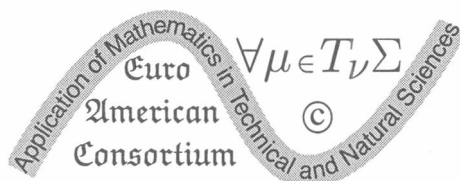


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# BOOK OF ABSTRACTS



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## The Dynamics of Two Linearly Coupled Goodwin Oscillators

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We study numerically a system of linearly coupled Goodwin oscillators for two regions, which, when isolated, differ in period and amplitude of regional business cycle. This system is described by the following equations [1]:

$$\begin{aligned} \varepsilon_1 \theta_1 \frac{d^2 y_1}{dt^2} + (\gamma_1 - \varphi_1) \frac{dy_1}{dt} + s_1 y_1 - \theta_1 \left( m_1 \frac{dy_1}{dt} - m_2 \frac{dy_2}{dt} \right) - m_1 y_1 + m_2 y_2 &= 0, \\ \varepsilon_2 \theta_2 \frac{d^2 y_2}{dt^2} + (\gamma_2 - \varphi_2) \frac{dy_2}{dt} + s_2 y_2 + \theta_2 \left( m_1 \frac{dy_1}{dt} - m_2 \frac{dy_2}{dt} \right) + m_1 y_1 - m_2 y_2 &= 0. \end{aligned}$$

Here  $y_i(t)$  = regional income,  $\varepsilon_i > 0$ ,  $\theta_i > 0$  = time-lag of the dynamic multiplier and the time-lag between investment decisions and the resulting outlays,  $s_i$  = marginal propensity to save,  $0 \leq s_i \leq 1$ ,  $\gamma_i = \varepsilon_i + \theta_i s_i$ ,  $m_i$  = marginal propensity to import,  $0 \leq m_i \leq 1$ ,  $\varphi_i(x)$  = induced investment function,  $\varphi_i(x) \geq 0$ ;  $\varphi_i(0) = 0$ ;  $\varphi_i'(0) = r_i > 0$ ;  $\varphi_i(\dot{y}) \rightarrow -I_{\min i}$  if  $x \rightarrow -\infty$ ;  $\varphi_i(x) \rightarrow I_{\max i}$  if  $x \rightarrow \infty$ ,  $i = 1, 2$ .

### References

- [1] T. Puu, *Nonlinear Economic Dynamics*, 4th edn, Springer-Verlag, Berlin and New York, 1997.

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