundary" between the „freedom" and the „indispensability", how the „freedom of will" is connected with the intra-personal and interpersonal conflicts, how to express in the terms of subjective analysis the difference between „freedom" and

„democracy"? as a conclusion let us present simply the enumeration of some basic concepts of the „freedom", which are reflected in the tabele .

This tabele does not reflect the logical connections between the concepts, although it is possible to visualize to our self, for example, following interpretation: if we would understand „internal freedom" *the* „freedom of choice" *of the tested alternatives (hypotheses), the right to distribute preferences and make a choice certain alternative, then should be referred as an "external freedom" the presence of the possibilities to realize the tuned selection.*



Then „*internal freedom*" is characterized by the fact of how „ethics" limits the possibilities of selection, i.e., by the relationship of the utilitarian and ethical preferences

$$\pi(\sigma_1), \pi(\sigma_2), \ldots, \pi(\sigma_N) \text{ and } \pi(I_1), \pi(I_2), \ldots, \pi(I_L)$$

and also the past preferences.

„External freedom" determines the final distribution the preferences of considering ethical limitations and the distribution of objective probabilities of solving problems on the set of alternatives S_a:

$$\pi(\sigma_1, S_l), \pi(\sigma_2, S_l), \ldots, \pi(\sigma_N, S_l) \text{ and } p(\sigma_1), p(\sigma_2), \ldots, p(\sigma_N).$$

Here is an analogy with the physical understanding of „freedom", for example, as „the number of degrees of freedom" as the presence of physical limitations.

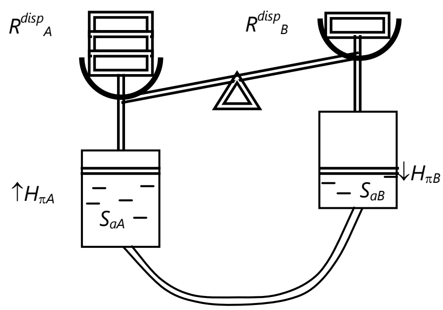
„Absolute freedom" and „free will" does not exist. Freedom is always limited, as the motion of mechanical system is limited by natural constraints on the range of change in the parameters, as well as by additional constraints, assigned in the particular task. Today we practically see any problem as the control problem.

External controlling action makes sense of coercion. We already mentioned about mental compensation being understood as the completion of a deficiency in the external physical freedom (or the selection), via the finding of internal „freedom", or more accurate - use of internal freedom for the spiritual reassignment.

So the slave of the „terrestrial" owner, voluntarily returns his „soul" to „celestial" owner („God's Slave") and, therefore, he takes away something from the first and he becomes spiritually free from him. In conclusion let us express the following consideration. The approaches of subjective analysis suggest, that the dimensionality of the object alternatives set S_a is the wider, the greater the available resources (R^{disp}) are. This relates both to the individual and to the group of individuals. The wider the set S_a, is the greater opportunities (freedom) of selection subject have. One can say descriptively that „a quantity of freedom" is proportional to the quantity of resources (wealth), which always, in any group, in any socium is limited, therefore, and *„a quantity of freedom" is always limited*. If the discussion goes on about socium, then it is possible, for example, to indicate that its "*freedom*" is limited to protect from the action of the uncontrollable forces of nature. If we have in mind separate individual, then „his wealth" determines his „*internal*" and „*external freedom*" to select objectives and to reach them.

The maxim asserts itself: *„poor cannot be free*", and what is more - possess „*external freedom*".

Fig. 7.20 represents redistribution of „freedom" between two subjects *A* and *B* as the consequence of the redistribution of resources. Sets S_{aA} and S_{aB} are depicted in the form of coupled vessels, suspended from the trays of the scale, at which the available resources are located. An increase in the resources *A* due to *B* leads to an increase *in the relative* freedom *A*.



$$R^{disp} = R_A^{disp'} + R_A^{disp} = \text{const}$$

Fig. 7.20

The absence of „external freedom" necessary leads to contraction of „*internal freedom*"; moreover the realization of one's poverty is not equivalent to the sensation „freedom as the known necessity". The problem of the freedom distribution tightly adjoins „the theory of collective welfare", which was discussed above [113]. Since the subjective analysis makes it possible to connect the theory of utility with such a subjective concepts as „freedom", it is possible to interpret the conditions of this theory from the „freedom" guarantee point of view as the collective selection by considering „freedom" the essential component of welfare.

For example, it is possible to correlate the Pigou - Dalton principle to the problem of „distribution of freedom" in socium, which usually refers to the utilities allocation problem:

Each deed of conveyance of „freedom" from *A* to *B* must not leave less „freedom" for *A* than it is expected for *B* in the result of transfer.

This is also a component of social justice in the sense of Pigou - Dalton.

From the subjective analysis point of view it is convenient to speak about „subjective freedom" and „objective freedom" - the inaccurate analogs of the „internal freedom" and the „external freedom". In this sense it is clear that the characterization of „freedom" through the preferences, the subjective entropies and the correlations, opens way to the analysis of „freedom" as the information category. There is special interest in the study of „freedom" concept in the dynamics.

The task of further formalization of different concepts of freedom in the terms of subjective analysis is important and exceptionally interesting.

7.6. Evolution of the conflicts

In this paragraph the results of the simulation of conflict evolution between subjects are given, when there are two alternatives S_α: (σ_1, σ_2) and conflict is developed between two subjects, i.e. S_ξ: (Σ_1, Σ_2). The relaxation model, presented in point 5.13.3 is used as the simulation model, which describes the dynamics of preferences. The preferences of the *j-th* subject, are the solution of system of equations

$$\frac{d\pi_j(\sigma_1)}{dt} = -k_j\left[\pi_j(\sigma_1) - \pi_{j\mathrm{opt}}(\sigma_1)\right]; \tag{7.35}$$

$$\pi_j(\sigma_2) = 1 - \pi_j(\sigma_1);\ (j \in \overline{1,2}).$$

Values $\pi_{j\mathrm{opt}}(\sigma_i)$ are defined as canonical preferences and are functions of resources.

Let at the beginning the canonical distributions depend on the ratio of the expected resources $R_j^{exp}(\sigma_i)$ to the required resources $R_j^{req}(\sigma_i)$ that is to the expenditures:

$$\pi_j^+(\sigma_i) = \frac{e^{\beta_j \bar{r}_{ji}}}{e^{\beta_j \bar{r}_{j1}} + e^{\beta_j \bar{r}_{j2}}}, \tag{7.36}$$

where $\bar{r}_{ji} = \left(R_j^{\exp}\left(\sigma_i\right)\right)^{-1}\left(R_j^{req}\left(\sigma_i\right)\right)$ in this case plays the role of the utility of the corresponding alternative.

Besides preferences $\pi_j(\sigma_i)$ ($j \in \overline{1,2}; i \in \overline{1,2}$) the relative entropy of each subject is calculated

$$\bar{H}_j = -\frac{1}{\ln 2}\left(\pi_j\left(\sigma_1\right)\ln\pi_j\left(\sigma_1\right) + \pi_j\left(\sigma_2\right)\ln\pi_j\left(\sigma_2\right)\right), \tag{7.37}$$

the correlation coefficient of the subject's preferences

$$\rho_\Sigma = \frac{\left(\pi_1\left(\sigma_1\right)-0,5\right)\left(\pi_2\left(\sigma_1\right)-0,5\right)+\left(\pi_1\left(\sigma_2\right)-0,5\right)\left(\pi_2\left(\sigma_2\right)-0,5\right)}{\sqrt{\left(\left(\pi_1\left(\sigma_1\right)-0,5\right)^2+\left(\pi_1\left(\sigma_2\right)-0,5\right)^2\right)\left(\left(\pi_2\left(\sigma_1\right)-0,5\right)^2+\left(\pi_2\left(\sigma_2\right)-0,5\right)^2\right)}}, \tag{7.38}$$

and also the values

$$K_1 = \rho_\Sigma\left(1-\bar{H}_1\right)^\delta\left(1-\bar{H}_2\right)^\delta; \qquad K_2 = \rho_\Sigma - K_1, \tag{7.39}$$

which can be considered as the indices of the conflict „sharpness", if it occurs, or as the exponent of concordance, if conflict is absent? Exponent δ is selected basing on the condition, so K_1 (and K_2) would possess a sufficient sensitivity to entropies change. The mentioned values vary within the limits

$$0 \le \bar{H}_j \le 1; \quad -1 \le \rho_\Sigma \le +1; \quad -1 \le K_1 \le +1; \quad -1 \le K_2 \le +1.$$

If both distributions of preferences are singular and both subjects with complete certainty prefer one and the same alternative, then

$$\bar{H}_1 = \bar{H}_2 = 0; \ \rho_\Sigma = +1; \ K_1 = +1.$$

In the case if preferred alternative is „single-sited ", a maximally „sharp" consonant conflict occurs. When the distributions of preferences are not singular, but the order of preferences in both subjects is identical, conflict occurs, but it is not maximally stressed ($K_1 > 0$):

$$\Sigma_1: \sigma_1 \rangle \sigma_2; \ \Sigma_2: \sigma_1 \rangle \sigma_2.$$

If in the case of „single-sited" alternatives the order of preferences proves to be opposite:

$$\Sigma_1: \sigma_1 \rangle \sigma_2; \ \Sigma_2: \sigma_1 \langle \sigma_2,$$

than $\rho_\Sigma = -1$, and $K_1 < 0$.

With the complete agreement (concordance) $K_1 = -1$. An index K_2 is used conveniently in the case, when one of the alternatives is corporate. Then the proximity of the entropies $\bar{H}_1$ and $\bar{H}_2$ values to one indicates the high degree of the subject's uncertainty and uncertainty in the choice of alternative. As it was noted earlier, in the state of proximity to the indifference or the uncertainty, the time for decision making can prove to be large. This situation can be treated as cognitive discord of one or both subjects, respectively internal conflicts, of which are transformed into the intersubject conflict.

If at least one of the subjects experiences fluctuations, indecision, then his entropy $\bar{H}_j$ is close to 1, and K_2 respectively approaches either to +1, or to –1 depending on that, coincide or not the orders of preferences which are little different in quantitative sense.

Index K_1 in this case is close to zero. $K2 \to +1$, if $\rho_\Sigma = +1$, and $\bar{H}_1$ or $\bar{H}_2 \to 1$. In this case it is possible to speak about the consonant intersubject conflict, on basis of which underlies the dissonant auto - conflict (internal) of one or both subjects. If $K \to -1$, then with the same conditions we will consider that the dissonant intersubjective conflict occurs, caused by the internal dissonant conflict. Some results of the dynamics of conflict simulation with above-indicated assumptions are given further.

Fig. 7.21 presents the case, when required resources for the first subject grow slowly:

$$R_1^{req}(\sigma_1) = 8(1 + 0{,}001t); \quad R_1^{req}(\sigma_2) = 4(1 + 0{,}001t),$$

and for the second subject they remain constant $R_2^{req}(\sigma_1) = 8 R_2^{req}(\sigma_2) = 4$. The expected resources (the „prizes") are constant, but for the first and second subjects they are different

$$R_1^{exp}(\sigma_1) = R_1^{exp}(\sigma_2) = 9; \quad R_2^{exp}(\sigma_1) = R_2^{exp}(\sigma_2) = 12.$$

The orders of preferences coincide: for Σ_1 and Σ_2: $\sigma_2 \rangle \sigma_1$, i.e. alternative σ_2 appear more preferable. If it is the „single-sited", then conflict occurs.

Fig. 7.21 the following designations are introduced $\pi_1^+(\sigma_1) = x_0$, $\pi_2^+(\sigma_2) = x_2$, $\pi_2^+(\sigma_1) = x_1$, $\pi_2^+(\sigma_2) = x_3$.

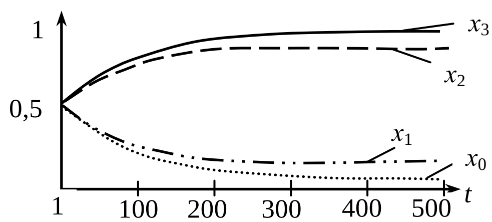


Fig. 7.21

The process of changing the preferences begins from the state of the complete indifference: $\pi_1(\sigma_1) = \pi_2(\sigma_1) = \pi_1(\sigma_2) = \pi_2(\sigma_2) = 0{,}5$. In the course of time the preferences approach theoretical optimum values, which in the general case experience the drift, caused by a change in the resources. In this case both entropies decrease, certainty will grow (Fig. 7.22).

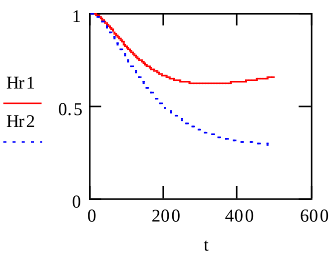
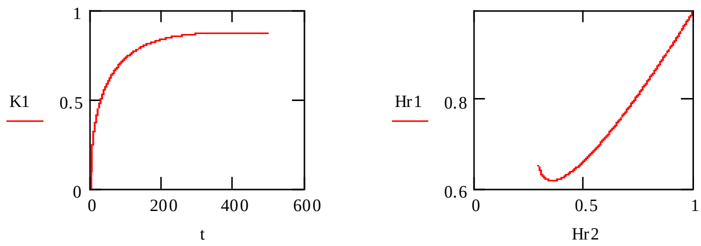


Fig. 7.22



a *b*

Fig. 7.23

It is obvious that ρ_Σ is constant and it is equal to +1, index K_1 increases, what testifies the growth of the conflict sharpness (Fig. 7.23, *a)*. Entropy map is shown on Fig. 7.23, *b*. The different version is presented on Fig. 7.24.

In this case the expected resources remain constant and are equal accordingly

$$R_1^{exp}\left(\sigma_1\right)=8;\quad R_1^{exp}\left(\sigma_2\right)=6;\quad R_2^{exp}\left(\sigma_1\right)=10;\quad R_2^{exp}\left(\sigma_2\right)=10,$$

and required resources vary with time, moreover they decrease for the realization of σ_1, and increase for σ_2

$$R_1^{req}\left(\sigma_1\right)=8\left(1-0{,}005t\right);\quad R_1^{req}\left(\sigma_2\right)=4\left(1+0{,}005t\right);$$

$$R_2^{req}\left(\sigma_1\right)=8\left(1-0{,}005t\right);\quad R_2^{req}\left(\sigma_2\right)=5\left(1+0{,}005t\right).$$

It follows from Fig. 7.23 that in the moment's t_1 and t_2 the inversion of the orders of preferences occurs.

Fig. 7.24 show entropies $\bar{H}_1$, $\bar{H}_2$, ρ_Σ, K_1, K_2 .

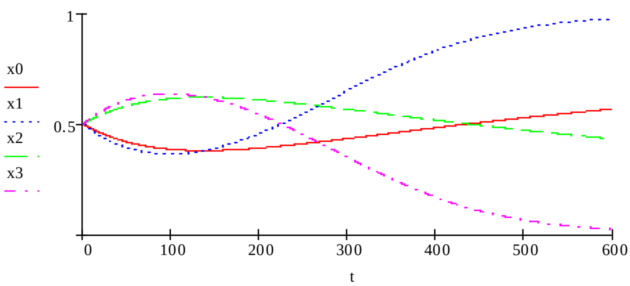


Fig. 7.24

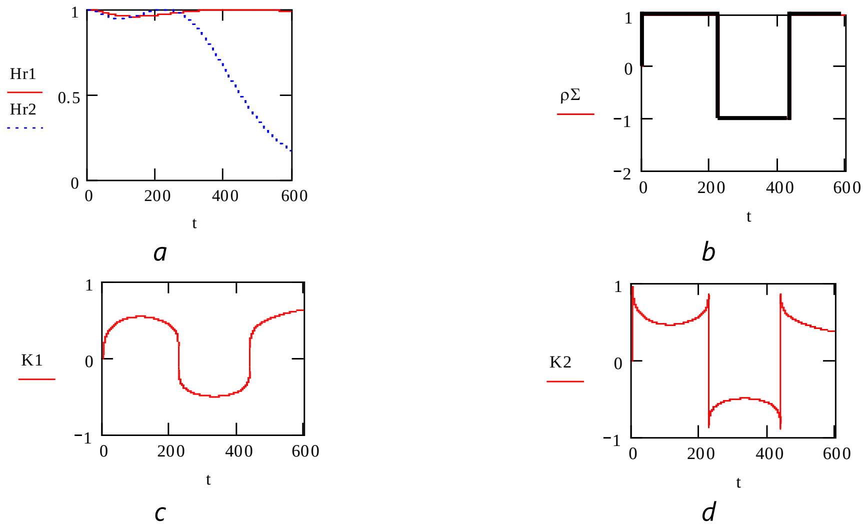


Fig. 7.25

In the interval $t \in [0, t_1)$ $\rho_\Sigma = +1$ and, taking into account that σ_2 is „single-sited", we conclude that the consonant conflict occurs. Subsequently as a result of a change in the required resources and the inversion of the preferences of the second subject connected with this the concordance state appears, which continues in the interval $t \in [t_1, t_2)$. Now for the first subject the new inversion of the preferences returns the state of conflict when $t \geq t_2$. Fig. 7.25 is demonstrates the degree of conflict sharpness with $t \in [0, t_1)$ and $t \geq t_2$ and the degree of concordance with $t \in [t_1, t_2)$. Entropy map is shown on Fig. 7.26.

The following version represents the case, when all required resources grow simultaneously: for the both alternatives and both subjects:

$$R_1^{req}(\sigma_1) = 8(1 + 0{,}005t); \quad R_1^{req}(\sigma_2) = 4(1 + 0{,}005t);$$
$$R_2^{req}(\sigma_1) = 8(1 + 0{,}005t); \quad R_2^{req}(\sigma_2) = 4(1 + 0{,}005t). \tag{7.40}$$

This can be treated as the simultaneous and proportional growth of prices on the market. The expected resources remain constant. It follows from Fig. 7.26 that the preferences of both subjects disperse at first from the state of indifferences and relative entropies decrease. Then, however, further increase in the required resources leads to the leveling of the preferences, and entropies approach to the maximum values.

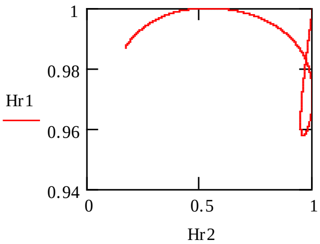


Fig. 7.26

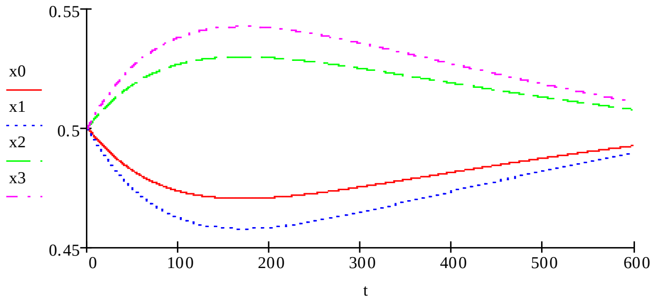
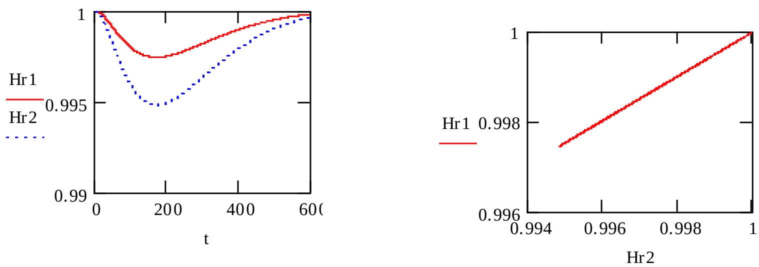


Fig. 7.27

It is seen from Fig. 7.27, that the sharpness of conflict at first increases, and then decreases.



a *b*

Fig. 7.28

Let us assume that preferences are determined by the ratio of required resources to available resources:

$$\pi_j^-(\sigma_i) = \frac{e^{-\beta_j \bar{r}_{ji}}}{e^{-\beta_j \bar{r}_{j1}} + e^{-\beta_j \bar{r}_{j2}}}, \tag{7.41}$$

where $\bar{r}_{ji} = \left(R_j^{disp}(\sigma_i)\right)^{-1}\left(R_j^{req}(\sigma_i)\right)$.

Fig. 7.29 shows the case, when required resources for all alternatives and for all subjects simultaneously grow in the identical proportion, and available resources of both subjects diminish also in the identical proportion. For the model (7.41) and single-sited alternatives the aggravated conflict occurs (Fig. 7.29 and 7.30).

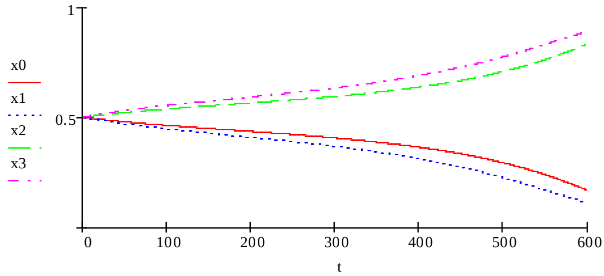


Fig. 7.29

The results of the simulation of the very narrow class of conflicts are shown above as an example. In this case we were limiting ourselves to the case of conflict between two subjects with the presence of two alternatives only.

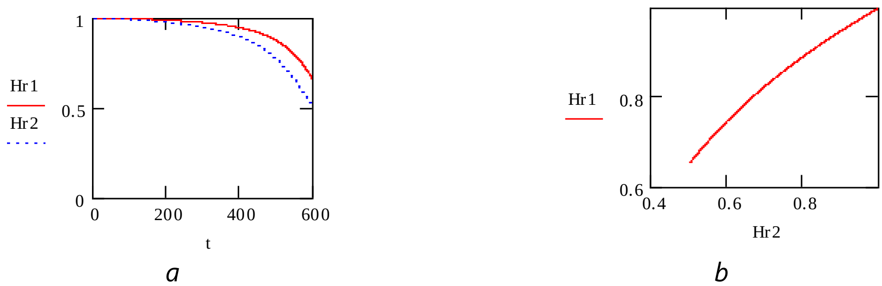


Fig. 7.30

More complete analysis assumes the simulation not only in the conflict „preparatory" stage, but also process of conflict „resolution". It is obvious that the conflict passes into the stage of „resolution ", when the certain index of the conflict sharpness exceeds critical value, for example: $K_1 \geq K_1^*$. It is also clear that the threshold value K_1^* (or any another criterion) is individual for each subject and appears as one additional index of individual psyche. In this case the connection of this index with the canonical distributions of preferences is important and, correspondingly, with the possibility of quantitative assessments estimations. The forms of conflict resolution can be different depending on the type of the binary relations between the conflicting parties, for example, – of the „antagonistic" or „competitory" relations.

In the present paragraph we do not examine completely the conflicts of rating preferences, or the conflicts of rating distributions with the rank distributions. Out of the examination remained possible conflicts in the hierarchical systems. All these questions require design of the corresponding models, the road to which opens technologies, based on the usage of canonical distributions, and undoubtedly, they represent the significant, independent interest.

7.7. Dynamics of the „living points"

Models examined in this chapter describe the behavior of hypothetical subject, conditionally called „living point". They convert the internal motives, expressed through the preferences, into the motion in the space of the exogenous parameters, which, in the particular case, can appear as the space coordinates x, y,... Preferences are simulated with the aid of the canonical distributions, which follow from the entropy optimal principle.

The proposed models can be considered as an attempt of interpretation of the subject motions, which are not obeying the law of mechanics, more precisely speaking, not caused by external forces exclusively.

Actually, how to describe the motion of player on the football field or the flight of the aircraft, performing acrobatic maneuvers. The motions of aircraft are described by flight dynamics open equations, since each time it is necessary to define forces, including controlling actions of pilot. In a certain sense it is possible to consider, that the presented below analysis is an attempt of the exogenous manifestations simulation both of intrapersonal and interpersonal conflicts.

In the previous paragraph we examined the isolated dynamics of preferences due to the conditions of the conflict origin and growth. Both the change in the preferences and the influence of these changes on the subject dynamics in the exogenous space is simultaneously simulated here.

Exogenous manifestations are understood as motion, the visible behavior of subjects, under the action of the internal processes of the preferences reconstruction, which in turn, depend on the fact, what occurs with the subject in the exogenous space.

We do not solve the problem of „external", moreover, optimal control (for example, through the resources). However, the constructed models contain everything necessary for the setting and solution of these problems: controlled and controlling variables, the object of control and the corresponding mathematical relationships.

Certainly, the models given below are only the first step with the large number of simplifying assumptions and a high level of idealization. Basically, what we should here emphasize, lies in the fact that control of active system is performed indirectly as control of the active system subject preferences, whose mathematical models are considered as known. The given treatment clarifies including in the division materials presented in the chapter, dedicated to conflicts.

The subject exogenous parameters x, y,... can have an arbitrary sense, including possible to consider them space coordinates.

The subject in this given model is considered as the „point", i.e., his position in the exogenous space is determined by the collection of the instantaneous values of the *x*, *y*,... parameters. The laws of mechanics in the exogenous space are performed exactly, and when we speak about „internal" forces, we have in mind the reasons, which force subject to perform those or other actions, in particular, the mechanical motion.

It is assumed that the canonical preferences form in the subject consciousness of the unique „forces", which are converted into the speeds and the accelerations of change in the state parameters. In turn, the preferences change in dependence on exogenous factors, and also because of other reasons, first of all of preferences relaxation (or adaptation), which has already been discussed in chapter 5.

Let us recall, that the discussion deals about the fact that, according to the made assumption, the psyche is not „tuned" to the appearing situation instantaneously - the optimum preferences distribution did not appears instantly.

It takes a certain time of adaptation, the „tuning time" is required. The term „living point" or „inspired point", reflects conditionality and limitedness of model. On the other side, it indicates a certain analogy with the material point in the mechanics. Essential difference lies in the fact that in this case the compelling „forces" appear in the consciousness of subject himself and in this sense have internal origin.

Let us assume that the „living point" motion occurs in the plane and the parameters *x*, *y* - are the instantaneous coordinates of point.

Then $V_x = \dfrac{dx}{dt}$; $V_y = \dfrac{dy}{dt}$ are the projections of the point speed. Let us define alternatives $\sigma_i \in S_a$ as the „prizes", located on the plane *OXY* at points (x_i, y_i) and which have the value for the subject equal to R_i^e.

For example, this can be the additional resources, which the subject obtains, reaching a position (x_i, y_i). At the beginning of process the subject has the certain available resources, which are expended proportional to the passing distance.

We will consider the „mental force" proportional to the value of preference $\pi(\sigma_i \mid \sigma)$, where σ is determined by the instantaneous position of subject, and directed to the point of the arrangement of alternative „prize" (x_i, y_i) (Fig. 7.31).

$$\left.\begin{aligned} f_{ix} &= k\pi\left(\sigma_i \mid \sigma\right)\frac{x_i - x}{d_i};\\ f_{iy} &= k\pi\left(\sigma_i \mid \sigma\right)\frac{y_i - y}{d_i}. \end{aligned}\right\} \qquad (7.42)$$

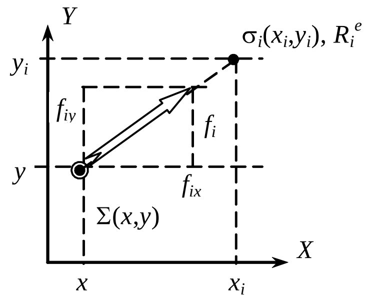


Fig. 7.31

Composite force acting along the axes *OX* and *OY*, are respectively equal:

$$\left.\begin{aligned} f_x &= k\sum_{i=1}^{N}\pi\left(\sigma_i \mid \sigma\right)\frac{x_i - x}{d_i}; \\ f_y &= k\sum_{i=1}^{N}\pi\left(\sigma_i \mid \sigma\right)\frac{y_i - y}{d_i}. \end{aligned}\right\} \tag{7.43}$$

Here $d_i = \sqrt{\left(x_i - x\right)^2 + \left(y_i - y\right)^2}$.

Taking into account of the above aforesaid we can write the following model of the „living particle" dynamics:

$$\frac{dV_x}{dt} + \frac{1}{s}V_x = k\sum_{i=1}^{N}\pi\left(\sigma_i \mid \sigma\right)\frac{x_i - x}{d_i}; \tag{7.44}$$

$$\frac{dV_y}{dt} + \frac{1}{s}V_y = k\sum_{i=1}^{N}\pi\left(\sigma_i \mid \sigma\right)\frac{y_i - y}{d_i}; \tag{7.45}$$

$$\frac{d\pi\left(\sigma_i \mid \sigma\right)}{dt} = -m\left(\pi\left(\sigma_i \mid \sigma\right) - \pi^{\circ}\left(\sigma_i \mid \sigma\right)\right);\ \left(i \in \overline{1,N}\right). \tag{7.46}$$

$$\frac{dx}{dt} = V_x; \tag{7.47}$$

$$\frac{dy}{dt} = V_y; \tag{7.48}$$

$$\frac{dR^d}{dt} = -n\sqrt{V_x^2 + V_y^2}. \tag{7.49}$$

In equations (7.44—(7.49) *k*, *s*, *m*, *n* — are the constant structural parameters.

Last equation describes a change in the available resources and reflects the assumption that the expenditure of resources is proportional to the passable road: $dR^d = -ndl = -n\sqrt{V_x^2 + V_y^2}dt.$

The group of N equations (7.46) describes the adaptation process of the preferences distribution to the optimum $\pi°(\sigma_i \mid \sigma)$, which changes in the course of time as well. Some other forms of the equations of adaptation are, including the case of variable normalization was described in chapter 5. Optimum (canonical) distribution $\pi°(\sigma_i \mid \sigma)$ depends on the selection of the effectiveness function.

Let us assume that $\pi°(\sigma_i \mid \sigma)$ has the form:

$$\pi^{\circ}\left(\sigma_i \mid \sigma\right) = \frac{R_i^e\left(t\right)e^{-\beta\sqrt{\left(x_i - x\right)^2 + \left(y_i - y\right)^2}R^{d^{-1}}\left(x, y\right)}}{\sum_{j=1}^{N} R_j^e\left(t\right)e^{-\beta\sqrt{\left(x_i - x\right)^2 + \left(y_i - y\right)^2}R^{d^{-1}}\left(x, y\right)}}. \tag{7.50}$$

Here $R_i^e(t)$ is the „cost" (utility) of the „prize", which is located at point with the coordinates x_i, y_i and which corresponds to alternative σ_i (Fig. 7.32), R^d (x, y) - the value of the residual available resources at the moment, when the subject is located at point (x, y). Distribution (7.50) is so arranged, that $\pi°(\sigma_i \mid \sigma)$ increases as we approach to a point (x_i, y_i), i.e. to the „prize" R_i^e and tends to 1, if the point (x_i, y_i) is the nearest. In this case other preferences for the more distant points are tending to zero. This means, that in proportion to the „development" of the available resources (in given model) the entropy of the preferences approaches zero.

In the limit, with $R_i^d \to 0$, the subject preserves non-zero preference to the „nearest" alternative and loses any interest in the remaining alternatives. It is easy to perceive this fact from the following reasoning's. Let there be two alternatives and preferences are determined by the formulas:

$$\pi_1 = \frac{ae^{-\beta x_1 y^{-1}}}{ae^{-\beta x_1 y^{-1}} + be^{-\beta x_2 y^{-1}}}; \quad \pi_2 = \frac{be^{-\beta x_2 y^{-1}}}{ae^{-\beta x_1 y^{-1}} + be^{-\beta x_2 y^{-1}}}; \; x_1 \geq 0; x_2 \geq 0.$$

Let us write them in the form:

$$\pi_1 = \frac{1}{1 + \bar{b}e^{-\beta\left(x_2 - x_1\right)y^{-1}}}; \quad \pi_2 = \frac{1}{1 + \bar{a}e^{-\beta\left(x_1 - x_2\right)y^{-1}}}; \qquad \bar{b} = \frac{b}{a}; \qquad \bar{a} = \frac{a}{b}.$$

It is easy to note that, if $x_2 > x_1$, then

$$\lim_{y \to 0} \pi_1 = 1; \; \lim_{y \to 0} \pi_2 = 0$$

and, on the contrary, if $x_2 < x_1$, then

$$\lim_{y \to 0} \pi_1 = 0; \; \lim_{y \to 0} \pi_2 = 1.$$

It is also assumed that the motion of the „point" ceases with the complete exhaustion of resources. Thus the „life on credit" must be excluded.

Simulation was also performed for another form of canonical distribution, namely

$$\pi^{\circ}\left(\sigma_i \mid \sigma\right) = \frac{R_i^e\left(t\right)e^{-\beta\left(\sqrt{\left(x_i - x\right)^2 + \left(y_i - y\right)^2}\right)^{-1} R^d}}{\sum_{j=1}^{N} R_j^e\left(t\right)e^{-\beta\left(\sqrt{\left(x_i - x\right)^2 + \left(y_i - y\right)^2}\right)^{-1} R^d}}. \tag{7.51}$$

This distribution is arranged so, that as we approach one of the goals (with the non-zero residual resources R^d), i.e. with $d_i = \sqrt{\left(x_i - x\right)^2 + \left(y_i - y\right)^2} \to 0$, the corresponding preference tends to 1, and remaining preferences are tending to zero. This is evident from the following calculation.

$$\pi_1 = \frac{ae^{\beta\frac{y}{x_1}}}{ae^{\beta\frac{y}{x_1}} + be^{\beta\frac{y}{x_2}}}; \quad \pi_2 = \frac{be^{\beta\frac{y}{x_2}}}{ae^{\beta\frac{y}{x_1}} + be^{\beta\frac{y}{x_2}}}.$$

Converting, we find:

$$\pi_1 = \frac{1}{1 + \bar{b}e^{\beta\left(\frac{y}{x_2} - \frac{y}{x_1}\right)}}; \quad \pi_2 = \frac{1}{1 + \bar{a}e^{\beta\left(\frac{y}{x_1} - \frac{y}{x_2}\right)}}.$$

If $x_1 \to 0$ with $x_2 \neq 0$; $x_2 > 0$, $x_1 > 0$ and $y \neq 0$

$$\lim_{x_1 \to 0} \pi_1 = 1; \quad \lim_{x_1 \to 0} \pi_2 = 0, ,$$

and vice versa with $x_2 \to 0$ with $x_1 \neq 0$; ; $y \neq 0$

$$\lim_{x_2 \to 0} \pi_1 = 0; \quad \lim_{x_2 \to 0} \pi_2 = 1.$$

From equations (7.44), (7.45) it follows that, if the „forces" f_{ix} and f_{iy} are not equal to zero, then the „point" has non-zero speed, on the other hand the „forces" f_{ix} and f_{iy} cannot be equal to zero in view of the $\pi^{\circ}(\sigma_i \mid \sigma)$ distribution normalization conditions and limited passages reasoning's given above.

Thus, the possibility of the equilibrium positions presence in the model is excluded.

We modify equations (7.44) and (7.45) after including the „conservative" component in them. The following equations ensure the presence of equilibrium points:

$$\frac{dV_x}{dt} + \frac{1}{s}V_x + hx = k\sum_{i=1}^{N} \pi\left(\sigma_i \mid \sigma\right)\frac{x_i - x}{d_i}; \tag{7.52}$$

$$\frac{dV_y}{dt}+\frac{1}{s}V_y+hx=k\sum_{i=1}^{N}\pi\left(\sigma_i\mid\sigma\right)\frac{y_i-y}{d_i}; \qquad (7.53)$$

This model actually coincides with the material point equations of motion with the presence of „elastic element" in a medium with viscous resistance.

Calculations are performed for the case, depicted on the Fig. 7.33.

In the initial position (x = 40, y = 40) the state of the indifference occurred:

$$\pi\left(\sigma_i\mid\sigma\right)=\frac{1}{3}\ \ (\forall i\in\overline{1,3}).$$

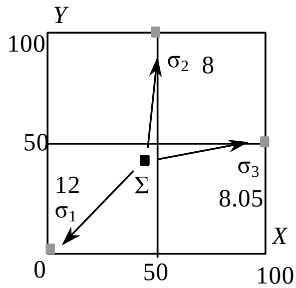


Fig. 7.32

Fig. 7.33, *a*, *b*, *c*, *d* presents the case, when subject is starting the motion in the direction σ_2 then sharply changes it and finally reaches the „prize" σ_3 at point with the coordinates $x = y_5 = 100$; $y = y_6 = 50$ (Fig. 7.33, *a*).

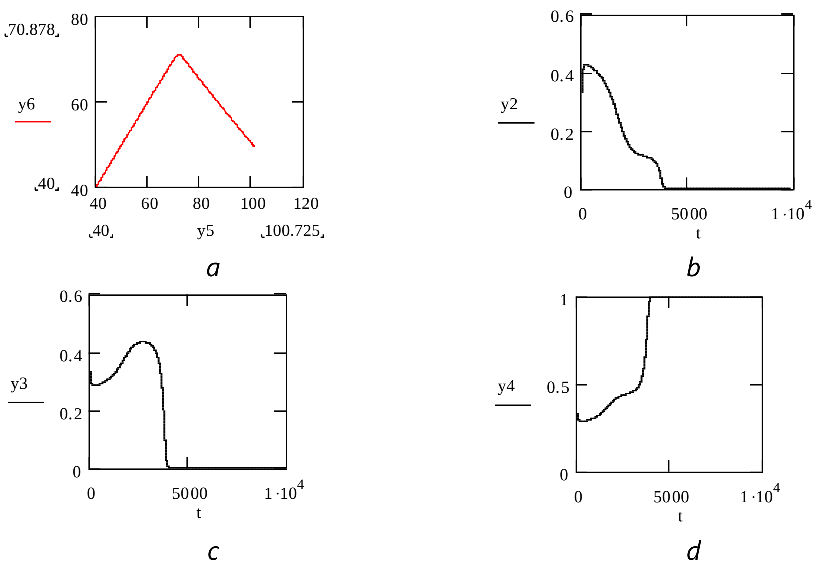


Fig. 7.33

Preferences $\pi(\sigma_1) = y_2$; $\pi(\sigma_2) = y_3$; $\pi(\sigma_3) = y_4$ in the initial moment are considered identical and equal $\frac{1}{3}$, then they change in such way, that $\pi(\sigma_1) \to 0$; $\pi(\sigma_2) \to 0$; $\pi(\sigma_3) \to 1$, the entropy $H_\pi \to 0$.

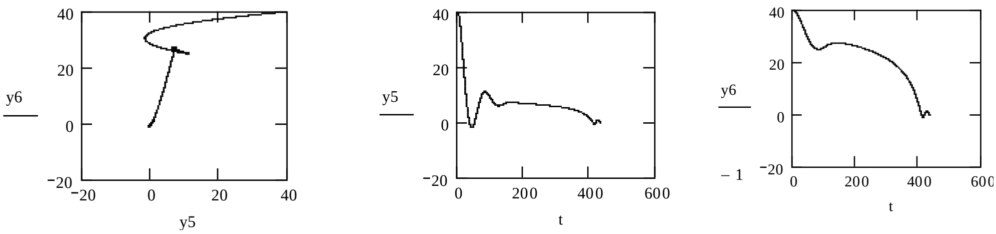
The different version of the solution, which corresponds to other values of the structural parameters is shown on Fig. 7.34, *a—i*.

Plan of „point" movement in the coordinates $x = y_5$, $y = y_6$ is presented on Fig. 7.34, *a* and it testifies, that the „point" achieves the first goal (σ_1) after some fluctuations. Change in the coordinates and speeds $V_x = y_0$; $V_y = y$, and also $\pi(\sigma_1) = y_2$; $\pi(\sigma_2) = y_3$; $\pi(\sigma_3) = y_4$ are shown on Fig. 7.34, *b—i*.

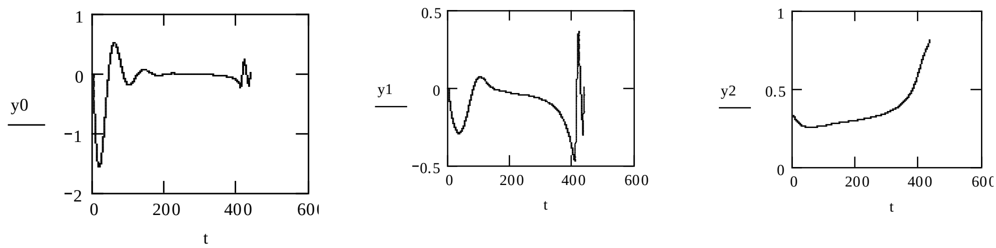
It is evident from Fig. 7.34 that for the moment of σ_1 reaching the initial resources (100 conditional units) is completely exhausted.

If on the field are two subjects – the players (two „points"), that have as the alternatives the same objects (S_{a1} coincides with S_{a2}), then the possibility appears to reflect in the model in any manner their interaction with each other.

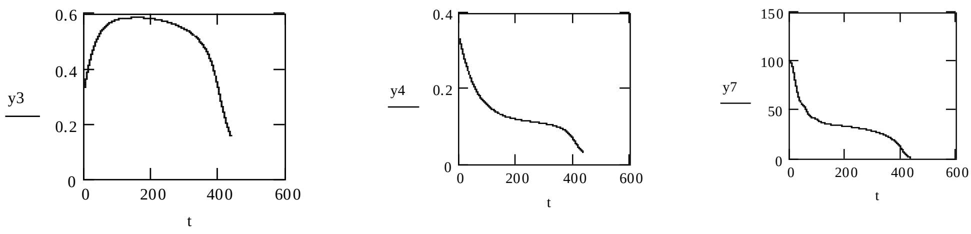
The spectrum of interactions is very wide: from the union and the mutual aid, to the conflict and the opposition.



a *b* *c*



d *e* *f*



g *h* *i*

Fig. 7.34

The group dynamics model of group, consisting of two subjects (M = 2) is the set of the equations, which describe the behavior of each of the subjects in one system of equations. In this case in equation of motion the terms, causing the interaction between the subjects, must be included. Speaking about the group dynamics, we will use the designation, which characterizes the dimensionality of the task: $M \times N$, where M is the number of subjects in the group, N is the dimensionality of extended set of alternatives

$$S_a = \bigcup_{j=1}^{M} S_{aj}.$$

Let us examine at first the model of 2×3 group of two subjects with presence of three alternatives, which have identical „value" for both subjects. The model of dynamics in this case consists of 16 differential equations of first order each.

As the „force" caused by interaction let us select the following expression:

$$\frac{\xi(j)-\xi(k)}{d_{jk}+\varepsilon}\frac{\vec{r}_k-\vec{r}_j}{d_{jk}+\varepsilon}, \ (\varepsilon > 0). \tag{7.54}$$

Here $\xi(j)$ — integral rating preferences, $\vec{r}_j$ — the radius-vectors of instantaneous position of the subjects - „points", d_{12} — distance between the „points" in the given moment. The sense of this expression is reduced to the following: the greater the difference in the ratings, the greater the „force", the nearer the „points" are located, the greater the force, moreover with $d_{jk} \to 0$ the force do not approaches infinity in view of the low positive constant ε presence in the denominator. If $\xi(j) > \xi(k)$, then the „force", which acts on subject k, is directed to the j side (see Fig. 7.35).

At the same time the „force", which acts on j, is directed to the k side.

Thus, the unique „pursuit" must appear: the subject with the higher rating pursues the subject with the lower rating.

This occurs against the background of mutual tendency to the alternative goals „prizes". Certainly, this is only quotient assumption. We can imagine such interaction, which cause the additional expenditures of the resources from each of the sides, or the „removal" of resources by one subject from another subject with fulfillment of the certain conditions.

Ratings in expression (7.55) are determined from the formula

$$\xi(j) = \frac{e^{\alpha R_j^d}}{\sum_{q=1}^{M} e^{\alpha R_q^d}}; \quad (M = 2), \qquad (7.55)$$

i.e., the greater resources have the subject in a given moment the greater they are.

Since the available resources in the process of motion are changing, than ratings are also changing, and, therefore, also the interaction forces. The scheme of initial arrangement of two players - „points" on „playfield" and three alternative „prizes" σ_1, σ_2, σ_3 is shown on Fig. 7.36.

Both players have the initial distributions of preferences, which correspond to the complete indifference. The plan of motion of both players on Fig. 7.37, *an* attests to the fact, that, the first player, who possesses the great resource possibilities (350 conditional units against 250 conditional units of the second player), achieves the third alternative „goal", simultaneously „repulsing" the second player from goal. On Fig. 7.37 $y_5 = x_1$; $y_6 = y_1$; $y_{13} = x_2$; $y_{11} = y_2$. From Fig. 7.38, *b*, *c* we perceive, that the resources of the first and second players ($R_1^d = y_7$, $R_2^d = y_{15}$) are not exhausted.

On Fig. 7.38, *a* the different version is shown, when with the equal other conditions the initial resources are distributed differently: $R_1^d(0) = y_7 = 150$ conditional units, $R_2^d(0) = y_5 = 250$ conditional units.

It follows from the plan of motion that the second player is richer and to the larger degree is absorbed by fight with the rival, he prevents his movement to the „goal", but in this case does not achieve the „goal" itself.

The first player is deprived of the possibility to reach any of the „goals" (Fig. 7.38, *b* $R_1^d = y_7$, $R_2^d = y_{15}$).

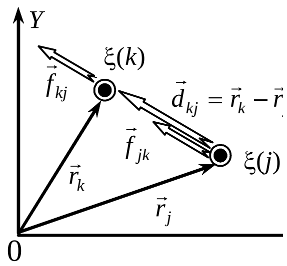


Fig. 7.35

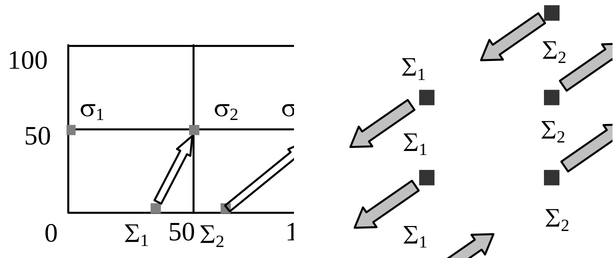
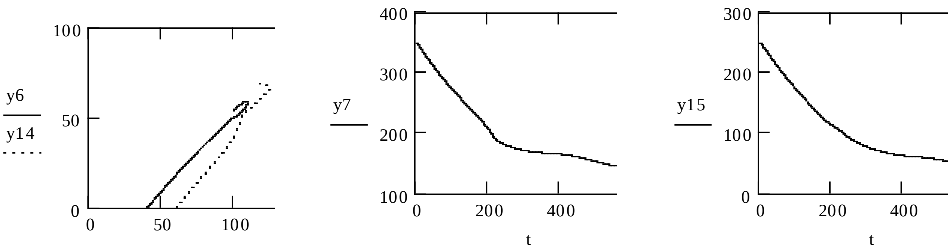


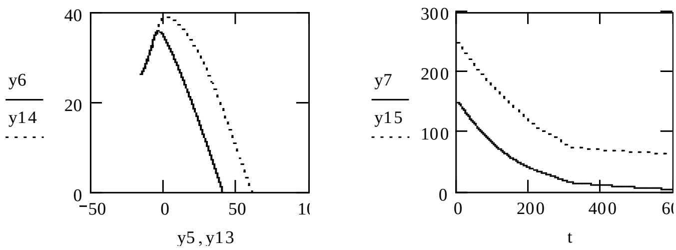
Fig. 7.36



a *b* *c*

Fig. 7.37

Fig. 7.39 shows even the more radical case, when „interrelations" between the players prevail above the desire to the „goal" so, that they both practically do not move to it, remaining near the zero (initial) level.



a *b*

Fig. 7.38

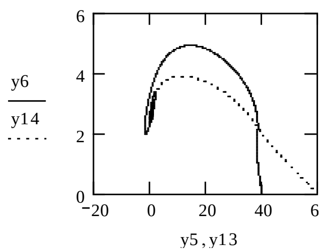


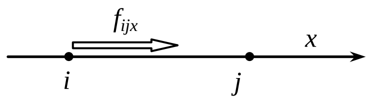
Fig. 7.39

The more wide variety of the versions of the situation development and some additional effects appear with the simulation of the group dynamics, which consists of four subjects.

There are the foundations for assuming that in this case the elements of corporate behavior can already appear. Let us examine the group dynamics of four subjects with the presence of three alternatives („4×3" system): S_{ξ}: ($j \in \overline{1,4}$), S_a: ($i \in \overline{1,3}$). Let us make additional assumptions referring to „endogenous" forces, which force each of the „points" to move. Besides the „forces" of attraction to the assigned target objects - „prizes" at each of the „points" act the „forces", caused by interaction between them.

In the given case these „forces" have a nature of attraction or repulsion. Entire association of „points" - subjects is divided into two classes of „rich" and „poor" with the aid of the boundary rating $\xi^*(R_*^d)$, determined by the appropriate boundary value of the available resources. If $\xi_j > \xi^*$, subject relates to the class of „rich", and if $\xi_j < \xi^*$ — to the class of „poor".

Let us designate through $\vec{f}_{ij}$ the „force" acting on subject „i" from the side of subject j.



Let f_{ijx} is projection of „force" $\vec{f}_{ij}$ on the OX axis.

The formulas given below are built so that subjects, who belong to one class are attracted mutually:

$$f_{ijx} = \xi_j \frac{\left(\xi_i - \xi^*\right)\left(\xi_j - \xi^*\right)}{\left|\xi_i - \xi^*\right|\left|\xi_j - \xi^*\right|} \frac{x_j - x_i}{d_i^2 + \varepsilon}\left(\left|\xi_i - \xi_j\right| + \varepsilon\right)^{\varphi_{ij}}. \tag{7.56}$$

We see that the „force", which acts on „i" from the „j" side is proportional to the rating valueξ_j. The direction of „force" is determined by the second cofactor - the fraction, which takes the values ±1.

Coefficient $\dfrac{x_j - x_i}{d_i} = \cos\left(\vec{r}_j - \vec{r}_i, vx\right)$, the low parameter $\varepsilon > 0$ have purely computational sense and is included on for the purpose to „go around" possible zeros in the denominator, $d_i = \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2}$.

The multiplier $(d_i + \varepsilon)^{-1}$ in any case provide the increase of force of interaction with increasing of "geometrical" proximity of subjects finally the last cofactor

$$\left(\left|\xi_i - \xi_j\right| + \varepsilon\right)^{\varphi_{ij}} \tag{7.57}$$

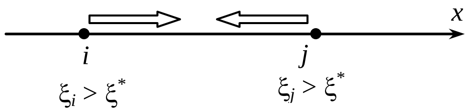
characterizes the influence of rating proximity. The index φ_{ij} is selected in the form:

$$\varphi_{ij} = -\mu \frac{\left(\xi_i - \xi^*\right)\left(\xi_j - \xi^*\right)}{\left|\xi_i - \xi^*\right|\left|\xi_j - \xi^*\right|} = \begin{cases} +\mu; \\ -\mu, \end{cases} \quad \mu > 0 \tag{7.58}$$

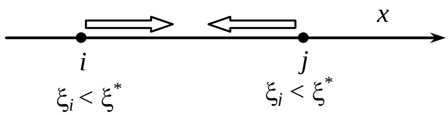
and it works according to the following rule:
1. Both subjects are „rich” $\xi_i > \xi^*$, $\xi_j > \xi^*$, then $\varphi_{ij} = -\mu$ and last cofactor in (7.56) takes the form:

$$\frac{1}{\left(\left|\xi_i - \xi j\right| + \varepsilon\right)^{\mu}} \tag{7.59}$$

and, therefore, the smaller is the difference in ratings ξ_i and ξ_j, the greater the „attraction”



2. Both subjects are „poor” $\xi_i < \xi^*$, $\xi_j < \xi^*$, then also $\varphi_{ij} = -\mu$ and „attraction” occurs again



3. Subjects belong to different „classes”: one is „poor”, another is „rich”, for example, $\xi_i < \xi^*$, $\xi_j > \xi^*$, then also $\varphi_{ij} = \mu > 0$, and into expression for the force enters the cofactor

$$\left(\left|\xi_i - \xi_j\right| + \varepsilon\right)^{\mu}. \tag{7.60}$$

The „repulsion” occurs, and the stronger it is the greater the difference in ratings ξ_i and ξ_j is, and equal 0 if $\xi_i = \xi_j = \xi^*$,. Ratings are normalized : $\sum_{j=1}^{M} \xi_j = 1$

and this condition is satisfied for any moment of time. In the models the relaxation scheme s are used both for the object preferences $\pi_j(\sigma_k)$, and for the rating preferences.

$$\frac{d\pi_j\left(\sigma_k\right)}{dt} = -m_j\left(\pi_j\left(\sigma_k\right) - \pi_j^{\circ}\left(\sigma_k\right)\right); \quad j \in \overline{1,4}; \quad k \in \overline{1,3}.. \tag{7.61}$$

$$\frac{d\xi_j}{dt} = -p_j\left(\xi_j - \xi_j^{\circ}\left(R_j^d, \ldots\right)\right). \tag{7.62}$$

Here m_j and p_j is parameters, which determine the relaxation „rate”. The level of boundary rating ξ^* which divides association into two classes is considered equal to the average rating $\overline{\xi} = \frac{1}{M}\sum_{j=1}^{M}\xi_j = \frac{1}{M}$. If all ratings $\xi_j = \xi^* = \frac{1}{M}$, there is a complete ratings equality, and in accordance with the accepted form of distribution for ratings, the equality of resources. In this case $\pi_j^{\circ}(\sigma_k)$ and $\xi_j^{\circ}(R_j^d,\ldots)$ are canonical distributions. The following models are accepted:

$$\xi_j^{\circ} = \frac{e^{\alpha_j R_j^d}}{\sum_{q=1}^{M} e^{\alpha_q R_q^d}}; \tag{7.63}$$

$$\pi_j^{\circ}\left(\sigma_k\right) = \frac{R_k e^{-\beta_j\sqrt{\left(x_j-x_k\right)^2+\left(y_i-y_k\right)^2}}\left(R_j^d\right)^{-1}}{\sum_{s=1}^{N} R_s e^{-\beta_j\sqrt{\left(x_j-x_s\right)^2+\left(y_i-y_s\right)^2}}\left(R_j^d\right)^{-1}}. \tag{7.64}$$

Last relationship coincides with that accepted in the previous tasks. The motion of the „living” points, as earlier, occurs in the plane *OXY*. Let $\vec{V}_y = \left(V_{jx}, V_{jy}\right)$ - the velocity vector of point „*j*”. For the projections of the speed V_{jx} the following equations are accepted:

$$\frac{dV_{jx}}{dt} = -\frac{1}{\tau_j}V_{jx} - b_j x_j + k_j\sum_{r=1}^{3}\pi_j\left(\sigma_r\right)\frac{x_r - x_j}{\sqrt{\left(x_j - x_r\right)^2 + \left(y_j - y_r\right)^2}} + \tag{7.65}$$

$$+n_i\sum_{i=1}^{4}\xi_i\frac{\left(\xi_j - \xi^*\right)\left(\xi_i - \xi^*\right)}{\left|\xi_j - \xi^*\right|\left|\xi_i - \xi^*\right|}\frac{x_i - x_j}{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2 + \varepsilon}\left(\left|\xi_i - \xi_j\right| + \varepsilon\right)^{\varphi_{ij}};$$

$$\frac{dx_j}{dt} = V_{jx}. \tag{7.66}$$

The analogous form has equations for the coordinate yj and projections of speed Vjy. Complete system of equations for the system of „4×3” contains 52 differential equations of first order. The results of calculations for the different variants of the structural parameters numerical realization are shown on Fig.7.42.

Two tendencies are manifested explicate: the tendency of each subject to Reach one of the alternative goals and the tendency to choke competition rivals, to prevent them form realize the goal.

The motion must be stopped with the complete exhaustion of resources, in this case none of the goals can't be achieved. Simulation was performed for the initial state of the „4×3" system, presented on Fig. 7.40.

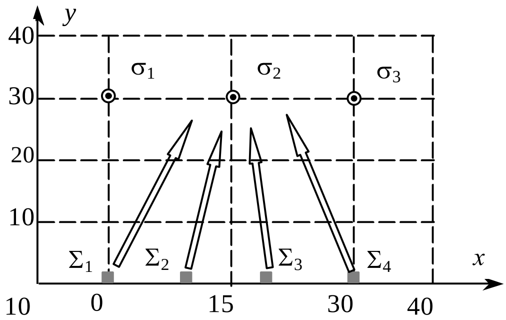


Fig. 7.40

The values of „prizes" at points σ_1, σ_2, σ_3 in the different variants were different, the initial values of the available resources also varied from one version to the next. Fig. 7.41 shows the motion plan of 4 „points" in the direction to goals. However, fight with each other proves to be more attractive in comparison with the desire to obtain prize. To this case corresponds the partitioning of group equally into the two classes: the „poor" and the „rich", in this case R1d(0) > Rd*; R2d(0) > Rd*; R3d(0) < Rd*; R4d(0) < Rd*. It was assumed that Rd* = 10 conditional units, R1d(0) = R2d(0) = 5 conditional units, R3d(0) = R4d(0) = 15 conditional units.

Fig. 7.42 presents the case, when the third and fourth subjects are the „rich". The structural coefficients of model are selected so that tendency to the goals conquers. The advance of „riches" to the goal σ2 occurs with a certain delay, in this case they are finally united and reject the „poors", which are also united stepping back (their trajectory converges).

Fig. 7.43 demonstrates the situation, when both groups, the „riches" and the „poors" reach area of the goal σ_2, tendencies to the association and the fight are not so strong far from the goal; however, in the goals environment the intensive struggle appears, on which the substantial part of the available resources is spent.

Fig. 7.44 presents the case, when there is only one alternative σ3 (goals at points σ1 and σ2 are equal to zero). All subjects are also to reach the area of the goal σ3 and arrange intensive „brawl" in the goal area.

In this case as the complementary factor the different rate of the resources expense is introduced, the „rich" one are expending resources along the trajectory two times more rapidly than „poor", and also the term, that imitates the „spring" effect is included.

The version, shown on Fig. 7.45, differs from the previous cases regarding the fact that three subjects are „poor" (Σ_1, Σ_2, Σ_3) and one is „rich" (Σ_4) and there exist only two „non-zero" alternatives σ_1 and σ_3.

The „rich" is pulled into the fight, but he loses to the „poor" ones acting in concert. Let us note that in this version the corporate behavior of the „poors" actions is manifested, whose trajectories have qualitatively identical form. Together they reject the „rich" player out of the playfield limits, on the opposite side to the goals side. The small virtual „revolution" occurs. In this case, however, no one achieves the goal.

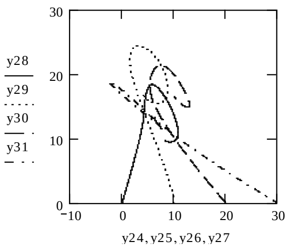


Fig. 7.41

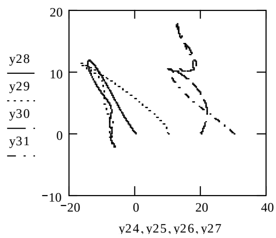


Fig. 7.42

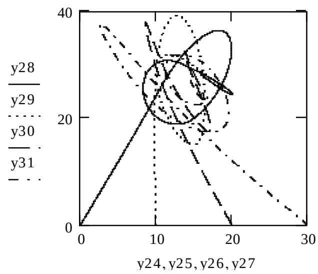


Fig. 7.43

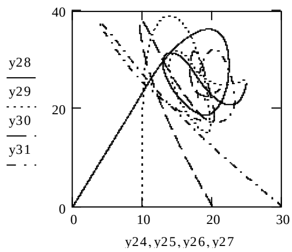


Fig. 7.44

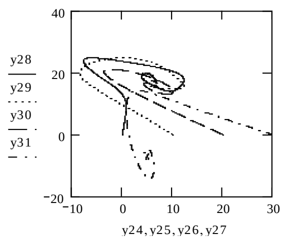


Fig. 7.45

The given models are very particular, the large number of structural parameters is contained in them, whose identification on the basis of experimental data is, at least thus far, unreal task. By the selection of these parameters certain value it is possible to attain, the manifestation of those or other effects. The role of similar models the author sees in the fact, that they make possible to perform qualitative analysis of the subjects and groups of subject's behavior and to find effects corresponding to the common sense.

The ways of further modifications and generalizing the models are obvious. For example, it is possible to forego the „immobility" of goals - alternatives, to use more compound schematics of the expense of resources, obtaining of incomes distributed in time.

It is possible to examine the schemes, in which the coordinates of subjects have another - not space-dimensional sense. In particular, they can be the economic indices, and models will acquire the economic content.

However, main improvements, evidently, must relate to the construction of the „internal forces”, including the „forces of interaction”. There is a possibility to test with this method the diverse forms of the canonical distributions, presented in the previous chapters.

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