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NATIONAL AVIATION UNIVERSITY

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Optics

Manual

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У той час, коли Україна вступає до «Європи знань», видання англомовних навчальних посібників є вкрай необхідним. Даний посібник розроблено для використання на всіх формах занять з курсу загальної фізики в умовах кредитно-модульної системи.

Модуль «Оптика» складається з навчальних елементів, які містять теоретичне ядро, задачі для аудиторної та індивідуальної роботи, а також лабораторний практикум. Розглянуто програмні питання з основ хвильової оптики.

Для студентів напряму підготовки 6.050901 «Радіотехніка».

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Having joined to “Europe of knowledge” Ukraine requires essentially creating new manuals on physics in English. The book is developed for all forms of studying physics on the Credit-based Modular System basis in higher school.

Physics. Module “Optics” presents the essential principles of wave optics. It contains Study Units which include theoretical information, test questions, sample problems, laboratory works and individual home tasks.

It is designed for students of engineering specialities.

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PREFACE

Foreword to Module M 5 "Optics"

This book is the fifth module of the discipline "Physics". It helps to elucidate essential principles of wave optics.

As a result of studying this module, students must ***know*** the definitions of such concepts as light ray, coherence, basic regularities of light propagation and such phenomena as interference, diffraction, polarization, dispersion.

Students must get ***skills*** to research and apply theoretical and experimental methods of wave optics, plot graphs, estimate errors of physical measurements and use theoretical knowledge for solving practical problems.

It is necessary to ***understand***, that such phenomena as interference, diffraction, polarization, dispersion are based on the wave nature of light.

The differential and integral calculus is widely used in the module but for the first year students' level.

The module "Optics" consists of the following **Study Units** (**SU**):

SU 1 — Electromagnetic properties of light;
SU 2 — Interference of light;
SU 3 — Diffraction of light;
SU 4 — Polarization of light;
SU 5 — Dispersion of light;
SU 6 — Laboratory works;
SU 7 — Individual home tasks;
Supplementary SU — Key words, Help tables.

The Preliminary unit contains the basic concepts and laws of electricity and magnetism and oscillations and waves that are necessary to study efficiently this module and a glossary with explanations of mathematics and physics terminology.

"Study Units 1–5" include theoretical material, test questions, sample problems, as well as problems for work in class. "Study Unit 6" gives instructions on how to perform laboratory works. "Study unit 7" contains problems to be solved by students on their own. "Supplementary Units" are aimed at facilitating the module study.

For effectiveness, we advise using self-check questions. Each question is provided with information where to find an answer. Concepts, which are studied in the module, are basic for all engineering fields of study; they are used in aeronavigation, radiolocation, technical electrodynamics etc.

ELECTROMAGNETIC PROPERTIES OF LIGHT

Optics is a science about light phenomena. Historically it has two stages of development. The first stage corresponds to classical or *wave optics* (till 1900) and is based on the wave nature of light; the second one is connected with a discovery of photons — quanta of electromagnetic energy (so cold *quantum optics*).

In this manual we shall treat the wave (classical) optics that considers light as electromagnetic waves.

As it was pointed in Module 4 “Oscillation and Waves”, Maxwell established that *light is an electromagnetic wave*. So, light phenomena must be described by the same equations that express the origin and propagation of electromagnetic waves including their interaction with substances.

According to the Maxwell’s electromagnetic theory, we have to regard three characteristics of substance: permittivity ε, permeability μ and conductivity σ. Conductivity σ determines absorption of waves, and permittivity ε and permeability μ determine the phase velocity of the electromagnetic waves propagation in a medium

$$v = 1/\sqrt{\varepsilon\varepsilon_0\mu\mu_0}\ .$$

As the phase velocity of light in vacuum is:

$$c = 1/\sqrt{\varepsilon_0\mu_0}\ (\varepsilon = 1,\ \mu = 1),\text{ so } v = c/\sqrt{\varepsilon\mu}\ .$$

The ratio of the speed of light in vacuum c to its phase velocity in a medium v is called the *absolute refractive index n*:

$$n = \frac{c}{v},$$

where $n = \sqrt{\varepsilon\mu}$.

For the majority of transparent substances $\mu = 1$, therefore $n = \sqrt{\varepsilon}$.

The refractive index characterizes the optical density of the medium: a medium with $n = \text{const}$ is called *optically homogeneous*, a medium with a greater n is called *optically denser*.

A line along which light energy propagates is called a *ray*. In optically homogeneous medium for a plane or spherical wave the rays are straight. In isotropic medium the rays are perpendicular to the wave surfaces, in anisotropic medium they are not.

1.1. Reflection and Refraction of Plane Electromagnetic Waves at Interface Between Two Dielectrics

Experiments show that if a light wave falls on the interface between two dielectrics, it is divided into two waves: one of them is reflected on the interface and is propagated in the first medium, and the second wave is refracted and propagated in the second medium. It may be shown that the *frequencies* of the reflected and refracted waves *coincide* with that of the falling (incident) wave.

1.1.1. Constancy of Wave Frequency at Reflection and Refraction

Let us regard a plane electromagnetic wave that falls on the infinite interface between two homogeneous isotropic dielectrics with the refractive indexes n_1 and n_2. Let us determine the direction of propagation by means of the wave vector $\vec{k}$ for the incident wave, the wave vector $\vec{k}'$ for the reflected wave and the wave vector $\vec{k}''$ for the refracted wave. The behavior of the wave at the interface where free charges and currents are absent is determined by the boundary conditions:

$$E_{\tau 1} = E_{\tau 2}, \quad H_{\tau 1} = H_{\tau 2}, \tag{1.1}$$

where $E_{\tau 1}$, $E_{\tau 2}$ and $H_{\tau 1}$, $H_{\tau 2}$ are the tangential components of electric and magnetic field intensities in the first and second media (see Module 3 “Electricity and Magnetism”, subsections 1.5 and 4.3).

The electric field intensity of the incidence wave that propagates in the direction of vector $\vec{k}$ may be presented in the form:

$$\vec{E} = \vec{E}_0 e^{i(\omega t - \vec{k}\vec{r})}. \tag{1.2}$$

According to the superposition principle (see subsection. 2.1) the electric field intensity in the first medium is determined by the intensities of the incident and reflected waves:

$$\vec{E}_1 = \vec{E} + \vec{E}' = \vec{E}_0 e^{i(\omega t - \vec{k}\vec{r})} + \vec{E}'_0 e^{i(\omega' t - \vec{k}'\vec{r})}, \tag{1.3}$$

and in the second medium by the intensity of the refracted wave only:

$$\vec{E}_2 = \vec{E}_0'' e^{i(\omega'' t - \vec{k}'' \vec{r})}. \qquad (1.4)$$

Here $E_0 = |E_0| e^{i\alpha}$, $E_0' = |E_0'| e^{i\alpha'}$, $E_0'' = |E_0''| e^{i\alpha''}$ are the complex amplitudes of the incident, reflected and refracted waves; α, α', α'' are the initial phases of these waves correspondingly; $\vec{r}$ is the position vector that starts arbitrary and ends at the wave falling point at the interface of dielectrics.

According to the equation (1.1), the tangential components at the interface must be the same:

$$E_{0\tau} e^{i(\omega t - \vec{k}\vec{r})} + E_{0\tau}' e^{i(\omega' t - \vec{k}'\vec{r})} = E_{0\tau}'' e^{i(\omega'' t - \vec{k}''\vec{r})}. \qquad (1.5)$$

For this equality at any time and at any point at the interface such conditions are necessary and sufficient:

$$\omega t = \omega' t = \omega'' t \Rightarrow \omega = \omega' = \omega''. \qquad (1.6)$$

$$\vec{k}\vec{r} = \vec{k}'\vec{r} = \vec{k}''\vec{r} \Rightarrow k_r r = k_r' r = k_r'' r \Rightarrow k_r = k_r' = k_r'', \qquad (1.7)$$

where k_r, k_r', k_r'' are the projections of the wave vectors onto the vector $\vec{r}$.

It follows from the equation (1.6), that the *frequency of the electromagnetic wave at reflection and refraction does not change.*

1.1.2. Relation between Angles of Incidence, Reflection and Refraction

Let us regard a plane electromagnetic wave that falls on the interface between two homogeneous isotropic dielectrics with the refractive indexes n_1 and n_2 (Fig. 1.1).

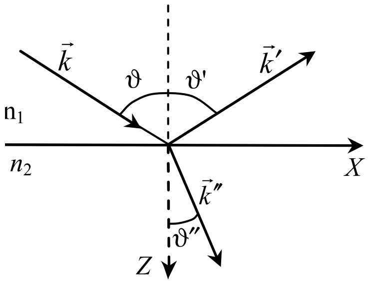


Fig. 1.1

Let us determine the direction of propagation with the aid of the wave vector $\vec{k}$ for the incident wave, the wave vector $\vec{k}'$ for the reflected wave and the wave vector $\vec{k}''$ for the refracted wave. The angles ϑ, ϑ' and ϑ'' that are counted from the normal *Z*, are called the *angle of incidence* (ϑ), the *angle of reflection* (ϑ') and the *angle of refraction* (ϑ'').

Law of reflection of light: the reflected and incident rays and the normal to the point of incidence lie in one plane; the angle of reflection equals the angle of incidence:

$$\vartheta = \vartheta' . \tag{1.8}$$

Law of refraction of light (Snell's law): the refracted and incident rays and the normal to the point of incidence lie in one plane; the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for given substances and is equal to the relative refractive index of these substances:

$$\frac{\sin\vartheta}{\sin\vartheta''} = n_{12} . \tag{1.9}$$

The relative refractive index of the second substance with respect to the first one equals the ratio of their absolute refractive indices $n_1 = c / v_1$ and $n_2 = c / v_2$. Therefore,

$$n_{12} = \frac{n_2}{n_1} = \frac{v_1}{v_2} . \tag{1.10}$$

In the general case a refractive index depends on a wave length, temperature and pressure.

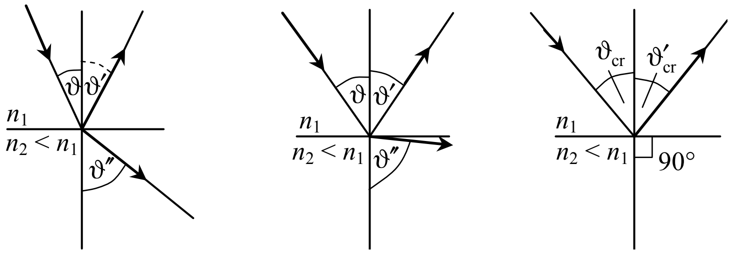
Transforming the equations (1.9) and (1.10) as:

$$n_1 \sin\vartheta = n_2 \sin\vartheta'' ,$$

we may understand, that when light passes from an optically *less denser* medium to an optically *denser* one ($n_1 < n_2$), the angle of incidence is greater then the angle of refraction $\vartheta > \vartheta''$ (Fig. 1.1). On the contrary, if light passes from an optically *denser* medium to an optically *less denser* one ($n_1 > n_2$), the angle of refraction is greater then the angle of incidence $\vartheta < \vartheta''$ and the refracted ray moves away from a normal to the interface of the media (Fig. 1.2, *a*).

If the angle of incidence ϑ increases, the angle of refraction ϑ'' grows even more rapidly (Fig. 1.2, *b*) and, at so called the *critical (limit) angle* of incidence ϑ_{cr}, the angle of refraction becomes equal $\vartheta'' = \pi/2$ (Fig. 1.2, *c*). It is clear that

$$\sin\vartheta_{\text{cr}} = n_2 / n_1 = n_{21}, \quad (n_1 > n_2). \tag{1.11}$$

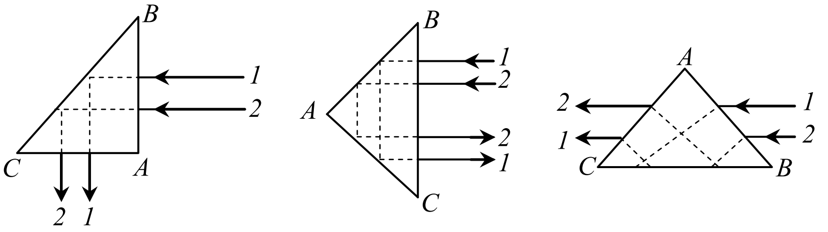


a *b* *c*

Fig. 1.2

For the angles of incidence $\vartheta > \vartheta_{\text{cr}}$ (i.e. from ϑ_{cr} to $\pi/2$), the light wave penetrates into the second medium to a distance of the order of a wavelength λ and then returns to the first medium. This phenomenon is called *total internal reflection*.

Phenomenon of total internal reflection is used, for example, in a right-angle prism to return the direction of rays on 90° and 180° or to overturn the image (Fig. 1.3 *a*, *b*, *c*).



a *b* *c*

Fig. 1.3

1.1.3. Fresnel's Formulas

For a complete description of reflection and refraction of light it is necessary to know the relation between the amplitudes and phases of electromagnetic waves at the interface of two media. These relations were derived first by Fresnel (1823) and they are called *Fresnel's formulas*. Using Maxwell's electromagnetic theory it is possible to obtain these formulas.

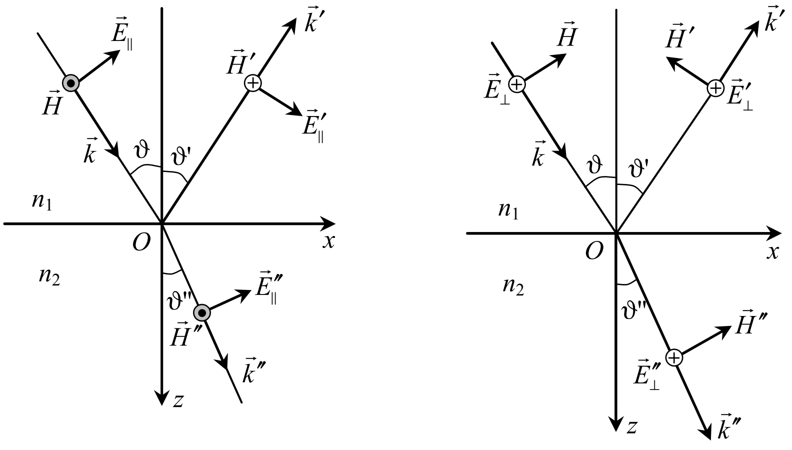
We know that what oscillates in an electromagnetic wave are the vectors $\vec{E}$ and $\vec{H}$. But the most actions of light (photoelectrical, photochemical, physiological, etc.) are due to the oscillations of the electric field intensity vector $\vec{E}$. Therefore we shall regard in the following the behavior of this *electric vector* $\vec{E}$ (sometimes it is called the *light vector*) remembering, that the magnetic vector $\vec{H}$ is always perpendicular to it.

Assume that a plane electromagnetic wave falls on the interface of two transparent homogeneous and isotropic media. As in the general case light is natural, the vector $\vec{E}$ (and $\vec{H}$) oscillations occur in all planes and change with time. But at any moment of time each of these vectors may be resolved on two components, directed parallel and perpendicular to the plane of incidence. Therefore we shall consider two cases:

1) the electric vector lies in the plane of incidence (||) (and the magnetic vector is perpendicular to it);

2) the electric vector is perpendicular to the plane of incidence $(\perp)$ (and the magnetic vector lies in it).

These two cases are shown in Fig. 1.4, *a* and *b*.



a *b*

Fig. 1.4

As it follows from Maxwell's equations, there is a relation for the amplitudes of a plane electromagnetic wave: $|E_0|\sqrt{\varepsilon_0\varepsilon} = |H_0|\sqrt{\mu_0\mu}$ (see Module 4 "Oscillations and Waves", subsection 4.2). For transparent dielectrics $\mu \approx 1$ (in an optical region of spectrum), so the absolute refractive index is $n = \sqrt{\varepsilon}$, and the magnetic field intensity is:

$$|H_0| = |E_0|\sqrt{\frac{\varepsilon_0\varepsilon}{\mu_0}} = \frac{n|E_0|}{\sqrt{\mu_0/\varepsilon_0}} = \frac{n|E_0|}{Z}, \tag{1.12}$$

where the constant value $Z = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$ is called the *wave resistance of vacuum*. As at a certain space point the vectors $\vec{E}$ and $\vec{H}$ oscillate in phase, the connection between the amplitudes [the equation (1.12)] holds also for the instantaneous values:

$$H = \frac{nE}{Z}. \tag{1.12a}$$

For the case shown in Fig. 1.5, *a*, according to the boundary conditions the equation (1.1) and the equation (1.12a), we get:

$$E_{\parallel}\cos\vartheta + E'_{\parallel}\cos\vartheta = E''_{\parallel}\cos\vartheta''; \quad n_1E_{\parallel} - n_1E'_{\parallel} = n_2E''_{\parallel}. \tag{1.13}$$

According to the equations (1.6) and (1.7), at the interface of two dielectrics the phase factor $\exp(\omega t - \vec{k}\vec{r}_\tau)$ is the same for the incident, reflected and refracted waves. Therefore the ratio of the instantaneous values of electric field intensities of these waves is equal to the ratio of their amplitudes. The ratio $E'_{\parallel}/E_{\parallel} = E'_{0\parallel}/E_{0\parallel}$ we denote as $r_{\parallel}$, and $E''_{\parallel}/E_{\parallel} = E''_{0\parallel}/E_{0\parallel}$ — as $t_{\parallel}$.

Here $E_{0\parallel}$, $E'_{0\parallel}$ and $E''_{0\parallel}$ are, in general, the complex amplitudes of the plane wave that is polarized parallel to the plane of incidence.

The values $r_{\parallel}$ and $t_{\parallel}$ are called the *amplitude coefficients of reflection and transmission* for a plane wave, polarized in the plane of incidence.

Now the equation (1.13) may be rewritten as:

$$\cos\vartheta + r_{\parallel}\cos\vartheta = t_{\parallel}\cos\vartheta''; \quad n_1 - n_1r_{\parallel} = n_2t_{\parallel}. \tag{1.14}$$

Solving this system of equations using the law of refraction $\sin\vartheta/\sin\vartheta'' = n_2/n_1$, we obtain:

CONTENTS