UDC 514

THE VOLUME OF TORUS

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A torus is a solid shape resembling a donut, formed by rotating a circle with radius r and centered at (R, 0) around the y-axis. Our goal is to determine the volume of the torus using the washer method. The equations for the inner and outer radii of the torus are as follows:

inner radius $x = R - \sqrt{r^2 - R^2}$, outer radius $x = R + \sqrt{r^2 - R^2}$.

Hence, employing the washer method, the cross-sectional area is as follows:

$$S(y) = \pi\left(\left(R + \sqrt{r^2 - R^2}\right)^2 - \left(R - \sqrt{r^2 - R^2}\right)^2\right) = 4\pi R\sqrt{r^2 - R^2}$$

Next, the lowest cross-section will happen at $y = -r$ and the highest cross-section will happen at $y = r$ and so the limits for the integral will be $r \le y \le -r$. Therefore, the integral giving the volume is as follows:

$$V = \int_{-r}^{r} 4\pi R\sqrt{r^2 - R^2}\,dy = 8\pi R\int_{0}^{r} \sqrt{r^2 - R^2}\,dy$$

Result

To solve the integral we will use the substitution:

$$y = r\sin\theta$$

by substituting into the integral we get

$$\int_{0}^{r} \sqrt{r^2 - R^2}\,dy = \int_{0}^{\pi/2} r^2\cos^2\theta d\theta = 1/4\pi r^2$$

Therefore, the volume of the torus is

$$V = 8\pi R\int_{0}^{r} \sqrt{r^2 - R^2}\,dy = 2R\pi^2 r^2$$

Refrences:

1. James Stewart (2015), Calculus, Engage Learning; 8th edition