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FINITE VOLUME METHOD TO SOLUTION OF NAVIER-STOKES EQUATIONS FOR VERTICAL AXIS WIND TURBINES

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Abstract*—*Computational finite volume method to solution of Navier–Stokes equations for vertical axis wind turbines was presented. Discrete forms of these equations were obtained that brought to nonlinear equations system.

Index Terms—Aerodynamics; incompressible; viscous.

I. INTRODUCTION

Development and improvement of alternative energy sources is an urgent problem for the Energy of Ukraine. One of the most promising ways to solve this problem applies to wind energy. Two- and three-bladed horizontal-axis (HA) propeller type wind turbines (windmills) are widespread in the world. This is due to the high rate of wind energy use. As for the vertical-axis (VA) wind turbines only a Darrieus rotor has the close values of the power efficiency.

Increasing of the power of wind turbines and increasing of wind energy utilization makes very important the task of selecting the efficient aerodynamic shape of the rotor. The leading role in the wind turbine plays unsteady aerodynamic processes, so the main focus of research should be the development of new generic methods of calculation of transient processes in the flow stream of wind turbines.

II. REVIEW OF METHODS TO SOLVE NAVIER–STOKES EQUATIONS

To create a discrete analog of the original equations is necessary to choose calculated grid method. The term "grid" refers to a preliminary decomposition of the computational area into a finite number of elements and its relationships (computational grid). There are 3 main grid methods.

1) Finite differences method.
2) Finite elements method.
3) Finite volumes method (FVM).

Finite volumes method [1] – [3] has two important advantages in applying to describe flow of compressible and incompressible fluids. Firstly, it has good conservation properties (conservation of mass, etc.). Secondly it allows to sample out complicated computational areas to simpler forms.

III. PROBLEM STATEMENT

Processes of aerodynamics and dynamics of the wind turbine are described by Reynolds-averaged Navier–Stokes equations of incompressible fluid flow and of rigid body rotation about a fixed axis

$$\left.\begin{aligned} &\frac{\partial u_j}{\partial x_j}=0,\\ &\frac{\partial u_i}{\partial t}+\frac{\partial (u_j u_i)}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}\\ &\qquad+\frac{\partial}{\partial x_j}\left[\nu_{\text{eff}}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right],\\ &I_z\frac{d\omega}{dt}=Q-Q_{\text{ld}}-Q_{\text{fr}},\end{aligned}\right\} \quad (1)$$

where x_i, $i = 1, 2$ are Cartesian coordinates (x, y); t is time; u_i is the Cartesian vector components of average speed (u, v); p is the pressure; ρ is the density; ν_{eff} is the effective turbulent kinematic coefficient of viscosity; I_z is the rotor moment of inertia; ω is the angular velocity of rotation; Q is torque determined from the solution of the problem of aerodynamics; Q_{ld} is payload moment attached to the shaft of an electric generator; Q_{fr} is the resulting friction moment in the electromechanical system of wind turbine.

$$\nu_{\text{eff}}=\nu+\nu_t,$$

where ν and ν_t are molecular and turbulent kinematic coefficients of viscosity, respectively.

We have to obtain the torque (aerodynamic moment) using the relations:

$$Q=\oiint_S\left[(x-x_0)F_y-(y-y_0)F_x\right]dS,$$

$$F_x=-p\cos\left(\vec{n},\vec{i}\right)+\tau\cos\left(\vec{t},\vec{i}\right),$$

$$F_y=-p\cos\left(\vec{n},\vec{j}\right)+\tau\cos\left(\vec{t},\vec{j}\right),$$

where F_x, F_x is the components of the aerodynamic forces on the axes of Cartesian coordinates, referred to elemental area; x, y are Cartesian coordinates; x_0,

y_0 are Cartesian coordinates of the axis of rotation about which the moment characteristics is determined; S is surface of the rotor blades; $\tau = \mu\left(\partial U_\tau / \partial l_n\right)$ is the tangential stress; μ is the dynamic coefficient of viscosity; U_τ is the tangential component of the velocity vector; l_n is the distance along the normal to the surface of the rotor; $\bar{\mathbf{n}}$ is normal vector to the surface of the rotor; $\bar{\mathbf{t}}$ is tangent vector to the surface of the rotor; $\bar{\mathbf{i}}, \bar{\mathbf{j}}$ is unit orts of the Cartesian coordinates.

After reaching a certain angular velocity, the rotor is able to perform useful work.

The moment of payload (useful torque that applied to the shaft of the electric generator) is determined from the condition of maintaining a predetermined angular speed ω_{st}

$$Q_{ld} = \begin{cases} 0, \omega < \omega_{\mathrm{st}}, \\ Q_{ld}, \omega \geq \omega_{\mathrm{st}}. \end{cases}$$

The resulting friction moment in the electromechanical system of wind turbine is given by a quadratic function of the angular velocity of rotation

$$Q_{\mathrm{fr}} = A\omega^2 + B\omega + C,$$

where A, B, C are the empirical coefficients that depend on the specific parameters of the wind turbine.

IV. Building of Adaptive Computational Grid

One of the important stages of the numerical solution of three-dimensional problems of mechanics is the stage of construction of the computational grid. Construction of the computational grid is the process of sampling of an investigated region with the aim of transforming the original differential Navier–Stokes equation to the equivalent system of algebraic equations, for which you can use the computer. At the same time the sampling algorithm of computational domain should be maximally automated.

For certain types of tasks automatic grid generation can be carried out directly on the partition of discrete area on elements, but in most cases it is more convenient a different approach, which consists of the construction of pre-cubic grid and its subsequent deformation. At this stage, the initial physical area represented as a cubic computational domain, which is initially evenly (or unevenly) is crushed on three basic directions of three-dimensional space. The resulting cubic cell in the grid structure is a “parent” cell, which can contain eight cubic cells “descendants”.

The structure of such network is a tree shown in Fig. 1.

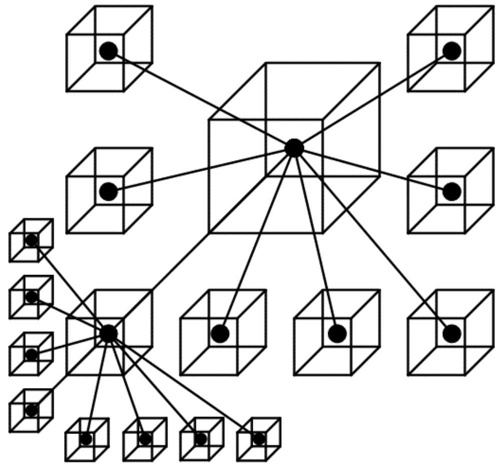

Fig. 1. Sample of tree structure for the cubic grid

As a result of the partition of the cubic “parent” cell, it's formed eight “descendants” cubic cells of the same size. It is necessary to take into account that new heights of “descendants” cells are located in the middle of the sides and edges of the “parent” cell. The process of creating of adaptive cubic grid is realized step by step by adaptation levels. The first level of adaptation those cells that contain the elements of the considered body surface (rotor blades) and do not have a “descendants” (Fig. 2). After all of the considered level “parent” cells have been adapted, we move to the next level of adaptation (Fig. 3). Results of construction of the grid are the coordinates of the nodes that allow to receive a discrete model of the investigated area.

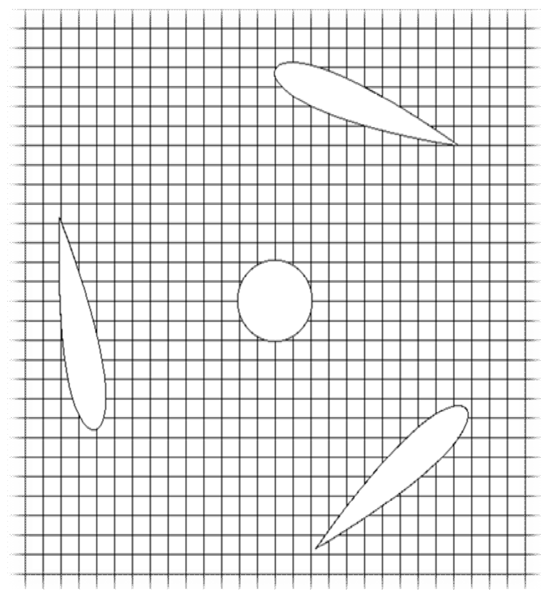

Fig. 2. Uniform grid for the blades of Darrieus rotor

V. Initial and Boundary Conditions

As initial conditions the parameters of the undisturbed flow in the whole computational domain are defined.

For the time t parameters of the undisturbed flow:

$$U = U_\infty; \quad V = 0; \quad p = p_\infty,$$

where U, V are the components flow velocity to the coordinates x and y respectively; p is the pressure.

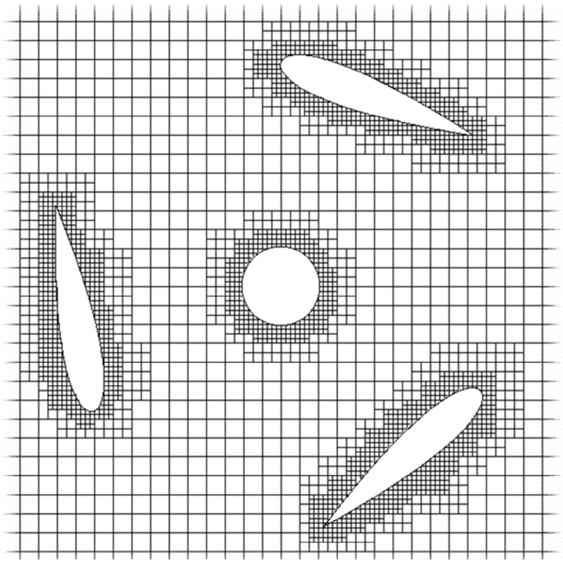

Fig. 3. Uniform grid for the blades of Darrieus rotor with the adaptation

On the streamlined body surface:

$$\mathbf{U} = \boldsymbol{\omega} \times \mathbf{r}; \quad \frac{\partial p}{\partial n} = 0.$$

where $\mathbf{r}$ is the radius-vector of the point; $\mathbf{n}$ is the unit normal to the surface.

Inflow boundary (inlet).

For the input border the following boundary conditions are imposed:

$$U = U_\infty; \quad V = 0; \quad p = p_\infty.$$

Outflow boundary (outlet).

Neumann condition is placed on outflow boundary:

$$\frac{\partial U}{\partial x} = 0; \quad \frac{\partial V}{\partial y} = 0; \quad \frac{\partial p}{\partial n} = 0,$$

where $\mathbf{n}$ is the normal to the outflow boundary.

The initial angular speed of the rotor is assumed to be zero.

VI. Approximation of Navier–Stokes Equations

Because of the nonlinearity of the Navier–Stokes equations solving problems can be obtained only numerically. As a method of numerical solution it is selected the FVM due to its unique ability to use irregular grids of all shapes and effective approximation of curved boundaries.

Rewrite the source equations (1) in a form:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \frac{\partial \mathbf{M}}{\partial x} + \frac{\partial \mathbf{N}}{\partial y},$$

where

$$\mathbf{q} = \begin{Bmatrix} U \\ V \\ 0 \end{Bmatrix}, \quad \mathbf{F} = \begin{Bmatrix} U^2 + \dfrac{p}{\rho} \\ UV \\ U \end{Bmatrix}, \quad \mathbf{G} = \begin{Bmatrix} UV \\ V^2 + \dfrac{p}{\rho} \\ V \end{Bmatrix},$$

$$\mathbf{M} = \begin{Bmatrix} S_{xx} \\ S_{yx} \\ 0 \end{Bmatrix}, \quad \mathbf{N} = \begin{Bmatrix} S_{xy} \\ S_{yy} \\ 0 \end{Bmatrix}, \qquad (2)$$

$$S_{xx} = 2\nu_{\text{eff}} \frac{\partial U}{\partial x}, \quad S_{xy} = \nu_{\text{eff}} \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right),$$

$$S_{xy} = \nu_{\text{eff}} \left(\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} \right), \quad S_{yx} = 2\nu_{\text{eff}} \frac{\partial V}{\partial y}.$$

Finite volumes method is to apply the method of subdomains to each equation (2) in a finite volume *ABCD* (Fig. 4). To calculate the first derivatives apply the Green theorem [1]:

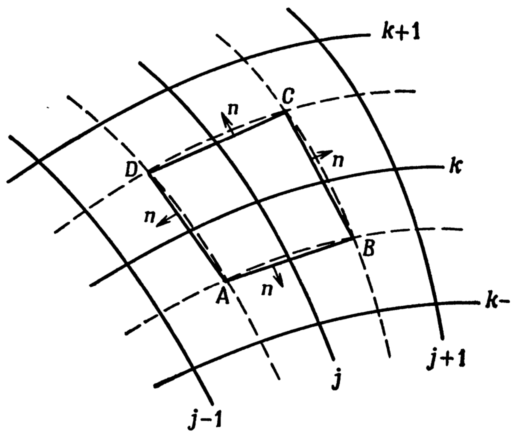

Fig. 4. The finite volume of deformed grid

$$\int_{ABCD} \left(\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} \right) dxdy = \frac{d}{dt} \int \mathbf{q} dV + \int_{ABCD} \mathbf{H} \cdot \mathbf{n} ds,$$

where $\mathbf{H} = (F; G)$. In Decartes coordinates:

$$\mathbf{H} \cdot \mathbf{n} ds = F dy - G dx.$$

For our case:

$$S_{ABCD} \frac{dq}{dt} + \sum_{AB}^{DA} \left(F \Delta y - G \Delta x \right),$$

where S_{ABCD} is the area of quadrilateral *ABCD*. We obtain:

$$S_{ABCD}\frac{dq_{j,k}}{dt}+\left(F\Delta y-G\Delta x\right)_{AB}+\left(F\Delta y-G\Delta x\right)_{BC}$$
$$+\left(F\Delta y-G\Delta x\right)_{CD}+\left(F\Delta y-G\Delta x\right)_{DA}, \quad (3)$$

where for example

$$F_{AB}=\frac{F_{j,k-1}+F_{j,k}}{2}, \Delta y_{AB}=y_B-y_A,$$
$$G_{AB}=\frac{G_{j,k-1}+G_{j,k}}{2}, \Delta x_{AB}=x_B-x_A,$$
$$x_A=\frac{x_{j-1,k-1}+x_{j-1,k}+x_{j,k-1}+x_{j,k}}{4}.$$

As for the second derivatives apply again the method of subdomains in a finite volume *ABCD* (Fig. 5) apply the Green theorem also. For example we have:

$$\int_{ABCD}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right)dxdy+\int_{ABCD}\mathbf{H}\cdot\mathbf{n}ds,$$

where

$$\mathbf{H}\cdot\mathbf{n}ds=\frac{\partial U}{\partial x}dy-\frac{\partial U}{\partial y}dx.$$

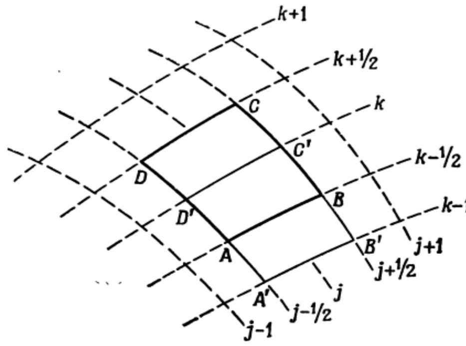

Fig. 5. The finite volume of deformed grid

Similarly to (3) obtain again:

$$\left(\frac{\partial U}{\partial x}\Delta y-\frac{\partial U}{\partial y}\Delta x\right)_{AB}+\left(\frac{\partial U}{\partial x}\Delta y-\frac{\partial U}{\partial y}\Delta x\right)_{BC}$$
$$+\left(\frac{\partial U}{\partial x}\Delta y-\frac{\partial U}{\partial y}\Delta x\right)_{CD}+\left(\frac{\partial U}{\partial x}\Delta y-\frac{\partial U}{\partial y}\Delta x\right)_{DA},$$

where for example

$$\left(\frac{\partial U}{\partial x}\right)_{AB}=\left(\frac{\partial U}{\partial x}\right)_{j,k-1/2}$$
$$=\frac{1}{S_{A'B'C'D'}}\iint\left(\frac{\partial U}{\partial x}\right)dxdy=\frac{1}{S_{A'B'C'D'}}\int Udy$$
$$\approx\frac{U_{j,k-1}\Delta y_{A'B'}+U_B\Delta y_{B'C'}+U_{j,k}\Delta y_{C'D'}+U_A\Delta y_{D'A'}}{S_{A'B'C'D'}}.$$

Finally we obtain equations system of the form

$$\mathbf{b}U+\mathbf{n}V=\mathbf{m}P.$$

where **b, n, m** are the matrix coefficients to unknown functions *U*, *V*, *P*.

The solution of equations is produced separately by coordinates:

1) Solve the system, taking into account that derivatives to *y* are constants.

2) Solve the system, taking into account that derivatives to *x* are constants.

RESULTS

Dynamic characteristics were calculated for the three bladed Darrieus rotor. The diameter and the length of the rotor were 0.26 m and 0.4 m respectively. Revolutions per minute (RPM) versus the wind speed were presented in Fig. 6. The rotor cut in at wind speed of about 2.5 m/s.

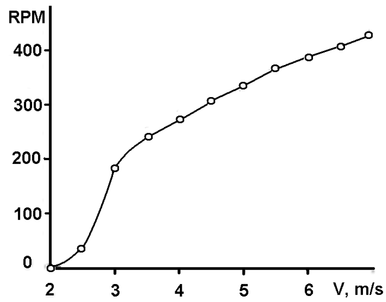

Fig. 6. Dependance of RPM versus the wind speed

CONCLUSIONS

Computational FVM to solution of Navier–Stokes equations for vertical axis wind turbines was presented.

Discrete forms of these equations were obtained that brought to nonlinear equations system.

The solution of equations with respect to velocities and the pressure was produced separately by coordinates.

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В. М. Синєглазов, А. А. Зіганшин. Метод скінченних об'ємів до розв'язання рівнянь Нав'є–Стокса для вертикально-осьових вітротурбін

Представлено числовий метод скінченних об'ємів для розв'язання рівнянь Нав'є–Стокса стосовно до вертикально-осьових вітрових турбін. Отримано дискретні форми цих рівнянь, які призводять до системи нелінійних рівнянь.

Ключові слова: аеродинаміка; нестискувальний; в'язкий.

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В. М. Синеглазов, А. А. Зиганшин. Метод конечных объемов к решению уравнений Навье–Стокса для вертикально-осевых ветротурбин

Представлен численный метод конечных объемов для решения уравнений Навье-Стокса применительно к вертикально осевым ветровым турбинам. Получены дискретные формы этих уравнений, которые приводят к системе нелинейных уравнений.

Ключевые слова: аэродинамика; несжимаемый; вязкий.

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