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Divergent bifurcations of stationary motion modes of wheeled vehicle model with controlled wheel module

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Summary. An alternative approach of the determining of conditions of safe stability loss of rectilinear motion of a wheeled vehicle model with controlled wheel module in the sense of N.N. Bautin is considered. The slipping forces are presented accurate within cubic expansion terms in the skid angles. Terms and conditions of safe stability loss depend on the ratio between the coefficients of resistance to the skid, the adhesion coefficients in the transverse direction of the axes and the parameter of torsional stiffness of the controlled wheel module.

The presented approach to the analysis of real bifurcations related to the divergent loss of rectilinear motion mode stability has a clear geometric pattern: if in the vicinity of rectilinear motion at subcritical speed, there are additionally two unstable circular stationary states, then the stability limit is of dangerous nature in the sense of N.N. Bautin; if two circular stationary modes exist at supercritical speed, the limit of the stability loss in the parameter space of the longitudinal velocity is safe in the sense of N.N. Bautin. Analysis of the number of stationary modes in the vicinity of the critical velocity of rectilinear motion is performed for the obtained determining equation - cubic binomial.

Key words: wheel module, stability, adhesion coefficient, slipping forces, divergent bifurcation.

INTRODUCTION

A variety of operating conditions of vehicles made for their widespread specialization. Automobiles differ by specific properties that enable their use in specific terms with the greatest efficiency. One group of specific properties is the performance characteristics of the automobile, which include: traction and speed properties (dynamics), braking performance, fuel economy, steerability, stability, passability, travelling comfort. Stability is considered to be an important component of the vehicle performance properties, the lack of which is one of the most common causes of road accidents.

Stability is the property of the system to return to the original steady state after leaving it in the result of any external influence.

A considerable amount of work are dedicated to the study of common issues of automobile motion stability [1, 9, 16, 18, 20]. The processes of the stability of individual automobiles and trucks at a steady motion mode, when driving in a straight line, while moving along a curved path, when braking, near the stability limit and so forth are under study [11, 14, 21, 25, 28]. Besides, as pointed by [15, 27], the same movement can be stable with respect to one variable and unstable with respect to the other variable. Therefore, there are differences in the formulation of the parameters of automobile motion stability. Two aspects to assess the parameters of automobile motion stability were singled out [5].

According to the first aspect, the concept describes the movement of the automobile as a solid body in all its degrees of freedom, except for the direction perpendicular to the bearing surface and direction coinciding with the longitudinal axis of the vehicle. However, the formulation of motion stability criteria for each of the degrees of freedom is vague and does not correspond to the classic notion of stability.

According to the second aspect, the concept of stability of the automobile characterizes its behavior only in the movement along the given course. The formulation of the concept of motion stability in this parameter is given in full compliance with the definitions adopted in the general theory of mechanical systems stability.

Thus, despite the fact that currently the problem of determining the conditions of automobile stability as a dynamic system has been studied thoroughly enough, however, as practice shows, the issues of determining the system behavior nature as for instability and identifying the causes of its emergence remain relevant. Success in solving similar problems depends on how well chosen is the mathematical model and its relevant parameters that describe the real automobile behavior [24].

The stability of the vehicle and the conditions of turnability of a nonlinear model with a fixed steering is considered in [6], and the conditions of safe stability loss of rectilinear motion of two-axle vehicle model with oversteer - in [5].

Self-oscillations of a single wheel unit are investigated in [7].

One of the well-established mathematical models of the automobile is based on a flat bicycle scheme and includes the main non-linear properties of the wheel skid [10].

This article is dedicated to the development of the methods for the analysis of controllability and stability of a nonlinear automobile model with rigid wheel module, first introduced in the works by Ya. M. Pevzner [17].

The proposed methods for determining the stationary motion modes of the automobile models, complemented by the algorithms of the construction of the bifurcation set in conjunction with the Poincare Index method, make it possible to conduct a preliminary analysis of the number of stationary modes and to determine the stability boundaries in the plane of the controlled parameters.

THE ANALISIS OF RECENT RESEARCHES AND PUBLICATIONS

Divergent stability loss of the rectilinear stationary motion mode of the vehicle in the simplest case is related to the implementation of the assembly bifurcation. From the origin of coordinates at a critical speed either a couple of stable stationary states is born, or a couple of unstable steady states comes to the origin and merges with stable rectilinear mode. Research results of Troger H., Scheidl R., Stribersky A., Kacani V., Zeman K. [3, 4, 10, 21, 27] were based on numerical method for the continuation of two parameters, and it makes difficult the determination of the conditions of safe stability loss of rectilinear motion mode in the space of parameters according to N.N. Bautin [2].

In the case of symmetric vehicle with absolutely rigid steering control, substantial "internal" parameters affecting the nature of the loss of stability are adhesion coefficients on the axes: at reduction of the adhesion coefficient on the front axle, the nature of the danger of the region border of stability changes due to the realization of butterfly catastrophe; bifurcation set in this case has a characteristic cross-section with three critical points (cusps) [22].

The problem of stability loss (secure-insecure) of the rectilinear motion of a wheeled vehicle model with a controlled has already been considered in [22]. A formalized approach to the analysis of the stability boundary security, based on the assessment of the number of stationary modes in the vicinity of the rectilinear motion mode, is offered in [23].

OBJECTIVES

Obtaining sufficient conditions for a safe loss of stability of rectilinear motion of a wheeled vehicle model with controlled wheel module.

THE MAIN RESULTS THE RESEARCH

Figure 1 shows the design diagram of the vehicle model with controlled wheel module.

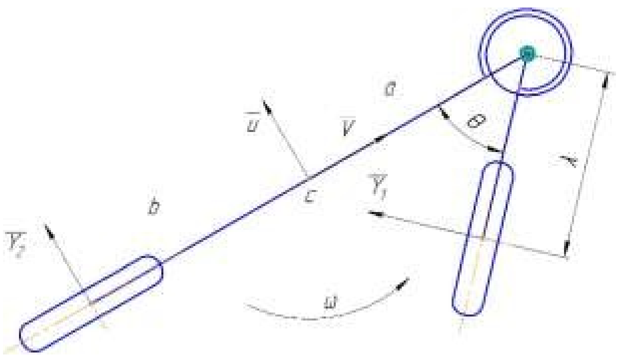

Fig. 1. Design diagram of the vehicle model

The controlled module is pivotally connected to the vehicle case, the corner between the longitudinal axis of the case and the vertical longitudinal plane of the wheel is θ, the case-wheeled module is affected by the elastic restoring moment tending to return the system to the position $\theta = \theta_0$, where θ_0 is the angle of rotation of forward row wheels positioned by the driver; Y_1, Y_2 are the equivalent cross forces (skid forces) operating in a spot of contact of wheels with a basic surface that are defined according to I.Rocard's axiomatics [19]; the axis of a wheel is removed from an axis of hinged connection at the distance of offset λ (λ <0 in the case when the axis of a wheel is ahead of a point of hinged connection of links); v is a longitudinal component of speed of the case mass center (it is kept constant); a , b are distance from the center of mass of the controlled wheel module to the point of fastening of the forward (controlled) axis and the back axis respectively; the reduced factor of rigidity of the controlled module *k*; the damping coefficient on the rotation angle of the controlled module *h*, the positioned rotation angle of the back row wheels is θ_1 .

The weight and moment of the case inertia relative to the central vertical axis are m, J ; m_1, J_1 are the weight and moment of inertia of the controlled wheel module relative to the central vertical axis (we assume that it passes through the wheel axis).

The system of the differential equations of the movement of the vehicle model with controlled wheel module (phase variables $u, \omega, \theta, \Theta$) is received at the assumptions accepted in [12], in which: U is the derivative of a cross component of speed of the center of masses; Ω is the derivative of angular speed; Θ is the speed of change of the rotation angle of the controlled module; *TT* is angular speedup of the controlled module:

$$\begin{cases} -m(U+\omega v)+m_1\cos\theta\,\lambda(\Omega+TT)-m_1(a\Omega+\omega v+ \\ \quad +U)-m_1\sin\theta\,\lambda(\omega^2+\Theta^2+2\omega\Theta)+Y_1\cos\theta+ \\ \quad +X_1\sin\theta+Y_2\cos\theta_1-X_2\sin\theta_1=0; \\ -J\Omega+a\,m_1\cos\theta\,\lambda(\Omega+TT)-a\,m_1\sin\theta\,\lambda(2\omega\Theta+\omega^2+ \\ \quad +\Theta^2)-a\,m_1(a\Omega+\omega v+U)+h\Theta+k(\theta-\theta_0)+ \\ \quad +Y_1\,a\cos\theta+X_1\,a\sin\theta-Y_2\,b\cos\theta_1-X_2\,b\sin\theta_1=0; \\ (J_1+m_1\lambda^2)TT+(J_1+m_1\lambda(\lambda-a\cos\theta))\Omega- \\ \quad +m_1\lambda\cos\theta(U+v\omega)-m_1\omega\sin\theta\,\lambda(u+a\,\omega)+h\Theta+ \\ \quad +k(\theta-\theta_0)+Y_1\lambda=0. \end{cases} \tag{1}$$

In (1) the slipping forces are approximated by a monotonic dependence of the nature of the saturation curve:

$$Y_i=\frac{k_i\delta_i}{\sqrt{1+\left(\frac{k_i\delta_i}{\kappa_i^2 Z_i}\right)^2}},$$

where: k_i – coefficients of resistance to the slip, κ_i – adhesion coefficients in the lateral direction, Z_i – vertical reactions on the axes.

We assume that the reduced axial force on the front axle in the system (1) is negligible (X_1 = 0), the rotation angle of the back row of wheels $\theta_1 = 0$.

Condition of divergent stability loss of rectilinear mode was found in general form [12]:

$$v_{kp}^2=\frac{k_1k_2\,k(a+b-\lambda)^2}{-m\,k_1k_2\lambda b+m\big(k\left(k_1(a-\lambda)\right)\big)-k m_1k_2(a+b-\lambda)},$$

and safe-dangerous loss of stability conditions were studied in [2, 8].

CONSTRUCTION OF THE BIFURCATION SET OF THE AUTOMOBILE MODEL WITH CONTROLLED WHEEL MODULE.

The stationary states of the system (1) (special points of the phase space) satisfy the system of finite equations

$$\begin{aligned} &E_1(U=0,\Omega=0,TT=0,\Theta=0,u,\,\omega,\,\theta,v,\theta_0)=0,\\ &E_2(U=0,\Omega=0,TT=0,\Theta=0,u,\,\omega,\,\theta,v,\theta_0)=0,\\ &E_3(U=0,\Omega=0,TT=0,\Theta=0,u,\,\omega,\,\theta,v,\theta_0=0; \end{aligned} \tag{2}$$

where: E_i – left-hand sides of system equations (1). System (2) has two controlling parameters (v,θ_0). Parameter continuation method proposed by Shinohara [8] makes it possible to identify different branches of the equilibrium curve and to evaluate the maximum number of stationary modes in the final field of the control parameters.

In [10] the problem of the evolution of stationary states when changing one of the control parameters is considered. The variety of bifurcation values of parameters v^*, θ_0^*, to which the multiple stationary modes of the movement (u^*,ω^*,θ^*) of system (2) correspond, can be found on the basis of two parameters continuation method. The condition for the implementation of multiple stationary mode (u^*,ω^*,θ^*) is the equality $E_4\,|_{(u^*,\omega^*,\theta^*,v^*,\theta_0^*)}=0$ (Jacobian determinant of the system (2) vanishes):

$$E_4(u,\,\omega,\,\theta,v,\theta_0)=\begin{Vmatrix} \frac{\partial E_1}{\partial u} & \frac{\partial E_1}{\partial \omega} & \frac{\partial E_1}{\partial \theta} \\ \frac{\partial E_1}{\partial u} & \frac{\partial E_1}{\partial \omega} & \frac{\partial E_1}{\partial \theta} \\ \frac{\partial E_1}{\partial u} & \frac{\partial E_1}{\partial \omega} & \frac{\partial E_1}{\partial \theta} \end{Vmatrix}=0.$$

System (2) with the last equation defines the critical set of stationary modes. The method of continuation on two parameters leads to the auxiliary system of differential equations:

$$u'=\frac{D_1}{\sqrt{D_1^2+D_2^2+D_3^2+D_4^2+D_5^2}},$$

$$\omega'=\frac{D_2}{\sqrt{D_1^2+D_2^2+D_3^2+D_4^2+D_5^2}},$$

$$\theta'=\frac{D_3}{\sqrt{D_1^2+D_2^2+D_3^2+D_4^2+D_5^2}},$$

$$v'=\frac{D_4}{\sqrt{D_1^2+D_2^2+D_3^2+D_4^2+D_5^2}},$$

$$\theta_0'=\frac{D_5}{\sqrt{D_1^2+D_2^2+D_3^2+D_4^2+D_5^2}},$$

where: $D_1=\frac{D(E_1,E_2,E_3,E_4)}{D(\omega,\theta,v,\theta_0)}$,

$$D_2=\frac{D(E_1,E_2,E_3,E_4)}{D(u,\theta,v,\theta_0)},$$

$$D_3=\frac{D(E_1,E_2,E_3,E_4)}{D(u,\omega,v,\theta_0)},$$

$$D_4=\frac{D(E_1,E_2,E_3,E_4)}{D(u,\omega,\theta,\theta_0)},$$

$$D_5=\frac{D(E_1,E_2,E_3,E_4)}{D(u,\omega,\theta,v)}.$$

Starting point for realization of continuation method (0,0,0,$v_{кр}$, 0), where $v_{кр}$ is the critical speed of the rectilinear mode of the vehicle movement, is defined from the solution of the equation $E_4\,|_{(u=0,\,\omega=0,\,\theta=0,\,\theta_0=0)}=0$.

In Fig. 2 the bifurcation set obtained by numerical integration of auxiliary system of the differential equations at the following values of design data is presented: a=1,45 m; b=1,55 m; λ =0,0043 m; h=0; m=2090 kg; κ_1=0,7; κ_2=0,8; k_1=91500 H; k_2= 61000 H; J_1=3,22 kg m^2.

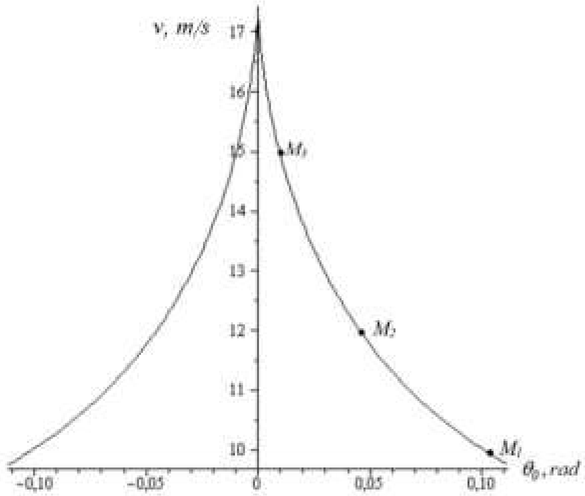

Fig. 2. Critical set of longitudinal velocity control parameters v and the rotation angle of the controlled wheel module *(k* = 400 H·m)

Points M_1, M_2, M_3 in Figure 3 correspond to the velocity of 10 m/s, 12 m/s and 15 m/s. They are pivot points in which the loss of stability of the circular stationary modes occurs. They are the elements of the bifurcation set (Figure 2).

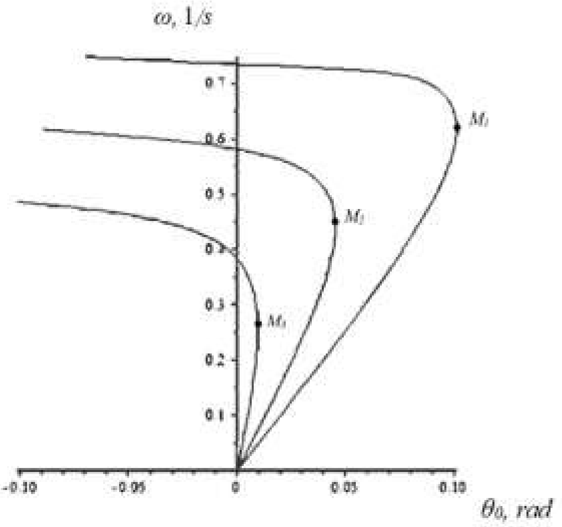

Fig. 3. Equilibrium curves of the corresponding angular speeds ω of the stationary modes at the change of the rotation angle of the controlled wheel module θ_0; (k=400 Nm; v=15; 12; 10 m/s)

The bifurcation set divides the plane of the control parameters (v, θ_0) into areas with various quantity of the stationary modes: in internal area - 3, in external - 1. At the points of the bifurcation set (Figure 2) there is a real bifurcation of the folds (merger-birth) of a pair of stationary modes - stable and unstable. At the fixed speed $v < v_{kp}$ and θ_0 parameter increase from zero to θ_0* symmetrical stable stationary mode moves along the equilibrium curve, in the pivot point (v_i*, θ_{0i}*) the divergent stability loss of the circular stationary mode occurs (Figure 3).

With the decrease in the value of torsional rigidity the nature of the bifurcation, set changes greatly (Figure 4).

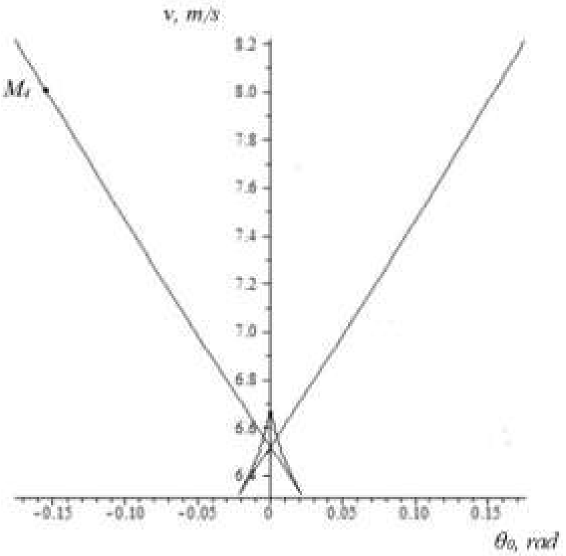

Fig. 4. Critical set of parameters of longitudinal velocity control v and rotation angle of the controlled wheel module θ_0 (k=40 Hм)

The pivot point M_4 (corresponding to the velocity of 8 m/s) divides the equilibrium curve into stable and unstable parts - stable from 0 to M_4 (Figure 5). Equilibrium curve corresponding to v = 6 m/s has no pivot point, divergent stability loss of the motion circular modes does not occur here.

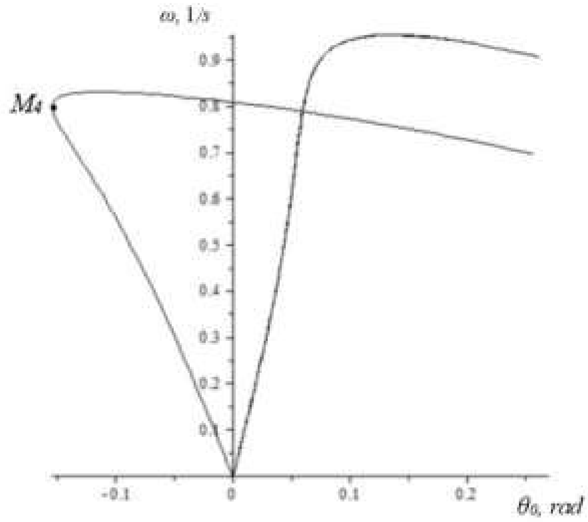

Fig. 5 Equilibrium curves of the corresponding angular velocities ω of stationary modes while changing the pivot point of the controlled wheel module θ_0; (k=40 Nm; v=6; 8 m/s)

ANALYTICAL RESULTS OF THE ANALYSIS OF (DANGEROUS-SAFE) LOSS OF STABILITY OF RECTILINEAR MOTION MODE.

Estimating the number of stationary modes of the system (1) at subcritical speed parameter value and supercritical value allows to determine the conditions of safe-dangerous stability loss of rectilinear stationary motion mode.

The system determining the set of stationary modes has the form (3) (skid forces are represented up to cubic expansion terms)

$$\begin{cases} -\dfrac{m\cdot v^2\cdot(\theta+\delta_2-\delta_1)}{l}+k_2\cdot\delta_2-\dfrac{1}{2}\dfrac{k_2^3\cdot\delta_2^3}{\kappa_2^2\cdot Z_2^2}+k_1\cdot\delta_1- \\ -\dfrac{1}{2}\dfrac{k_1^3\cdot\delta_1^3}{\kappa_1^2\cdot Z_1^2}-\dfrac{1}{2}\theta^2\cdot k_1\cdot\delta_1=0, \\ a\cdot\left(k_1\cdot\delta_1-\dfrac{1}{2}\dfrac{k_1^3\cdot\delta_1^3}{\kappa_1^2\cdot Z_1^2}\right)+k\cdot\theta-b\cdot\left(k_2\cdot\delta_2-\dfrac{1}{2}\dfrac{k_2^3\delta_2^3}{\kappa_2^2\cdot Z_2^2}\right)- \\ -\dfrac{1}{2}a\cdot\theta^2\cdot k_1\cdot\delta_1=0, \\ k\theta+\lambda k_1\delta_1=0. \end{cases} \quad (3)$$

From the third equation, we express θ as function δ_1:

$$\theta=-\frac{\lambda k_1\delta_1}{k}$$

and put it in the first two equations of the system (3). We obtain the system of two equations relative to the variables δ_1 and δ_2. Turn to the dimensionless skid coefficients on the front and rear axles:

$$kk=\frac{k}{|\lambda|mg\dfrac{b}{l}},\quad kk_1=\frac{k_1}{mg\dfrac{b}{l}},\quad kk_2=\frac{k_2}{mg\dfrac{a}{l}}.$$

System (3) in the dimensionless form becomes:

$$\begin{cases} -\dfrac{v^2\left(-\dfrac{kk_1\cdot\delta_1}{kk}+\delta_2-\delta_1\right)}{g}+a\cdot kk_2\cdot\delta_2-\dfrac{1}{2}\dfrac{a\cdot kk_2^3\cdot\delta_2^3}{\kappa_2^2}+ \\ +b\cdot kk_1\cdot\delta_1-\dfrac{1}{2}\dfrac{b\cdot kk_1^3\cdot\delta_1^3}{\kappa_1^2}=0, \\ kk_1\cdot\delta_1-\dfrac{1}{2}\dfrac{kk_1^3\cdot\delta_1^3}{\kappa_1^2}-\dfrac{\lambda\cdot kk_1\cdot\delta_1}{a}-kk_2\cdot\delta_2+\dfrac{1}{2}\dfrac{kk_2^3\cdot\delta_2^3}{\kappa_2^2}=0. \end{cases} \quad (4)$$

The expression for the square of the critical velocity of rectilinear motion (skid and stiffness coefficients are presented in dimensionless form):

$$v_{кр}^2=\frac{kk_1\cdot kk\cdot a\cdot kk_2\cdot g\cdot(a+b-\lambda)}{-a\cdot kk_2\cdot kk_1-a\cdot kk_2\cdot kk+kk_1\cdot kk\cdot a-kk_1\cdot kk\cdot\lambda}. \quad (5)$$

Figure 6 shows the dependence of the critical speed $v_{кр}$ of rectilinear motion of a wheeled vehicle model with controlled wheel module as a function of the dimensionless parameter of torsional stiffness.

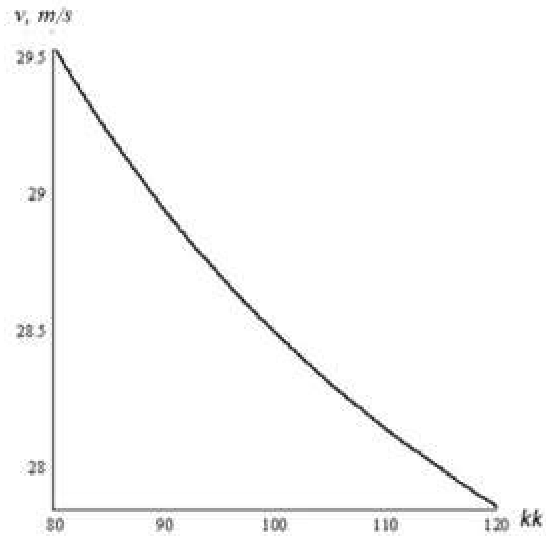

Fig. 6. Dependence of the critical speed v of rectilinear motion of a wheeled vehicle model from the dimensionless parameter kk of torsional stiffness

From the system (4) one can go to a single defining equation for δ_1:

$$\left(-\frac{kk_1^3}{\kappa_1^2}+\frac{kk_2\cdot l\cdot kk_1^3\cdot g}{\kappa_1^2\cdot v^2}+\frac{kk_2^3\cdot\left(\dfrac{kk_1}{kk}+1+\dfrac{g\cdot(l-\lambda)\cdot kk_1}{v^2}\right)^3}{\kappa_2^2}\right)\cdot\frac{\delta_1^3}{2}+$$
$$+\left(kk_1-\frac{\lambda\cdot kk_1}{a}-kk_2\cdot\left(\frac{kk_1}{kk}+1+\frac{g\cdot(l-\lambda)\cdot kk_1}{v^2}\right)\right)\cdot\delta_1=0. \quad (6)$$

The number of solutions of this equation determines the number of stationary states of the system in the vicinity of rectilinear motion.

The condition of the critical velocity of rectilinear motion is as follows :

$$g_1=-kk_1\cdot kk_2-kk_2\cdot kk+kk_1\cdot kk>0. \quad (7)$$

The condition for safe stability loss of rectilinear motion earlier obtained in [23] is $(\lambda\to 0)$:

$$g_2=-\kappa_2^2\cdot kk_1\cdot kk_2-\kappa_2^2\cdot kk_2\cdot kk+kk_1\cdot kk\cdot\kappa_1^2<0. \quad (8)$$

By combining (7) and (8) we have:

$$\frac{kk_1 \cdot kk_2}{kk_1 - kk_2} < kk < \frac{kk_1 \cdot kk_2 \cdot \kappa_2^2}{kk_1 \cdot \kappa_1^2 - kk_2 \cdot \kappa_2^2}. \tag{9}$$

From the analysis of the coefficients signs of the determining equation at subcritical and supercritical speeds, we find the condition (refined) of safe stability loss of rectilinear stationary motion:

$$\begin{aligned} g_3 = & kk_1 \cdot kk \cdot a^3 \kappa_2^2 \cdot \lambda - l \cdot a^3 \kappa_2^2 \cdot kk_2 \cdot kk_1 - l \cdot a^3 \kappa_2^2 \cdot kk_2 \cdot kk - \\ & - l \cdot a^2 \kappa_2^2 \cdot kk_1 \cdot kk \cdot \lambda + kk_1 \cdot kk \cdot \kappa_1^2 a^3 l - 3 \cdot kk_1 \cdot kk \cdot \kappa_1^2 a \lambda^2 l - \\ & - 3 \cdot kk_1 \cdot kk \cdot \kappa_1^2 a^2 \lambda \cdot l + 3 \cdot kk_1 \cdot kk \cdot \kappa_1^2 a^2 \lambda^2 + kk_1 \cdot kk \cdot \kappa_1^2 a^3 \lambda - \\ & - 3 \cdot kk_1 \cdot kk \cdot \kappa_1^2 a \lambda^3 - kk_1 \cdot kk \cdot \kappa_1^2 \lambda^3 \cdot l + kk_1 \cdot kk \cdot \kappa_1^2 \cdot \lambda^4 < 0. \end{aligned} \tag{10}$$

The condition of the critical velocity of rectilinear motion ($\lambda \neq 0$):

$$-a \cdot kk_2 \cdot kk_1 - a \cdot kk_2 \cdot kk + kk_1 \cdot kk \cdot a - kk_1 \cdot kk \cdot \lambda > 0. \tag{11}$$

Conditions (10), (11) can be represented as a double inequality:

$$\begin{aligned} & \frac{a \cdot kk_1 \cdot kk_2}{a \cdot kk_1 - a \cdot kk_2 - \lambda \cdot kk_1} < kk < \\ & < \frac{l \cdot a^3 \cdot \kappa_2^2 \cdot kk_2 \cdot kk_1}{(l-\lambda) \cdot (a-\lambda)^3 \cdot \kappa_1^2 \cdot kk_1 - l \cdot a^3 \cdot \kappa_2^2 \cdot kk_2 - (l-a) \cdot \lambda \cdot a^2 \cdot kk_1 \cdot \kappa_2^2}. \end{aligned} \tag{12}$$

Figure 7 is a graphic interpretation of the safe conditions of stability loss of rectilinear motion of the vehicle with the controlled wheel module. The appropriate range of the parameter torsional stiffness changes is determined .

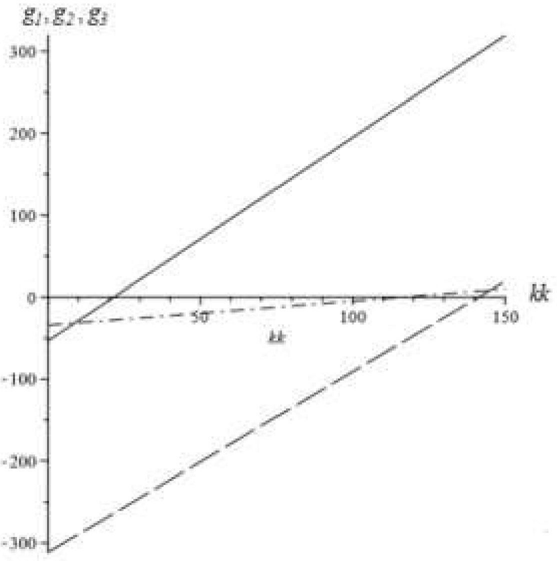

Fig. 7. Graphic illustration of the conditions of safe stability loss

The left boundary of the interval is determined by the intersection of the solid line (corresponding to the condition (10)) and the right border - by the intersection of the dotted line (corresponding to the condition (11)) with the x- axis; 21,64 <*kk* <141,28.

Thus, the interval of *kk* torsional stiffness parameter change, providing safety of vehicle loss of stability, coincides with the result obtained earlier at $\lambda \to 0$ [13]: 21,42< *kk* <116,18. The right border *(kk* = 116,18*)* corresponds to the intersection of the dash-dot line with the x-axis and corresponds to the condition (8). Expansion of range of *kk* parameter change was 21,6%.

CONCLUSIONS

1. Numerical and analytical analysis of the real bifurcations in the vicinity of the rectilinear mode of vehicle movement associated c divergent loss of stability was conducted.
2. On the basis of the proposed approach the system of finite equations defining a set of stationary modes is reduced to one determining equation (cubic binomial).
3. The analysis of the number of solutions of the defining equation makes it possible to draw conclusions about the safe-dangerous border of rectilinear motion mode in space of system design parameters.
4. Specified sufficient conditions of safe loss of stability of rectilinear motion mode of the wheeled vehicle model with controlled wheel module were obtained.
5. Numerical estimates for the torsional stiffness parameter interval, providing a safe loss of stability of rectilinear motion mode of the wheeled vehicle model are presented.

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ДИВЕРГЕНТНЫЕ БИФУРКАЦИИ СТАЦИОНАРНЫХ РЕЖИМОВ ДВИЖЕНИЯ МОДЕЛИ КОЛЕСНОГО ЭКИПАЖА С УПРАВЛЯЕМЫМ КОЛЕСНЫМ МОДУЛЕМ

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Аннотация. Рассмотрен альтернативный подход определения условий безопасной потери устойчивости прямолинейного движения модели колесного экипажа с управляемым колесным модулем в смысле Н.Н. Баутина. Силы увода представлены с точностью до кубических членов разложения по углам увода. Условия безопасной потери устойчивости зависят от соотношения между коэффициентами сопротивления уводу, коэффициентами сцепления в поперечном направлении осей и параметром крутильной жесткости управляемого колесного модуля.

Представленный подход анализа вещественных бифуркаций, связанных с дивергентной потерей устойчивости прямолинейного режима, имеет наглядную геометрическую картину: если в окрестности прямолинейного движения при докритической скорости дополнительно существуют два неустойчивых круговых стационарных состояния, то граница устойчивости имеет опасный характер в смысле Н.Н. Баутина; если два круговых стационарных режима существуют при закритической скорости, то граница потери устойчивости в пространстве параметра продольной скорости движения имеет безопасный характер в смысле Н.Н. Баутина. Анализ количества стационарных режимов в окрестности критической скорости прямолинейного движения выполняется для определяющего уравнения – кубического двучлена.

Ключевые слова: колесный модуль, коэффициент сцепления, силы увода, устойчивость, дивергентная бифуркация.