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Robust adaptive stabilization of multivariable static systems with unknown square gain matrices and bounded disturbances having unknown bounds

This work deals with designing the adaptive robust control systems containing linear discrete-time multivariable static plants which have unknown and possibly singular square gain matrices in the presence of arbitrary bounded disturbances whose bounds are unknown. The asymptotical properties of these systems are established.

The problem of stabilizing linear multivariable (interconnected) systems stated several decades ago in the paper [1] remains actual up to now [2]. It is important problem from both theoretical and practical point of view.

Since the seventies, the so-called internal model method becomes popular among other methods dealing with an improvement of the control system by exploiting the different types of plant and disturbances models. A perspective modification of the internal model control principle is the model inverse approach [2]. Unfortunately, this approach is quite unacceptable when the systems to be controlled are square but singular because they become noninvertible. It turned out that the so-called generalized inverse (pseudoinverse) model approach can be exploited to cope with the nonivertibility of singular system [3].

The common feature of the works [1, 3] dealing with the control of multivariable plants is that their parameters are assumed to be known. Usually, the adaptive approach is used to cope with the parametric uncertainty [4, 5]. Namely, the inverse model approach was before utilized in [6] for controlling a multivariable static plant whose gain matrix is unknown but nonsingular.

Different adaptive methods have recently been advanced in literature [7, 8]. However, they do not make it possible to avoid the crucial assumption with respect to the nonsingularity of the gain matrix introduced before in the book [5, item 4.2.3°] and also in the textbook [6, item 5.2.3].

A new method for the adaptive robust control of multivariable static plant whose gain matrix may be singular in the presence of arbitrary bounded disturbances has been presented in [9]. The essential assumption made in [9] is that their bounds are known. This paper is an extension of [9] to deal with the adaptive stabilization of multivariable static plants which haves any square gain matrix assuming the bounds on these disturbances are unknown.

Consider a linear multivariable static plant described by

$$y_n = Bu_n + v_n, \tag{1}$$

where $y_n = [y_n^{(1)}, \ldots, y_n^{(N)}]^{\mathrm{T}}$ is the N-dimensional output vector to be measured at nth time instant, $u_n = [u_n^{(1)}, \ldots, u_n^{(N)}]^{\mathrm{T}}$ is the N-dimensional vector of unmeasurable disturbances.

$$B = \begin{pmatrix} b_{11} & \dots & b_{1N} \\ \dots & \dots & \dots \\ b_{N1} & \dots & b_{NN} \end{pmatrix}$$

is an arbitrary transfer $N \times N$ gain matrix. It is assumed that its elements are all unknown. However, there are some interval estimates

$$\underline{b}_{ik} \le b_{ik} \le \overline{b}_{ik}, \quad i, k = 1, \dots, N \tag{2}$$

with the known upper and lower bounds. This implies that *B* in (1) may be ill-conditioned or even singular, in general.

Suppose $\{v_n^{(i)}\}$ represents the *i*th unmeasurable bounded disturbance sequence satisfying

$$| v_n^{(i)} | \le \varepsilon_i < \infty \quad \forall\, i = 1, \dots, N, \tag{3}$$

where ε_i is an upper bound on $v_n^{(i)}$. We assume that ε_is are unknown and it is essential.

Let

$$y^0 = [y^{0(1)}, \dots, y^{0(N)}]^{\mathrm{T}}$$

denote the desired output vector whose components satisfy $| y^{0(1)} | + \dots + | y^{0(N)} | \ne 0$.

The problem is to design the adaptive controller guaranteeing

$$\lim_{n \to \infty} \sup\, (\| y_n \| + \tau \| u_n \|) < \infty, \quad \tau > 0 \tag{4}$$

provided the assumptions (2) and (3).

Basic idea proposed by the authors in [9] is the transaction from the adaptive identification of the true plant having the singular transfer matrix $\tilde{B}$ to the adaptive identification of a fictitious plant with the nonsingular transfer matrix of the form

$$\tilde{B} = B + \delta_0 I, \tag{5}$$

where *I* denotes the identity matrix and δ_0 is a fixed quantity determined later.

Although $\tilde{B}$ as well as *B* remain unknown, the requirement

$$\det \tilde{B} \ne 0 \tag{6}$$

can always be satisfied by the suitable choice of δ_0 in (5).

The equation of the fictitious plant is derived by adding the quantity $\delta_0 u_{n-1}$ to the both sides of (1). Then, due to (5) we obtain the equation

$$\tilde{y}_n = \tilde{B} u_{n-1} + v_n, \tag{7}$$

equivalent to the true plant equation for

$$\tilde{y}_n = y_n + \delta_0 u_{n-1}. \tag{8}$$

We observe from (7) that the true and fictitious plants have the same inputs u_n and are subjected to the same unmeasurable disturbance v_n. It is essential that output $\tilde{y}_n$ of the fictitious plant, by virtue of (8), can always be measured.

Following to [9], define

$$\beta_{\min}^{(i)} := \underline{b}_{ii} - \sum_{k=1,\, k\neq i}^{N} \max\{|\,\underline{b}_{ik}\,|, |\,\overline{b}_{ik}\,|\},$$
$$\beta_{\max}^{(i)} := \overline{b}_{ii} + \sum_{k=1,\, k\neq i}^{N} \max\{|\,\underline{b}_{ik}\,|, |\,\overline{b}_{ik}\,|\}. \qquad (9)$$

Then, δ_0 required to ensure $\det \tilde{B} \neq 0$ is chosen to satisfy the conditions

$$\delta_0 > -\beta_{\min} \ \text{ for } \ |\,\beta_{\min}\,| < |\,\beta_{\max}\,|, \quad \delta_0 < -\beta_{\max} \ \text{ for } \ |\,\beta_{\min}\,| > |\,\beta_{\max}\,|, \qquad (10)$$

where

$$\beta_{\min} := \min\{\beta_{\min}^{(1)},\ldots,\beta_{\min}^{(N)}\}, \quad \beta_{\max} := \max\{\beta_{\max}^{(1)},\ldots,\beta_{\max}^{(N)}\}. \qquad (11)$$

As in [8, item 4.2], the adaptive control law is designed in the form

$$u_{n+1} = u_n + \tilde{B}_n^{-1}\tilde{e}_n, \qquad (12)$$

where instead of the current estimate B_n, another $\tilde{B}_n$ is exploited, where the output error

$$e_n = y^0 - y_n$$

is replaced by

$$\tilde{e}_n = y^0 - \tilde{y}_n. \qquad (13)$$

with $\tilde{y}_n$ given by the expression (8).

The adaptive identification algorithm used to determine the estimates $\tilde{B}_n$ may be taken as the iterative procedure

$$\tilde{b}_n^{(i)} = \tilde{b}_{n-1}^{(i)} - \gamma_n^{(i)} \frac{f(\tilde{e}_n^{*(i)}, \varepsilon_{n-1}^{(i)})}{1+\|\nabla u_n\|^2} \nabla u_n \,\operatorname{sign} \tilde{e}_n^{*(i)}, \quad i = 1,\ldots,N, \qquad (14)$$

proposed in [8, item 4.2]. In this algorithm, the notation $\tilde{b}_n^{(i)} := [\tilde{b}_{i1}(n),\ldots,\tilde{b}_{iN}(n)]^{\mathrm{T}}$ is introduced.

$$f(e,\overline{\varepsilon}) = \begin{cases} 0, & \text{if } |e| \le \overline{\varepsilon}, \\ |\,e\,| - \overline{\varepsilon} & \text{otherwise} \end{cases} \qquad (15)$$

represents the dead-zone function depending on the identification error

$$\tilde{e}_n^{*(i)} = \nabla\tilde{y}_n^{(i)} - \tilde{b}_{n-1}^{(i)\mathrm{T}}\nabla u_n \qquad (16)$$

with $\nabla u_n := u_n - u_{n-1}$, $\nabla\tilde{y}_n^{(i)} := \tilde{y}_n^{(i)} - \tilde{y}_{n-1}^{(i)}$ and also on the current estimate $\varepsilon_{n-1}^{(i)}$ found at the $(n$–1)th time instant. The coefficients $\gamma_n^{(i)}$ are chosen as

$$0 < \gamma' \le \gamma_n^{(i)} \le \gamma'' < 2 \qquad (17)$$

to satisfy (6).

Similar to [8, item 4.2], the algorithm for estimating $\varepsilon_n^{(i)}$ is designed in the form of the following iterative procedure:

$$\varepsilon_n^{(i)} = \varepsilon_{n-1}^{(i)} + \gamma_n^{(i)} \frac{f(\tilde{e}_n^{*(i)}, \varepsilon_{n-1}^{(i)})}{1+\|\nabla u_n\|^2}, \quad i = 1, \ldots, N \tag{18}$$

with

$$\varepsilon_0^{(i)} = 0 \quad \forall i = 1, \ldots, N. \tag{19}$$

The asymptotical behavior of the adaptive control system is established in the following theorem (the main result).

Theorem. Determine δ_0 using the formulas (9)–(11) and choose an arbitrary initial $\tilde{B}_0$ according to the expression (5) with $B_0 = \{b_{ik}(0)\}$ whose elements satisfy

$$\underline{b}_{ik} \le b_{ik}(0) \le \overline{b}_{ik}.$$

Subject to the assumptions given in the inequalities (2), (3) with known $\underline{b}_{ik}$s, $\overline{b}_{ik}$s and ε_is the adaptive control algorithm described in the equations (12) to (19) when applied to the plant (1) yields:

i) the matrix sequence $\{\tilde{B}_n\} := \tilde{B}_1, \tilde{B}_2, \ldots,$ induced by the identification procedure converges, i.e.,

$$\lim_{n\to\infty} \tilde{B}_n = \tilde{B}_\infty;$$

ii) there exists a limit

$$\lim_{n\to\infty} \varepsilon_n^{(i)} = \varepsilon_\infty^{(i)}, \quad i = 1, \ldots, N;$$

iii) the requirement (4) is satisfied.

The proof follows the lines of the books [6, 8]. (Due to space limitation, details are omitted.)

It can be observed that the sequences $\varepsilon_n^{(i)}$ is non-decreasing.

Note that there is no guarantee that the ultimate performance index is the same as in the nonadaptive control system containing the generalized inverse model.

Conclusion

It was established, that there is the possibility to design the adaptive robust controller stabilizing the linear multivariable plant whose gain matrix is singular in the presence of the bounded disturbances with unknown bounds.

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