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TROPOSPHERE PARAMETERS CALCULATION AND RESIDUAL ERROR MODELING FOR GNSS LANDING SYSTEM

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Abstract. The paper shows the results of simulation of troposphere residual errors in the satellite based aircraft landing system. For such simulation the troposphere MOPS model has been used. The presented simulated troposphere parameters include: dry and wet components of the refraction index and troposphere scale height. Beside of that the simulated residual value of troposphere error for navigation satellites in view of satellite landing system has been presented.

Keywords: global navigation satellite systems (GNSS), landing system, GBAS, troposphere, tropospheric delay, modelling.

Introduction

The growth of aviation and the urgent need to reduce fuel consumption and emissions demand increased airspace and airport capacity and a focus on providing the preferred trajectory (route and altitude) to each aircraft. Aircraft operators also require efficiency gains via approaches with the lowest possible minima and the significant safety benefits of vertical guidance. In fact, controlled flight into terrain (CFIT), in the absence of vertical guidance, is still a frequent accident category, at least for some segments of the aviation community. Another key goal is to reduce the effects of airport noise on populated areas. GNSS-based services can meet these goals and have already provided significant safety and efficiency benefits to aircraft operators. ICAO Global Air Navigation Plan acknowledges that "GNSS centered Performance Based Navigation enables a seamless, harmonized and cost-effective navigational service from departure to final approach that will provide benefits in safety, efficiency and capacity".

The existing core satellite constellations alone however do not meet strict aviation requirements. To meet the operational requirements on accuracy, integrity, continuity and availability for various phases of flight, the core satellite constellations require augmentation in the form of aircraft-based augmentation system (ABAS), satellite-based augmentation system (SBAS) and/or ground-based augmentation system (GBAS).

GBAS augments the existing Global Positioning System (GPS) used in airspace by providing corrections to aircraft in the vicinity of an airport in order to improve the accuracy of, and provide integrity for, these aircrafts' GPS navigational position. It uses ground monitoring stations to verify the validity of satellite signals and calculate differential corrections to enhance accuracy, then this information via VHF data broadcast (VDB) is delivered from a ground station to the aircraft.

Methodology of troposphere parameters calculation

Though it is considered that in differential mode of coordinates definition tropospheric delay is compensated in a maximal way, but using the differential mode at aircraft landing has some peculiarities when comparing with ground applications. The specific is that the troposphere delay depends significantly on a height. Therefore the tropospheric delay of signal from navigation satellite in airborne and ground subsystems will be different. This difference, the tropospheric correction, is modelled by airborne GNSS based landing subsystem for a given satellite according to the following formula (ICAO Annex 10 Vol. I, RTCA DO-245A):

$$TC = N_R h_0 \frac{10^{-6}}{\sqrt{0.002 + sin^2(\Theta)}} \left(1 - e^{\frac{-\Delta h}{h_0}}\right), \qquad (1)$$

where: $TC[m]$ – tropospheric correction for a given satellite; N_R – troposphere refractivity index transmitted by ground subsystem (Type 2 message), in millimeters per kilometer; h_0 – troposphere scale height transmitted by the

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ground subsystem (Type 2 message), in meters; Θ – elevation angle of the satellite; Δh – difference in altitude between airborne and ground subsystems, in meters (which is a height of the aircraft above the GBAS reference point).

The residual tropospheric uncertainty is defined as a following:

$$\sigma_{tropo} = \sigma_n \cdot h_0 \cdot \frac{10^{-6}}{\sqrt{0.002 + sin^2(\Theta)}} \cdot \left(1 - e^{-\frac{\Delta h}{h_0}}\right), \quad (2)$$

where: σ_n – the refractivity uncertainty that is transmitted by the ground subsystem in a Type 2 message.

Parameters N_R and h_0 are defined by the ground subsystem and are transmitted to an airborne subsystem through communication channel. Let's consider the methods of N_R and h_0 parameters definition in a ground subsystem. First of all we'll show the relations between these parameters.

It is necessary to define first a zenith troposphere delay of ground subsystem. It is a delay that occurs in ground subsystem when the satellite is in zenith in relation to the antenna of ground subsystem. This delay is defined as:

$$\Delta Z(H) = 10^{-6} \int_h^{\infty} N_R(H) dH = 10^{-6} N_R(H) h_0(H), \quad (3)$$

where: h – height of ground subsystem; H – length of signal path (height).

The troposphere refraction consists of dry and wet components:

$$N_R(H) = N_d(H) + N_w(H), \quad (4)$$

and, in the same manner zenith troposphere delay consists of dry and wet components:

$$\Delta Z(H) = Z_d(H) + Z_w(H), \quad (5)$$

which can be defined as following:

$$Z_d(H) = 10^{-6} \int_h^{\infty} N_d(H) dH = 10^{-6} N_d(H) h_d(H), \quad (6)$$

$$Z_w(H) = 10^{-6} \int_h^{\infty} N_w(H) dH = 10^{-6} N_w(H) h_w(H), \quad (7)$$

where: $h_d(H)$ – height of dry component of troposphere; $h_w(H)$ – height of wet component of troposphere.

Model for defining of refraction index at the sea level is known from the 50-ies of XX century and according to (Hofmann-Wellenhof *et al.* 2008) or (Schüler 2001) has a following view:

$$N_d(0) = k_1 \frac{P(0)}{T(0)}, \quad (8)$$

$$N_w(0) = k_2 \frac{e(0)}{T^2(0)}, \quad (9)$$

where: $P(0)$ $[hPa]$ – pressure at the sea level; $T(0)$ $[K]$ – absolute temperature at the sea level; $e(0)$ $[hPa]$ – partial pressure of water vapor; $k_1 = 77.64 \left[\frac{K}{hPa}\right]$ $k_2 = 3.718 \cdot 10^5 \left[\frac{K^2}{hPa}\right]$ – experimentally defined coefficients.

Partial pressure of water vapor can be defined from the following equation:

$$e(0) = e^* \frac{RH}{100\%} EXP^{\frac{L}{R_e}\left(\frac{1}{T^*} - \frac{1}{T(0)}\right)}, \quad (10)$$

where: RH – percentage of relative humidity; e^*= 6.11 $[hPa]$ – particular pressure of water vapor at the temperature of water freezing; T^*= 273.15 $[K]$ – the temperature of water freezing; $L = 2.83 \cdot 10^6 \frac{J}{kg}$ – residual heat of vapor creation above the flat water surface; $R_e = 461 \frac{J}{K \cdot kg}$ – gas constant for water vapor.

There are few models for defining of wet and dry refraction indexes, namely Hopfield model (Hopfield 1969), Saastamoinen model (Saastamoinen 1972), Black model, model of New Bruswick University, GCAT and MOPS models. These models can be viewed in more detailed in (Schüler 2001) and (Pershin 2009). Let's consider here Hopfield and MOPS models.

Hopfield model defines the dry component of troposphere refraction as the following:

$$\mathrm{N_d}(H) = N_d(0)\left[1 - \frac{h}{h_d(H)}\right]^4, \quad (11)$$

where: $h_d(H) = 40136 + 148.72 \cdot (T(H) - 273.16)$ $[m]$ – is a height of dry component of troposphere, and the wet component of troposphere refraction is defined as:

$$\mathrm{N_w}(H) = N_{w,0}(0)\left[1 - \frac{h}{h_w(H)}\right]^4, \quad (12)$$

where: $h_w(h) = 7508 + 0.002421 \cdot EXP^{\frac{T(h) - 273.16}{22.9}}$ $[m]$ – is a height of wet component of troposphere.

Let's consider the MOPS in more detailed. Though according to (Schüler 2001) this model has worse accuracy characteristics, its advantage is the absence of measured meteorological data.

Instead of measured meteorological data these meteorological parameters in MOPS model are defined:

$$\xi(\varphi, DoY) = \{P(0), \quad T(0), \quad e(0), \quad \beta, \quad \lambda\},$$

where: φ – latitude of ground subsystem; DoY – day of year; β $\left[\frac{K}{m}\right]$ – temperature lapse rate (coefficient of temperature rate according to height); λ – coefficient of water vapor partial pressure rate according to height.

First of all it's necessary to choose from the table the values of $\xi_0(\varphi)$ and $\Delta\xi(\varphi)$ which are the closest to the latitude φ. Then these meteorological parameters are corrected according to latitude:

$$\xi_0^*(\varphi)=\xi_0(\varphi_i)+\left[\xi_0(\varphi_{i+1})-\xi_0(\varphi_i)\right]\cdot\frac{\varphi-\varphi_i}{\varphi_{i+1}-\varphi_i}, \quad (13)$$

$$\Delta\xi^*(\varphi)=\Delta\xi(\varphi_i)+\left[\Delta\xi(\varphi_{i+1})-\Delta\xi(\varphi_i)\right]\cdot\frac{\varphi-\varphi_i}{\varphi_{i+1}-\varphi_i}. \quad (14)$$

The following the correction according the Day of Year is applied

$$\xi(\varphi,DoY)=\xi_0^*(\varphi)-\Delta\xi^*(\varphi)\cdot cos\left[\frac{2\pi(DoY-DoY_0)}{365.25}\right], \quad (15)$$

where: DoY_0 – is the coolest day of year, for the northern semi sphere it equals 28, and for the southern 211.

Let us define now the meteorological parameters for the specified height.

The temperature in the troposphere can be approximated linearly using the temperature lapse rate (Schüler 2001):

$$T(H)=T(0)-\beta H. \quad (16)$$

Now let us define the pressure. The equation for hydrostatic equilibrium follows from the ideal gas laws of Gay-Lussac and Boyle–Mariotte and can be expressed in differential form as:

$$dP=-g\cdot\rho\cdot dH. \quad (17)$$

The density is related with a pressure in the following way:

$$P=\rho\cdot R_d\cdot T, \quad (18)$$

where: P – pressure; dP – differential change in pressure; T – temperature; g – gravity; ρ – density of dry air; dH – differential change in height; $R_d=287.054\left[\frac{J}{kg\cdot K}\right]$ – specific gas constant of dry air.

This leads to differential equation:

$$\frac{dP}{P}=-\frac{g}{R_d T}\cdot dH. \quad (19)$$

Taking into account (16), the differential equation (20) can be solved by integration parameters according to the heights of sea level and ground subsystem:

$$\int_{P(0)}^{P(H)}\frac{1}{P}\cdot dP=-\frac{g}{R_d}\int_0^H\frac{1}{T(0)-\beta H}\cdot dH \Rightarrow$$

$$P(H)=P(0)\cdot\left(\frac{T(0)-\beta H}{T(0)}\right)^{\frac{g}{R_d\beta}}. \quad (20)$$

Taking into account the abovementioned calculations we obtain the following expression for partial pressure of water vapor definition:

$$e(H)=e(0)\cdot\left(\frac{P(H)}{P(0)}\right)^{\lambda+1}=e(0)\cdot\left(\frac{T(0)-\beta H}{T(0)}\right)^{\frac{g(\lambda+1)}{R_d\beta}}. \quad (21)$$

After substitution the obtained above meteorological parameters in equations (8) and (9) it is possible to define dry and wet refraction indices:

$$N_d=k_1\frac{P(0)}{T(0)}\left(1-\frac{\beta H}{T(0)}\right)^{\left(\frac{g}{R_d\beta}-1\right)}, \quad (22)$$

$$N_w=k_2\frac{e(0)}{T^2(0)}\left(1-\frac{\beta H}{T(0)}\right)^{\left(\frac{g(\lambda+1)}{R_d\beta}-2\right)}. \quad (23)$$

Computer simulation of troposphere parameters

The computer simulation has been performed for the parameters defined according equations (22) and (23) for the whole year period and latitude ranges 0–90 °. The results of simulation are presented at the Figs 1 and 2.

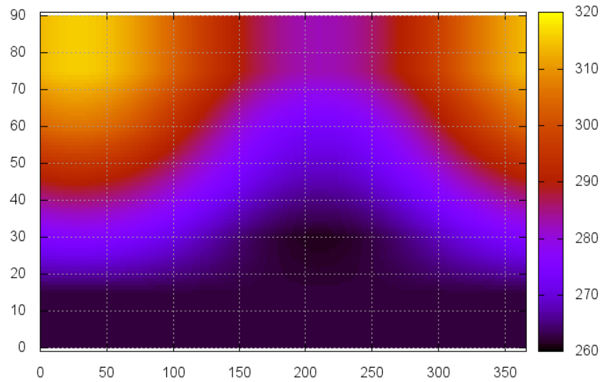

Fig. 1. Model of dry component of troposphere refraction index: abscissa – Day of Year; ordinate – latitude (degrees); gradient – dry component of troposphere refraction index (mm/km)

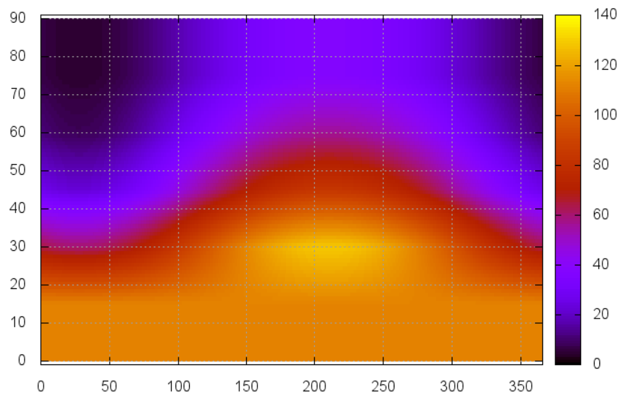

Fig. 2. Model of wet component of troposphere refraction index: abscissa – Day of Year; ordinate – latitude (degrees); gradient – wet component of troposphere refraction index (mm/km)

As it can be seen from the Fig. 1, the parameter N_d takes the maximal value, which is about 320 mm/km for the latitudes 80–90 ° in late January, and minimal value, which is about 260 mm/km for the latitudes 20–30 ° in early August. Near the equator and up to 15 ° the parameter N_d keeps low values. And on the contrary, the parameter N_w (Fig. 2) takes the peak minimal value, which is about 0 mm/km for the latitudes 80–90 ° in late January, and maximal value 140 mm/km for the latitudes 20–30 ° in early August. Near the equator and up to 15 ° the parameter N_w keeps high values.

Methodology of troposphere delay residual error calculation

Let's integrate the equation (6) after substitution in it the received previously N_d using equation (22) to obtain dry troposphere delay:

$$Z_d = 10^{-6} k_1 \frac{P(0) R_d}{g} \left(1 - \frac{\beta}{T(0)} H\right)^{\frac{g}{R_d \beta}}. \quad (24)$$

Then we define a height of dry component of troposphere using equation (6):

$$h_d = \frac{Z_d}{10^{-6} N_d} = \frac{10^{-6} k_1 P(0) R_d T(0) \left(1 - \frac{\beta}{T(0)} H\right)^{\frac{g}{R_d \beta}}}{10^{-6} g k_1 P(0) \left(1 - \frac{\beta}{T(0)} H\right)^{\frac{g}{R_d \beta} - 1}}. \quad (25)$$

After simplification we obtain the following:

$$h_d = \frac{R_d}{g} \left(T(0) - \beta H\right). \quad (26)$$

Let's integrate the equation (7) after substitution in it received in (23) N_w to obtain wet troposphere delay:

$$Z_w = 10^{-6} k_2 \frac{e(0) R_d}{T(0)\left(g(\lambda+1) - R_d \beta\right)} \left(1 - \frac{\beta}{T(0)} H\right)^{\frac{g(\lambda+1)}{R_d \beta} - 1}. \quad (27)$$

Then we define a height of wet component of troposphere using equation (7), and after some simplifications we obtain the following:

$$h_w = \frac{R_d}{g(\lambda+1) - R_d \beta} \left(T(0) - \beta H\right). \quad (28)$$

Computer simulation of troposphere delay residual error

The computer simulation has been performed for the parameters h_d and h_w for the whole year period and latitude ranges 0–90 °. The results of simulation are presented at the Figs 3 and 4.

As it can be seen from the Fig. 3, the parameter h_d takes the peak minimal value, which is about 7200 m for the latitudes 80–90 ° in late January, and maximal value 9000 m for the latitudes 20–30 ° in early August. Near the equator and up to 15 ° the parameter h_d keeps high values. And on the contrary, the parameter h_w (Fig. 4) takes the peak maximal value, which is about 3600 m for the latitude 60 ° in late January, and minimal value 2000 m for the latitude 30 ° in early August.

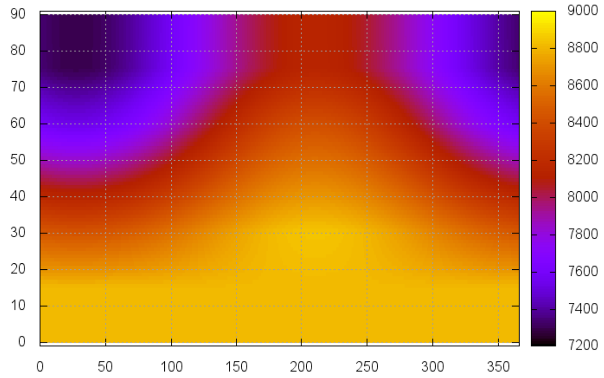

Fig. 3. Model of dry component of troposphere scale height: abscissa – Day of Year; ordinate – latitude (degrees); gradient – dry component of height of troposphere (m)

It is possible to compose the following equation, using (3) (5) (6) (7):

$$10^{-6} N_R(H) h_0(H) = 10^{-6} N_d(H) h_d(H) + 10^{-6} N_w(H) h_w(H). \quad (29)$$

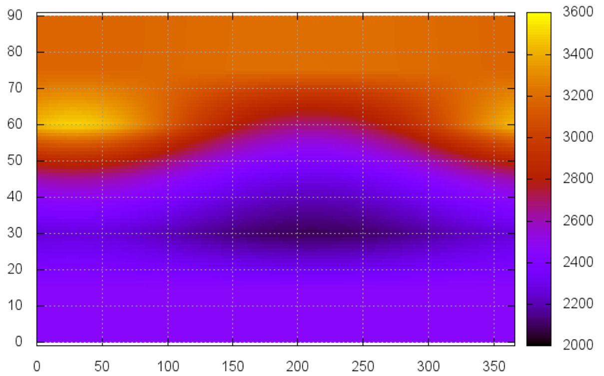

Fig. 4. Model of wet component of troposphere scale height: abscissa – Day of Year; ordinate – latitude (degrees); gradient – wet component of height of troposphere (m)

Then with help of (4) and after some simplifications we obtain:

$$\left(N_d(H) + N_w(H)\right) h_0(H) = N_d(H) h_d(H) + N_w(H) h_w(H). \quad (30)$$

From which it is possible to get the equation for the troposphere scale height:

$$h_0(H) = \frac{N_d(H) h_d(H) + N_w(H) h_w(H)}{N_d(H) + N_w(H)}. \quad (31)$$

The computer simulation has been performed for the parameters $N_R(H)$ and $h_0(H)$ for the whole year period and latitude ranges 0–90 °. The results of simulation are presented at the Figs 5 and 6.

As it can be seen from the Fig. 5, the parameter $N_R(H)$ takes the peak minimal value, which is about

3100 mm / km at the latitude 45 ° in late January, and maximal value 400 mm / km at the latitude 30 ° in early August. And on the contrary, the parameter h_w (Fig. 6) takes the peak maximal value, which is about 7600 m for the latitude 45 ° in late January, and minimal value 6500 m for the latitude 30 ° in early August.

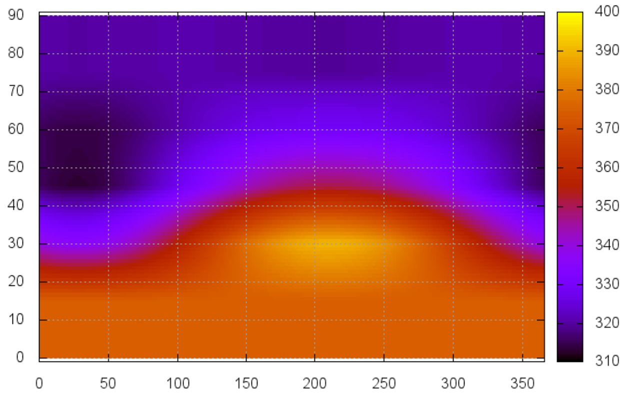

Fig. 5. Model troposphere refractivity index: abscissa – Day of Year; ordinate – latitude (degrees); gradient – index of troposphere refraction (mm / km)

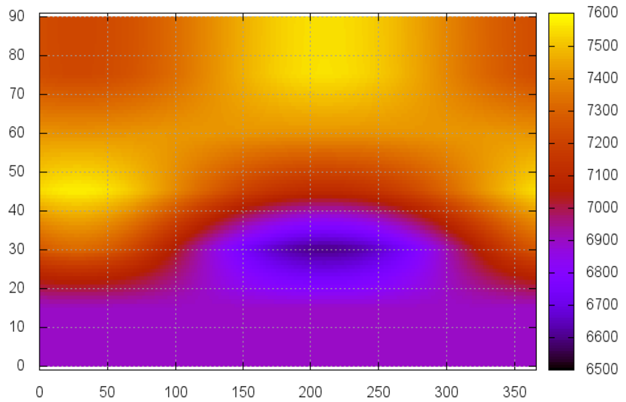

Fig. 6. Model of troposphere scale height: abscissa – Day of Year; ordinate – latitude (degrees); gradient – height of troposphere scale (m)

The computer simulation of troposphere correction defined according to the equation (1) has been performed. The value of troposphere correction has been specified in the model for the typical configuration of navigation satellites. The specified configuration has been obtained during few hours measurements recording of navigation GNSS receiver Dl-4plus of Novatel inc. (Canada). The measurements recording have been performed in mid-May. The GNSS receiver physically was in Kyiv, geographical coordinates 50 ° 26' N Latitude and 30 ° 25' E Longitude. The height of the airborne subsystem according to the ground subsystem has been chosen arbitrary and has been set to be equal Δh = 5000 m. The results of simulation are presented at the Fig. 7.

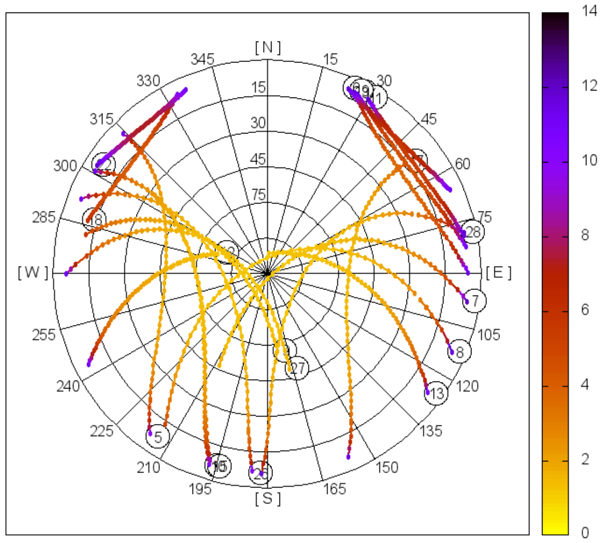

Fig. 7. Model of troposphere correction in meters, GBAS mode

The tracks of GPS satellites are represented at the Fig. 7, where the gradient shows the value of troposphere correction in meters, which has been obtained in simulation. As it can be seen, the value of troposphere correction essentially depends on elevation angle of GPS satellite and takes values starting from zero in zenith and up to 14 m near the horizon.

Conclusions

ICAO adopted concept of satellite based landing system of aircraft involves determining the troposphere correction in differential mode. This paper addresses the method of troposphere correction estimation using MOPS model, which is widely used to determine it in normal functioning mode of satellite navigation systems. The proposed method differs from the traditional ones by the absence of necessity to use experimental meteorological data.

The results of computer simulations of troposphere correction in differential mode are presented here together with intermediate parameters used for its calculation, such as troposphere refraction index, troposphere scale height, dry and wet components.

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