VIVERE!
VINCERE!
CREARE!

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
National Aviation University

CONTINUING AIRCRAFT AIRWORTHINESS (ICAO Doc 9760)

SELF-STUDY METHOD GUIDE

Part II

Application of the Multi-Optional Functions Entropy Doctrine to Assess the Aircraft Maintenance Process Improvements for the Students of the Field of Study 27 "Transport", Specialty 272 "Aviation Transport", Specialization 01 "Aircraft and Aero-Engines Maintenance and Repair"

Kyiv 2018

Ministry of Education and Science of Ukraine
National Aviation University

CONTINUING AIRCRAFT AIRWORTHINESS (ICAO Doc 9760)

SELF-STUDY METHOD GUIDE

Part II

Application of the Multi-Optional Functions Entropy Doctrine to Assess the Aircraft Maintenance Process Improvements for the Students of the Field of Study 27 “Transport”, Specialty 272 “Aviation Transport”, Specialization 01 “Aircraft and Aero-Engines Maintenance and Repair”

Kyiv 2018

УДК 629.73:656.017:629.7.017(076.5)
С76

Compiler *A. V. Goncharenko*

Reviewers:
V. V. Panin, Doctor of engineering sciences, professor (State University of Infrastructure and Technology);

S. R. Ignatovich, Doctor of engineering sciences, professor (National Aviation University);

English language adviser *I. S. Kozeletska*, senior lecturer (NAU)

Approved by the Methodical and Editorial Board of the National Aviation University (Minutes № 2/18 of 19.06.2018).

Містять декілька рекомендацій для самостійної роботи щодо застосування ентропійної доктрини багатоопційних функцій для оцінки удосконалення процесу технічного обслуговування повітряного судна в рамках елементів наукового дослідження.

Для студентів 1-го курсу галузі знань 27 «Транспорт», спеціальності 272 «Авіаційний транспорт», спеціалізації 01 «Технічне обслуговування та ремонт повітряних суден і авіадвигунів».

С76 **Continuing Aircraft Airworthiness (ICAO Doc 9760)** : Self-Study Method Guide . Part II . Application of the Multi-Optional Functions Entropy Doctrine to Assess the Aircraft Maintenance Process Improvements / compiler A. V. Goncharenko. – K. : NAU, 2018. – 48 p.

The METHOD GUIDE contains a few recommendations on the Self-Study in regards with the Multi-Optional Functions Entropy Doctrine application to assess the aircraft maintenance process improvements in the framework of the scientific research elements.

Designed for the 1st year students of the Field of Study 27 "Transport", Specialty 272 "Aviation Transport", Specialization 01 "Maintenance and Repair of Aircraft and Aircraft Engines".

CONTENTS

INTRODUCTION

This **METHOD GUIDE ON THE SELF-STUDY** (SS) is contemplated in response to the needs of our students in more detailed elaborations concerning problems stated, set or given for the students' independent work, possibly used in their course projecting, further graduation papers or even Ph.D. studies. The whole material is split into portions. Each portion is intended to cover a fraction of probable applications aimed at **CONTINUING AIRCRAFT AIRWORTHINESS** or its retaining measures.

The presented in the, second, **PART II** of the **METHOD GUIDE ON THE** SS problems are dedicated, and a special attention is drawn here, to the scientific component of the SS work. Specifically, the objectives of the **PART II** are to help students cope with the challenging problems relating to the **AIRCRAFT** (A/C) technical operation in regards with the aeronautical engineering **MAINTENANCE** (M/T) optimal periodicities.

The set of the considered problems is based upon the **RECOMMENDED LITERATURE SOURCES** (the list is presented, but not limited to it). The **LIST OF LITERATURE** at the end of the **METHOD GUIDE** is basic (major) and compiled in the alphabetic order with respect to the matter of supposed (assumed) importance.

The **REFERENCES LIST** is selected, set in the order [1-144], does not pretend for completeness, but instead it is aimed at developing the students' abilities of thinking and to analyze, contemplate in the specified directory rather than their abilities to know and memorize. However, these are very significant too. Actually, in the contemporary informative boom world, the needed or required data can easily be retrieved from the internet, found in multiple references, guidance materials [1, 69, 71, 74, 111, 112, 134, 135, 137], studies [2, 66-68, 75-88, 113-116, 119-133, 136, 139-144], dictionaries [70, 110], comprehensive books [3, 72] or monographs [64, 65, 86-88, 113, 114, 117, 118, 123] etc. The **METHOD GUIDE** is designed for the 1st year students of the Field of Study: 27 "Transport", Specialty: 272 "Aviation Transport", Specialization: 01 "Maintenance and Repair of Aircraft and Aircraft Engines". It includes detailed solutions for obtaining reliability objective measures allowing assessing the improvements of the A/C functional system M/T process considered in reference [112].

AERONAUTICAL ENGINEERING MAINTENANCE OPTIMAL PERIODICITY DETERMINATION

The principal theoretical provisions can be found out in references [136, 141-143, 3, 72, 76, 79, 82, 84, 87, 110, 114, 116, 122, 124-133, 139, 140].

1. Determination of the periodicity via a graph construction and with the use (help) of the Laplace transformations

The presented case study centers the idea of the aeronautical engineering reliability probabilistic characteristics determination based on the related graph of the A/C given functional system functioning and the system's from state to state transitioning construction. The initial modeling is suggested (proposed) for the consideration with the use (help) of the graph represented in Fig. 1, as in the *Theory of Mass Service*, [136].

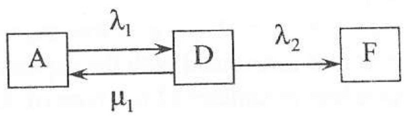

Fig. 1. Graph of three states of an aircraft functional system with a possible transition from the normal into damaged and backwards as well as from the damaged into failure state

Here, in Fig. 1, "A" designates the up state of the system; "D" – damage; "F" – failure. Accepting, for instance, the law of the damages appearance times distribution, as well as of the damages restoration times to the up state and the damages development times to the failure happening, as the exponential ones with the corresponding rates of λ_1, μ_1, and λ_2 (see Fig. 1), [143, p. 171], that is

$$f_1(t)=\lambda_1 e^{-\lambda_1 t}, \qquad f_2(t)=\lambda_2 e^{-\lambda_2 t}, \qquad f_3(t)=\mu_1 e^{-\mu_1 t}, \qquad (1)$$

where t – time, there is a possibility to write down the corresponding, to the graph of Fig. 1, system of the differential equations by *Erlang* in the following view of the three ordinary linear equations of the first order of (2):

$$\left.\begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A + \mu_1 P_D; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - (\lambda_2 + \mu_1) P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D, \end{aligned}\right\} \tag{2}$$

where P_A, P_D, and P_F – probabilities of the corresponding states (Fig. 1).

The *Theory of Mass Service* [136, 82] provisions imply that the flow of events occurring in the system, which graph of states and transitions is described with the graph portrayed in Fig. 1, obeys the *Poisson Law*.

It means that for the *Poisson Flow* the number of the events getting into any time interval is distributed in accordance with the *Poisson Law*: [136, § 1.4, pp. 17, 18, (1.4.2)].

And for the *Simplest* (*Stationary Poisson*) *Flow* the time interval between any two neighboring events is distributed with the exponential law in the kind of (1) with the corresponding parameter of the type of λ or μ: [136, § 1.4, p. 19, (1.4.7)].

It results in a *Markovian Random Process* with the discrete states and continuous time going on in the corresponding *Poisson System* (see the graph of such system shown in Fig. 1): [136, § 2.2, pp. 51-53].

Such kinds of processes and systems are pretty (quite, relatively, fairly, rather) often (or at least not seldom) encountered in practical.

One of the methods of the system of Eq. (2) solution is presented with the *Laplace transformations* in the *operational calculus* [140, Chapter XIX, pp. 400-432].

The system of Eq. (2) is transformed with [140, Chapter XIX, § 1, p. 401, (4)]:

$$F(p) = \int_0^{+\infty} e^{-pt} f(t) dt, \tag{3}$$

where p – complex parameter (variable).

The function $F(p)$ is called the *Laplace transformant* (*image*) of the function $f(t)$, which is called the *initial function*, or *original*. The indication is [140, Chapter XIX, § 1, pp. 401, 402, (7)]:

$$L\{f(t)\} = F(p). \tag{4}$$

In accordance with the theorem for transformants of derivatives [140, Chapter XIX, § 8, p. 409, (27)], the system of Eq. (2) will have the corresponding algebraic system

$$\left.\begin{aligned} &\left(pF_A(p) - \left[P_A|_{t_0=0} = 1\right] = L\left\{\frac{dP_A}{dt}\right\}\right) = \\ &= -\lambda_1 [F_A(p) = L\{P_A\}] + \mu_1 [F_D(p) = L\{P_D\}]; \\ &\left(pF_D(p) - \left[P_D|_{t_0=0} = 0\right] = L\left\{\frac{dP_D}{dt}\right\}\right) = \\ &= \lambda_1 [F_A(p) = L\{P_A\}] - (\lambda_2 + \mu_1)[F_D(p) = L\{P_D\}]; \\ &\left(pF_F(p) - \left[P_F|_{t_0=0} = 0\right] = L\left\{\frac{dP_F}{dt}\right\}\right) = \\ &= \lambda_2 [F_D(p) = L\{P_D\}]. \end{aligned}\right\}$$

$$\left.\begin{aligned} pF_A(p) - 1 &= -\lambda_1 F_A(p) + \mu_1 F_D(p); \\ pF_D(p) - 0 &= \lambda_1 F_A(p) - (\lambda_2 + \mu_1) F_D(p); \\ pF_F(p) - 0 &= \lambda_2 F_D(p). \end{aligned}\right\}. \tag{5}$$

The obtained algebraic equations system, Eq. (5), solving is possible in different ways.

One of them is a matrix-vector.

Let us rewrite the system of Eq. (5) in the following style

$$\left.\begin{matrix} (p+\lambda_1)F_A & -\mu_1 F_D & +0 = 1; \\ -\lambda_1 F_A & +(p+\lambda_2+\mu_1)F_D & +0 = 0; \\ 0 & -\lambda_2 F_D & +pF_F = 0. \end{matrix}\right\} \tag{6}$$

The matrix for the transformation of the system of Eq. (6) will be [140, Chapter XXI, § 1, p. 510, (5)]:

$$\mathbf{M}=\begin{Vmatrix} p+\lambda_1 & -\mu_1 & 0 \\ -\lambda_1 & p+\lambda_2+\mu_1 & 0 \\ 0 & -\lambda_2 & p \end{Vmatrix}. \tag{7}$$

The needed (unknown/wanted/sought) vector-column of transformants is

$$\mathbf{F}=\begin{Vmatrix} F_A \\ F_D \\ F_F \end{Vmatrix}. \tag{8}$$

Then the transformation of the system of Eq. (6) is [140, Chapter XXI, § 8, p. 522, (5)]:

$$\begin{Vmatrix} p+\lambda_1 & -\mu_1 & 0 \\ -\lambda_1 & p+\lambda_2+\mu_1 & 0 \\ 0 & -\lambda_2 & p \end{Vmatrix}\cdot\begin{Vmatrix} F_A \\ F_D \\ F_F \end{Vmatrix}=\begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix}. \tag{9}$$

Or, to make it shorter it is [140, Chapter XXI, § 8, p. 523, (6)]:

$$\mathbf{M}\cdot\mathbf{F}=\mathbf{B}. \tag{10}$$

where $\mathbf{B}$ – vector-column of the free members of the system of Eq. (6):

$$\mathbf{B}=\begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix}. \tag{11}$$

The required solution of Eq. (8) will be [140, Chapter XXI, § 9, p. 523, (2)] found with the use of the inverse matrix $\mathbf{M}^{-1}$:

$$\mathbf{F}=\mathbf{M}^{-1}\cdot\mathbf{B}. \tag{12}$$

The last equation (12) with taking into account [140, Chapter XXI, § 7, p. 521, (5)]:

$$\mathbf{M}^{-1}=\frac{1}{\Delta(\mathbf{M})}\cdot\widetilde{\mathbf{M}}, \tag{13}$$

where, $\Delta(\mathbf{M})$ – determinant of matrix $\mathbf{M}$, Eq. (7), [140, Chapter XXI, § 7, p. 520, (2)]; $\widetilde{\mathbf{M}}$ – adjacent matrix to matrix $\mathbf{M}$, [140, Chapter XXI, § 7, p. 521, (4)]; can be written as [140, Chapter XXI, § 9, p. 523, (3)]:

$$\mathbf{F}=\frac{1}{\Delta(\mathbf{M})}\cdot\widetilde{\mathbf{M}}\cdot\mathbf{B}, \tag{14}$$

or, in the developed view [140, Chapter XXI, § 9, p. 523, (4)]:

$$\begin{Vmatrix} F_A \\ F_D \\ F_F \end{Vmatrix}=\frac{1}{\Delta(\mathbf{M})}\cdot\begin{Vmatrix} M_{11} & M_{21} & M_{31} \\ M_{12} & M_{22} & M_{32} \\ M_{13} & M_{23} & M_{33} \end{Vmatrix}\cdot\begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix}, \tag{15}$$

where M_{ij} – algebraic addition of the element of m_{ij}, [140, Chapter XXI, § 2, p. 512], of the initial matrix Eq. (7).

Fulfilling multiplying of the matrixes in the right hand part of Eq. (15) we will obtain [140, Chapter XXI, § 9, p. 523, (5)]:

$$\begin{Vmatrix} F_A \\ F_D \\ F_F \end{Vmatrix}=\frac{1}{\Delta(\mathbf{M})}\cdot\begin{Vmatrix} M_{11}\cdot 1+M_{21}\cdot 0+M_{31}\cdot 0 \\ M_{12}\cdot 1+M_{22}\cdot 0+M_{32}\cdot 0 \\ M_{13}\cdot 1+M_{23}\cdot 0+M_{33}\cdot 0 \end{Vmatrix}=\frac{1}{\Delta(\mathbf{M})}\cdot\begin{Vmatrix} M_{11} \\ M_{12} \\ M_{13} \end{Vmatrix}. \tag{16}$$

In accordance with the initial matrix Eq. (7)

$$M_{11}=(-1)^{1+1}\cdot[(p+\lambda_2+\mu_1)p+\lambda_2\cdot 0]=(p+\lambda_2+\mu_1)p. \tag{17}$$

$$M_{12}=(-1)^{1+2}\cdot[(-\lambda_1 p)-0\cdot 0]=\lambda_1 p. \tag{18}$$

$$M_{13}=(-1)^{1+3}\cdot[\lambda_1\lambda_2-0\cdot(p+\lambda_2+\mu_1)]=\lambda_1\lambda_2. \tag{19}$$

$$\Delta=\Delta(\mathbf{M})=(p+\lambda_1)(p+\lambda_2+\mu_1)p+(-\lambda_1)(-\lambda_2)\cdot 0+(-\mu_1)\cdot 0\cdot 0-$$
$$-(p+\lambda_2+\mu_1)\cdot 0\cdot 0-(-\mu_1)(-\lambda_1)p-(p+\lambda_1)(-\lambda_2)\cdot 0. \tag{20}$$

Finally

$$\Delta=(p+\lambda_1)(p+\lambda_2+\mu_1)p-\lambda_1\mu_1 p=[(p+\lambda_1)(p+\lambda_2+\mu_1)-\lambda_1\mu_1]p. \tag{21}$$

It is comparable with the determinant of the characteristic equation adequate to the determinant of equation (7), in accordance with [140, Chapter XIII, § 30, pp. 108-113], the characteristic equation for system (2) will be similarly (likewise) [140, Chapter XIII, § 30, p. 109, (5)].

Now, applying the matrix-vector approach of Eq. (6)-(21) it yields for transformants

$$F_A=\frac{(p+\lambda_2+\mu_1)p}{[(p+\lambda_1)(p+\lambda_2+\mu_1)-\lambda_1\mu_1]p}=\frac{p+\lambda_2+\mu_1}{(p+\lambda_1)(p+\lambda_2+\mu_1)-\lambda_1\mu_1}. \tag{22}$$

$$F_A=\frac{p+\lambda_2+\mu_1}{p^2+\lambda_2 p+\mu_1 p+\lambda_1 p+\lambda_1\lambda_2+\lambda_1\mu_1-\lambda_1\mu_1}. \tag{23}$$

$$F_A = \frac{p+\lambda_2+\mu_1}{p^2+(\lambda_1+\lambda_2+\mu_1)p+\lambda_1\lambda_2} = \frac{p+\lambda_2+\mu_1}{(p-k_1)(p-k_2)}, \quad (24)$$

where roots k_1 and k_2 of the denominator of Eq. (24) (compare with the determinant of the characteristic equation adequate to the determinant of equation (7), in accordance with [140, Chapter XIII, § 30, pp. 108-113], the characteristic equation for system (2) will be similarly (likewise) [140, Chapter XIII, § 30, p. 109, (5)]) are

$$k_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}; \qquad k_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}; \quad (25)$$

where $a=1$, $b=\lambda_1+\lambda_2+\mu_1$, $c=\lambda_1\lambda_2$ – corresponding coefficients of the denominator of Eq. (24).

Other transformant

$$F_D = \frac{\lambda_1 p}{[(p+\lambda_1)(p+\lambda_2+\mu_1)-\lambda_1\mu_1]p} = \frac{\lambda_1}{(p-k_1)(p-k_2)}. \quad (26)$$

The third transformant

$$F_F = \frac{\lambda_1\lambda_2}{[(p+\lambda_1)(p+\lambda_2+\mu_1)-\lambda_1\mu_1]p} = \frac{\lambda_1\lambda_2}{(p-k_1)(p-k_2)p}. \quad (27)$$

The other way of getting solutions for transformants from the system of algebraic Eq. (5) is as follows.

From the second equation of the system of Eq. (6) we have

$$-\lambda_1 F_A + (p+\lambda_2+\mu_1)F_D = 0; \qquad F_A = \frac{p+\lambda_2+\mu_1}{\lambda_1}F_D. \quad (28)$$

Substituting Eq. (28) for its value into the first Eq. (6) we get

$$(p+\lambda_1)F_A - \mu_1 F_D = 1; \qquad (p+\lambda_1)\frac{p+\lambda_2+\mu_1}{\lambda_1}F_D - \mu_1 F_D = 1. \quad (29)$$

Solving Eq. (29) for F_D we obtain

$$F_D\left[\frac{(p+\lambda_1)(p+\lambda_2+\mu_1)}{\lambda_1} - \mu_1\right] = 1;$$

$$F_D\frac{(p+\lambda_1)(p+\lambda_2+\mu_1)-\lambda_1\mu_1}{\lambda_1} = 1. \quad (30)$$

Using the expressions for the denominators of Eq. (22)-(27) the result yields

$$F_D\frac{(p-k_1)(p-k_2)}{\lambda_1} = 1; \qquad F_D = \frac{\lambda_1}{(p-k_1)(p-k_2)}. \quad (31)$$

Thus, we have got the same to Eq. (26) result. Substituting it, Eq. (31), into Eq. (28) gives also the found Eq. (24):

$$F_A = \frac{p+\lambda_2+\mu_1}{\lambda_1}\cdot\frac{\lambda_1}{(p-k_1)(p-k_2)}; \qquad F_A = \frac{p+\lambda_2+\mu_1}{(p-k_1)(p-k_2)}. \quad (32)$$

At last, applying the third equation of the system of Eq. (6) and expression (31) we are having

$$-\lambda_2 F_D + pF_F = 0; \qquad -\frac{\lambda_1\lambda_2}{(p-k_1)(p-k_2)} + pF_F = 0;$$

$$F_F = \frac{\lambda_1\lambda_2}{p(p-k_1)(p-k_2)}. \quad (33)$$

Eq. (33) is also the same as Eq. (27) result. This means that we have checked all the previous derivations for their correctness (accuracy).

Now, we may look for original functions of the sought probabilities at the reference books and materials [79, p. 439], [122, pp. 234-242]. From the literature sources tables [79, p. 439] we choose

$$\frac{1}{(p+a)(p+b)} = L\left\{\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)\right\}, \quad (34)$$

here a and b – another designation of the corresponding roots of the transformants fractions (24)-(27), (31)-(33); at the reference book [79].

Or from [122, p. 235, # 12]

$$\frac{1}{(s-a)(s-b)} = L\left\{\frac{1}{a-b}\left(e^{at}-e^{bt}\right)\right\}, \quad (35)$$

here s – another designation of the complex parameter (variable) of the Laplace transformations in the operational calculus at the reference book [122]; a and b – also another designation of the corresponding roots of the transformants fractions (24)-(27), (31)-(33); at the reference book [122].

Thus, for the probability of the "normal" state, we have that value from Eq. (24) or (32), [79, p. 439] or [122, p. 235, # 13] together with Eq. (34) or (35)

$$\frac{p}{(p+a)(p+b)} = L\left\{\frac{1}{a-b}\left(ae^{-at} - be^{-bt}\right)\right\};$$

or

$$\frac{s}{(s-a)(s-b)} = L\left\{\frac{1}{a-b}\left(ae^{at} - be^{bt}\right)\right\}. \qquad (36)$$

For the "damage" – Eq. (26) or (31), we apply Eq. (34) or (35). And for "failure" – Eq. (27) or (33), we use [79, p. 439]:

$$\frac{1}{p(p+a)(p+b)} = L\left\{\frac{1}{ab(a-b)}\left[(a-b) + be^{-at} - ae^{-bt}\right]\right\}; \qquad (37)$$

or [122, p. 235, # 14]:

$$\frac{1}{(s-a)(s-b)(s-c)} = L\left\{-\frac{(b-c)e^{at} + (c-a)e^{bt} + (a-b)e^{ct}}{(a-b)(b-c)(c-a)}\right\}, \qquad (38)$$

for the given case of the stated problem setting $c = 0$; or [122, p. 242, # 1.7]:

$$\frac{1}{s(s-a)(s-b)} = L\left\{Ae^{at} + Be^{bt} + k\right\}; \qquad (39)$$

where

$$A = \frac{1}{a(a-b)}; \qquad B = \frac{1}{b(b-a)}; \qquad k = \frac{1}{ab}. \qquad (40)$$

Computer simulation results are represented in Fig. 2. In Fig. 2 y0, y1, and y2 represent numerical integration of the differential equations system (2); $\lambda_1 = 5 \cdot 10^{-3}$ h–1; $\lambda_2 = 1 \cdot 10^{-3}$ h–1; $\mu_1 = 2 \cdot 10^{-3}$ h–1; at the initial conditions: $t_0 = 0$; $P_A\big|_{t=t_0} = 1$; $P_D\big|_{t=t_0} = 0$; $P_F\big|_{t=t_0} = 0$.

Symbols "0", "1", "2" depict the system states of "A", "D", and "F" respectively.

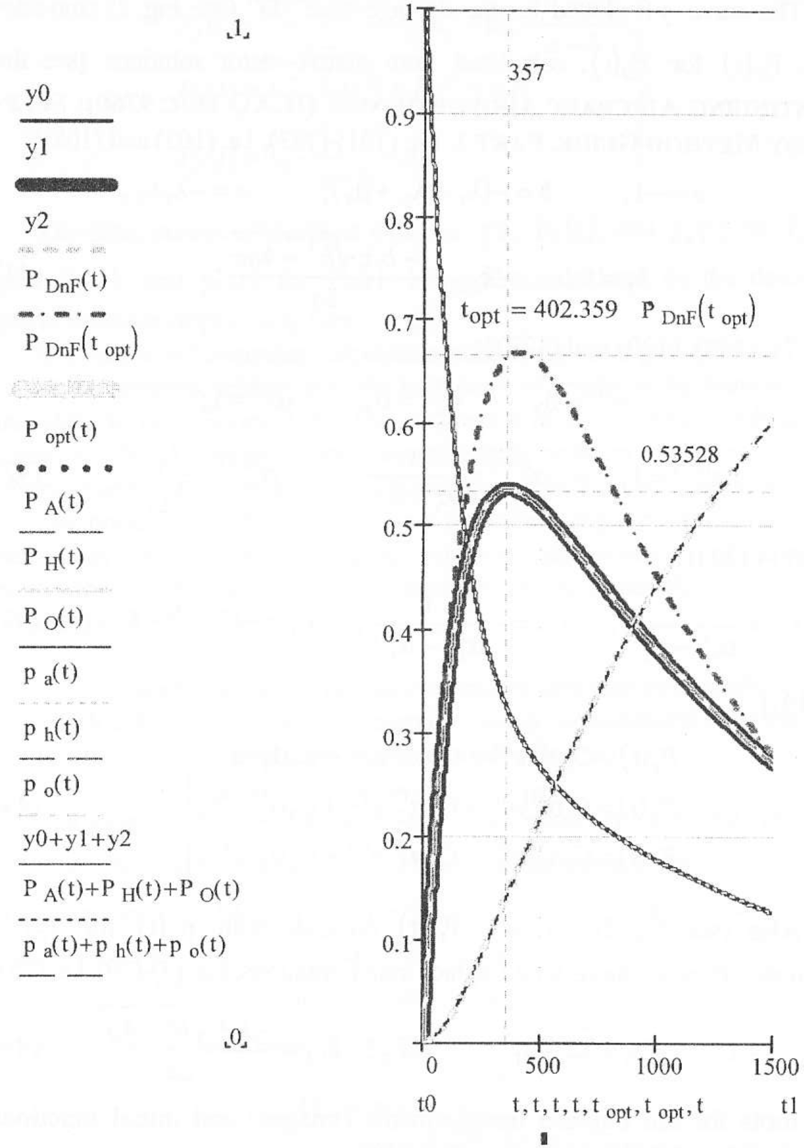

Fig. 2. Results of computer calculation experiments on the MathCad platform

The curve y1 plotted for the damage state "D" (see Fig. 2) coincides with $\mathrm{P_H}(t)$ for $P_D(t)$, calculated from matrix-vector solutions {see the **CONTINUING AIRCRAFT AIRWORTHINESS (ICAO DOC 9760): SELF-STUDY METHOD GUIDE. PART I,** Eq. (101)-(133), i.e. (105) and (108)}:

$$a=-1, \qquad b=-(\lambda_1+\lambda_2+\mu_1), \qquad c=-\lambda_1\lambda_2,$$

$$k_1=0, \qquad k_{2,3}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}; \tag{41}$$

{(117), (119), (120) and (127)}:

$$\alpha_2^{(1)}=0, \qquad \alpha_1^{(1)}=0, \qquad \alpha_3^{(1)}=1;$$

$$\alpha_2^{(2,3)}=1; \qquad \alpha_1^{(2,3)}=\frac{\mu_1}{\lambda_1+k_{2,3}}; \qquad \alpha_3^{(2,3)}=\frac{\lambda_2}{k_{2,3}}; \tag{42}$$

{(131)-(133)}:

$$C_3=\frac{1}{\alpha_1^{(3)}-\alpha_1^{(2)}}, \qquad C_2=-\frac{1}{\alpha_1^{(3)}-\alpha_1^{(2)}}, \qquad C_1=-C_2\alpha_3^{(2)}-C_3\alpha_3^{(3)}; \tag{43}$$

{(114)}:

$$\left.\begin{aligned} P_A(t)&=C_1\alpha_1^{(1)}e^{k_1t}+C_2\alpha_1^{(2)}e^{k_2t}+C_3\alpha_1^{(3)}e^{k_3t};\\ P_D(t)&=C_1\alpha_2^{(1)}e^{k_1t}+C_2\alpha_2^{(2)}e^{k_2t}+C_3\alpha_2^{(3)}e^{k_3t};\\ P_F(t)&=C_1\alpha_3^{(1)}e^{k_1t}+C_2\alpha_3^{(2)}e^{k_2t}+C_3\alpha_3^{(3)}e^{k_3t}.\end{aligned}\right\} \tag{44}$$

Also (see Fig. 2) y1 and $\mathrm{P_H}(t)$ coincide with $\mathrm{p_h}(t)$ for $P_D(t)$, calculated from solutions with Laplace transformations, Eq. (3)-(39), i.e. (25):

$$a=1, \quad b=\lambda_1+\lambda_2+\mu_1, \quad c=\lambda_1\lambda_2, \quad k_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}; \tag{45}$$

the roots for the Laplace transformants (images) and initial functions (originals) and their parameters (34)-(40):

$$a=k_1, \quad b=k_2, \quad A=\frac{1}{a(a-b)}, \quad B=\frac{1}{b(b-a)}, \quad k=\frac{1}{ab}; \tag{46}$$

$$\left.\begin{aligned} P_A(t)&=\frac{ae^{at}-be^{bt}}{a-b}+(\lambda_2+\mu_1)\frac{e^{at}-e^{bt}}{a-b};\\ P_F(t)&=\lambda_1\lambda_2\left(Ae^{at}+Be^{bt}+k\right);\\ P_D(t)&=\lambda_1\frac{e^{at}-e^{bt}}{a-b}.\end{aligned}\right\} \tag{47}$$

The other curves coincide as well, i.e. y0, $\mathrm{P_A}(t)$, and $\mathrm{p_a}(t)$ for $P_A(t)$; y2, $\mathrm{P_O}(t)$, and $\mathrm{p_o}(t)$ for $P_F(t)$; being also calculated by the described above methods respectively (see Fig. 2).

The *MathCad* computer calculation platform gives a powerful tool for a variety of problem settings with the help (use) of graphs in the framework of the *Mass Service Theory* [136]. The next period of this manual is dedicated to some (a few, several) more complicated problem versions on the improvement of the A/C given functional system M/T process modeling.

The presented methods can be used for scheduling (appointing) the M/T periodicity of aviation engineering products, the reliability of which is predetermined by the failures that develop slowly (gradually) not quickly (sharply) [143, p. 172].

2. Determination of the maintenance optimal periodicity with taking into account (consideration) economical indexes (indicators, measures, criteria)

The given criterion ensures the maximal reliability of a unit (product) work at a between-scheduled period $P(t_p)$ at the minimal value of a labor-spending for its M/T T_{TO} (for execution of scheduled works and troubleshooting (elimination/removal of failures)), [143, p. 172]. The periodicity of M/T t_p in the considered case is determined on condition of achieving the maximal value of the ratio of [143, p. 172]:

$$\Pi_{t_p}=\max\left\{\frac{P(t_p)}{T_{TO}}\right\}. \tag{48}$$

In some cases the maximal value of this ratio of (48) can be determined at subjecting (imposing, enforcing, putting into effect, implementing, applying) some constraints (limitations): minimal labor-consumption (labor capacitance) T_{TO} at the predetermined (given, specified, required) reliability $P(t_p)$ level – $P(t_p)_{req}$; maximal reliability $P(t_p)_{\max}$ at the predetermined (given, specified, required) labor-consumption (labor capacitance) T_{TO} level – $(T_{TO})_{req}$. Usually, the diagrams (graphics) of $P(t_p)$, T_{TO}, and Π_{t_p} dependencies changes on time t are plotted in the process of calculations, [143, p. 172, Fig. 15.3], and the M/T optimal periodicity is determined by them.

All the presented methods of the M/T periodicity determination are based upon the statistical data (material) of aeronautical engineering failures and damages, as well as of their runs times, accumulated at the process of operation. Besides (moreover, in addition, furthermore), one should have the data of the labor-spending for malfunctioning (troubleshooting) detection and removal, as well as for the data on the M/T carrying out (execution). Cards-orders for M/T, orders for defects detection and classification (determination), aeronautical engineering reliability level control checks, technological instructions and prescriptions for scheduled works carrying out (execution), results of expert inspections of M/T techniques (technological) processes can be used as the initial information, [143, p. 172].

The M/T periodicity of A/C certain units (products) calculated by any method is reckoned (considered, thought, supposed, deemed, guessed) to be a desired (preferred, most wanted) periodicity, since it (the determined, defined periodicity) may change at the grouping of fixed (separate, specific, exact, definite, particular) works into the scheduled forms for the A/C as an entirety, [143, p. 173].

3. Grouping works into the scheduled maintenance optimal forms

The task of multiple (numerous, many, several, various) separate (fixed, specific, exact, definite, particular) works grouping with their own carrying out (executions) periodicities into the optimal scheduled M/T forms is to determine the number of the M/T forms and point out their basic periodicities for the A/C as an entirety, [143, pp. 162-174, Chapter 15, especially Sub-Chapter 15.5, pp. 173, 174].

The works grouping by the labor-consuming forms of scheduled M/T can be fulfilled (carried out, realized, completed, performed) in different ways, [143, Sub-Chapter 15.5, pp. 173, 174].

1. The periodicity of the scheduled M/T forms is reckoned (considered, thought, supposed, deemed, guessed) to be known (it may be predetermined in technical requirements for the given A/C type). In this case, each work, completion of which influences flight safety, belongs (pertains, is relevant, relates) to the scheduled M/T form, the periodicity of which is less than the desired periodicity of that work completion and is closer to that periodicity than the all other works periodicities. The periodicity of the rest of the works is formed with taking into account (consideration) economical criteria [143, Sub-Chapter 15.5, p. 173].

2. The periodicities of the different scheduled M/T forms are not predetermined (given, prescribed) in advance. In this case, the number of the M/T forms, scope and periodicity of works for each form are prescribed (appointed, assigned, selected, chosen) on condition of the minimal labor-spending for the M/T per one flight hour assurance (providing, guaranteeing, insurance), on the condition that the given safety indexes (measures, criteria) having been ensured as well [143, Sub-Chapter 15.5, p. 173].

The essence of the method is as the following. All works in regards with the M/T process of the aeronautical engineering products are divided into two groups [143, Sub-Chapter 15.5, p. 173].

The works relating with the dangerous failures of the products prevention are included into the first group (M_1). For such works, it is considered to be known the assigned periodicity $t_{1,i}$ of each of which completion [143, Sub-Chapter 15.5, p. 173].

The works which optimal periodicity of completion $t_{2,i}$ can be assigned by the criterion of the minimum specific expenses for the A/C M/T are included into the second group (M_2) [143, Sub-Chapter 15.5, p. 173].

In calculations, each work of the groups of M_1 and M_2 is marked with the two indicators (indexes, subscripts): the first characterizing the number of the group of the works (1 or 2), the second – the ordinal number of the work $(i=1,2,\ldots,n_1$ or $i=1,2,\ldots,n_2)$ by value of its periodicity completion growth: $t_{1,1}<t_{1,2}<\ldots<t_{1,n_1}$. The value of i_1 takes all (every of the, each of the) values of the work group of M_1, i.e. (that is) $i_1 \in M_1$; and $i_2 \in M_2$. It is supposed to be known the direct costs $y_{1,i}$ for the one time completion of each of the group works M_1; and for each of the second group M_2 works – the specified (given) dependencies of the specific M/T costs upon the

periodicities of the M/T: $y_{2,i} = f(t_{2,i})$. In the process of the works formation there may happen to be a few (several) groups m of the works having the same (identical) localized periodicity t_i. For each of those groups of the works it is necessary to know the mean (average) duration B_i of the group the works completion [143, Sub-Chapter 15.5, p. 173].

Having known the periodicity of the each work carrying out as well as labor expenses for the works onetime performance (or the dependence of the specific M/T costs upon the periodicity of the M/T), the each works group completion duration, it is possible to obtain (get, find) the M/T forms for the scheduled M/T as the entire integrity. The task of the works grouping into the optimal forms of the scheduled M/T is that for changing the values of the works periodicities of $t_{1,i}$ and $t_{2,i}$ in order to find such appropriate series (sequence, chain) of the basis M/T periodicity for the A/C as the entire integrity, at which the value of the annual (yearly) M/T and operational repair (restoration, overhaul) expenditures C_{TO}, with taking into account the losses due to the M/T standing is minimized [143, Sub-Chapter 15.5, p. 174]:

$$C_{TO} = \sum_{\substack{i=1 \\ i \in M_1}}^{n_1} y_{1,i} \left(\frac{T_\text{r}}{t_{1,i}} - 1 \right) + \sum_{\substack{i=1 \\ i \in M_2}}^{n_2} y_{2,i} T_\text{r} + \sum_{\substack{i=1 \\ i \in M}}^{m} B_i \left(\frac{T_\text{r}}{t_i} - 1 \right), \quad (49)$$

where T_r – yearly (annual) flight hours per an A/C, m – number of the groups of the works having the identical (same) localized periodicity t_i, B_i – average (mean) duration of the each group of works carrying out at the A/C [143, Sub-Chapter 15.5, p. 174].

In the cases when the presented above dependencies of $y_{1,i}$, $y_{2,i}$, and B_i are unknown, there are being used some more elaborated (complex, complicated) methods of the scheduled M/T optimization, for example, the linear programming method [143, Sub-Chapter 15.5, p. 174].

The detailed solutions, in the view of Eq. (1)-(49), including those for obtaining the reliability objective measures, allow successfully assessing the improvements of the A/C functional system M/T process considered in reference [112] on the basis of [136, 140].

It may be concluded. The obtained solutions results described with the mathematical expressions of (1)-(133) create a possibility for the students to model the situations relating with the reliability objective measures allowing successfully assessing the improvements of the A/C functional system M/T

process considered in reference [112] on the basis of the methods discussed in [136, 140] in application to [1-144].

Further **PARTS** of the **METHOD GUIDE ON THE** SS problems are going to be intended for several other scientific components of the SS work on the academic subject. Background readings of the new theories [4-60, 64, 65, 89-109, 117, 118, 138] are welcome here in respect with the **CONTINUING AIRCRAFT AIRWORTHINESS (ICAO DOC 9760)** [1-144].

It is very attractive to use the calculus of variations theories [64, 65, 80, 85, 88, 95, 96, 117, 118] emphasizing the uncertainties of different kinds [4-60, 64, 65, 89-109, 117, 118, 138] for optimization of the problems stated in all references of [1-144].

MODELLING ON THE BASIS OF THE MULTI-OPTIONAL UNCERTAINTY CONDITIONAL OPTIMIZATION DOCTRINE

The principal theoretical provisions can be found out in references [61-65, 117, 118, 9, 12, 13, 31, 38, 44-46, 79, 122, 136, 139, 140, 143].

A modelling in regards to aeronautical engineering maintenance optimal periodicity determination can be performed with the use of the *Multi-Optional Hybrid Effectiveness Functions Entropy Conditional Optimization Doctrine*, [9, 12, 13, 31, 38, 44-46].

The optimal values of aeronautical engineering maintenance periodicities illustrated in Fig. 2 can be obtained not only in the entire probabilistic way, but also in a hybrid optional way, [9, 12, 13, 31, 38, 44-46].

The essence of the method is to consider the process developing in the system from the position of some hybrid optional functions distribution optimality with making allowance for the hybrid multi-optional functions distribution uncertainty measured by the entropy of the hybrid multi-optional effectiveness functions, [9, 12, 13, 31, 38, 44-46].

1. The simplest case of the optimal maintenance periodicity determination

First, let us consider a graph which corresponds to the simplest problem. It is represented in Fig. 3.

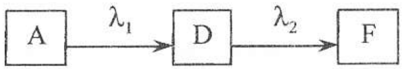

Fig. 3. Graph of three states of an aircraft functional system

Here, in Fig. 3, there are used the designations coinciding with those ones of the Fig. 1.

Starting with the simplest case, mathematically described with the expressions similar to the procedures of Eq. (1)-(47), as well as discussed above and represented by the graph for the three states of an aircraft functional system shown in Fig. 1, we might consider the corresponding intensities of λ_1 and λ_2, for the given problem setting (see Fig. 3), as some

certain parameters of the multi-"optionality" (options). Therefore, we may use the apparatus of the *Multi-Optional Hybrid Effectiveness Functions Entropy Conditional Optimization Doctrine* [9, 12, 13, 31, 38, 44-46], which is a kind of an evolution of the *Subjective Preferences Functions* (*Subjective Analysis, Subjective Entropy Paradigm, Subjective Entropy Maximum Principle*) *Theory* [64, 65, 117, 118] developed for solving the problems of the multi-alternativeness of the individual human being choice in different situations with a given set of attainable (achievable, reachable) alternatives; the *Theory* [64, 65, 117, 118] emerged (come out, appeared, materialized, originated) from the well-known in theoretical statistical physics *Jaynes' Entropy Maximum Principle* [61-63]. According with that *Doctrine* [9, 12, 13, 31, 38, 44-46] the system states options are "A" – up state of the system, "D" – damaged state, and "F" – failure.

Suppose there are certain functions expressing some kind of the system functioning effectiveness and related to the options. The objectively existing optimality has to be reflected somehow in the connected functions distribution, thus, having attributes of the optimality on their own. In order to find optimal functions, the methods of *Calculus of Variations* [80, 85, 88] are widely used and it is generally accepted.

Let us try an attempt to find the extremals for the presented research. The considered options are logically to be chosen as the three possible states, that is the number of the options equals three. However, there is a state without an exit, which means the problem becomes a two-optional one, i.e. $i=2$. The optional effectiveness functions F_i pertaining with the options are $F_i(\lambda_i)$ – relevant to the intensities of the transitions λ_1 and λ_2. The hybrid functions $h_i[F_i(\lambda_i)]$ are related with effectiveness functions of the options, depending upon an uncertainty of the situation.

Assuming the uncertainty is measured with the entropy of the optional hybrid functions, we synthesize (construct, compile) the purpose (objective) functional for the three hypothetically possible options in the postulated in the *Subjective Analysis Theory* [64, 65, 117, 118] way [9, 12, 13, 31, 38, 44-46]:

$$\Phi_h = -\sum_{i=1}^{3} h_i[F_i(\cdot)] \ln h_i[F_i(\cdot)] + \beta \sum_{i=1}^{3} h_i[F_i(\cdot)] F_i(\lambda_i) + \gamma \left\{ \sum_{i=1}^{3} h_i[F_i(\cdot)] - 1 \right\}, \quad (50)$$

where

$$-\sum_{i=1}^{3} h_i[F_i(\cdot)]\ln h_i[F_i(\cdot)] = H_h \quad (51)$$

– entropy of the optional (options) hybrid functions; β – system's optimization internal parameter; γ – normalizing function (coefficient);

$$\left\{\sum_{i=1}^{3} h_i[F_i(\cdot)] - 1\right\} \quad (52)$$

– normalizing condition.

The sign "+" in front of $\beta > 0$ means that higher values are prescribed to the hybrid optional functions connected with the higher values of the options' effectiveness functions. The sign "–" in front of $\beta > 0$ means vice versa that higher values are attached to the hybrid optional functions related with the lower values of the options' effectiveness functions.

Once again, in the case pictured in Fig. 3 the synthesized view functional of Eq. (50) drops out one of the three options' i.e. the third optional effectiveness function F_3, pertaining with the option: $F_3(\lambda_3)$, as it is not relevant to the intensities of the transitions λ_1 and λ_2. The hybrid functions $h_{1,2}[F_{1,2}(\lambda_{1,2})]$ are related with effectiveness functions of the options $F_{1,2}(\lambda_{1,2})$, depending upon an uncertainty of the situation.

The options of the situation may be considered as related to, connected with, the "A" and "D" states except for the "F" state since it has no exit. However, for some other considerations the other two states of the three might be essential. Anyway, in the stated problem setting, there are just the two optional parameters: the intensities of the transitions λ_1 and λ_2, the two optional effectiveness function F_i pertaining with the options numbered, $i \in \overline{1,2}$: $F_{1,2}(\lambda_{1,2})$, and the two hybrid functions $h_{1,2}[F_{1,2}(\lambda_{1,2})]$ related with effectiveness functions of the options $F_{1,2}(\lambda_{1,2})$, depending upon an uncertainty of the situation.

Thus, for the system with no opportunity of going out of, let us say, the third state, we must synthesize the two option functional:

$$\Phi_h = -\sum_{i=1}^{2} h_i[F_i(\cdot)]\ln h_i[F_i(\cdot)] + \beta\sum_{i=1}^{2} h_i[F_i(\cdot)]F_i(\lambda_i) + \gamma\left\{\sum_{i=1}^{2} h_i[F_i(\cdot)] - 1\right\}. \quad (53)$$

Now, for the functional of expression (53) to undergo an extremum with respect to the hybrid functions $h_i[F_i(\lambda_i)]$ the necessary conditions are as follows:

$$\frac{\partial \Phi_h}{\partial h_i} = 0. \quad (54)$$

In case that there must be an optimal time between M/T

$$t_p^* = \beta \quad (55)$$

– the periodicity of the scheduled M/T works performance, it gives

$$\Phi_h = -\sum_{i=1}^{2} h_i(\cdot)\ln h_i(\cdot) + t_p^*\sum_{i=1}^{2} h_i(\cdot)F_i(\cdot) + \gamma\left\{\sum_{i=1}^{2} h_i(\cdot) - 1\right\}. \quad (56)$$

And

$$\frac{\partial \Phi_h}{\partial h_i} = -\ln h_i(\cdot) - 1 + t_p^* F_i(\cdot) + \gamma = 0, \qquad \forall i \in \overline{1,2}. \quad (57)$$

Then

$$\ln h_1(\cdot) - t_p^* F_1(\cdot) = \gamma - 1 = \ln h_2(\cdot) - t_p^* F_2(\cdot). \quad (58)$$

From where

$$\ln h_1(\cdot) - t_p^* F_1(\cdot) = \ln h_2(\cdot) - t_p^* F_2(\cdot). \quad (59)$$

After that we have got the *Law of Subjective Conservatism* [55] on one hand and on the other hand

$$\ln h_1(\cdot) - \ln h_2(\cdot) = t_p^*[F_1(\cdot) - F_2(\cdot)]. \quad (60)$$

At last

$$t_p^* = \frac{\ln h_1(\cdot) - \ln h_2(\cdot)}{F_1(\cdot) - F_2(\cdot)}. \quad (61)$$

In case the options related effectiveness function is expressed explicitly with respect to λ_i in the way of

$$F_i(\lambda_i) = \lambda_i, \quad \text{and} \quad h_i[F_i(\lambda_i)] = h_i(\lambda_i) = x\lambda_i, \quad (62)$$

where x – unknown, uncertain multiplier in type of the Lagrange one, we obtain with the help of the procedure considered through (50)-(62) the needed optimal periodicity in the sense of the probability $P_D(t)$ maximum [143, p. 171].

Indeed substituting Eq. (62) into Eq. (61)

$$t_p^* = \frac{\ln \frac{x\lambda_1}{x\lambda_2}}{\lambda_1 - \lambda_2} = \frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}. \tag{63}$$

The sense of the uncertain multiplier x becomes obvious with the use of the normalizing condition (likewise Eq. (52)) of the initial functional Eq. (56). That is

$$\sum_{i=1}^{2} h_i[F_i(\lambda_i)] = 1. \tag{64}$$

With respect to Eq. (62)

$$x\lambda_1 + x\lambda_2 = 1 \quad \Rightarrow \quad x = \frac{1}{\lambda_1 + \lambda_2}. \tag{65}$$

On objective functional, formulae (53), (55), and (56), extremum existence conditions of Eq. (54), (57)

$$\ln h_i(\cdot) = \gamma - 1 + t_p^* F_i(\cdot), \qquad \forall i \in \overline{1,2}. \tag{66}$$

Hence

$$h_i(\cdot) = \exp[\gamma - 1 + t_p^* F_i(\cdot)] = \exp[\gamma - 1]\exp[t_p^* F_i(\cdot)]. \tag{67}$$

Because of the normalizing conditions likewise equations of (52) or (64)

$$\sum_{j=1}^{2} h_j[F_j(\lambda_j)] = 1 = \sum_{j=1}^{2} \exp[\gamma - 1]\exp[t_p^* F_j(\cdot)] = \exp[\gamma - 1]\sum_{j=1}^{2} \exp[t_p^* F_j(\cdot)]. \tag{68}$$

Therefore

$$\exp[\gamma - 1] = \frac{1}{\sum_{i=1}^{2} \exp[t_p^* F_i(\cdot)]}. \tag{69}$$

This yields for the optimal distribution of the hybrid-optional functions

$$h_i^{(\text{opt})}[F_i(\lambda_i)] = \frac{\exp[t_p^* F_i(\cdot)]}{\sum_{j=1}^{2} \exp[t_p^* F_j(\cdot)]}. \tag{70}$$

Thus, finally we obtained the known in Subjective Analysis the Canonical Distribution of the Subjective Preferences, [64, 65, 117, 118].

However in this work, we interpret it, Eq. (70), as the optional hybrid functions distribution [9, 12, 13, 31, 38, 44-46] since we do not consider any active elements or subjects (persons, individuals, or human beings) in the system. Instead we deal with the objectively existing optimal quality of the

system, corresponding with the system intrinsic nature, rather than subjectively preferred (although might be also essential, indispensable) matter.

Because of the assumptions expressed with the equations of (50), (53), (55), (56), and (62), as well as due to the derivations presented with the procedures following Eq. (50)-(65)

$$h_1[\cdot] = \frac{\lambda_1}{\lambda_1 + \lambda_2}, \qquad h_2[\cdot] = \frac{\lambda_2}{\lambda_1 + \lambda_2}. \tag{71}$$

Because of the same assumptions, as well as due to the derivations presented with the procedures following Eq. (50)-(57) and (66)-(70)

$$h_1^{(\text{opt})}[\cdot] = \frac{\exp[t\lambda_1]}{\exp[t\lambda_1] + \exp[t\lambda_2]}, \qquad h_2^{(\text{opt})}[\cdot] = \frac{\exp[t\lambda_2]}{\exp[t\lambda_1] + \exp[t\lambda_2]}. \tag{72}$$

Furthermore, accordingly with [64, 65, 117, 118], for functional of Eq. (56), t – time is the independent variable for now, although.

And in any case whether t_p^* Eq. (63) is obtained on conditions of Eq. (56)-(62) or from the related probability $P_D(t)$ extremization

$$h_1^{(\text{opt})}(t_p^*) = \frac{\exp[t_p^*\lambda_1]}{\exp[t_p^*\lambda_1] + \exp[t_p^*\lambda_2]}, \qquad h_2^{(\text{opt})}(t_p^*) = \frac{\exp[t_p^*\lambda_2]}{\exp[t_p^*\lambda_1] + \exp[t_p^*\lambda_2]}. \tag{73}$$

Now, there is a necessity to prove the identities of Eq. (71) to Eq. (73) for the considered problem setting.

Indeed.

Substituting the Eq. (63) value into Eq. (73)

$$h_1^{(\text{opt})}(t_p^*) = \frac{\exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}\lambda_1\right]}{\exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}\lambda_1\right] + \exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}\lambda_2\right]},$$

$$h_2^{(\text{opt})}(t_p^*) = \frac{\exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}\lambda_2\right]}{\exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}\lambda_1\right] + \exp\left[\frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}\lambda_2\right]}. \tag{74}$$

Then, for the nominator of the first Eq. (74)

$$\exp\left[\frac{\ln\lambda_1-\ln\lambda_2}{\lambda_1-\lambda_2}\lambda_1\right]=\exp\left[\ln\frac{\lambda_1}{\lambda_2}\frac{\lambda_1}{\lambda_1-\lambda_2}\right]=\left(\exp\left[\ln\frac{\lambda_1}{\lambda_2}\right]\right)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}=$$

$$=\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}. \qquad (75)$$

The first Eq. (74) turns out

$$\frac{\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}}{\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}+\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}}=\frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}\left[\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}+\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}\right]}=$$

$$=\frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}\left[\left(\frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}}\right)+\left(\frac{(\lambda_1)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}}{(\lambda_2)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}}\right)\right]}=$$

$$=\frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}\left[\frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}(\lambda_2)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}+(\lambda_2)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}(\lambda_1)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}}{(\lambda_2)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}(\lambda_2)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}}\right]}=$$

$$=\frac{(\lambda_1)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}(\lambda_2)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}}{(\lambda_1)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}(\lambda_2)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}+(\lambda_2)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}(\lambda_1)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}}=$$

$$=\frac{1}{1+\frac{(\lambda_2)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}(\lambda_1)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}}{(\lambda_1)^{\frac{\lambda_1}{\lambda_1-\lambda_2}}(\lambda_2)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}}}=\frac{1}{1+\frac{(\lambda_2)^{\frac{\lambda_1-\lambda_2}{\lambda_1-\lambda_2}}}{(\lambda_1)^{\frac{\lambda_1-\lambda_2}{\lambda_1-\lambda_2}}}}=\frac{1}{1+\frac{(\lambda_2)^1}{(\lambda_1)^1}}=\frac{\lambda_1}{\lambda_1+\lambda_2}. \qquad (76)$$

Thus, we have got the second Eq. (62) on condition of Eq. (65) for the first hybrid effectiveness multi-optional function:

$$h_1^{(\mathrm{opt})}\left(t_p^*\right)=\frac{\exp\left[t_p^*\lambda_1\right]}{\exp\left[t_p^*\lambda_1\right]+\exp\left[t_p^*\lambda_2\right]}=\frac{\lambda_1}{\lambda_1+\lambda_2}, \qquad (77)$$

as we presupposed above.

The obtained result expressed with Eq. (77) is obvious because the optimal interval for the M/T periodicity t_p^* has been found on the basis of the previously postulated optimality with taking into account entropy paradigm for the hybrid optional functions $h_i(\cdot)$; in their turn the hybrid optional functions have been modified (hybridized) with the unknown (uncertain) multiplier x (likewise implanted extra chromosome). Therefore optimal, with taking into consideration the uncertainty in the view of their entropy, hybrid optional functions distribution returns the assumed optimal hybrid optional functions values in accordance with the previously supposed view and accepted effectiveness (see Eq. (56)-(62)).

2. Probabilistic approach solution as a proof for the proposed Doctrine possible applications

The probabilistic approach solution in the stated problem setting pertaining with the graph shown in Fig. 3 is a partial case solution for the process described in the first section (see and compare Fig. 1 and Fig. 3 also analyze Eq. (1)-(47)).

For such partial solution obtaining, there is just a need of the $\mu_1=0$ acceptance.

This returns the following modifications in the Eq. (1)-(47), we distinguish just the necessary (indispensable, essential, important) ones.

Distribution densities come up without that for the restoration transition (see Eq. (1)):

$$f_1(t)=\lambda_1 e^{-\lambda_1 t}, \qquad f_2(t)=\lambda_2 e^{-\lambda_2 t}. \qquad (78)$$

The system of the differential equations by *Erlang* is modified in the way of simplification (see Eq. (2)):

$$\left.\begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - \lambda_2 P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D. \end{aligned}\right\} \quad (79)$$

Next up is the method different from the *Laplace transformations* in the *operational calculus* [140, Chapter XIX, pp. 400-432].

From the first equation of the system (79) we find

$$\frac{dP_A}{dt} = -\lambda_1 P_A; \quad \frac{dP_A}{P_A} = -\lambda_1 dt; \quad \int \frac{dP_A}{P_A} = -\int \lambda_1 dt + C, \quad (80)$$

where C – constant of integration, it will be found from the initial conditions:

$$t_0 = 0; \quad P_A|_{t=t_0} = 1; \quad P_D|_{t=t_0} = 0; \quad P_F|_{t=t_0} = 0, \quad (81)$$

Eq. (80) yields

$$\ln P_A = -\lambda_1 t + C, \quad (82)$$

from where

$$\ln 1 = -\lambda_1 \cdot 0 + C, \quad 0 = -0 + C, \quad C = 0. \quad (83)$$

Thus,

$$P_A(t) = e^{-\lambda_1 t}. \quad (84)$$

The same result as [143, pp. 170-172], which is correct because state "A" means not transition into the state of damage "D", the probability is $P_A(t) = P_{\overline{D}}(t)$, the designation for such event is in the subscript of the probability of $P_{\overline{D}}(t)$ with the negation mark: $\overline{D}$; and transition from "A" strait to the state of failure "F" is impossible: $P_{\overline{D}F}(t) = 0$, as well as any of the backward transitions. Therefore

$$P_A(t) = P_{\overline{D}}(t) = P_{\overline{D}F}(t) + P_{\overline{D}\overline{F}}(t) = 0 + e^{-\lambda_1 t} = e^{-\lambda_1 t}. \quad (85)$$

Substituting the result of Eq. (84) or (85) for the corresponding value of $P_A(t)$ in the second equation of the system (79) we get

$$\frac{dP_D}{dt} = \lambda_1 e^{-\lambda_1 t} - \lambda_2 P_D. \quad (86)$$

Thus, we have got a linear not uniform (homogeneous) differential equation of the first order. It can easily be solved with the method represented in [139, 140]. Exactly, accordingly to [140, Chapter XIII, especially § 7, pp. 30-33] the Eq. (86) has the view of [140, Chapter XIII, § 7, p. 30, (1)]

$$\frac{dy}{dx} + P(x)y = Q(x). \quad (87)$$

where $P(x)$ and $Q(x)$ – given continuous functions of x (or constants).

T h e s o l u t i o n o f t h e l i n e a r e q u a t i o n (8 7) . We will be finding the solution of the equation (87) in the view of a product of two functions of x [140, Chapter XIII, § 7, p. 30, (2)]:

$$y = u(x)v(x). \quad (88)$$

One of these functions can be taken arbitrary, the other one will be determined on the basis of Eq. (87).

Differentiating both parts of Eq. (88), we find, [140, Chapter XIII, § 7, p. 31],

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}. \quad (89)$$

Substituting the obtained expression of the derivative of $\frac{dy}{dx}$ into Eq. (87), we will have, [140, Chapter XIII, § 7, p. 31],

$$u\frac{dv}{dx} + v\frac{du}{dx} + Puv = Q, \quad (90)$$

or, [140, Chapter XIII, § 7, p. 31, (3)],

$$u\left(\frac{dv}{dx} + Pv\right) + v\frac{du}{dx} = Q. \quad (91)$$

Let us choose function v as such as, [140, Chapter XIII, § 7, p. 31, (4)],

$$\frac{dv}{dx} + Pv = 0. \quad (92)$$

Dividing variables in this differential equation with respect to function v, we find, [140, Chapter XIII, § 7, p. 31],

$$\frac{dv}{v} = -Pdx. \quad (93)$$

Integrating, we get, [140, Chapter XIII, § 7, p. 31],

$$-\ln|C_1|+\ln|v|=-\int Pdx, \quad (94)$$

where C_1 – constant of integration; or, [140, Chapter XIII, § 7, p. 31],

$$v=C_1 e^{-\int Pdx}. \quad (95)$$

Since it is enough to have some different from zero solution of Eq. (92), then we will take [140, Chapter XIII, § 7, p. 31, (5)] as the function of $v(x)$, assuming and accepting $C_1=1$:

$$v(x)=e^{-\int Pdx}, \quad (96)$$

where $\int Pdx$ – some antiderivative (counterderivative). It is obviously that $v(x)\neq 0$.

Substituting the found value of $v(x)$ into Eq. (91), [140, Chapter XIII, § 7, p. 31, (3)], we will get (taking into account that $\left(\frac{dv}{dx}+Pv=0\right)$, [140, Chapter XIII, § 7, p. 31],

$$v(x)\frac{du}{dx}=Q(x), \quad (97)$$

or, [140, Chapter XIII, § 7, p. 31],

$$\frac{du}{dx}=\frac{Q(x)}{v(x)}, \quad (98)$$

from where, [140, Chapter XIII, § 7, p. 31],

$$u=\int\frac{Q(x)}{v(x)}dx+C, \quad (99)$$

where C – constant of integration.

Substituting u and v into the formula of Eq. (88), [140, Chapter XIII, § 7, p. 30, (2)], we finally get

$$y=v(x)\left[\int\frac{Q(x)}{v(x)}dx+C\right], \quad (100)$$

or, [140, Chapter XIII, § 7, p. 32, (6)],

$$y=v(x)\int\frac{Q(x)}{v(x)}dx+Cv(x). \quad (101)$$

In the considered problem setting (see, compare, and substitute the corresponding functions, expressions, variables, and values of Eq. (86), (87), (101))

$$\frac{dP_D}{dt}+\lambda_2 P_D=\lambda_1 e^{-\lambda_1 t}, \qquad \frac{dy}{dx}+P(x)y=Q(x),$$

$$y=P_D, \quad x=t, \quad P(x)=\lambda_2, \quad Q(x)=\lambda_1 e^{-\lambda_1 t}, \quad v(x)=e^{-\int\lambda_2 dt}=e^{-\lambda_2 t}. \quad (102)$$

Thus

$$P_D=e^{-\lambda_2 t}\int\frac{\lambda_1 e^{-\lambda_1 t}}{e^{-\lambda_2 t}}dt+Ce^{-\lambda_2 t}. \quad (103)$$

Then

$$P_D=e^{-\lambda_2 t}\lambda_1\int e^{(\lambda_2-\lambda_1)t}dt+Ce^{-\lambda_2 t}=\frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2 t}\left[e^{(\lambda_2-\lambda_1)t}\right]+Ce^{-\lambda_2 t}. \quad (104)$$

From the initial conditions, expressions (81) $t_0=0$; $P_D|_{t=t_0}=0$. It yields

$$0=\frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2\cdot 0}\left[e^{(\lambda_2-\lambda_1)\cdot 0}\right]+Ce^{-\lambda_2\cdot 0}=\frac{\lambda_1}{\lambda_2-\lambda_1}\cdot 1\cdot 1+C\cdot 1. \quad (105)$$

Therefore

$$C=-\frac{\lambda_1}{\lambda_2-\lambda_1}. \quad (106)$$

Then

$$P_D=\frac{\lambda_1}{\lambda_2-\lambda_1}\left[e^{-\lambda_1 t}\right]-\frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2 t}=\frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t}-e^{-\lambda_2 t}\right). \quad (107)$$

Final result Eq. (107) is the same as [143, p. 171, (15.1)]. It is the probability of the damage state "D" of the system (see Fig. 3).

The probability of the failure state "F" will be obtained as the solution from the third equation of the system (79), i.e.

$$\frac{dP_F}{dt}=\lambda_2 P_D=\frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t}-e^{-\lambda_2 t}\right). \quad (108)$$

Integration yields

$$P_F=\frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left[\int e^{-\lambda_1 t}dt-\int e^{-\lambda_2 t}dt\right]+C, \quad (109)$$

where C – constant of integration.

Then

$$P_F = \frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left[\left(-\frac{1}{\lambda_1}\right)e^{-\lambda_1 t} + \frac{1}{\lambda_2}e^{-\lambda_2 t}\right] + C. \qquad (110)$$

From the initial conditions, expressions (81) $t_0 = 0$; $P_F\big|_{t=t_0} = 0$. It yields

$$0 = \frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left[\left(-\frac{1}{\lambda_1}\right)e^{-\lambda_1\cdot 0} + \frac{1}{\lambda_2}e^{-\lambda_2\cdot 0}\right] + C = \frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) + C =$$

$$= \frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left(\frac{\lambda_1-\lambda_2}{\lambda_2\lambda_1}\right) + C = -1 + C \quad \Rightarrow \quad C = 1. \qquad (111)$$

Therefore

$$P_F = \frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left[\frac{e^{-\lambda_2 t}}{\lambda_2} - \frac{e^{-\lambda_1 t}}{\lambda_1}\right] + 1 = \frac{\lambda_2\lambda_1}{\lambda_2-\lambda_1}\left[\frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2\lambda_1}\right] + 1 =$$

$$= \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2-\lambda_1} + 1 = \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2-\lambda_1} + 1 + e^{-\lambda_1 t} - e^{-\lambda_1 t} =$$

$$= 1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2-\lambda_1} + e^{-\lambda_1 t} =$$

$$= 1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_1 t} - \lambda_1 e^{-\lambda_1 t}}{\lambda_2-\lambda_1}.$$

Finally

$$P_F = 1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_1 e^{-\lambda_1 t}}{\lambda_2-\lambda_1} = 1 - e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right). \qquad (112)$$

The result (112) is the same as [143, pp. 170-172].

Now we will return to the *Laplace transformations* in the *operational calculus* [140, Chapter XIX, pp. 400-432].

The system (2), now (79), after the *Laplace transformations* in accordance with the theorem for transformants of derivatives [140, Chapter XIX, § 8, p. 409, (27)], will get the view of the corresponding algebraic equations system (see Eq. (5) and on):

$$\left.\begin{array}{l} pF_A(p) - 1 = -\lambda_1 F_A(p); \\ pF_D(p) - 0 = \lambda_1 F_A(p) - \lambda_2 F_D(p); \\ pF_F(p) - 0 = \lambda_2 F_D(p). \end{array}\right\} \qquad (113)$$

$$\left.\begin{array}{llll} (p+\lambda_1)F_A & +0 & +0 = 1; \\ -\lambda_1 F_A & +(p+\lambda_2)F_D & +0 = 0; \\ 0 & -\lambda_2 F_D & +pF_F = 0. \end{array}\right\} \qquad (114)$$

$$\mathbf{M} = \left\|\begin{array}{ccc} p+\lambda_1 & 0 & 0 \\ -\lambda_1 & p+\lambda_2 & 0 \\ 0 & -\lambda_2 & p \end{array}\right\|, \ \mathbf{F} = \left\|\begin{array}{c} F_A \\ F_D \\ F_F \end{array}\right\|, \ \left\|\begin{array}{ccc} p+\lambda_1 & 0 & 0 \\ -\lambda_1 & p+\lambda_2 & 0 \\ 0 & -\lambda_2 & p \end{array}\right\| \cdot \left\|\begin{array}{c} F_A \\ F_D \\ F_F \end{array}\right\| = \left\|\begin{array}{c} 1 \\ 0 \\ 0 \end{array}\right\|. \qquad (115)$$

$$\mathbf{M}\cdot\mathbf{F} = \mathbf{B}, \ \mathbf{B} = \left\|\begin{array}{c} 1 \\ 0 \\ 0 \end{array}\right\|, \ \mathbf{F} = \mathbf{M}^{-1}\cdot\mathbf{B}, \ \mathbf{M}^{-1} = \frac{1}{\Delta(\mathbf{M})}\cdot\widetilde{\mathbf{M}}, \ \mathbf{F} = \frac{1}{\Delta(\mathbf{M})}\cdot\widetilde{\mathbf{M}}\cdot\mathbf{B}. \qquad (116)$$

$$\left\|\begin{array}{c} F_A \\ F_D \\ F_F \end{array}\right\| = \frac{1}{\Delta(\mathbf{M})}\cdot\left\|\begin{array}{ccc} M_{11} & M_{21} & M_{31} \\ M_{12} & M_{22} & M_{32} \\ M_{13} & M_{23} & M_{33} \end{array}\right\| \cdot \left\|\begin{array}{c} 1 \\ 0 \\ 0 \end{array}\right\|. \qquad (117)$$

$$\left\|\begin{array}{c} F_A \\ F_D \\ F_F \end{array}\right\| = \frac{1}{\Delta(\mathbf{M})}\cdot\left\|\begin{array}{c} M_{11}\cdot 1 + M_{21}\cdot 0 + M_{31}\cdot 0 \\ M_{12}\cdot 1 + M_{22}\cdot 0 + M_{32}\cdot 0 \\ M_{13}\cdot 1 + M_{23}\cdot 0 + M_{33}\cdot 0 \end{array}\right\| = \frac{1}{\Delta(\mathbf{M})}\cdot\left\|\begin{array}{c} M_{11} \\ M_{12} \\ M_{13} \end{array}\right\|. \qquad (118)$$

$$M_{11} = (-1)^{1+1}\cdot[(p+\lambda_2)p + \lambda_2\cdot 0] = (p+\lambda_2)p. \qquad (119)$$

$$M_{12} = (-1)^{1+2}\cdot[(-\lambda_1 p) - 0\cdot 0] = \lambda_1 p. \qquad (120)$$

$$M_{13} = (-1)^{1+3}\cdot[\lambda_1\lambda_2 - 0\cdot(p+\lambda_2)] = \lambda_1\lambda_2. \qquad (121)$$

$$\Delta = (p+\lambda_1)(p+\lambda_2)p. \qquad (122)$$

$$k_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}, \quad a = 1, \quad b = \lambda_1+\lambda_2, \quad c = \lambda_1\lambda_2. \qquad (123)$$

$$k_{1,2} = \frac{-(\lambda_1+\lambda_2) \pm \sqrt{(\lambda_1+\lambda_2)^2 - 4\lambda_1\lambda_2}}{2}. \qquad (124)$$

$$k_{1,2} = \frac{-(\lambda_1+\lambda_2) \pm \sqrt{(\lambda_1)^2 + (\lambda_2)^2 - 2\lambda_1\lambda_2}}{2}. \qquad (125)$$

$$k_{1,2} = \frac{-(\lambda_1+\lambda_2) \pm \sqrt{(\lambda_1-\lambda_2)^2}}{2}, \qquad k_{1,2} = \frac{-(\lambda_1+\lambda_2) \pm (\lambda_1-\lambda_2)}{2}.$$

(126)

$$k_1 = -\lambda_2, \qquad k_2 = -\lambda_1. \tag{127}$$

$$F_D = \frac{\lambda_1 p}{[(p+\lambda_1)(p+\lambda_2)]p} = \frac{\lambda_1}{(p-k_1)(p-k_2)}. \tag{128}$$

$$\frac{1}{(s-a)(s-b)} = L\left\{\frac{1}{a-b}\left(e^{at} - e^{bt}\right)\right\}, \qquad a = k_1 = -\lambda_2, \qquad b = k_2 = -\lambda_1. \tag{129}$$

$$P_D = \frac{\lambda_1}{\lambda_1 - \lambda_2}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right). \tag{130}$$

The result of Eq. (130) is the same as of Eq. (107).

And for the $P_D(t)$ Eq. (130) extremum existence

$$\frac{dP_D(t)}{dt} = \frac{\lambda_1}{\lambda_1 - \lambda_2}\left(\lambda_1 e^{-\lambda_1 t} - \lambda_2 e^{-\lambda_2 t}\right) = 0. \tag{131}$$

$$\lambda_1 e^{-\lambda_1 t} - \lambda_2 e^{-\lambda_2 t} = 0, \qquad \lambda_1 e^{-\lambda_1 t} = \lambda_2 e^{-\lambda_2 t}, \qquad \frac{\lambda_1}{\lambda_2} = e^{(\lambda_1 - \lambda_2)t}. \tag{132}$$

$$\ln\frac{\lambda_1}{\lambda_2} = (\lambda_1 - \lambda_2)t, \qquad t_{\text{opt}} = \frac{\ln\lambda_1 - \ln\lambda_2}{\lambda_1 - \lambda_2}. \tag{133}$$

The result of Eq. (133) is the same as of Eq. (63). However Eq. (63) is obtained in the *Multi-Optional Hybrid Effectiveness Functions Entropy Conditional Optimization Doctrine* [9, 12, 13, 31, 38, 44-46] way Eq. (50)-(63) rather than in the demonstrated herewith this sub-chapter entirely probabilistic method.

Thus, the identity is proven and the optimum is visible in Fig. 2.

References

1. **Airworthiness** Manual: ICAO Doc 9760, 3rd Edition. – Montreal, Quebec, Canada: ICAO, 2014. – 281 p.
2. **Continuing** aircraft airworthiness: manual / S. Dmitriev, O. Tuharinov, Yu. Kazarinov, O. Rugain. – Kyiv, Ukraine: Publishing House of the National Aviation University, 2015. – 104 p.
3. **Dhillon B. S.** Maintainability, maintenance, and reliability for engineers / B. S. Dhillon. – New York: Taylor & Francis Group, 2006. – 214 p.
4. **Goncharenko A.** Aircraft operation depending upon the uncertainty of maintenance alternatives / A. Goncharenko // Aviation. – 2017. Vol. 21(4). – pp. 126-131.
5. **Goncharenko A. V.** A concept of ballast water treatment on the basis of multi-alternativeness / A. V. Goncharenko, V. A. Evdokimova // Сучасні інформаційні та інноваційні технології на транспорті (MINTT-2014) [Збірка матеріалів VI Міжнародної науково-практичної конференції. (27-29 травня 2014 р., Херсон)]. – Херсон: Херсонська державна морська академія, 2014. – pp. 18-21.
6. **Goncharenko A. V.** A concept of entropy approach to the problem of multi-alternative operational modes control / A. V. Goncharenko // Науковий вісник ХДМА. – 2013. – № 2(9). – pp. 26-34.
7. **Goncharenko A. V.** A concept of multi-optional optimality at modeling ideal gas isothermal processes / A. V. Goncharenko // Electronics and control systems. – 2017. – № 2(52). – pp. 94-97.
8. **Goncharenko A. V.** A diagnostics problem of a-posterior probability determination via Bayes' formula obtained in the multi-optional hybrid functions entropy conditional optimization way / A. V. Goncharenko // Problems of friction and wear. – 2017. – № 4(77). – pp. 95-99.
9. **Goncharenko A. V.** A hybrid approach to the optimal aeronautical engineering maintenance periodicity determination / A. V. Goncharenko // Proceedings of the NAU. – 2017. – № 3(72). – pp. 42-47.
10. **Goncharenko A. V.** A hybrid pseudo-entropy function for a decision making in conditions of uncertainty / A. V. Goncharenko // Інтелектуальні системи прийняття рішень та проблеми обчислювального інтелекту: міжнародна наукова конференція, Залізний Порт, Україна, 25-28 травня 2015 р.: матеріали конф. – Херсон: Видавництво ХНТУ, 2015. – pp. 174-176. (ISBN 978-966-2207-24-8)
11. **Goncharenko A. V.** A particular case of a variational problem of control in an active aviation system / A. V. Goncharenko // Transactions of the institute of aviation. – 2013. – № 228, pp. 3-12.
12. **Goncharenko A. V.** Aeronautical and aerospace materials and structures damages to failures: theoretical concepts / A. V. Goncharenko //

International Journal of Aerospace Engineering. – Volume 2018 (2018), Article ID 4126085, 7 pages https://doi.org/10.1155/2018/4126085; 2018. – pp. 1-7.

13. **Goncharenko A. V.** Aeronautical engineering maintenance periodicity optimization with the help of subjective preferences distributions / A. V. Goncharenko // Proceedings of the NAU. – 2017. – № 2(71). – pp. 51-56.

14. **Goncharenko A. V.** Aircraft maximal distance horizontal flights in the conceptual framework of subjective analysis / A. V. Goncharenko // Proceedings of the NAU. – 2013. – № 4(57). – pp. 56-62.

15. **Goncharenko A. V.** Alternativeness of control and power equipment repair versus purchasing according to the preferences of the options / A. V. Goncharenko // Electronics and control systems. – 2016. – № 4(50). – pp. 98-101.

16. **Goncharenko A. V.** An alternative method of the main psychophysics law derivation / A. V. Goncharenko // Clin. and Exp. Psychol. – 2017. – 3: 155. – pp. 1-5. doi: 10.4172/2471-2701.1000155. (ISSN: 2471-2701)

17. **Goncharenko A. V.** An example of an alternative method of the normal distribution density derivation via a concept of a multi-optional optimality / A. V. Goncharenko // Electronics and control systems. – 2017. – № 3(53). – pp. 95-99. DOI: 10.18372/1990-5548.53.12149 (ISSN: 1990-5548)

18. **Goncharenko A. V.** An optional hybrid functions method of an ideal gas adiabatic process equation derivation / A. V. Goncharenko // Electronics and control systems. – 2017. – № 4(54). – pp. 109-112.

19. **Goncharenko A. V.** Applicable aspects of alternative UAV operation / A. V. Goncharenko // 2015 IEEE 3rd International Conference "Actual Problems of Unmanned Aerial Vehicles Developments (APUAVD)" Proceedings. October, 13-15, 2015, Kyiv, Ukraine. – К.: Освіта України, 2015. – pp. 316-319.

20. **Goncharenko A. V.** Artificial versus natural intellect in control of optimality / A. V. Goncharenko // Інтелектуальні системи прийняття рішень та проблеми обчислювального інтелекту: міжнародна наукова конференція, Євпаторія, 20-24 травня 2013 р.: матеріали конф. – Херсон: ХНТУ, 2013. – pp. 20-22. (ISBN 978-966-8912-70-2)

21. **Goncharenko A. V.** Bayes criterion modified with subjective preferences functions densities distributions used at the choosing of the decision making thresholds / A. V. Goncharenko // Матеріали XIII міжнародної науково-технічної конференції "АВІА-2017". (19-21 квітня 2017 р., Київ). – К.: НАУ, 2017. – pp. 17.17-17.21.

22. **Goncharenko A. V.** Concentrations formula conditional optimality with respect to their entropy / A. V. Goncharenko // Problems of friction and wear. – 2018. – № 1(78). – pp. 85-88. (ISSN 0370-2197)

23. **Goncharenko A. V.** Conflictability of operational situations in terms of entropy paradigm / A. V. Goncharenko // Сучасні інформаційні та

інноваційні технології на транспорті (MINTT-2013) [Збірка матеріалів V Міжнародної науково-практичної конференції. У 2-х тт. (28-30 травня 2013 р., Херсон)]. – Т. 1. – Херсон: Херсонська державна морська академія, 2013. – pp. 115-118.

24. **Goncharenko A. V.** Control of flight safety with the use of preferences functions / A. V. Goncharenko // Electronics and control systems. – 2013. – № 3(37). – pp. 113-119. (ISSN: 1990-5548)

25. **Goncharenko A. V.** Distinguishing minimal engineering diagnosis risks via preferences functions / A. V. Goncharenko // Proceedings of The Seventh World Congress "Aviation in the XXI-st Century" "Safety in Aviation and Space Technologies". Kyiv, Ukraine, September 19-21, 2016: матеріали конгр. – Київ, NAU; 2016. – pp. 1.2.6-1.2.10.

26. **Goncharenko A. V.** Expediency of an improvement for a diesel-geared propulsion with respect to subjectively preferred operational factors / A. V. Goncharenko // Науковий вісник ХДМІ. – 2011. – № 1(4). – pp. 30-41.

27. **Goncharenko A. V.** Expediency of unmanned air vehicles application in the framework of subjective analysis / A. V. Goncharenko // 2013 IEEE 2nd International Conference "Actual Problems of Unmanned Air Vehicles Developments" Proceedings. October, 15-17, 2013, Kyiv, Ukraine. – К.: Освіта України, 2013. – pp. 129-133.

28. **Goncharenko A. V.** Exponential distribution density derived with the help of the multi-optional hybrid functions entropy conditional optimization / A. V. Goncharenko // Problems of friction and wear. – 2017. – № 4(77). – pp. 90-94. (ISSN 0370-2197)

29. **Goncharenko A. V.** Extremality of control and preferences distributions "goodness" / A. V. Goncharenko // Electronics and control systems. – 2014. – № 4(42). – pp. 84-90. (ISSN: 1990-5548)

30. **Goncharenko A. V.** Fuel oil atomization characteristics smoothed by a logarithm normal distribution for marine diesel engines / A. V. Goncharenko // Двигатели внутреннего сгорания. – 2010. – № 2. – pp. 34-40. (ISSN 0419-8719)

31. **Goncharenko A. V.** Generalization for the degrading state maximal probability in the framework of the hybrid-optional entropy conditional optimality doctrine / A. V. Goncharenko // Problems of friction and wear. – 2018. – № 1(78). – pp. 89-92. (ISSN 0370-2197)

32. **Goncharenko A. V.** Horizontal flight for maximal distance at presence of conflict behavior (control) of the aircraft control system active element / A. V. Goncharenko // Матеріали XI міжнародної науково-технічної конференції "АВІА-2013". (21-23 травня 2013 р., Київ). – Т. 4. – К.: НАУ, 2013. – pp. 22.30-22.33.

33. **Goncharenko A. V.** Human factor aspect applicably to aeronautical engineering maintenance / A. V. Goncharenko // Матеріали XIII міжнародної

науково-технічної конференції "ABIA-2017". (19-21 квітня 2017 р., Київ). – К.: НАУ, 2017. – pp. 17.9-17.13.

34. **Goncharenko A. V.** Mathematical modeling of the ship's main engine random operational process / A. V. Goncharenko // Двигатели внутреннего сгорания. – 2012. – № 2. – pp. 117-125. (ISSN 0419-8719)

35. **Goncharenko A. V.** Measures for estimating transport vessels operators' subjective preferences uncertainty / A. V. Goncharenko // Науковий вісник ХДМА. – 2012. – № 1(6). – pp. 59-69.

36. **Goncharenko A. V.** Measures for estimating transport vessels operators' subjective preferences uncertainty / A. V. Goncharenko // Transactions of the institute of aviation. – 2013. – № 228 – pp. 13-21.

37. **Goncharenko A. V.** Modeling aviation legislation influence upon airworthiness support technologies via preferences functions / A. V. Goncharenko // Proceedings of The Seventh World Congress "Aviation in the XXI-st Century" "Safety in Aviation and Space Technologies". Kyiv, Ukraine, September 19-21, 2016: матеріали конгр. – Київ, NAU; 2016. – pp. 1.2.11-1.2.15.

38. **Goncharenko A. V.** Multi-optional hybrid effectiveness functions optimality doctrine for maintenance purposes / A. V. Goncharenko // 14th IEEE International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET-2018). – February, 20-24, 2018, Lviv-Slavske, Ukraine. – 2018. – pp. 771-775.

39. **Goncharenko A. V.** Navigational alternatives, their control and subjective entropy of individual preferences / A. V. Goncharenko // 2014 IEEE 3rd International Conference "Methods and Systems of Navigation and Motion Control (MSNMC)" Proceedings. October, 14-17, 2014, Kyiv, Ukraine. – К.: Освіта України, 2014. – pp. 99-103.

40. **Goncharenko A. V.** One theoretical aspect of entropy paradigm application to the problems of tribology / A. V. Goncharenko // Problems of friction and wear. – 2017. – № 1(74). – pp. 78-83. (ISSN 0370-2197 print)

41. **Goncharenko A. V.** Operational reliability measures for marine propulsion diesel engines / A. V. Goncharenko // Сучасні енергетичні установки на транспорті і технології та обладнання для їх обслуговування: Всеукраїнська науково-практична конференція, Херсон, 12-14 жовтня 2011 р.: матеріали конф. – Херсон, 2011. – pp. 23-27.

42. **Goncharenko A. V.** Optimal controlling path determination with the help of hybrid optional functions distributions / A. V. Goncharenko // Radio Electronics, Computer Science, Control. – 2018. – № 1(44). – pp. 149-158.

43. **Goncharenko A. V.** Optimal dividing between purchasing and fabrication / A. V. Goncharenko // Сучасні енергетичні установки на транспорті, технології та обладнання для їх обслуговування: Республіканська науково-практична конференція, Херсон, 5-7 жовтня 2010 р.: матеріали конф. – Херсон, 2010. – pp. 54-55.

44. **Goncharenko A. V.** Optimal maintenance periodicity for aeronautical engineering operation determined on the theoretical platform of subjective analysis / A. V. Goncharenko // XIIIth International Conference "AVIA-2017". (April 19-21, 2017, Kyiv). – Kyiv: National Aviation University, 2017. – pp. 17.29-17.33.

45. **Goncharenko A. V.** Optimal managerial and control values for active operation / A. V. Goncharenko // Electronics and control systems. – 2016. – № 3(49). – pp. 112-115. (ISSN: 1990-5548)

46. **Goncharenko A. V.** Optimal UAV maintenance periodicity obtained on the multi-optional basis / A. V. Goncharenko // 2017 IEEE 4th International Conference "Actual Problems of Unmanned Aerial Vehicles Developments (APUAVD)" Proceedings. Section A. Unmanned aerial vehicle design. – October, 17-19, 2017, Kyiv, Ukraine. – 2017. – pp. 65-68.

47. **Goncharenko A. V.** Preferences distributions densities for a common continuous alternative / A. V. Goncharenko // Науковий вісник ХДМА. – 2014. – № 2(11). – pp. 22-27. (ISSN 2313-4763)

48. **Goncharenko A. V.** Prospects of alternative sources of energy and engines used in ships propulsion and power plants / A. V. Goncharenko // Сучасні енергетичні установки на транспорті, технології та обладнання для їх обслуговування: Республіканська науково-практична конференція, Херсон, 5-7 жовтня 2010 р.: матеріали конф. – Херсон, 2010. – pp. 115-116.

49. **Goncharenko A. V.** Rational modes of operation for a four-arm tiller electro-hydraulic steering gear with respect to multi-alternativeness and preferences / A. V. Goncharenko // Науковий вісник ХДМА. – 2013. – № 1(8). – С. 28-34. (ISSN 2077-3617)

50. **Goncharenko A. V.** Research of operational effectiveness changes / A. V. Goncharenko // Сучасні енергетичні установки на транспорті, технології та обладнання для їх обслуговування: Республіканська науково-практична конференція, Херсон, 5-7 жовтня 2010 р.: матеріали конф. – Херсон, 2010. – pp. 20-23.

51. **Goncharenko A. V.** Safe maneuvering of a ship in a multi-alternative operational situation / A. V. Goncharenko // Bezpieczeństwo na lądzie, morzu i w powietrzu w XXI wieku. – 2014. – pp. 207-210. (ISBN 978-83-61520-02-3)

52. **Goncharenko A. V.** Safety and its entropy measures of certainty or uncertainty / A. V. Goncharenko // Безпека життєдіяльності на транспорті і виробництві – освіта, наука, практика (SLA-2014) [збірник матеріалів Міжнародної науково-практичної конференції. (18-19 вересня 2014 р., Херсон)]. – Херсон: Херсонська державна морська академія, 2014. – pp. 44-46.

53. **Goncharenko A. V.** Several models of artificial intelligence elements for aircraft control / A. V. Goncharenko // 2016 IEEE 4th International Conference "Methods and Systems of Navigation and Motion Control (MSNMC)" Proceedings. October, 18-20, 2016, Kyiv, Ukraine. – К.: Освіта України, 2016. – pp. 224-227.

54. **Goncharenko A. V.** Several models of physical exercise subjective preferences / A. V. Goncharenko // Clin. and Exp. Psychol. – 2016. – 2: 121. – pp. 1-6. doi:10.4172/2471-2701.1000121. (ISSN: 2471-2701 CEP)

55. **Goncharenko A. V.** Some identities of subjective analysis derived on the basis of the subjective entropy extremization principle by Professor V. A. Kasianov / A. V. Goncharenko // Automatic Control and Information Sciences. – 2014. – Vol. 2, No. 1. – pp. 20-25.

56. **Goncharenko A. V.** Subjective entropy extremization principle as a tool of an aircraft maximal duration horizontal flight control / A. V. Goncharenko // Авиационно-космическая техника и технология. – 2013. – Вып. 8(105). – pp. 229-234.

57. **Goncharenko A. V.** Subjective entropy maximum principle for preferences functions of alternatives given in the view of logical conditions / A. V. Goncharenko // Штучний інтелект. – 2013. – № 4(62). – 1 G. pp. 4-9.

58. **Goncharenko A. V.** Subjective preferences for optimal economy continuous rating of MaK 9M453C / A. V. Goncharenko // Сучасні інформаційні та інноваційні технології на транспорті: III Міжнародна науково-практична конференція, Херсон, 23-25 травня 2011 р.: матеріали конф. У 2-х тт. Том 2. – Херсон: Видавництво Херсонського державного морського інституту, 2011. – pp. 114-119.

59. **Goncharenko A. V.** The optimal commercial speed of a transport vessel with respect to operators' subjective preferences / A. V. Goncharenko // Науковий вісник ХДМІ. – 2011. – № 2(5). – pp. 12-20.

60. **Goncharenko A. V.** The optimal internal "shadow" taxation on condition of a firm external economic activity / A. V. Goncharenko, O. A. Zaporozchenko // Proceedings of the NAU. – 2013. – № 2(55). – pp. 251-257.

61. **Jaynes E. T.** Information theory and statistical mechanics / E. T. Jaynes // Physical review. – U.S.A. – 1957. – Vol. 106, № 4. – pp. 620-630.

62. **Jaynes E. T.** Information theory and statistical mechanics. II / E. T. Jaynes // Physical review. – U.S.A. – 1957. – Vol. 108, № 2. – pp. 171-190.

63. **Jaynes E. T.** On the rationale of maximum-entropy methods / E. T. Jaynes // Proceedings of the IEEE. – 1982. – Vol. 70. – pp. 939-952.

64. **Kasianov V.** Subjective entropy of preferences. Subjective analysis: monograph / V. Kasianov. – Warsaw, Poland: Institute of Aviation Scientific Publications, 2013. – 644 p. (ISBN 978-83-63539-08-5)

65. **Kasianov V. A.** Extremal principle of subjective analysis. Light and shadow. Proportions of shadow economy. Entropy approach: monograph / V. A. Kasianov, A. V. Goncharenko. – Kyiv, Ukraine: Publishing House "Kafedra", 2017. – 90 p. (ISBN 978-617-7301-41-6).

66. **Kroes M. J.** Aircraft maintenance and repair: 7th ed. / M. J. Kroes, W. A. Watkins, F. Delp, R. Sterkenburg. – New York, New York, USA, McGraw-Hill, Education, 2013. – 736 p.

67. **Kroes M. J.** Aircraft powerplants: 7th ed. / M. J. Kroes, T. W. Wild. – New York, New York, USA: GLENCOE Macmillan/McGraw-Hill, International Editions, 1994. – 694 p.

68. **Lind D. A.** Basic statistics for business and economics: 2nd ed. / D. A. Lind, R. D. Mason. – Boston, Massachusetts, USA: Irwin/McGraw-Hill, 1997. – 597 p.

69.**Manuals** on flight operations of aircraft.

70. **Random** House Webster's unabridged dictionary: 2nd ed. – New-York, USA: Random House, 1999. – 2230 p.

71.**Schedules** and technological guidance materials on aircraft scheduled works performances.

72. **Smith D. J.** Reliability, maintainability and risk. Practical methods for engineers / D. J. Smith. – London: Elsevier, 2005. – 365 p.

73. **Sushchenko O.** Design of robust systems for stabilization of unmanned aerial vehicle equipment / O. Sushchenko, A. Goncharenko // International Journal of Aerospace Engineering. – Volume 2016 (2016), Article ID 6054081. – 2016. – pp. 1-10.

74.**Technical** descriptions of aircraft.

75. **Wild T. W.** Aircraft powerplants: 8th ed. / T. W. Wild, M. J. Kroes. – New York, New York, USA: McGraw-Hill, Education, 2014. – 756 p.

76. **Барлоу Р.** Математическая теория надежности: пер с англ. под. ред. Б. В. Гнеденко / Р. Барлоу, Ф. Прошан. – М.: Сов. Радио, 1969. – 288 с.

77. **Баруча-Рид А. Т.** Элементы теории марковских процессов и их приложения / А. Т. Баруча-Рид. – М.: Наука, 1969. – 551 с.

78. **Боровков А. А.** Вероятностные процессы в теории массового обслуживания / А. А. Боровков. – М.: Наука, 1972. – 367 с.

79. **Бронштейн И. Н.** Справочник по математике для инженеров и учащихся втузов / И. Н. Бронштейн, К.А. Семендяев. – М.: Наука, 1986. – 544 с.

80. **Буслаев В. С.** Вариационное исчисление: учеб. пособие / В. С. Буслаев. – Л.: Изд-во Ленингр. ун-та, 1980. – 288 с.

81. **Введение** в математическое моделирование: учебное пособие / В. Н. Ашихмин и др. Под ред. П. В. Трусова. – М.: Интермет Инжиниринг, 2000. – 275 с.

82. **Вентцель Е. С.** Теория вероятностей / Е. С. Вентцель. – М.: В. Школа, 2000. – 458 с.

83. **Вирченко Н. А.** Графики функций: справочник / Н. А. Вирченко, И. И. Ляшко, К. И. Швецов. – К.: Наукова думка, 1979. – 320 с.

84. **Волков Л. И.** Надежность летательных аппаратов: учеб. пособие для авиационных вузов / Л. И. Волков, А. М. Шишкевич. – М.: В. Школа, 1975. – 296 с.

85. **Гельфанд И. М.** Вариационное исчисление / И. М. Гельфанд, С. В. Фомин. – М.: Государственное издательство физико-математической литературы, 1961. – 228 с.

86. **Гнеденко Б. В.** Введение в теорию массового обслуживания / Б. В. Гнеденко, И. Н. Коваленко. – М.: URSS, 2013. – 400 с.

87. **Гнеденко Б. В.** Математические методы в теории надежности / Б. В. Гнеденко, Ю. К. Беляев, А. Д. Соловьев. – М.: Наука, 1965. – 524 с.

88. **Гноенский Л. С.** Математические основы теории управляемых систем / Л. С. Гноенский, Г. А. Каменский, Л. Э. Эльсгольц. – М.: Наука, 1969. – 512 с.

89. **Гончаренко А. В.** Аналіз гвинтової характеристики двигуна 5 ДКРН 70/226,8 за питомою витратою палива / А. В. Гончаренко // Науковий вісник ХДМІ. – 2009. – № 1(1). – С. 16-24.

90. **Гончаренко А. В.** Аналіз параметричних досліджень реологічних властивостей водовугільних суспензій для застосування у суднових двигунах внутрішнього згоряння / А. В. Гончаренко // Авиационно-космическая техника и технология. – 2009. – № 8(65). – С. 90-95.

91. **Гончаренко А. В.** Варіанти вибору стратегії підтримання безпеки функціонування транспортної системи / А. В. Гончаренко // Вісник НАУ. – 2009. – № 2(39). – С. 30-35.

92. **Гончаренко А. В.** Вибір оптимальної комерційної швидкості транспортного судна / А. В. Гончаренко // Науковий вісник ХДМІ. – 2010. – № 1(2). – С. 41-49.

93. **Гончаренко А. В.** Вплив суб'єктивних переваг на показники роботи суднової енергетичної установки / А. В. Гончаренко // Автоматика, автоматизация, электротехнические комплексы и системы. – 2008. – № 2(22). – С. 105-111.

94. **Гончаренко А. В.** Дослідження характеристики двигуна стосовно зміни оптимальних значень / А. В. Гончаренко // Сучасні інформаційні та інноваційні технології на транспорті (MINTT-2009). [Збірка наукових праць у п'яти томах. (25-27 травня 2009 р., Херсон)]: Матеріали Міжнародної науково-практичної конференції. – Том 4. – Херсон:

Видавництво Херсонського державного морського інституту, 2009. – С. 45-48.

95. **Гончаренко А. В.** Експлуатація активних транспортних систем в умовах багатоальтернативності та невизначеності: автореф. … докт. техн. наук: 05.22.20 / А. В. Гончаренко. – К.: НАУ, 2016. – 39 с.

96. **Гончаренко А. В.** Експлуатація активних транспортних систем в умовах багатоальтернативності та невизначеності: дис. … докт. техн. наук: 05.22.20 / А. В. Гончаренко. – К., 2016. – 328 с.

97. **Гончаренко А. В.** Керування підтриманням безпеки польотів через технічні та витратні чинники: автореф. … канд. техн. наук: 05.13.03 / А. В. Гончаренко. – К.: НАУ, 2005. – 20 с.

98. **Гончаренко А. В.** Керування підтриманням безпеки польотів через технічні та витратні чинники: дис. … канд. техн. наук: 05.13.03 / А. В. Гончаренко. – К., 2005. – 198 с.

99. **Гончаренко А. В.** Методичні рекомендації до дипломного проектування / А.В. Гончаренко. – Херсон: Видавництво ПП Тріфонов, друкарня «Графіка», 2010. – 63 с. (ISBN: 978-966-2997-06-4)

100. **Гончаренко А. В.** Моделювання впливу ентропії суб'єктивних переваг на прийняття рішень стосовно ремонту суднової енергетичної установки / А. В. Гончаренко // Автоматика, автоматизация, электротехнические комплексы и системы. – 2009. – № 1(23). – С. 123-131.

101. **Гончаренко А. В.** Моделювання впливу профілактичних замін на показники безпеки польотів / А. В. Гончаренко // Вісник НАУ. – 2004. – № 3(21). – С. 74-77.

102. **Гончаренко А. В.** Обоснование величины инвестиций в безопасность полетов с учетом коэффициента технического использования / А. В. Гончаренко // наука і молодь. Прикладна серія: Збірник наукових праць. – К.: НАУ, 2004. – Вип. 4. – С. 15-18.

103. **Гончаренко А. В.** Обоснование величины инвестиций в безопасность полетов с учетом коэффициента технического использования / А. В. Гончаренко // ПОЛІТ: Матеріали IV Міжнародної наукової конференції студентів та молодих учених. – К.: НАУ, 2004. – С. 19.

104. **Гончаренко А. В.** Оптимальне внутрішнє тіньове оподаткування за умови зовнішньоекономічної діяльності фірми / А. В. Гончаренко, О. А. Запорожченко // Вісник НАУ. – 2013. – № 2(55). – С. 251-257.

105. **Гончаренко А. В.** Показники безпеки функціонування транспортної системи в умовах зростання цін на пальне / А. В. Гончаренко // Вісник НАУ. – 2009. – № 1(38). – С. 35-39.

106. **Гончаренко А. В.** Постановка задачі про вибір оптимального рівня витрат на підтримання безпеки польотів / А. В. Гончаренко // НАУКА І МОЛОДЬ: Матеріали міжнародної наукової конференції. – К.: НАУ, 2001. – С. 110.

107. **Гончаренко А. В.** Принципові питання змісту та методики виконання дипломної роботи за спеціальністю «Експлуатація суднових енергетичних установок» усіх форм навчання: навчальний посібник для ВНЗ / А. В. Гончаренко. – Херсон: Видавництво ПП Тріфонов, друкарня «Графіка», 2010. – 165 с. (ISBN: 978-966-2997-08-8)

108. **Гончаренко А. В.** Типи задач рекомендованих до опрацювання при виконанні дипломної роботи за спеціальністю «Експлуатація суднових енергетичних установок» усіх форм навчання: навчальний посібник для ВНЗ / А. В. Гончаренко. – Херсон: Видавництво ПП Тріфонов, друкарня «Графіка», 2010. – 192 с. (ISBN: 978-966-2997-07-1)

109. **Гончаренко В. С.** Моделювання процесу гасіння пожеж у машинно-котельних відділеннях / В. С. Гончаренко // Сучасні інформаційні та інноваційні технології на транспорті (MINTT-2009). [Збірка наукових праць у п'яти томах. (25-27 травня 2009 р., Херсон)]: Матеріали Міжнародної науково-практичної конференції. – Том 2. – Херсон: Видавництво Херсонського державного морського інституту, 2009. – С. 36-40.

110. **ГОСТ 27.002–89.** Надежность в технике. Основные понятия. Термины и определения. – введ. 01.07.1990. – М.: Изд-во стандартов, 1990. – 36 с.

111. **Единые** нормы летной годности гражданских транспортных самолетов. – М.: ЦАГИ, 1985. – 470 с.

112. **Збереження** льотної придатності повітряних суден: методичні вказівки до виконання курсового проекту / уклад.: В. Г. Докучаєв, Ю. Г. Кордянін, І. М. Новіков, та ін. – К.: НАУ, 2003. – 56 с.

113. **Ивченко Г. И.** Теория массового обслуживания / Г. И. Ивченко, В. А. Каштанов, И. Н. Коваленко. – М.: URSS, 2015. – 304 с.

114. **Игнатов В. А.** Элементы теории оптимального обслуживания технических изделий / В. А. Игнатов, Г. Г. Маньшин, В. В. Костановский. – Минск: Наука и техника, 1974. – 192 с.

115. **Камке Э.** Справочник по обыкновенным дифференциальным уравнениям / Э. Камке. – М.: Наука, 1961. – 703 с.

116. **Капур К.** Надежность и проектирование систем: пер. с англ. Е. Г. Коваленко; под ред. д-ра техн. наук, проф. И. А. Ушакова / К. Капур, Л. Ламберсон. – М.: Мир, 1980. – 604 с.

117. **Касьянов В. А.** Вариационные принципы субъективного анализа. Модифицированный вариационный принцип Эйлера-Лагранжа.

Энтропийный подход: монография / В. А. Касьянов, А. В. Гончаренко. – К.: ДП НВЦ «Приорітети», 2015. – 112 с. (ISBN 978-966-8809-67-5)

118. **Касьянов В. А.** Свет и тень. Пропорции теневой экономики. Энтропийный подход: монография / В. А. Касьянов, А. В. Гончаренко. – К.: Кафедра, 2013. – 86 с. (ISBN 978-966-2705-36-2)

119. **Кельберт М. Я.** Вероятность и статистика в примерах и задачах. Основные понятия теории вероятностей и математической статистики / М. Я. Кельберт, Ю. М. Сухов. – М.: изд. МЦНМО, 2007. – 455 с.

120. **Коваленко И. Н.** Анализ редких событий при оценке эффективности и надежности систем / И. Н. Коваленко. – М.: Сов. радио, 1980. – 209 с.

121. **Коваленко И. Н.** О двухциклической системе обслуживания / И. Н. Коваленко // Кибернетика и системный анализ. – 2015. – № 1. – С. 59-65.

122. **Корн Г.** Справочник по математике для научных работников и инженеров. Определения, теоремы, формулы: 5-е изд. / Г. Корн, Т. Корн. – М: Наука, 1984. – 831 с. [**Korn G. A.** Mathematical handbook for scientists and engineers. Definitions, theorems and formulas for reference and review: second, enlarged and revised edition / G. A. Korn, T. M. Korn. – USA: McGraw-Hill Book Company, 1968.]

123. **Кунцевич В. М.** Управление в условиях неопределенности: гарантированные результаты в задачах управления и идентификации: монография / В. М. Кунцевич. – К.: Наукова думка, 2006. – 264 с.

124. **Математическая** энциклопедия; гл. ред. И. М. Виноградов, в 5-ти томах. – М.: Советская Энциклопедия, 1984. – Т. 5. – 1248 стб.

125. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1986. – Т. 1: Методология. Организация. Терминология / под ред. А. И. Рембезы. – 224 с.

126. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1987. – Т. 2: Математические методы в теории надежности и эффективности / под ред. Б. В. Гнеденко. – 280 с.

127. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1988. – Т. 3: Эффективность технических систем / под общ. ред. В. Ф. Уткина, Ю. В. Крючкова. – 328 с.

128. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1988. – Т. 5: Проектный анализ надежности / под ред. В. И. Патрушева и А. И. Рембезы. – 316 с.

129. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1989. – Т. 6: Экспериментальная отработка и испытания / под общ. ред. Р. С. Судакова, О. И. Тескина. – 376 с.

130. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1990. – Т. 8: Эксплуатация и ремонт / под ред. В. И. Кузнецова и Е. Ю. Барзиловича. – 320 с.

131. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1990. – Т. 9: Техническая диагностика / под общ. ред. В. В. Клюева, П. П. Пархоменко. – 1987. – 352 с.

132. **Надежность** и эффективность в технике: справочник: в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. – М.: Машиностроение, 1990. – Т. 10: Справочные данные по условиям эксплуатации и характеристикам надежности / под ред. В. А. Кузнецова. – 1990. – 336 с.

133. **Надежность** машин: учеб. пособие; под ред. Д. Н. Решетова / Д. Н. Решетов, А. С. Иванов, В. З. Фадеев. – М.: В. Школа, 1988. – 238 с.

134. **Наставление** по производству полетов в гражданской авиации (НПП ГА-85). – М.: Воздушный транспорт, 1985. – 286 с.

135. **Наставление** по штурманской службе ГА. – М.: Воздушный транспорт, 1985. – 254 с.

136. **Овчаров Л. А.** Прикладные задачи теории массового обслуживания / Л. А. Овчаров. – М.: Машиностроение, 1969. – 324 с.

137. **Организация** и проведение полетов самолетов ГА. – М.: Воздушный транспорт, 1986. – 232 с.

138. **Пат. 94181** Україна, МПК B63H 25/00. Спосіб вибору оптимальної комбінації режимів експлуатації суднової рульової машини / А. В. Гончаренко; заявник та власник патенту Національний авіаційний університет. – № u 2013 09054; заявл. 19.07.2013; опубл. 10.11.2014, Бюл. № 21.

139. **Пискунов Н. С.** Дифференциальное и интегральное исчисления для втузов / Н. С. Пискунов, т. 1: Учебное пособие для втузов. 13-е изд. – М.: Наука, Главная редакция физико-математической литературы, 1985. – 432 с.

140. **Пискунов Н. С.** Дифференциальное и интегральное исчисления для втузов / Н. С. Пискунов, т. 2: Учебное пособие для втузов. 13-е изд. – М.: Наука, Главная редакция физико-математической литературы, 1985. – 560 с.

141. **Смирнов Н. Н.** Методические указания к курсовому проектированию по дисциплине «Техническая эксплуатация летательных аппаратов» для студентов специальности 1610 / Н. Н. Смирнов, Ю. М. Чинючин. – М.: РИО МИИГА, 1985. – 56 с.

142. **Смирнов Н. Н.** Обслуживание и ремонт авиационной техники по состоянию / Н. Н. Смирнов, А. А. Ицкович. – М.: Транспорт, 1987. – 272 с.

143. **Техническая** эксплуатация летательных аппаратов: учебник для вузов / под ред. Н. Н. Смирнова. – М.: Транспорт, 1990. – 423 с.

144. **Эльсгольц Л. Э.** Дифференциальные уравнения / Л. Э. Эльсгольц. – М.: Государственное издательство технико-теоретической литературы, 1957. – 271 с.

Навчальне видання

ПІДТРИМАННЯ ЛЬОТНОЇ ПРИДАТНОСТІ ПОВІТРЯНИХ СУДЕН (ICAO Doc 9760)

Частина II

ЗАСТОСУВАННЯ ЕНТРОПІЙНОЇ ДОКТРИНИ БАГАТООПЦІЙНИХ ФУНКЦІЙ ДЛЯ ОЦІНКИ УДОСКОНАЛЕННЯ ПРОЦЕСУ ТЕХНІЧНОГО ОБСЛУГОВУВАННЯ ПОВІТРЯНОГО СУДНА

Методичні рекомендації
до виконання самостійної роботи
для студентів 1-го курсу галузі знань
27 «Транспорт», спеціальності 272 «Авіаційний
транспорт», спеціалізації 01 «Технічне
обслуговування та ремонт повітряних
суден і авіадвигунів»

(Англійською мовою)

Укладач ГОНЧАРЕНКО Андрій Вікторович

В авторській редакції

Технічний редактор *А. І. Лавринович*
Комп'ютерна верстка *Н. С. Ахроменко*

Підп. до друку 27.06.2018. Формат 60х84/16. Папір офс.
Офс. друк. Ум. друк. арк. 2,79. Обл.-вид. арк. 3,0.
Тираж 100 пр. Замовлення № 92-1.

Видавець і виготівник
Національний авіаційний університет
03680. Київ-58, проспект Космонавта Комарова, 1

Свідоцтво про внесення до Державного реєстру ДК № 977 від 05.07.2002