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SPECTRAL-TIME MODELS OF DATA SIGNALS UNDER THE ACTION OF INTERFERENCES IN THE TASKS RELATED TO ELECTRIC AND MAGNETIC VALUES MEASURING

Spectral-time models based on the use of linear and harmonizable random processes allow to describe and study a wide range of data signals and influencing interferences during measuring of the parameters and characteristics of electric and magnetic values not only in terms of correlation theory but taking into account higher-order moments.

Key Words: spectral-time model, linear random processes, data signals, mathematical models.

While measuring mechanical, light, heat, acoustical and other physical values their conversion into electrical and magnetic values always occurs. That is why functioning of many information measuring systems is based on the measuring of electric and magnetic values parameters and characteristics [6, 8]. It is known, that the achieved measurement precision is limited for a number of reasons, including the action of interferences. Further increase of measurement precision is connected with both, systems enhancement and deeper analysis of signals and interferences based on the development of new mathematical models and their description.

The given study is devoted to the analysis of spectral-time models that allow describing all the range of data signals and influencing interferences used in measurement technology in practice. They include discrete and continuous, stationary and non-stationary, Gaussian and non-Gaussian ones.

Models based on the use of linear random process and a field are called time models; spectral models considered to be used for harmonizable random process and a field.

Definition: Hilbert random processes $\{\xi(t) \in L_2(-\infty;\infty)\}$ that simultaneously allow view representation

$$\xi(t) = \int_{-\infty}^{\infty} \varphi(\tau, t) d\eta(\tau), \quad t \in T \tag{1}$$

and

$$\xi(t) = \int_{0}^{\infty} \cos 2\pi f t dR_C(f) + \int_{0}^{\infty} \sin 2\pi f t dR_S(f), \quad t \in T \tag{2}$$

create the class of LG-models.

The view representations (1) in which non-random function $\varphi(\tau, t) \in L_2$ at all conditions and $\{\eta(\tau),\ \eta(0), \tau \in (-\infty;\infty)\}$ is a non-uniform random process with independent increments and infinitely divisible distribution law create the class of linear random processes [4, 5]. The view representations (2) where correlation functions of random functions $R_C(f)$ and $R_S(f)$ have limited variation create the class of harmonizable random processes [1, 3]. In this study we will consider only one-dimensional case.

Thereby, the class of LG-models is formed by the intersection of linear and harmonizable random processes classes. Using the results of study [1, 3-5, 7] we can define the main statistical characteristics of LG-models and consider the examples of their use for the description of particular signals in measurement technology.

1. Multidimensional characteristic function of a non-stationary non-Gaussian linear process [1] received with the use of the results [4,5] is described by the following expression:

$$f_\xi(U_1,\ldots,U_n;t_1,\ldots,t_n)=\exp\left\{i\sum_{j=1}^{n}U_j\int_{-\infty}^{\infty}\varphi(\tau,t_j)d\mu(\tau)-\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}U_jU_k\times\right.$$
$$\times\int_{-\infty}^{\infty}\varphi(\tau,t_j)\varphi(\tau,t_k)dD(\tau)+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[e^{ix\sum_{j=1}^{n}U_j\varphi(\tau_1 t_j)}-1-\frac{ix}{1+x^2}\times\right.$$
$$\left.\left.\times\sum_{j=1}^{n}U_j\varphi(\tau,t_j)\right]\frac{1+x^2}{x}d_x d_\tau G(\tau,x)\right\},\qquad(3)$$

where $\mu(\tau)$, $D(\tau)$, $G(\tau,x)$ are τ-function continuous. They categorize completely divisible distribution law of generating random process with $\eta(\tau)$ independent increments.

The expression (3) clearly describes the structure of the linear process. Gaussian and Poisson components of the generating process $\eta(\tau)=\omega(\tau)+\pi(\tau)$ form the correspondent components of the linear process. If Poisson component is not found, i.e. when $G(\tau,x)=0$ the linear process is considered to be Gaussian one. In the case, when generating process $\eta(\tau)$ doesn't include Gaussian component, i.e. when $D(\tau)\equiv 0$, the linear process will not have it too. Realizations of Gaussian linear process are continuous with probability one. The behaviour of realization of the linear process $\xi(t)$ with only Poisson component of the process $\pi(\tau)$ is defined by the functions $G(\tau,x)$ and $\varphi(\tau,t)$ behaviour in any particular case.

For the solution of practical tasks the following formulae for defining of compound cumulants of n-degree $(n>2)$ linear process is of considerable importance

$$\chi_n\left[\xi^{k_1}(t_1)\ldots\xi^{k_n}(t_n)\right]=\int_{-\infty}^{\infty}\prod_{j=1}^{n}\varphi^{k_j}(\tau,t_j)d\mathrm{H}_n(\tau),\quad t_j\in T\qquad(4)$$

where k_j are integer nonnegative numbers that fit the equations $\sum_{j=1}^{n}k_j=n,$ and

$$dH_n(\tau)=\begin{cases} d\mu(\tau)+\int\limits_{-\infty}^{\infty} x^2 d_\tau d_x G(\tau,x), & n=1 \\ dD(\tau)+\int\limits_{-\infty}^{\infty} x(1+x^2) d_\tau d_x G(\tau,x), & n=2 \\ \int\limits_{-\infty}^{\infty} x^{n-1}(1+x^2) d_\tau d_x G(\tau,x), & n>2 \end{cases}$$

Linear processes as time models of signals are widely used for the tasks of signal conversion analysis and design in linear and non-linear transmission channels of measuring systems.

2. As a basis of spectral representation of random processes is the representation (2) that is considered to be the limit of integral sum of elementary harmonic vibrations with random amplitudes and phases. The analysis of harmonizable processes is usually conducted in terms of two initial moments. We can use the results [3] to demonstrate that correlation function of non-stationary harmonizable process (2) at a condition $\mathrm{M}\{R_C(f)\}=\mathrm{M}\{R_S(f)\}=0$ is described by the following expression

$$\begin{aligned} R(t_1,t_2)=\int\limits_0^\infty\int\limits_0^\infty \big[&(\cos 2\pi f_1 t_1 \cos 2\pi f_2 t_2) d_{f_1} d_{f_2} Q(f_1,f_2)+(\cos 2\pi f_1 t_1 \sin f_2 t_2)\times \\ &\times d_{f_1} d_{f_2} D_{cs}(f_1,f_2)+(\sin 2\pi f_1 t_1 \cos 2\pi f_2 t_2) d_{f_1} d_{f_2} D_{cs}(f_1,f_2)+ \\ &+(\sin 2\pi f_1 t_1 \sin f_2 t_2) d_{f_1} d_{f_2} I(f_1,f_2)\big], \end{aligned} \tag{5}$$

where

$$\mathrm{M}\{R_c(f_1)R_c(f_2)\}=Q(f_1,f_2),\ \mathrm{M}\{R_s(f_1)R_s(f_2)\}=I(f_1,f_2),$$
$$D_{cs}(f_1 f_2)=\mathrm{M}\{R_c(f_1)R_s(f_2)\}=D_{sc}(f_2,f_1).$$

Functions $Q(f_1,f_2)$ and $I(f_1,f_2)$, are also called spectral ones, and $D_{cs}(f_1 f_2)$, $D_{sc}(f_2,f_1)$ are called mutual spectral functions of generating random functions $R_C(f)$, $R_S(f)$ of the harmonizable process (2).

Practical importance of the results of a harmonizable process analysis considerably increases when we define a narrower subset with additional properties from this class of processes.

The example is the definition of a subset of non-stationary intermittently correlated random process (ICRP) the use of which allowed to develop new mathematical models of physical processes, as well as electric and magnetic signals with two initial periodic moments of functions (ensemble average, dispersion and correlation function) [1,2,7,10]. So, in the study [7] the implementation of ICRP for describing of magnetic noises in ferromagnetics during their magnetization reversal, hysteresis characteristics in magnetic materials, the load of power installations per day, shot noise in semiconducting and electronic devices with variable average current allowed to develop the procedure of their experimental research and statistic characteristics measuring.

Thereby, the given representations and characteristics of LG-models are received for linear and harmonizable random processes, non-stationary and non-Gaussian in general case. It should be noted, that stationary, Gaussian and discrete cases that are very important for practical tasks are regarded to as special cases.

3. Here are the examples of electric and magnetic signals described by LG-models.

Modulated signals with harmonic carrier are most widely used for the transmission of measurable signals. It is assumed that modulated desired signals, as well as the value of frequency carriers and other parameters of the signal are already known.

So, amplitude-modulated signal that is described by the expression

$$x(t) = A[1 + m_{AM} a_1(t)]\cos(2\pi f_н t + \theta), \quad t \in T$$

on the output of a transmitter will represent the signal of another type, namely $\xi(t)$ on a receiver with the action of

$$y(t) = A\{1 + m_{AM}[a_1(t) + \xi(t)]\}\cos(\omega_н(t) + \theta), \quad t \in T$$

interference.

$y(t) = x(t) + \xi(t)$ or $y(t) = x(t)\xi(t)$, where $\alpha(t)$ is a modulated signal, random in general case, and $\xi(t)$ is an interference that allow the description by the representation (1) or (2) , i.e. by LG-models. A, θ, ω_H , m_{AM} – are numerical coefficients.

At angle modulation

$$x(t) = A\cos(\varpi_н t + m_{YM}\int_{t_1}^{t_2} a(\tau)d\tau + \theta), \quad t \in T$$

we receive respectively

$$y(t) = A\cos\left(\omega_н t + m_{YM}\int_{t_1}^{t_2}[a(\tau) + \xi(\tau)]d\tau + \theta\right), \quad t \in T$$

or depending on the initial conditions in combinations specified for amplitude-modulated signal.

At associated amplitude and angle modulations of a signal

$$x(t) = A\left[1 + m_{AM} a_1(t)\cos\left(\omega_н t + m_{YM}\int_{t_1}^{t_2} a_2(\tau)d\tau + \theta\right)\right], \quad t < T$$

different combinations of a signal and an interference can occur on the receiver. For example,

$$y(t) = A\{1 + m_{AM}[a_1(t) + \xi_1(t)]\}\cos\left(\omega_н t + m_{YM}\int_{t_1}^{t_2}[a_2(\tau) + \xi_2(\tau)]d\tau + \theta\right), \quad t \in T$$

$y(t) = x(t) + \xi(t)$ or $R(t) = y(t)| + \xi(t)$ or $R(t) = y(t)\xi(t)$, where linear and harmonizable random processes $\xi(t)$, $\xi_1(t)$, $\xi_2(t)$ can be both non-correlated (independent) and dependent.

Modulated signals with noise carrier have more noise immunity as compared to signals with harmonic carrier, and the task of transmission and reception of modulated signals research is more physically based, as all the measuring devices generate signals with the definite number of harmonic components or continuous spectrum.

Receiving of the noise carrier as the random process can be described in two ways:

a) as a result of linear filtering

$$\xi(t) = \int_{-\infty}^{\infty} \varphi(t - \tau)\, d\eta(\tau), \quad t \in T$$

representing stationary linear process, where homogeneous process with independent increments $\eta(\tau)$ is considered to be generating one;

b) stationary harmonizable process

$$\xi(t) = \int_0^{\infty} \cos 2\pi f t dZ_c(f) + \int_0^{\infty} \sin 2\pi f t dZ_s(f), \quad t \in T,$$

where $Z_c(f)$ and $Z_s(f)$ are random functions with non-correlated (orthogonal) increments, and spectral densities of both representations coincide.

Noise processes occurring in physical medium (magnetic noises in ferromagnetic materials, electromagnetic noises in semiconductors or insulators), electronic systems (shot noise, thermal noise, flicker noise) and other noises can be described by a unified mathematical model represented by linear mathematical process [4,5,7,9]. The application of this process for noise processes description allows using of initial physical prerequisites of process forming more widely if to compare it with harmonizable process. So, describing shot noise in semiconducting and electronic devices we can define three modes of a noise source and choose three kinds of generating process $\eta(\tau)$ respectively:

saturation mode is characterized by the similar charge carrier velocities (generating process is Poisson one);

saturation mode is characterized by the considerably different charge carrier velocities (complex generic generating process);

extreme case, when the average number of charge carriers increases indefinitely (Wiener Gaussian process).

Function $\varphi(\tau, t)$ as the kernel of integral representation (1) describing the thermal noise by the linear stationary Gaussian process is defined by the kind of desired spectral density $S(f)$ at continual Wiener generating process $\omega(\tau)$. So, if $S(f)$ is constant in all the frequency band (white noise) , then $\varphi(t)$ is delta function. For band noise $S(f)$ is constant and differs from zero only in finite frequency interval $f \in [-f_0, f_0)$ function $\varphi(t)$ is $\dfrac{\sin 2\pi f_0 t}{2\pi f_0 t}$.

Reverberatory process that characterizes distribution of electromagnetic vibrations in conducting continuum connected with reflection, refraction and reradiating from radiators discretely distributed in medium can be described by the discrete linear process

$$\xi(t) = \sum_{\{j|\tau_j(t)\}} \alpha(\tau_j) s(\tau_j, t), \quad t \subset T,$$

where $\{\alpha(\tau_j),\, j = \ldots, -1, 0, 1, \ldots\}$ — is a sequence of random quantities, that characterize the state of j radiators in the moment of time τ_j; $s(\tau_j, t)$ is elementary diffuse signal caused by radiated non-stochastic signal $s(t)$ and received by the receiving point in τ_j moment.

The latter expression can be represented as (1) i.e.

$$\xi(t) = \int_{-\infty}^{\infty} s(\tau, t) d\pi(\tau), \quad t \subset T,$$

where Poisson process $\pi(\tau)$ is generating one.

Impulse signals represent a wide range of data signals in measurement technology. Whereas the description of non-stochastic video and radio impulses sequence is conventional, there are no structural models for describing random signal impulses that overlap in time.

The use of linear processes allows solving the problem, notably using the representation (1) as mathematical models of random impulse signals where Poisson component is a generating process in a common case of a heterogeneous stochastically continuous random process with independent increments $\eta(\tau)$ and infinitely divisible laws of distribution, i.e. Gaussian component $\omega(\tau)$ is missed. The kernel of integral representation (1) can selectively describe:

impulse transition function of a linear channel of distribution of an explored signal from its source to the input of a receiver;

impulse transition function of a linear system during the conversion of an impulse signal;

analytic description of an elementary impulse form.

Particularly, linear process describes impulse interferences of reception caused by lightings, industrial-scale plants, ignition systems of internal-combustion engines, atmospheric interferences, and output electric signals of rectifying and gated installations at a random action.

Electric signals at vibroacoustic noises measuring received from the output of acoustic and vibration sensing elements are also described by LG-models. The type of process is defined by the conditions of vibroacoustic noise generation. While measuring the noise of mechanisms with cyclic mode of operation (electric motors, generators, and propellers) the model of the harmonizable process has more discrete physical interpretation because of harmonic components in the noise. Vibroacoustic noise of frictionless bearings conditioned by strokes of balls onto the surface of rings with roughness during rolling (impulse action) is represented as a feedback of a linear system that consists of a set of RLC tanks is described by a linear process.

Measurement errors of electric and magnetic values are fluctuating and represent cumulative effect of different factors superimposition. It allows considering them as random models with completely divisible distribution law.

Thereby, suggested spectral-time models based on the use of linear and harmonizable random processes can be used for describing a wide range of data signals and acting interferences in measuring electric and magnetic values. LG-models class enables to receive more grounded integral representations of signals and interferences as all the conditions of generating are used in there definition, as well as time and frequency responses and the characteristics of analysed signals. Received characteristics of LG-models allow analysing not only in terms of correlation theory but taking into consideration higher-order moments.

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