LABORATORY WORKS

Instructions of laboratory works arrangement

LABORATORY WORK consists of the following stages:

- study the work instructions and corresponding theoretical material;
 - write a report on the work;
 - be admitted to carry out the work;
 - carry out experimental measurements;
 - process results of measurements and enter them into the report;
 - present the laboratory work to the teacher.

A LAB REPORT consists of the following parts:

- 1. The purpose and the tasks of the work.
- 2. Theoretical information.
- 3. Devices and equipment.
- 4. Measurement method and deduction of formulas for calculation.
- 5. Tables of measurements and calculations.
- 6. Processing the results of measurements, evaluation of errors and drawing graphs.
 - 7. Record of the results and the errors.
 - 8. Conclusions.

In conclusion, it is necessary to estimate the results in terms of laws of physics, to evaluate the accuracy of measurements and specify the cause of errors.

Laboratory work 1

DETERMINATION A LENS CURVATURE RADIUS AND LIGHT WAVELENGTH USING NEWTON'S RINGS

The purpose of the work: to study the method of obtaining interference fringes of equal thickness — Newton's rings; to determine a lens curvature radius and light wavelength by measuring the Newton's rings diameters.

Theoretical information and experimental equipment

The interference of light is called the phenomenon of the light intensity redistribution in some points of space at the superposition of the coherent waves at these points.

The coherent waves (sources) are called the waves (sources) with the equal frequency, with the fixed phase difference and the coinciding planes of vectors \vec{E} oscillation.

Light, from the classical electrodynamics viewpoint, represents the crossed electromagnetic waves spreading in vacuum with velocity of $c = 3.10^8$ m/c. The plane electromagnetic wave (polarized) spreading along axis X, for example (Fig. 6.1), is described by the following equations:

$$\vec{E} = \vec{E}_m \cos(\omega t - k_0 x + \alpha);$$

$$\vec{H} = \vec{H}_m \cos(\omega t - k_0 x + \alpha),$$

where \vec{E} is a vector of electric intensity; \vec{H} is a vector of magnetic intensity; $\omega = 2\pi v$ is a circular frequency (v is an oscillations frequency), t is time; α is an initial phase; $k = \frac{2\pi}{\lambda_0}$ is a wave number

 $(\lambda_0 \text{ is a wavelength in vacuum}).$

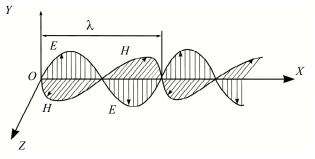


Fig. 6.1

The waves with the frequencies 10^{12} – 10^{17} Hz relate to the optical range of the electromagnetic waves. They are generated by the atoms of a body.

Visual light has a frequency range $0.75 \cdot 10^{15} - 0.40 \cdot 10^{15}$ Hz. The white (polychromatic) light is a plurality of monochromatic waves. The waves

with a frequency more than $0.75 \cdot 10^{15}$ Hz represent ultraviolet radiation, with a frequency less than $0.40 \cdot 10^{15}$ Hz represent infrared radiation.

The wavelength and the frequency are related as $\lambda = c/v$.

When the light waves propagate in some medium, the wavelength is changed (decreased), but its frequency remains the same. The light intensity I depends on the amplitude squared of a light wave: $I \sim A^2$.

In the case of monochromatic light, the oscillations of two waves can be presented by the equations:

$$E_1 = A_1 \cos (\omega t - k_0 x_1 + \alpha); \quad E_2 = A_2 \cos (\omega t - k_0 x_2 + \alpha).$$

The amplitude of the resulting oscillation in a given point is:

$$A^{2} = A_{1}^{2} + A_{2}^{2} + 2A_{1}A_{2}\cos(k_{0}x_{2} - k_{0}x_{1}), \qquad (6.1)$$

where $k_0(x_2 - x_1) = \Delta \varphi$ is a phase difference of the oscillations excited by two waves.

Taking into account that the light intensity I is proportional to the square of light wave amplitude A, we can receive from (6.1):

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta \varphi .$$

This formula shows that the light intensity I depends on the value of $\cos \Delta \varphi$: if $\Delta \varphi > 0$, the intensity value at the given point will exceed $I_1 + I_2$, if $\Delta \varphi < 0$ the intensity will be less than $I_1 + I_2$.

If
$$I_1 = I_2$$
, so $I = 2I_1 + 2I_1 \cos \Delta \varphi$.

In the points, where $\cos\Delta\varphi = 1$, it is observed that the maximum intensity $I = 4I_1$, and in the points, where $\cos\Delta\varphi = -1$, the intensity will be equal to zero.

Therefore, the maximum intensity is observed when $\Delta \varphi = 2k\pi$, and the minimum is observed when $\Delta \varphi = (2k + 1)\pi$. Taking into account that the path difference and phase difference $\Delta \varphi$ are related as:

$$\Delta \varphi = \frac{2\pi}{\lambda} \Delta x \; ,$$

it is possible to write down the conditions of maximum and minimum for the path difference:

$$\Delta x = 2k\frac{\lambda}{2}$$
, $k = 0, 1, 2...$ is the maximum condition;

$$\Delta x = (2k+1)\frac{\lambda}{2}$$
, $k = 0, 1, 2...$ is the minimum condition.

Here $\Delta x = x_2 - x_1$ is the path difference.

Generally, if the waves are propagated in mediums with the indexes of refraction n_1 and n_2 and they are added on the boundary of these mediums, the optical path difference is taken into account in the place of path difference:

$$\Delta = n_2 x_2 - n_1 x_1.$$

In practice, the observation of the light interference from two independent sources (two lamps) is impossible. It is explained by the fact that any two natural light sources cannot be coherent. To receive the coherent sources, it is necessary to split one source into two, for example, with the help of a bimirror, a biprism, two slots etc.

One of the methods of obtaining the coherent waves is splitting a wave reflecting on the top and the bottom surfaces of a thin transparent film. The interference pattern in the form of the alternating dark and bright concentric rings was observed by Newton when reflecting monochromatic light from the thin air clearance between the spherical surface of the plano-convex lens and the plane glass plate (Fig. 6.2).

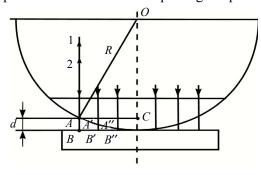


Fig. 6.2

Let a parallel beam of monochromatic light normally fall on a system of a plane glass plate and the plano-convex lens (Fig. 6.2). It is partially reflected at the points A, A', A''... on the top spherical surface of the air gap between the lens and the plate. A part of the beam goes further and is reflected at the points B, B', B''... on the glass plate surface.

When the ray incidence angle at the points B and A equals zero (i = 0) and the air index of refraction equals one, the optical path difference between the rays reflected at the points B and A equals:

$$\Delta = 2d + \lambda/2$$
.

The additional path difference $\lambda/2$ is added due to reflection of light on the point *B* (optically denser medium), i.e. the wave phase is changed by π upon reflection.

In the reflected monochromatic light, the maximum of intensity distribution in the interference pattern (bright ring) equals:

$$\Delta = 2d + \frac{\lambda}{2} = 2k\frac{\lambda}{2},\tag{6.2}$$

the minimum (dark ring) equals:

$$\Delta = 2d + \frac{\lambda}{2} = (2k+1)\frac{\lambda}{2}.$$
 (6.3)

The distance d can be determined from the triangle OCA:

$$r^{2} = R^{2} - (R - d)^{2} = 2Rd - d^{2}.$$
 (6.4)

As R >> d, so $r^2 = 2Rd$. If we replace r_i by $D_i/2$, the thickness of the air clearance under the dark ring number k equals:

$$d_i = D_i^2 / 8R. (6.5)$$

Substituting d from the equation (6.2) to the equation (6.4), we can find the bright ring radius:

$$r_{br} = \sqrt{R(2k-1)\frac{\lambda}{2}}.$$

Substituting d from the equation (6.3) to (6.4), we can find the dark ring radius $r_d = \sqrt{Rk\lambda}$, where k = 0, 1, 2,... is a ring number.

If the radii of the dark or bright rings and the wavelength of monochromatic light λ are known, it is possible to determine the the lens curvature radius R.

On the contrary, knowing R, we can find λ .

The dark spot is in the pattern center; therefore, it is better to determine the dark interference rings diameters to increase accuracy. The formula for the difference of two rings diameters D_k and D_m is:

$$\left(\frac{D_k}{2}\right)^2 - \left(\frac{D_m}{2}\right)^2 = (k-m)R\lambda,$$

where k and m are any two dark interference rings numbers.

Then the lens curvature radius is:

$$R = \frac{D_k^2 - D_m^2}{4\lambda(k - m)},\tag{6.6}$$

the monochromatic light wavelength is:

$$\lambda = \frac{D_k^2 - D_m^2}{4R(k - m)}. ag{6.7}$$

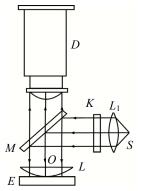


Fig. 6.3

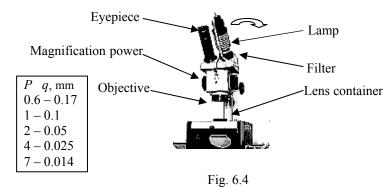
The device for measuring Newton's rings is shown on Fig. 6.3.

The beam goes from a lamp S through a lens L_1 and a filter K to a partly transparent (depending on the angle of inclination) plate M and partly reflects on the investigated lens L.

The rest of the beam reflects on the planeparallel plate E. Thus, the beam is doubled on two coherent rays that pass through the plate Mto the microscope objective.

Work procedure technique and data processing

- 1. Determination of a curvature radius of a lens
- 1.1. Put a container with a plane-convex lens and a plane-parallel plate onto the microscope table under the objective (Fig. 6.4). Turn aside the lamp and put a color filter with the known wavelength in the opening. Looking through the eyepiece and moving the lens container, find the interference image as the dark and bright concentric rings.



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1.2. Look into the eyepiece and keep under observation the left side of the fifth ring. Read outcomes of the micrometer inner scale, and then keep under observation the fourth, third, second and first rings in turn taking measurements. Passing the central spot, continue measuring to the right from the central spot and reach the right side of the fifth ring. Enter readings in Table 6.1.

Table 6.1

Color	(E	(Enter color and wavelength of your filter) $\lambda =$								
Ring Ring	Left-hand reading, N_l	Left-hand reading, N_l Right-hand reading, N_r	Ring diameter		n^2	Lens curvature radius R_i				
number	eft-} adin	ght- adin	I_i	u	², mm²					
	7 5 X 5	re R	ΔN_i	ΔN_i D_i	$D_i^{\cdot 2}$	1	2	3	4	5
1										
2										
3										
4										
5								Ī	$R_{av} =$	

1.3. To determine the diameter of the ring, take the difference between the right-hand and the left-hand readings for a given ring and multiply it by q (the least scale factor depending on the microscope magnification power P that is given on the handle) (Fig. 6.4):

$$D_i = q (N_{ri} - N_{li}) = q \Delta N_i.$$

- 1.4. Calculate D_i for each ring. Enter the results in Table 6.1.
- 1.5. Using the obtained readings, calculate the radius of the lens curvature for each combination according to the formula (6.6) combining rings in pairs. For example, k = 4, m = 1 then k = 5, m = 3 and so on. Enter the results in the right part of tab. 6.1 and then find R_{av} . Calculate absolute and ratio errors of R_{av} as direct measurement errors.
- Calculate absolute and ratio errors of R_{av} as direct measurement errors. 1.6. Determination of R by the graphical method. Using the ring diameters make a diagram (D_i^2 on the axis of ordinates and the ring number k on the abscissa axis). According to the formula $D_i^2 = 4R\lambda k$, the diagram should be a straight line (equation like y = ax, where $a = 4R\lambda$). The curvature radius of the lens is determined as an inclination tangent of the line to the abscissa axis ($\tan \alpha = y/x = a$).

For our case:

$$\tan \alpha = D_i^2/k = 4R\lambda$$
, $R = \tan \alpha/4\lambda$.

Compare R_{av} from Table 6.1 and the same obtained by the diagram.

- 2. Determination of light wavelength.
- 2.1. Replace the filter by a color filter with an unknown wavelength.
- 2.2. Repeat items 1.1–1.3. Enter the obtained results in Table 6.2.
- 2.3. Determine the length of an unknown light wave by the formula (6.7) up to 9 times and calculate λ_{av} similarly to R_{av} . Fill Table 6.2.
 - 2.4. Calculate absolute and ratio errors of λ .

Table 6.2

Ring number	ling, N_i , diamod in g, N_r , ling, N_r , ling N_r , line $N_$		lg her han her ha her han her han her ha her h	O_i^2 , mm ²	Light wavelength, λ					
	Left	Rig	ΔN_i	D_i , n	D_i^2	1	2	3	4	5
1										
2										
3										
4										
5							λ_{av}	=		

- 3. Determination of the air clearance thickness between the planoconvex lens and the plane glass plate.
- 3.1. Using the ring diameters D_i (see Table 6.1) and R_{av} , find the thickness of the air clearance d under each of the interference minimum by the formula (6.5).

Enter the obtained results in Table 6.3.

Table 6.3

Ring number	1	2	3	4	5
Ring diameter D_i , mm					
Thickness of air clearance d, mm					

Self-examination questions

- 1. What is called the phenomenon of light interference?
- 2. What is the condition for observing light interference?
- 3. What are the max and min conditions of light interference?
- 4. What is called optical path difference, phase difference? What is the connection between them?
- 5. What waves are called coherent?
- 6. What ways of obtaining coherent waves do you know?
- 7. How are the coherent waves received in this laboratory work?
- 8. Why is the interference pattern observed as rings?
- 9. Deduce formulas to determine R and λ .

Laboratory work 2

DETERMINATION OF THE MONOCHROMATIC LIGHT WAVELENGTH BY THE DIFFRACTION GRATING

The purpose of the work: to study the method of determining light wavelength with the help of diffraction grating; to determine grating constant, resolving power and angular dispersion of diffraction grating, wavelength of monochromatic light.

Theoretical information and experimental equipment

Diffraction of light is the name given to the totality of phenomena which come to bending light around small opaque obstacles, i.e. its deviation from the laws of geometrical optics. Diffraction takes place due to the wave nature of light and is observed during light propagation in the medium with sharply defined inhomogeneities (passing through the holes in opaque screens, near the opaque bodies boundaries, etc.).

Huygens' principle states that the position of the wavefront can be represented at any moment as the result of the superposition of all secondary (elementary) waves.

Every point of the wavefront becomes the source of the succeeding secondary waves. The Huygens principle enables to explain the laws of reflection and refraction of light. It cannot, however, explain the diffraction phenomena.

The *Huygens-Fresnel principle* states that a primary wave can be replaced by a system of the virtual coherent secondary waves that interfere upon superposition. Calculations of the secondary waves interference can be substantially simplified by means of a graphic geometrical method in which the wavefront is subdivided into the ringshaped sections called *Fresnel zones*. This division into zones is done so that the optical path difference of any two coherent secondary waves emitted from the boundaries (internal or external) of each pair of

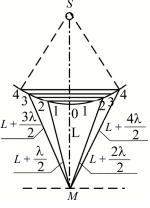


Fig. 6.5

adjacent zones to the point M is equal to $\lambda/2$. Therefore, these secondary waves reach the point M in opposite phases mutually weakening each other.

Fig. 6.5 illustrates the construction of Fresnel zones for a spherical wave excited by the source S. Section 101 of the wave surface is called the first (central) Fresnel zone, the ring-shaped section 21 is called the second zone, etc. Since $L >> \lambda$, the areas of the Fresnel zones are very small.

Two cases of diffraction of light are considered:

Fresnel diffraction phenomena are obtained when the source of light and the screen for observing the diffraction pattern are at a finite distance from the diffracting aperture or an obstacle. An analytical solution to these problems is usually rather difficult. In simpler cases, the appearance of the diffraction pattern can be established by applying the method of annular Fresnel zones.

Fraunhofer diffraction or diffraction of the parallel rays is obtained when the light source and the observation point are at the infinite distances from the obstacle causing diffraction. In practice, this type of diffraction can be observed by focusing the light on the screen by means of a converging lens. This type of diffraction is usually calculated analytically.

Diffraction by a narrow slit. Let monochromatic light fall on the slit normally to its plane (Fig. 6.6). Divide the unclosed part of the wave surface into the elementary zones of width h. The secondary waves sent by the adjacent zones in the different directions will be gathered by the lens in the different points of the screen.

Let us assume that the secondary waves going under the angle φ to the optic axis of the lens from the adjacent zones have optical path difference $\Delta = \lambda/2$ and are gathered in the point P. In the case,

the Fresnel zone width is: $h = \frac{\lambda/2}{\sin \varphi}$. The number of

zones that fit into the slit width a is:

$$m = \frac{a}{h} = \frac{2a\sin\varphi}{\lambda}$$
.

Thus, we have maximum of intensity in the point P if m is an odd number (2k + 1), and minimum, if m is an even number (2k).

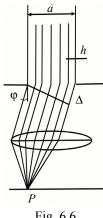


Fig. 6.6

So, in the case of diffraction by one slit, the condition of maxima is:

$$a \sin \varphi = \pm (2k+1)\frac{\lambda}{2}, (k=1, 2, 3...).$$

The condition of minima is:

$$a \sin \varphi = \pm 2k \frac{\lambda}{2}$$
, $(k = 1, 2, 3...)$.

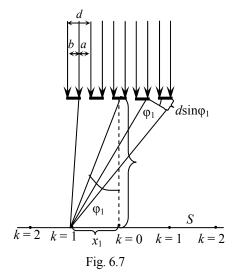
The width of diffraction maxima depends on the relation a/λ .

Diffraction by diffraction grating. Diffraction grating is a system of N equally spaced, identical parallel slits in a plane opaque screen.

Let monochromatic light fall on the diffraction grating normal to its plane (Fig. 6.7). The optical path difference between the waves coming from the corresponding points of the adjacent slits is equal to $\Delta = (a+b)\sin \varphi$, where d = a + b is the constant of the diffraction grating, a is the width of the slit and b is the width of the opaque intervals between the adjacent slits, φ is a diffraction angle. If this path difference is equal to a whole number of the wavelengths, the waves reinforce each other; so, the condition of the main maxima is:

$$d \sin \varphi = \pm k\lambda$$
.

As it can be seen, there are several such directions. If the wave falling on the grating is not monochromatic, the grating resolves the wave into its spectrum. Moreover, there will be several spectra rather than just a single spectrum.



The number k appearing in the above equation is called, therefore, the *order of the spectrum*. The number of k can be equal to zero (in the case of the undeflected ray) or be positive and negative ones. The constant of the diffraction grating is related with a number of slits by the ratio:

$$d = \frac{1}{n}$$
 ($n = \frac{N}{l}$, where N is the whole number of slits; l is the grating width).

Diffraction grating can be used as a spectrum device. The main characteristics of any spectrum

device are the dispersion D and the resolving power R.

Angular dispersion is:

$$D_{\varphi} = \frac{\partial \varphi}{\partial \lambda} \,. \tag{6.8}$$

Let us conclude the angular dispersion for the diffraction grating using the equation $d\sin\varphi = \pm k\lambda$ or $d\sin\varphi = \pm k\lambda$. Hence it appears:

$$D_{\varphi} = \frac{\partial \varphi}{\partial \lambda} = \frac{k}{d\cos\varphi}; \text{ when } \varphi \to 0, \ D_{\varphi} \approx \frac{k}{d}.$$
 (6.9)

The linear dispersion of a diffraction grating is: $D_l = D_{\phi} f$, where f is a lens focal distance.

The resolving power determines the width of spectrum maxima:

$$R = \frac{\lambda}{\partial \lambda}$$
.

Two waves are distinguishable if the center of the first wave maximum overlaps the end of the second wave maximum.

In the case of diffraction grating, the center position of k-max for a wavelength λ_1 is determined as:

$$d \sin \varphi = \pm k \lambda_1$$
.

The end of k-max for λ_2 is determined as:

$$d\sin\varphi = \left(k + \frac{1}{N}\right)\lambda_2$$
.

Taking $\lambda_2 = \lambda$, and $\lambda_1 = \lambda + \partial \lambda$ we obtain

$$k(\lambda + \partial \lambda) = \left(k + \frac{1}{N}\right)\lambda, \quad k\partial\lambda = \frac{\lambda}{N}, \quad \frac{\lambda}{\partial\lambda} = kN.$$

This implies:

$$R = kN. (6.10)$$

A monochromatic light wavelength can be determined by a diffraction grating. In the given laboratory work, the object of examination is a laser beam. We use an optical table for observation of a diffraction pattern and measuring diffraction angles φ_1 , φ_2 , φ_3 (Fig. 6.7).

Work procedure technique and data processing

- 1. The diffraction grating constant determination
- 1.1. Observe diffraction image as bright spots on the screen.
- 1.2. Measure the distance f between the diffraction grating and the screen.
- 1.3. Get the left reading x_{l1} of the first order spot on the screen. Obtain similar readings, having measured a right spot of the first order x_{r1} on the screen.
 - 1.4. Calculate the average value x_1 of the readings x_{l1} and x_{r1} as

$$x_1 = (x_{l1} + x_{r1})/2$$
.

Enter the obtained results in Table 6.4.

- 1.5. Carry out similar measurements for the spots of the second and third orders. Enter the obtained readings in Table 6.4 as well.
- 1.6. Calculate angles of diffraction ϕ_1 , ϕ_2 , ϕ_3 , taking into consideration that according to the triangle in Fig. 6.7

$$tg\varphi_i = \frac{x_i}{f} .$$

Do it for each order. Enter the obtained results in Table 6.4.

1.7. Knowing the wavelength of the laser light and the value of the diffraction angles for each order, determine the diffraction grating constant d according to the formula $d \sin \varphi = \pm k\lambda$.

Table 6.4

Spectrum order k	Left reading x_h , mm	Right reading x_n , mm	Average value x_i , mm	Diffraction angle φ _i	Distance f, mm	Constant d, mm	$d_{average}$, mm	Resolving power R
1								
2								
3								

- 1.8. As the value of d remains the same for a given diffraction grating, we can determine the average value of d_{av} .
- 1.9. Calculate the absolute and relative errors of the diffraction grating constant.
 - 2. Resolving power of diffraction grating determination
- 2.1. Measure the width of the diffraction grating l with the help of a ruler. Determine the number of the grating slits: $N = \frac{l}{d}$.
- 2.2. Determine the resolving power of the diffraction grating R for the first, second and third orders by formula (6.10).
 - 3. *Light wavelength determination (optionally)*.
- 3.1. Exchange the laser by an incandescent lamp with a light filter. Remove the light filter, observe and draw the diffraction spectrum of radiation of the incandescent lamp.
- 3.2. Set light filter and carry out measuring, similarly to item 1. Enter the obtained readings in the Table 6.5.

Table 6.5

Spectrum order k	Left reading x_{li} , mm	Right rea-ding x_{ri} , mm	Average value x_i , mm	Diffraction angle φ _i	Distance f, mm	Wavelength λ	laverage, mm
1							
2							
3							

- 3.3. Knowing the constant of the diffraction grating and using φ_{av} determine the filter wavelength for each order k of the spectrum.
- 3.4. Determine the average wavelength λ_{av} and the error of the red light measurement.

? Self-examination questions

- 1. What is the essence of light diffraction phenomenon?
- 2. What sorts of diffraction do you know?
- 3. Do you know the definition of the wave front?
- 4. Formulate Huygens-Fresnel principle.
- 5. What is the essence of Fresnel zones method?
- 6. What is the essence of diffraction by a narrow slit? Deduce the formula of maximum condition for diffraction by a narrow slit.
- 7. What is called a diffraction grating?
- 8. Deduce the formula for a diffraction grating.
- 9. What is called the resolving power of a diffraction grating?
- 10. What is called the angular and linear dispersion of a diffraction grating?
- 11. How is the phenomenon of diffraction used to study a body crystalline structure?

Laboratory work 3

PLANE POLARIZED LIGHT PROPERTY INVESTIGATION

The purpose of the work: to study the method of plane polarized light observation with the help of polarizers; to test Malus law; to observe double refraction and interference of the polarized rays.

Theoretical information

Theoretical information and experimental equipment

A drawing of the device for the plane polarized light properties study is shown in Fig. 6.8.

A source of light emits unpolarized (natural) light to the first *polarizer* (*P* is the polarizer polarization plane). The second polarizer is called *analyzer* (*A* is the analyzer polarization plane).

Analyzers are used to investigate the character and degree of the polarization of light. That is why the analyzers container has a scale to measure angles of rotation of the analyzer polarization plane.

We can observe on the screen the light intensity change during the analyzer rotation.

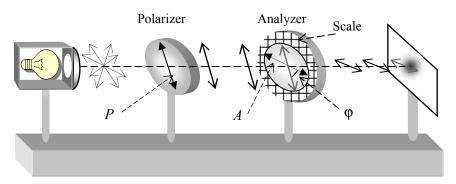


Fig. 6.8

Then the screen is replaced by a photosensitive device with a microammeter to measure photocurrent which is proportional to the light intensity. If the light wave entering an analyzer is linearly polarized, then the *Malus law* is valid for the intensity of the wave after the analyzer:

$$I = I_0 \cos^2 \varphi,$$

where I_0 is the intensity of the incident polarized light, φ is the angle between the polarizer plane of polarization P (the polarized light plane) and the analyzer plane of polarization A.

It is shown on Fig. 6.9.

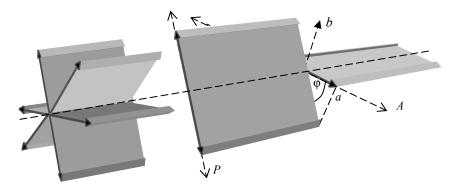


Fig. 6.9

The polarizer transmits light which oscillates in the polarization plane P of the polarizer only. Similarly, the analyzer transmits the projection a of the incident light only because it is in the analyzer plane of polarization.

On the contrary, the projection b cannot pass through the analyzer.

The analyzer and the polarizer are said to be *crossed* if the angle φ between their planes of polarization equals $\pi/2$, and *parallel* if $\varphi = 0$.

Upon the normal incidence of the linearly polarized wave on a uniaxial crystal plane-parallel plate, the following phase difference is observed between the ordinary and extraordinary waves as they pass through the plate:

$$\Delta\alpha = -\frac{2\pi}{\lambda}(n_0 - n_e)d.$$

Though these waves are coherent and are propagated in the same direction, they cannot interfere because they are polarized in mutually perpendicular planes. As a result of their superposition, elliptically polarized light is obtained.

The ordinary and extraordinary waves interference is obtained by using an analyzer to separate out the components that are polarized in a single plane and can interfere.

In the *interference with white light*, the phase difference $\Delta\alpha$ turns out to be different for light of various wavelengths.

If $\Delta \alpha = 0$, the screen is illuminated with a white light. In all other cases, the screen is illuminated with a colored light.

The change in angle φ from 0 to $\pi/2$ changes the color of the screen to the complementary color.

If the plate is of a variable thickness d, then the phase difference $\Delta\alpha$ varies for different points of the plate.

Upon illuminating the plate with monochromatic light, a system of dark and bright interference fringes is observed on the screen.

Each fringe corresponds to the points of equal thickness of the plate (fringes of equal thickness).

If a white light is used, differently colored equal-thickness fringes are observed.

Work procedure technique and data processing

- 1. Testing of polarized light.
- 1.1. Set a lamp, a polarizer, an analyzer and a screen on the optic bench. Rotating the analyzer, observe the changes of the polarized light intensity.
- 1.2. Substitute the photosensitive device with the microammeter for the screen. Switch the lamp on. Press the "ON-OFF" button on the control panel of the microammeter.
- 1.3. Rotating the analyzer from 0 to 360° with the step of 10°, enter the readings of the microammeter in the Table 6.6.

Table 6.6

Angle φ, degrees	Device reading I , μA	cos φ	$\cos^2 \phi$
0 10			
350 360			

- 1.4. Determine the value of trigonometric function $\cos \varphi$ and $\cos^2 \varphi$.
- 1.5. Plot the graph of the functional dependence $I = f(\varphi)$ in the polar coordinate.
 - 2. Malus law testing.
- 2.1. Using the obtained readings, draw the graph of the functional dependence $I = f(\cos^2 \varphi)$ for the angles 0...90°. This functional dependence must be linear.
 - 3. Double refraction observation.
- 3.1. Set a lamp, an objective and a screen on the optic bench. Place an Iceland spar crystal between the objective and the screen and observe the double refraction as two bright images.
- 3.2. Set an analyzer in front of the screen. Rotate the analyzer polarization plane and observe disappearance of the bright images in turn.

Calculate the angle of the analyzer plane rotation between disappearances of the first and second image.

This angle is the angle between the planes of oscillation of the ordinary and extraordinary rays.

- 4. Interference of polarized rays.
- 4.1. Set a crystal between the polarizer and the analyzer on the optic bench.
- 4.2. Rotating the analyzer, you can observe how the color of the spot on the screen changes. Explain the phenomenon.

7

Self-examination questions

- 1. What is the difference between natural and polarized light?
- 2. What types of polarized light do you know?
- 3. What methods of the polarized light obtaining do you know?
- 4. What is called a polarizer and an analyzer?
- 5. What is called the principal plane of crystal?
- 6. What is the difference between ordinary and extraordinary rays?
- 7. What is the Nicol prism?
- 8. What is the definition of Malus law?
- 9. What is the essence of double refraction?
- 10. What is the essence of Kerr effect?
- 11. What is the essence of rotation of the plane of polarization?
- 12. What are the conditions for observing interference of polarized rays?
- 13. What are the min and max conditions of interference of polarized rays?

INDIVIDUAL HOME TASKS

The order of performing individual home Tasks

The individual home tasks (IHT) are carried out in a separate writing-book or on the sheets of A4 format paper.

The term of handing-in of IHT is determined by the teacher.

The variants of IHT are presented in the table and given to the students according to their number on the list of the group.

Before carrying out IHT, a student should work through the corresponding lecture material, answer the SU questions, and be able to solve the problems, which are given at the end of the corresponding SU.

Every problem ought to have an explanation in the form similar to the samples in the corresponding IHT.

REQUIREMENTS FOR IHT CARING OUT:

- 1. Write on the title page "Individual homework on physics"; the number of your variant; your group, department; your surname, patronymic and a name; the date of carrying-out.
 - 2. Write the problem number and copy its full data.
- 3. Make a drawing in a case of need; solve the problem in the general way with the brief explanations, taking note of the laws, which are used.
 - 4. Check the measuring units.
- 5. Calculate the results and round off them according to the rules. The problem answers are to be given in the expanded form using the SI system of units.

Variants of IHT

Variants of IHT	Problem numbers							
1	1	21	41	10	40			
2	2	22	42	11	31			
2 3	3	23	43	12	32			
4 5	4	24	44	13	33			
5	5	25	45	14	34			
6	6	26	46	15	35			
7	7	27	47	16	36			
8 9	8	28	48	17	37			
9	9	29	49	18	38			
10	10	30	50	19	39			
11	11	31	41	30	21			
12	12	32	42	31	22			
13	13	33	43	32	23			
14	14	34	44	33	24			
15	15	35	45	34	25			
16	16	36	1	35	26			
17	17	37	2 3	36	27			
18	18	38	3	37	28			
19	19	39	4	38	29			
20	20	40	5	39	30			



Problems

- **1.** The wavelength of a monochromatic radiation is $0.6 \cdot 10^{-6}$ m in a vacuum. What is its frequency?
- **2.** A monochromatic light source (wavelength $\lambda = 4045 \cdot 10^{-10}$ m) illuminates two holes 1 mm apart. These holes produce a system of the interference fringes on a screen placed 1 m away from the holes. Calculate the distance between the two neighboring fringes.
- **3.** In the Young's experiment, two parallel slits are 0.5 mm apart. Calculate the wavelength of a monochromatic radiation used knowing that the interfringe distance obtained on a screen 2 m away from the slits is 2,2 mm.
- **4.** A white light falls at an angle of 45° onto a soap film (n = 1.33). At what minimum thickness of the film will the reflected rays be colored yellow ($\lambda = 6 \cdot 10^{-5}$ cm)?

- **5.** A vertical soap film forms a wedge. Interference is observed in reflected light through a red glass ($\lambda_1 = 6.31 \cdot 10^{-5}$ cm). The distance between the adjacent red bands is 3 mm. Then the same film is observed through a dark blue glass ($\lambda_2 = 4 \cdot 10^{-5}$ cm). Find the distance between the adjacent dark blue bands. Assume that the shape of the film does not change and the light falls onto the film normally.
- **6.** A beam of light ($\lambda = 5.82 \cdot 10^{-7}$ m) falls normally onto a glass wedge. The wedge angle is 20". What is the number of the dark interference bands per unit of wedge length? The refractive index of the glass is 1,5.
- **7.** A plant used to produce Newton's rings is illuminated by the monochromatic light. The radii of two adjacent dark rings are 4,0 mm and 4,38 mm. The radius of curvature of the lens is 6.4 m. Find the ordinal numbers of the rings and the wavelength of the incident light.
- **8.** Newton's rings are formed between a flat glass and a lens with a radius of curvature of 8.6 m. Monochromatic light falls normally. It has been found by measurement that the diameter of the fourth dark ring (assuming the central dark spot as a zero ring) is equal to 9 mm. Find the wavelength of the incident light.
- **9.** A plant for producing Newton's rings is illuminated by normally incident white light. Find: (1) the radius of the fourth dark-blue ring $(\lambda_1 = 4 \cdot 10^{-7} \text{ m})$, and (2) the radius of the third red ring $(\lambda_2 = 6.3 \cdot 10^{-7} \text{ m})$. The observations are made in the transmitted light. The radius of curvature of the lens is 5 m.
- **10.** The distance between the fifth and the twenty-fifth Newton's bright ring is 9 mm. The radius of curvature of the lens is 15 m. Find the wavelength of the monochromatic light. The observation is made in the transmitted light.
- 11. A plant for producing Newton's rings is illuminated by normally incident light of a mercury arc. The observation is made in the transmitted light. What bright ring (counting from the beginning) corresponding to the line $\lambda_1 = 5791$ Å coincides with the next bright ring which corresponds to the line $\lambda_2 = 5770$ Å?
- 12. The space between a lens and a glass plate of a plant used to observe Newton's rings is filled with liquid. Find the refractive index of the liquid if the radius of the third bright ring is equal to 3.65 mm. The observation is made in the transmitted light. The radius of curvature of the lens is 10 m. The wavelength of light is $5.89 \cdot 10^{-5}$ cm.

- 13. A plant used to observe Newton's rings is illuminated by normally incident monochromatic light with a wavelength of 6000 Å. Find the thickness of the air layer between a lens and a glass plate where the fourth dark ring is observed in the reflected light.
- **14.** A plant used to observe Newton's rings in reflected light is illuminated by normally incident monochromatic light with $\lambda = 5 \cdot 10^{-7}$ m. The space between a lens and a glass plate is filled with water. Find the thickness of the water layer between the lens and the glass plate where the third bright ring is observed.
- **15.** A plant used to observe Newton's rings in reflected light is illuminated by monochromatic light. If the space between a lens and a glass plate is filled with liquid, the radii of the dark rings diminish to 0.8 of the original ones. Find the liquid refractive index.
- **16.** A beam of a white light falls at the right angles onto a glass plate with a thickness of $d = 4 \cdot 10^{-7}$ m. The refractive index of the glass n = 1.5. What wavelengths lying within the limits of the visible spectrum (from $4 \cdot 10^{-4}$ to $7 \cdot 10^{-4}$ mm) are amplified in the reflected beam?
- 17. A thin film with a refractive index of $n_2 = 1.2$ (coating film) is applied to the surface of a lens ($n_1 = 1.5$). At what minimum thickness of this film will the reflected light be most attenuated in the middle of the visible spectrum at $\lambda = 5 \cdot 10^{-7}$ m?
- 18. A parallel beam of monochromatic light with a wavelength of $\lambda = 5890$ Å falls normally onto a slit $2 \cdot 10^{-6}$ m wide. Find the angles in which direction the minima of light will be observed.
- 19. A parallel beam of monochromatic light with a wavelength of $\lambda = 5 \cdot 10^{-5}$ cm normally falls onto a slit $2 \cdot 10^{-3}$ cm wide. Find the width of the slit image on a screen removed by l = 1 m from the slit. Take the width of the image to be the distance between the first diffraction minima located at both sides of the principal maximum of illumination.
- **20.** A parallel beam of monochromatic light with a wavelength of λ falls normally onto a slit. The slit width is 6λ . At what angle will the third diffraction minimum of light be observed?
- **21.** What is the constant of a diffraction grating if a telescope is set at an angle of 30° to the collimator axis to see a red line ($\lambda = 7 \cdot 10^{-7}$ m) in the second-order spectrum? What number of lines is there on 1 cm of the grating length? The light normally falls onto the grating.

- **22.** How many lines are there on 1 mm of a diffraction grating if a green line of mercury ($\lambda = 5461$ Å) is observed in the first-order spectrum at an angle of 19°8′?
- **23.** A beam of light normally falls onto a diffraction grating. The diffraction angle for the sodium line ($\lambda = 5890 \text{ Å}$) in the first-order spectrum is 17°8′. A certain line produces a diffraction angle of 24°12′ in the second-order spectrum. Find a wavelength of this line and the number of lines per millimeter of the grating.
- **24.** A beam of light from a discharge tube normally falls onto a diffraction grating. What should constant of the diffraction grating be for the maxima of the two lines $\lambda_1 = 6563$ Å and $\lambda_2 = 4102$ Å to coincide in the direction $\varphi = 41^{\circ}$?
- **25.** A beam of light normally falls onto a diffraction grating. When a goniometre is turned through an angle φ , the line $\lambda = 4.4 \cdot 10^{-4}$ mm in the third-order spectrum appears in the field of vision. Will any other spectral lines corresponding to the wavelengths within the visible spectrum (from $4 \cdot 10^{-4}$ to $7 \cdot 10^{-4}$ mm) be seen at the same angle φ ?
- **26.** A beam of light from a discharge tube filled with helium normally falls onto a diffraction grating. Onto what line in the third-order spectrum will the red line of helium ($\lambda = 6.7 \cdot 10^{-5}$ cm) of the second-order spectrum be superimposed?
- **27.** Light from a discharge tube filled with helium normally falls onto a diffraction grating. First a telescope is adjusted to see the violet lines ($\lambda = 3.89 \cdot 10^{-5}$ cm) at both sides of the central band in the first-order spectrum. The readings on the dial to the right from zero are 27°33′ and 36°27′, respectively. Then the telescope is adjusted to the red lines at both sides of the central band in the first-order spectrum. The readings on the dial to the right from zero show 23°54′ and 40°6′. Find the wavelength of the red line of the helium spectrum.
- **28.** Find the maximum order of a spectrum for the yellow line of sodium $\lambda = 5890$ Å if the constant of the diffraction grating is $2 \cdot 10^{-6}$ m.
- **29.** A beam of monochromatic light normally falls onto a diffraction grating. A maximum of the third order is observed at an angle of 36°48′ to the normal. Find the constant of the grating expressed in the wavelengths of the incident light.
- **30.** How many maxima are produced by the diffraction grating of the previous problem?

- **31.** A telescope of a goniometer with a diffraction grating is placed at an angle of 20° to a collimator axis, and the red line of the helium spectrum ($\lambda_1 = 6680 \text{ Å}$) is visible in its field of vision. What is the constant of the diffraction grating if a dark-blue line ($\lambda_2 = 4470 \text{ Å}$) of a higher order can be seen at the same angle? The maximum order of the spectrum which can be observed with this grating is 5. The light falls upon the grating normally.
- **32.** What is the constant of a diffraction grating if it can resolve in the first order the lines of the potassium spectrum $\lambda_1 = 4044$ Å and $\lambda_2 = 4047$ Å? The grating is 3 cm wide.
- **33.** What is the constant of a diffraction grating 2,5 cm wide for the sodium doublet $\lambda_1 = 5890$ Å and $\lambda_2 = 5896$ Å to be resolved in the first order?
- **34.** The constant of a diffraction grating 2,5 cm wide is $2 \cdot 10^{-6}$ m. What wavelength difference can be resolved by this grating in the region of the yellow rays ($\lambda = 6 \cdot 10^{-5}$ cm) in the second-order spectrum?
- **35.** Determine angular dispersion of a diffraction grating in the first-order spectrum for $\lambda = 5890$ Å. The grating constant is $2.5 \cdot 10^{-4}$ cm.
- **36.** Angular dispersion of a diffraction grating for $\lambda = 6680$ Å in a first-order spectrum is $2.02 \cdot 10^5$ rad/m. Find the period of the diffraction grating.
- **37.** Find linear dispersion (mm/Å) of the diffraction grating of the previous problem if the focal length of the lens which projects the spectrum onto a screen is 40 cm.
- **38.** At what distance from each other will two lines of a mercury arc $(\lambda_1 = 5770 \text{ Å} \text{ and } \lambda_2 = 5791 \text{ Å})$ be arranged on a screen in the first-order spectrum obtained by means of a diffraction grating with the period $2 \cdot 10^{-4}$ cm? The focal length of the lens projecting the spectrum onto the screen is 0.6 m.
- **39.** A beam of light normally falls onto a diffraction grating. A red line ($\lambda = 6300 \text{ Å}$) is visible in the third-order spectrum at an angle of $\varphi = 60^{\circ}$. (1) What spectral line is visible at the same angle in the fourth-order spectrum? (2) What number of lines is there on the diffraction grating per mm of the length? (3) What is the angular dispersion of this grating for the line $\lambda = 6300 \text{ Å}$ in the third-order spectrum?
- **40.** For what wavelength does a diffraction grating with the constant $d = 5 \cdot 10^{-6}$ m have an angular dispersion of $D = 6.3 \cdot 10^{5}$ rad/m in the third-order spectrum?

- **41.** Find the focal length of the lens which projects onto a screen a spectrum obtained with the aid of a diffraction grating so that the distance between two potassium lines 4044 Å and 4047 Å is 0.1 mm in the first-order spectrum. The diffraction grating constant is $2 \cdot 10^{-6}$ m.
- **42.** Determine the angle of complete polarization when the light is reflected on the glass with a refractive index of 1.57.
- **43.** The total internal reflection limit angle for a certain substance is 45°. What is the angle of complete polarization for this substance?
- **44.** At what angle to the horizon should the Sun be for its rays reflected from the surface of a lake to be polarized most completely?
- **45.** What is a refractive index of the glass if a beam reflected from it is completely polarized at an angle of refraction of 30°?
- **46.** A light beam passes through the liquid poured into a glass (n = 1.5) vessel and is reflected from the bottom. The reflected beam is completely polarized when it falls onto the bottom of the vessel at an angle of 42°37′. Find: (1) the refractive index of the liquid, (2) the angle at which a beam of light passing in this liquid should fall onto the bottom of the vessel to obtain the total internal reflection.
- **47.** A beam of a plane-polarized light with the wavelength 5890 Å in vacuum falls onto a plate of Iceland spar at the right angle to its optical axis. Find the wavelengths of the ordinary and extraordinary rays in the crystal. The Iceland spar refractive indexes are $n_{od} = 1.66$ for the ordinary ray and $n_{eod} = 1.49$ for the extraordinary one.
- **48.** The intensity of light transmitted through a polarizer and an analyzer is nine times less than the intensity of the light transmitted through a polarizer. Find the angle between the basic planes of the polarizer and the analyzer. Absorption of light is neglected.
- **49.** A sunlight passes through a polarizer and an analyzer. The angle between the basic planes of the polarizer and the analyzer is α . Both the polarizer and the analyzer absorb 8 % of the falling light. The intensity of light behind the analyzer is 9 % of the intensity of light falling onto the polarizer. Find the angle α .
- **50.** There is the necessity to make sunlight ten times less intensive. We have a polarizer and an analyzer at our disposal. Find the angle between the basic planes of the polarizer and the analyzer which should be mounted.



Answers

1. $5\cdot10^{-14}$ s⁻¹. **2.** $4.05\cdot10^{-4}$ m. **3.** $0.55\cdot10^{-6}$ m. **4.** $0.13\cdot10^{-6}$ m. **5.** 1.9 mm. **6.** 500 1/m. **7.** 5; 6; $5\cdot10^{-7}$ m. **8.** $589\cdot10^{-9}$ m. **9.** 2.8 mm; 3.1 mm. **10.** $675\cdot10^{-9}$ m. **11.** 275. **12.** 1.33. **13.** $1.2\cdot10^{-6}$ m. **14.** $0.47\cdot10^{-6}$ m. **15.** 1.56. **16.** $0.48\cdot10^{-6}$ m. **17.** $0.115\cdot10^{-6}$ m. **18.** 17° ; 36° ; 62° . **19.** 5 cm. **20.** 30° . **21.** $2.8\cdot10^{-6}$ m; 357. **22.** 600. **23.** $0.41\cdot10^{-6}$ m; 500. **24.** $5\cdot10^{-6}$ m. **25.** $0.66\cdot10^{-6}$ m. **26.** $0.447\cdot10^{-6}$ m. **27.** $0.7\cdot10^{-6}$ m. **28.** 3. **29.** 5. **30.** 10. **31.** $3.9\cdot10^{-6}$ m. **32.** $22\cdot10^{-6}$ m. **33.** $25.4\cdot10^{-6}$ m. **34.** $24\cdot10^{-12}$ m. **35.** $4.1\cdot10^5$ rad/m. **36.** $5\cdot10^{-6}$ m. **37.** $8.1\cdot10^{-3}$. **38.** $650\cdot10^{-6}$ m. **39.** $0.475\cdot10^{-6}$ m; 460; $2.76\cdot10^4$ rad/sm. **40.** $0.51\cdot10^{-6}$ m. **41.** 0.65 m. **42.** 57° . **43.** 55° . **44.** 37° . **45.** 1.73. **46.** 1.63; 67° . **47.** $0.355\cdot10^{-6}$ m; $0.395\cdot10^{-6}$ m. **48.** 70° . **49.** 62° . **50.** 72° .

APPENDIX

Refractive indices

Carbon bisulphide	1.63
Diamond	2.42
Glass	1.5–1.9
Ice	1.31
Turpentine	1.48
Water	1.33

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