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H_2/H_∞ OPTIMIZATION OF INERTIALLY STABILIZIED PLATFORMS

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Abstract—*The paper deals with H_2/H_∞-approach to design of the inertially stabilized platforms operated at vehicles of the different types including unmanned aerial ones. The formulation of the vector optimization problem is represented. The robust optimization algorithm and results of the synthesied system simulation are shown. Comparative analysis of the results of the parametrical optimization using the Nelder-Mead method and genetic algorithm are given. Proposed approach ensures functioning of the inertially stabilized platforms in the difficult conditions of real operation.*

Index terms—H_2/H_∞-approach; parametrical synthesis; robust systems; vector optimization; genetic algorithm.

I. INTRODUCTION

To keep an invariable orientation of the information and measuring devices in direction of the observation object is the complex problem if devices are mounted at a vehicle. Control of the device line-of-sight orientation by means of the inertial stabilization allows solving this complex problem. Such approach can be used for stabilization of the usual and infra-red cameras operating at ground and marine vehicles, aircrafts and spacecrafts. This ensures fulfilment of different applications such as the topographical survey and mapping with the high resolution of the obtained images. These possibilities could be technically implemented by means of the inertially stabilized platforms (ISPs) [1], [2].

The widespread application of ISPs is caused by the last achievements in the modern instrument-making. The progress in development of the gyro devices, actuators and electronic units ensures creating of the high-precision systems for stabilization and control by orientation of information and measuring devices mounted at the moving platforms. Furthermore, the modern mechanical constructions and new materials are able to provide the better balancing, strength and rigidity. Mechanisms with the greater rigidity have the higher frequencies of the mechanical resonances. This ensures the higher bandwidth of the system. Decrease of the noise of the modern gyros, actuators and electronic units leads to decrease vibration of the modern inertially stabilized platforms [1].

Depending on the specification of requirements, the ISPs are designed to point and stabilize one, two or more axes. The most applications require two orthogonal gimbals. However, some applications require three orthogonal gimbals to ensure the high precision of the stabilization and observation processes. But in some cases it is sufficient to carry out stabilization by one axis only.

Although requirements for ISPs vary widely depending on their application, they all have a common goal, which is to hold or control the direction of the line of sight in the inertial space.

The basic motivation of ISP development is the real necessity of its characteristics improvement. It should be noticed, that accuracy of the images obtained by cameras and their resolution are drastically improved in for last years. These high characteristics may not be achieved without appropriate progress of ISPs. And one of the important problems is the necessity to provide the possibility to operate in difficult conditions accompanied by action of the internal and external disturbances.

The parametric synthesis of robust systems insensitive to both variations of parameters during real operation and deviations of the model parameters from its real values is one of the widespread directions for creation of the modern ISPs. Synthesis of such systems is grounded on minimization of the H_∞-norm of the matrix transfer function of the closed loop system. Usage of thr H_∞-norm for ISP synthesis allows ensuring resistance of the system to external disturbances in conditions of its parametrical uncertainty.

It is known also an approach, which is based on minimization of the H_2-norm of the matrix transfer function of the closed loop system [3]. This norm characterizes the accuracy of a system. Methods of synthesis based on minimization of the H_2-norm ensure the high accuracy of the synthesized system but it stays sensitive to both external disturbances and plant parametrical ones.

Combination of the H_2/H_∞-optimization allows formulating of a problem of the synthesis of the stabilization system with the optimal performance level based on the quadratic criteria for deterministic and stochastic cases under condition of conservation this performance level during action of parametric disturbances [3] – [5].

II. Problem Statement

For the system parametric optimization it is necessary to choose the optimization criterion taking into consideration the various aspects of the system functioning. Both accuracy and stability in the presence of the internal and external disturbances are of great importance for systems of the studied type.

For calculation of the local performance indices of the robust stabilization systems the H_2-norms are used. It should be noted, that the H_2-norms represent the square roots from the integral quadratic performance criteria. In the general form these criteria look like:

– for the deterministic dynamic systems [6]

$$J_d = \int_0^\infty (\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u})dt, \qquad (1)$$

where $\mathbf{Q}, \mathbf{R}$ are the weighting matrices, which take into consideration the weights of the state variables and the external disturbances;

– for the stochastic dynamic systems [6]

$$J_s = \int_0^\infty (\mathrm{M}[\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u}])dt, \qquad (2)$$

where M is the symbol of the mathematical expectation.

For calculation of the stabilization systems robustness criteria the H_2-norm of the closed loop system complementary sensitivity matrix function $\mathbf{\Phi}_T(j\omega)$ is used [4]

$$H_\infty = \sup_{\omega\in[-\infty,\infty]} \overline{\sigma}(j\omega), \qquad (3)$$

where $\overline{\sigma}$ is the maximum singular value of the closed loop system complementary sensitivity matrix transfer function $\mathbf{\Phi}_T(j\omega)$.

The function $\overline{\sigma}(j\omega)$ in (3) is called the singular characteristic of the multi-dimensional system

$$\overline{\sigma}(j\omega) = \max_{i\in 1,\dots,n\ \forall\omega\in[-\infty,\infty]} \sqrt{eig_i \mathbf{\Phi}^*(j\omega)\mathbf{\Phi}(j\omega)}, \qquad (4)$$

where *eig* means operation of the eigenvalues determination. The H_2-norm characterizes the upper boundary of the system singular frequency characteristic maximum value.

Based on researches [5], [7] the criterion of the studied system the $\mathrm{H}_2/\mathrm{H}_\infty$-optimization may be represented in the following form

$$J_{\mathrm{H}_2/\mathrm{H}_\infty} = \boldsymbol{\lambda}_d \mathbf{J}^d_{\mathrm{H}_2} + \boldsymbol{\lambda}_s \mathbf{J}^s_{\mathrm{H}_2} + \boldsymbol{\lambda}_\infty \mathbf{J}_\infty + PF, \qquad (5)$$

where $\boldsymbol{\lambda}_d, \boldsymbol{\lambda}_s, \boldsymbol{\lambda}_\infty$ are the weighting coefficients; *PF* is the penalty function.

$$\mathbf{J}^d_{\mathrm{H}_2} = \begin{bmatrix} \|\Phi_{S1}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{\text{nom d}}\|_2 \\ \|\Phi_{S1}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)_1^{\text{par d}}\|_2 \\ \dots \\ \|\Phi_{S1}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)_n^{\text{par d}}\|_2 \end{bmatrix}; \quad \mathbf{J}^s_{\mathrm{H}_2} = \times \begin{bmatrix} \|\Phi_{S2}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{\text{nom s}}\|_2 \\ \|\Phi_{S2}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)_1^{\text{par s}}\|_2 \\ \dots \\ \|\Phi_{S2}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)_n^{\text{par s}}\|_2 \end{bmatrix}; \quad \mathbf{J}_\infty = \times \begin{bmatrix} \|\Phi_T(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{\text{nom}}\|_\infty \\ \|\Phi_T(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)_1^{\text{par}}\|_\infty \\ \dots \\ \|\Phi_T(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)_n^{\text{par}}\|_\infty \end{bmatrix};$$

$$\boldsymbol{\lambda}_d^{\mathrm{T}} = \begin{bmatrix} \lambda_2^{\text{nom d}} & \lambda_{21}^{\text{par d}} & \dots & \lambda_{2n}^{\text{par d}} \end{bmatrix}; \quad \boldsymbol{\lambda}_s^{\mathrm{T}} = \begin{bmatrix} \lambda_2^{\text{nom s}} & \lambda_{21}^{\text{par s}} & \dots & \lambda_{2n}^{\text{par s}} \end{bmatrix}; \quad \boldsymbol{\lambda}_\infty^{\mathrm{T}} = \begin{bmatrix} \lambda_\infty^{\text{nom s}} & \lambda_{1\infty}^{\text{par s}} & \dots & \lambda_{n\infty}^{\text{par s}} \end{bmatrix},$$

where $\mathbf{\Phi}_{S1}$ is the sensitivity function for the deterministic case; $\mathbf{\Phi}_{S2}$ is the sensitivity function for the stochastic case; $\mathbf{\Phi}_T$ is the complementary sensitivity function.

The local components of the global criterion (5) are conflicting. The problem of the $\mathrm{H}_2/\mathrm{H}_\infty$-optimization for the vector of the optimized parameters ***K*** looks like

$$\mathbf{K}_{\text{opt}} = \operatorname{argmin} J_{\mathrm{H}_2/\mathrm{H}_\infty}(\mathbf{K},\mathbf{Q},\mathbf{R},\Lambda,\mathbf{x},\mathbf{u},j\omega);$$

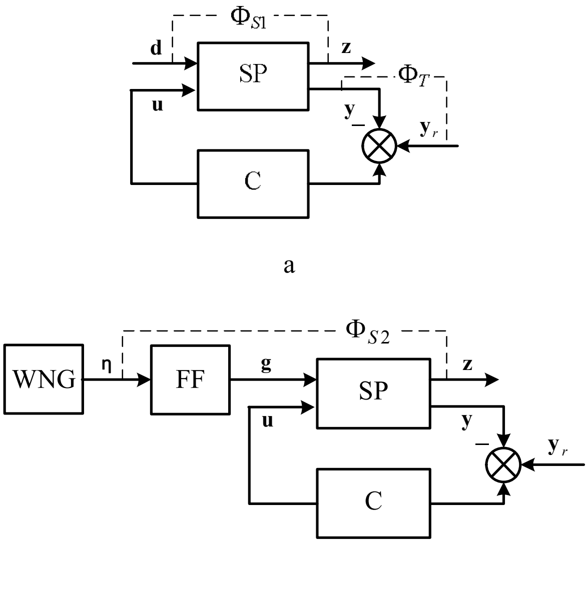
$$\mathbf{K}\in D,\ D: \mathrm{Re}[\mathrm{eig}_i(\mathbf{I}+\mathbf{L}(s))] < 0,\ i\in 1,\dots,i_0; \qquad (6)$$

$$\mathbf{L}(s) = \mathbf{P}(s)\mathbf{W}(s),$$

where D is the stability region in the space of designed parameters; $\mathbf{P}(s)$ is the transfer function of the controller; $\mathbf{W}(s)$ is the transfer function of the plant.

Requirements to control accuracy and robustness are conflicting. Therefore the problem of the robust $\mathrm{H}_2/\mathrm{H}_\infty$-optimization lies in searching a compromise between accuracy and robustness of the system. This compromise may be achieved by means of the vector criterion with the changeable weighting coefficients in the optimization criterion (5). Such approach allows decreasing or increasing of a measure of the accuracy and robustness depending on analysis of the system characteristics.

The structural schemes of the deterministic and stochastic stabilization systems are represented in Fig. 1.



a

b

Fig. 1. The deterministic (a) and stochastic (b) stabilization systems: SP is the stabilized plant; C is the controller; FF is the forming filter; WNG is the white noise generator; **d** is the input (command) signal; **z** is the observation signal; **u** is the control signal; **y** is the output signal; η is the white noise; **g** is the disturbance

The main stochastic disturbance, which influences on unmanned aerial vehicle (UAV), is the turbulent wind. The standard mathematical description of this disturbance may be obtained by means of Dryden model. In this case, the longitudinal, vertical and lateral components of the wind speed may be represented as stationary random processes with the spectral densities [8]

$$\begin{aligned}
&S_u(\omega)=\frac{2\sigma_u^2 L_u}{V_0\pi}\frac{1}{(1+\tau_u^2\omega^2)};\\
&S_v(\omega)=\frac{2\sigma_v^2 L_v}{V_0\pi}\frac{1+3\tau_v^2\omega^2}{(1+\tau_v^2\omega^2)};\\
&S_w(\omega)=\frac{2\sigma_w^2 L_w}{V_0\pi}\frac{1+3\tau_w^2\omega^2}{(1+\tau_w^2\omega^2)};\\
&\tau_u=\frac{L_u}{V_0};\quad \tau_v=\frac{L_v}{V_0};\quad \tau_w=\frac{L_w}{V_0},
\end{aligned} \tag{7}$$

where L_u, L_v, L_w are appropriate scales of the turbulence; V_0 is the constant flight speed; $\sigma_u, \sigma_v, \sigma_w$ are variances of the wind speed components.

To simulate the influence of disturbances on the studied system it is necessary to obtain the transfer function of the forming filter, which converts the white noise at its input into a random process with the given spectral density. For forming filters determination Dryden models may be used [8].

Usage of the vector criterion (5) for the robust parametrical optimization of the studied system allows obtaining solution, which will ensure the optimal compromise between the requirements to the accuracy and robustness of the system. Such approach to the optimization problem solution is called multi-objective, as it allows finding the compromise between the conflicting objectives [4], [5].

III. Mathematical Description of System

As a rule the motor and the stabilization plant are combined with each other by the elastic connection. Taking into consideration the experience of mathematical description development for stabilization systems, operated at vehicles of another type [9], the generalized model of the stabilization plant and motor (on example of one channel) may be represented in the following form

$$\begin{aligned}
&J_{\mathrm{m}}\ddot{\varphi}_{\mathrm{m}}=-M_{\mathrm{fr.\,m}}\operatorname{sign}\dot{\varphi}_{\mathrm{m}}+\frac{c_{\mathrm{m}}}{R_{\mathrm{w}}}U+c_{\mathrm{r}}\varphi_{\mathrm{m}}-c_r\varphi_{\mathrm{p}};\\
&J_{\mathrm{p}}\ddot{\varphi}_{\mathrm{p}}=-M_{\mathrm{fr.\,p}}\operatorname{sign}\dot{\varphi}_{\mathrm{p}}-M_{\mathrm{imb}}\cos\varphi_{\mathrm{p}}+c_{\mathrm{r}}\varphi_{\mathrm{m}}-c_{\mathrm{r}}\varphi_{\mathrm{p}};\\
&\dot{U}T_{\mathrm{arm}}+U=U_{\mathrm{con}}-c_{\mathrm{e}}\dot{\varphi}_{\mathrm{m}};\\
&U_{\mathrm{con}}=k_{\mathrm{r.\,g}}k_{\mathrm{amp}}\varphi_{\mathrm{p}}\left(k_{\mathrm{g}}+\frac{1}{T_{\mathrm{i}}s}\right)+U_{\mathrm{cons}}\\
&\qquad=k_{\mathrm{r.\,g}}k_{\mathrm{amp}}k_{\mathrm{g}}\dot{\varphi}_{\mathrm{p}}+\frac{K_{\mathrm{r.\,g}}k_{\mathrm{amp}}\varphi_p}{T_{\mathrm{i}}}+U_{\mathrm{cons}},
\end{aligned} \tag{8}$$

where J_{m} is the moment of the motor inertia; φ_{m} is an angle of the motor turn; $M_{\mathrm{fr.\,m}}$ is the moment of the dry friction of the motor; c_{m} is the constant of the load moment at the motor shaft; R_{w} is the resistance of the motor armature winding; U is the voltage of the motor armature circuit; J_{p} is the moment of the platform inertia; φ_{p} is an angle of the platform turn; $M_{\mathrm{fr.\,p}}$ is the moment of the dry friction at platform bearings; M_{imb} is the moment of the platform imbalance; c_{r} is the rigidity of elastic connection between the motor and a base, on which the plant is mounted; U_{con} is the voltage at the controller output; c_{e} is the coefficient of proportionality between the motor angular rate and the electromotive force; T_{arm} is the time constant of the motor armature circuit; $k_{\mathrm{r.\,g}}$ is the rate gyro transfer constant; k_g is the gain of the proportional part of the PI-controller; T_i is the integrator time constant of the integrating part of the PI-controller; k_{amp} is the amplifier gain; U_{cons} is the reference voltage.

Choice of the PI-controller is carried out in accordance with recommendations represented in [1].

Non-linear moments of the dry friction may be approximated by the linear dependences, in which the approximation coefficients may be determined as a ratio of the friction moment first harmonic amplitude to the angular rate amplitude [10]. In this case the relationships for determination of the friction moments

$$M_p = M_{frp}\text{sign}\dot{\varphi}_p, \quad M_m = M_{frm}\text{sign}\dot{\varphi}_m,$$

may be defined by the linear dependences $M_p = f_{frp}\dot{\varphi}_p$, $M_m = f_{frm}\dot{\varphi}_m$. Coefficients f_{frp}, f_{frm} will be defined by the expressions $f_{frp} = 4M_{frp}/(\pi\Omega_p)$, $f_{frm} = 4M_{frm}/(\pi\Omega_m)$ [10].

If the non-contact moment gearless drive is used, c_r in the model (8) will be believed equal to 1.

The model may be transformed to the model in the state space after introducing new variables and reduction of an order of the set of the equations (8). In this case, vectors of state, control, observation and matrices of the control, observation, state become

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \dot{\varphi}_m \\ \dot{\varphi}_p \\ \varphi_m \\ \varphi_p \\ U \end{bmatrix}; \quad \mathbf{u} = \begin{bmatrix} M_{imb} \\ U_{cons} \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} \dot{\varphi}_p \\ U \end{bmatrix};$$

$$\mathbf{B}^T = \begin{bmatrix} 0 & \frac{-1}{J_p} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{T_{arm}} \end{bmatrix}; \mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$\mathbf{A} = \begin{bmatrix} \frac{-f_{frm}}{J_m} & 0 & \frac{-c_r}{J_m} & \frac{c_r}{J_m} & \frac{c_m}{R_w J_m} \\ 0 & \frac{-f_{frp}}{J_p} & \frac{c_r}{J_p} & \frac{-c_p}{J_p} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \frac{-c_e}{T_{arm}} & k_{r.g}k_{amp}k_g & 0 & \frac{k_{r.g}k_{amp}}{T_i} & \frac{-1}{T_{arm}} \end{bmatrix}. \tag{9}$$

Restrictions of signals by level are widely used in the real apparatus. This corresponds to non-linear characteristic with saturation. Linearization of the model in this aspect is implemented by choice of the range and signal level.

IV. Characteristic of Genetic Algorithm

Imitation methods of research became widely spread in areas of control and began to be widespread for parametrical problems solution [11]. Now numerical optimization algorithms gradually replace the classical approaching methods of optimal parameters determination.

The experience in control algorithms research showed that optimization problems for simple control systems as a rule have one extremum. However, for complex control systems the presence of the bigger number of local extremes is more typical.

There are analytical and numerical algorithms for solution the one-extreme optimization problems. One of such algorithms is a simplex-method based on Nelder–Mead [12] deformable polyhedron.

The universal program realization of this algorithm is presented in work [13]. In addition, there are examples of its usage for different problems solution, including the parametrical optimization.

Nelder–Mead method [14] is used for minimization of the multivariable objective function, when other methods, for example, method of the gold section or the quadratic approximation can not be used. The algorithm of Nelder-Mead method in MatLab system is implemented by the function *Nelder*. For the multidimensional case (n >2) this algorithm may be repeated for every optimization sub-plane, as it is implemented by means of function *opt_Nelder*. Usually, in practical situations the function *fminsearch* is used, which allows minimization of the multi-variable objective function. The single-extreme optimization methods does not guarantee the solution optimality, therefore the necessity to use the global optimization methods is arisen.

Today the most preferable multi-parametric and multi-extreme optimization methods are considered to be the genetic algorithms. They implement the evolution theory postulates and the animal and plant selection experience. The optimal solution search strategy in genetic algorithms is based on selection hypothesis. Genetic algorithms represent the elementary models of the individual evolution process in nature. If we assume that every individual of a population is a point at an optimization problem coordinate space, the process of evolution in such a case is the motion of those points to the objective function optimal values. Just as the way the organism's inheritance information is presented in chromosomes in form of four-nucleotide different combinations linear sequence (A – adenine; C – cytosine; T – thymine; G – guanine), in genetic algorithm the variable vectors are also written in symbol chain form, using bi-, tri-, tetra-literal alphabet [5]. The chromosomal information representation allows using such genetic operators as crossover, mutation, and inversion.

The genetic operators are the simplified form of inheritance transfer. In nature they provide the evolution process and in genetic optimization algorithms – the coordinate points motion in direction to optimal value of an objective function.

V. ALGORITHM OF H_2/H_∞-OPTIMIZATION

Algorithm of the robust parametrical H_2/H_∞-optimization includes the following steps:

– creation of the full mathematical description of the system for stabilization of the information and measuring devices operated at vehicles, which maximally takes into consideration all the non-linearities inherent to the system;

– creation of the appropriate linearized mathematical model in the state space;

– forming the appropriate objective and penalty functions;

– creation of the procedure, which allows to simulate the external disturbances typical for vehicles operation, for example, using Dryden models in the case of UAVs;

– choice of the optimization method (Nelder–Mead method or genetic algorithm);

– creation of calculating procedures for parametrical synthesis execution based on the modern automated means for the optimal synthesis of the control systems (Control System and Robust Control Toolboxes of the system MATLAB);

– simulation and analysis of the obtained results;

– solution about termination of the parametrical synthesis or repetition of optimization procedures with the new initial data or weighting coefficients.

For creation of the calculating procedures for the robust system synthesis it is convenient to use the models in the state space. These models are characterized by the following important advantages [16]:

– transfer functions of models of the type "input–output" deal with the zero initial conditions only;

– a model in the form of the transfer function does not describe the behaviour of the system internal parameters, in particular, non-observable and non-controllable modes;

– the mathematical model in the state space is more convenient for the mathematical description of the multi-dimensional and non-linear systems;

– presence of models in the state space allows to use the automated means of the optimal control systems in the most rational way;

– the mathematical description in the state space may be widen on the non-stationary and non-linear systems.

The process of the problem solution represents a procedure of the multiple minimization of the criterion (5) by one of the known methods. The genetic algorithm may be used as such method. The advantage of the genetic algorithm lies in the possibility to find the global minimum in every concrete case.

For a system of the studied type it is expedient to use the transient process quality indices as the boundary conditions, which must be satisfied in any case, by means of their use in the penalty function. For systems of the studied type such characteristics as overshoot and regulation time are of great importance.

A type of the transients depends on the distribution of zeros and poles. The stabilization quality depends on the mutual location of zeros and poles of the disturbance representation. This defines the certain requirements to the distribution of the closed loop system transfer function poles. So, some area may be defined at the complex plane of the closed loop system poles location, which will satisfy the requirements to the system variability and speed of operation.

VI. RESULTS OF ROBUST PARAMETRICAL OPTIMIZATION

So, for the considered problem it is convenient to apply the genetic algorithm realized in MatLab Optimization Toolbox. The optimization criterion remains the same and is described by the expression (5). Unlike Nelder–Mead method, where the starting point is set, in genetic algorithm a number of variables and the initial and final values of every variable must be defined. The genetic algorithm program software also has other parameters, which are intended to modify it for the certain problem.

Doing the optimization by means of the genetic algorithm it is expedient to mention that this algorithm is universal, as it does not impose constraints for the criterion function of the definite type.

Otherwise, there are such situations, when it is necessary to terminate the algorithm due to such reasons:

– achievement of certain number of populations;

– evolution time expiration;
– population convergence.

First two criteria depend on the type of a problem, and sometimes there occurs a situation, when the algorithm could not find the function extreme or when the obtained after some number of populations results do not satisfy the given requirements. Under the population convergence one means that neither crossover nor mutation operations make the change into algorithm result during a few populations creation. Such a situation takes place either when reaching a region, at which the objective function does not change its value, or when the population falls into local extreme zone.

Results of the vector robust optimization are represented in Tables I, II. In Table I the optimal values of the stabilization law coefficients are given. Table II includes such performances of the optimized system as norms, margins by the amplitude and the phase.

TABLE I

OPTIMIZED PARAMETERS OF THE STABILIZATION SYSTEM

k_{amp}	k_{g}	T_{i}, s
10.234	5.21	0.001

TABLE II

PERFORMANCES OF THE OPTIMIZED SYSTEM

Parameters	Genetic Algorithm	Nelder–Mead Method
H_2-norm	0.207	0.399
H_∞-norm	0.632	0.793
Settling time (s)	0.59	0.727
Oscillation factor	3.5	2.91
Number of oscillations	3	3
Delay time (s)	0.0542	0.0543
Rise time (s)	0.0315	0.0314
ΔA, dB	61.1	59.4
$\Delta\varphi$, deg	91.5	91.1

Results of the synthesized system simulation are represented in Figs 2, 3, where the step and impulse response plots are represented.

Analysis of results represented in Figs 2, 3 shows that using of the genetic algorithm in H_2/H_∞-optimization procedure is more preferable, as the transient process provides motion to the equilibrium point more quickly and smoothly in comparison with Nelder–Mead method.

Confidence of the H_2/H_∞-optimization results is based on the principle of the guaranteed result [17], which does not depend on the spectral properties of the external disturbance and depends on its H_∞-norms only

$$\frac{\|\Phi(s)w(s)\|_\infty}{\|w(s)\|_\infty} < \gamma,$$

where $\Phi(s)$ is the closed loop system matrix transfer function; $w(s)$ is the Laplace transformation of the disturbance signal; γ is a small number.

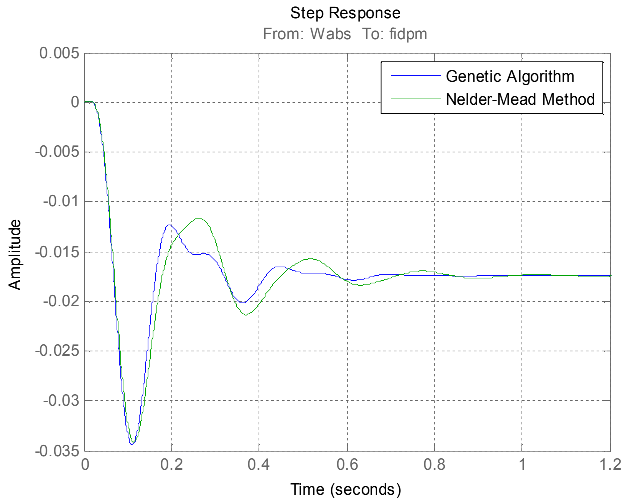


Fig. 2. Step responses of the optimal system obtained by means of Nelder–Mead method and genetic algorithm

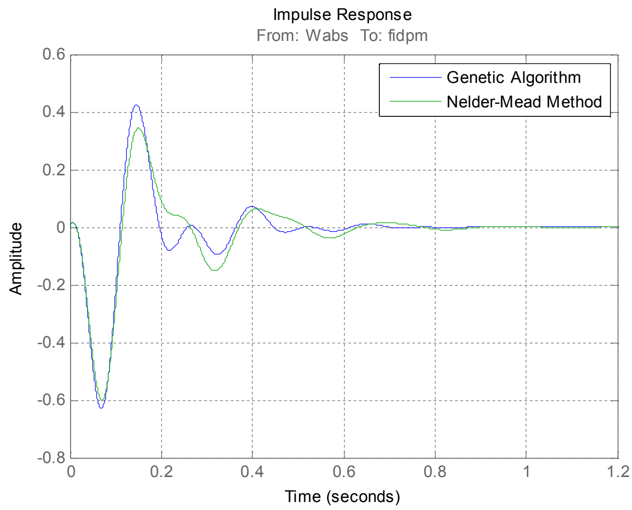


Fig. 3. Impulse responses of the optimal system obtained by means of Nelder–Mead method and genetic algorithm

Such approach is widely used in the practice of the robust systems design.

VII. CONCLUSIONS

The mathematical description (7), (8) of the system assigned for stabilization of information and measuring devices operated at UAV in difficult conditions under turbulent wind action is represented.

H_2/H_∞-approach to design of the system assigned for stabilization of observation devices operated at vehicles is suggested.

The algorithm of the vector robust parametrical optimization for the studied system is represented.

Comparative analysis of application in the optimization procedure of the Nelder–Mead method and genetic algorithm is given.

Results of the vector robust optimization and simulation of the synthesized system are given, that proves the requirements given to the transients of the studied system.

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О. А. Сущенко, О. В. Широкий. H_2/H_∞-оптимізація інерціальних стабілізованих платформ

Статтю присвячено H_2/H_∞-підходу до проектування інерціальних стабілізованих платформ, що функціонують на рухомих об'єктах різних типів, включаючи безпілотні літальні апарати. Представлено формалізовану постановку проблеми векторної оптимізації. Надано алгоритм робастної оптимізації та результати моделювання синтезованої системи. Представлено порівняльний аналіз результатів моделювання систем, синтезованих за

допомогою методу Нелдера Міда та генетичного алгоритму. Запропонований підхід забезпечує функціонування інерціальних стабілізованих платформ в складних умовах реальної експлуатації.
Ключові слова: H_2/H_∞-підхід; параметричний синтез; робастні системи; векторна оптимізація; генетичний алгоритм.

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О. А. Сущенко, А. В. Широкий. H_2/H_∞-оптимизация инерциальных стабилизированных платформ

Статья посвящена H_2/H_∞-подходу к проектированию инерциальных стабилизированных платформ, которые функционируют на подвижных объектах разных типов, включая беспилотные летательные аппараты. Представлена формализованная постановка проблемы векторной оптимизацию. Показаны алгоритм робастной оптимизации и результаты моделирования синтезированной системы. Представлен сравнительный анализ результатов моделирования систем, синтезированных с использованием метода Нелдера–Мида и генетического алгоритма. Предложенный подход обеспечивает функционирование инерциальных стабилизированных платформ в сложных условиях реальной эксплуатации.
Ключевые слова: H_2/H_∞-подход; параметрический синтез; робастные системы; векторная оптимизация; генетический алгоритм.

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