

Ministry of Education and Science of Ukraine  
National Aviation University

**CONTINUING AIRCRAFT  
AIRWORTHINESS  
(ICAO Doc 9760)**

**SELF-STUDY METHOD GUIDE**

**Part I**

**Reliability Measures to Assess  
the Aircraft Maintenance Process  
Improvements for the Students of the  
Field of Study 27 “Transport”,  
Specialty 272 “Aviation Transport”,  
Specialization 01 “Aircraft and  
Aero-Engines Maintenance and Repair”**

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Містять декілька рекомендацій для самостійної роботи щодо показників надійності для оцінки удосконалення процесу технічного обслуговування повітряного судна в рамках елементів наукового дослідження.

Для студентів 1-го курсу галузі знань 27 «Транспорт», спеціальності 272 «Авіаційний транспорт», спеціалізації 01 «Технічне обслуговування та ремонт повітряних суден і авіадвигунів».

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The **METHOD GUIDE** contains a few recommendations on the Self-Study in regards with the reliability measures to assess the aircraft maintenance process improvements in the framework of the scientific research elements.

Designed for the 1<sup>st</sup> year students of the Field of Study 27 "Transport", Specialty 272 "Aviation Transport", Specialization 01 "Maintenance and Repair of Aircraft and Aircraft Engines".

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## INTRODUCTION

This **METHOD GUIDE ON THE SELF-STUDY (SS)** is contemplated in response to the needs of our students in more detailed elaborations concerning problems stated, set or given for the students' independent work, possibly used in their course projecting, further graduation papers or even Ph.D. studies. The whole material is split into portions. Each portion is intended to cover a fraction of probable applications aimed at **CONTINUING AIRCRAFT AIRWORTHINESS** or its retaining measures.

The presented in this, first, **PART I** of the **METHOD GUIDE ON THE SS** problems are dedicated, and a special attention is drawn here, to the scientific component of the SS work. Specifically, the objectives of the **PART I** are to help students cope with the challenging problems relating to the **AIRCRAFT (A/C)** technical operation in regards with the aeronautical engineering **MAINTENANCE (M/T)** optimal periodicities.

The set of the considered problems is based upon the **RECOMMENDED LITERATURE SOURCES** (the list is presented, but not limited to it). The **LIST OF LITERATURE** at the end of the **METHOD GUIDE** is basic (major) and compiled in the alphabetic order with respect to the matter of supposed (assumed) importance.

The **REFERENCES LIST** is selected, set in the order [1-144], does not pretend for completeness, but instead it is aimed at developing the students' abilities of thinking and to analyze, contemplate in the specified directory rather than their abilities to know and memorize. However, these are very significant too. Actually, in the contemporary informative boom world, the needed or required data can easily be retrieved from the internet, found in multiple references, guidance materials [1, 69, 71, 74, 111, 112, 134, 135, 137], studies [2, 66-68, 75-88, 113-116, 119-133, 136, 139-144], dictionaries [70, 110], comprehensive books [3, 72] or monographs [64, 65, 86-88, 113, 114, 117, 118, 123] etc. The **METHOD GUIDE** is designed for the 1st year students of the Field of Study: 27 "Transport", Specialty: 272 "Aviation Transport", Specialization: 01 "Maintenance and Repair of Aircraft and Aircraft Engines". It includes detailed solutions for obtaining reliability objective measures allowing assessing the improvements of the A/C functional system M/T process considered in reference [112].

## THE SIMPLEST CASE OF THE MAINTENANCE OPTIMAL PERIODICITY DETERMINATION

*The principal theoretical provisions can be found out in references [141-143, 3, 72, 76, 82, 84, 87, 110, 114, 116, 124-133].*

### **1. Determination of the periodicity with the use (help) of the normative for the specified (predetermined) reliability level**

It is supposed, [143, pp. 162-174, Chapter 15, especially Sub-Chapter 15.4, pp. 169, 170], that on the basis of some directive documents or researches (investigations) the necessary or required reliability level of a certain engineering unit (product) is predetermined (given probability of the non-failure running, acceptable number of damages or failures per 1,000 flight hours or in-between scheduled maintenance period etc.), which must be ensured (guaranteed) in the process of operation. To that end, from the established (determined) normative reliability and with the help of the accepted model, the maintenance maximal periodicity is found that would ensure the specified reliability, [143, pp. 169, 170].

The process of the damages emergence on the units of aeronautical engineering in some cases can be considered as the simplest flow of uniform events, characterized with stationary, absence of after-action, and ordinary. Such a flow has a property that the probability of the damages emergence on the unit is predetermined by the *Poisson* law, [143, p. 170].

In the considered case, the probability that there will arise  $k$  number of damages in the given period between maintenances  $t_p$  is [143, Sub-Chapter 15.4, p. 170]:

$$P_k(t_p) = \frac{(\omega t_p)^k}{k!} e^{-\omega t_p}, \quad (1)$$

where  $\omega$  – parameter of the damages flow (damage intensity) in the between-scheduled maintenance period.

From the calculation results of the specified probabilities (Eq. (1)) depending upon the maintenance periodicity, for example, for the fuel-governing aggregate of the powerplant (Fig. 1) [143, p. 170, Fig. 15.1] it is visible that if the emergence of two damages on the aggregate is a danger (threat), then at the accepted probability level of their appearance  $P_2(t_p) = 0.05$  the maintenance periodicity of the aggregate should not exceed (go beyond) 383 h. The damage intensity (parameter of the damages flow) has been got by the statistical data,  $\omega = 1 \cdot 10^{-3} \text{ h}^{-1}$ , [143, p. 170].

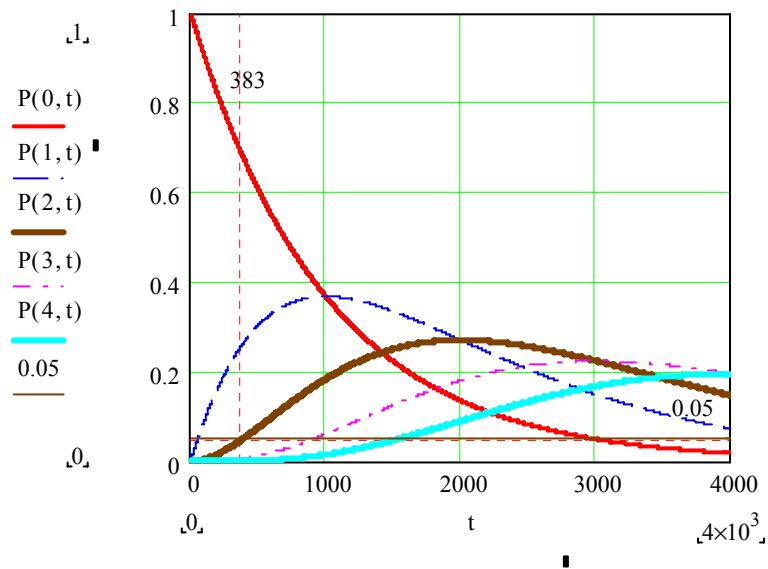


Fig. 1. Dependence of the fuel-governing aggregate malfunctioning coming out probability distribution upon running hours [143, p. 170, Fig. 15.1]

The given method can be used for complex products consisting of a great number of elements, failure of one or two of which does not influence upon the working state of the product as a whole, [143, p. 170].

## 2. Determination of the maintenance optimal periodicity with taking into account (consideration) the rate (speed) of the failure development

The essence of the method is that at the optimal timing intervals of the maintenance execution the probability of the conjoint event – emergence of a damage and the failure not coming out is maximized. It is supposed (assumed) that elimination (removal) of the damage in the predetermined time intervals prevents (prevails) the failures occurrences [143, pp. 170-172].

Let us say, [143, p. 170], that at the work of a product at a random moment of time  $\tau$  there happens a damage, the further development of which leads to the failure at the moment of time  $t$ . Maintenance has to be performed in the interval  $(\tau, t)$  at the moment  $t_p$ , Fig. 2.

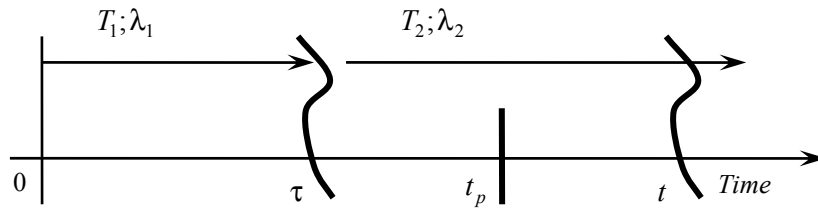


Fig. 2. Illustration of the failure after a damage development

Then, for the considered product, the probability that the damage will happen at the moment  $\tau$  but there will not occur the failure during the time interval of  $(t - \tau)$ , [143, p. 171],

$$dP(t, \tau) = f(\tau) d\tau P(t - \tau), \quad (2)$$

where  $f(\tau)$  – distribution density of running hours before the damage emergence;  $P(t - \tau)$  – probability of not materialization of the failure in the interval  $(t - \tau)$  (on condition that the damage appeared at the moment of time  $\tau$ ), [143, p. 171].

Integrating with respect to  $\tau$  from 0 up to  $t$  we will get, [143, p. 171],

$$P(t, \tau) = \int_0^t f(\tau) P(t - \tau) d\tau. \quad (3)$$

Accepting, for instance, the law of the damages appearance times distribution, as well as of their development times to the failure happening, as the exponential ones with the corresponding rates of  $\lambda_1$  and  $\lambda_2$ , [143, p. 171], that is

$$\begin{aligned} f_1(t) &= \lambda_1 e^{-\lambda_1 t}, & f_2(t) &= \lambda_2 e^{-\lambda_2 t}, & (4) \\ P_{\bar{F}|D} &= P(t - \tau) = 1 - Q(t - \tau) = 1 - \int_0^{t-\tau} f_2(x) dx = 1 - \int_0^{t-\tau} \lambda_2 e^{-\lambda_2 x} dx = \\ &= 1 - \left[ \lambda_2 \left( -\frac{1}{\lambda_2} \right) e^{-\lambda_2 x} \right]_0^{t-\tau} = 1 + e^{-\lambda_2(t-\tau)} - e^{-\lambda_2 \cdot 0} = 1 + e^{-\lambda_2(t-\tau)} - 1, \end{aligned}$$

where  $P_{\bar{F}|D}$  – conditional probability of the failure not happening, there is the “negation mark” over  $F$ , i.e.  $\bar{F}$ , on the condition that the damage appears;  $x$  – technical variable,

$$P_{\bar{F}|D} = P(t - \tau) = e^{-\lambda_2(t-\tau)}. \quad (5)$$

On the other hand

$$\begin{aligned} P_{\bar{F}|D} &= P(t - \tau) = \int_{t-\tau}^{\infty} f_2(x) dx = \int_{t-\tau}^{\infty} \lambda_2 e^{-\lambda_2 x} dx = \left[ \lambda_2 \left( -\frac{1}{\lambda_2} \right) e^{-\lambda_2 x} \right]_{t-\tau}^{\infty} = \\ &= -e^{-\lambda_2 \cdot \infty} + e^{-\lambda_2(t-\tau)} = e^{-\lambda_2(t-\tau)} - 0. \end{aligned}$$

Thus, the result of Eq. (5) is also obtained.

Then equation (3) yields, [143, p. 171],

$$\begin{aligned} P_{D\bar{F}} &= P(t, \tau) = P(t) = \int_0^t e^{-\lambda_2(t-\tau)} \lambda_1 e^{-\lambda_1 \tau} d\tau = \lambda_1 e^{-\lambda_2 t} \int_0^t e^{(\lambda_2 - \lambda_1)\tau} d\tau = \\ &= \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} \left[ e^{(\lambda_2 - \lambda_1)\tau} \right]_0^t = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} \left[ e^{(\lambda_2 - \lambda_1)t} - 1 \right]. \quad (6) \end{aligned}$$

or, [143, p. 171, (15.1)],

$$P_{D\bar{F}} = P(t, \tau) = P(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}). \quad (7)$$

The optimal time interval for the scheduled maintenance works performance will be found at

$$\frac{dP_{D\bar{F}}}{dt} = 0. \quad (8)$$



Accordingly to the condition of Eq. (8), it yields, [143, p. 171, (15.2)],

$$\frac{dP_{D\bar{F}}}{dt} = \frac{\lambda_1}{\lambda_2 - \lambda_1} (\lambda_2 e^{-\lambda_2 t} - \lambda_1 e^{-\lambda_1 t}) = 0. \quad (9)$$

Solving equation (9), we find the sought optimal periodicity of the scheduled maintenance works performance, [143, p. 171]:

$$\lambda_2 e^{-\lambda_2 t_p} - \lambda_1 e^{-\lambda_1 t_p} = 0, \quad \lambda_2 e^{-\lambda_2 t_p} = \lambda_1 e^{-\lambda_1 t_p}, \quad e^{(\lambda_1 - \lambda_2)t_p} = \frac{\lambda_1}{\lambda_2},$$

$$(\lambda_1 - \lambda_2)t_p = \ln \frac{\lambda_1}{\lambda_2}, \quad t_p = \frac{\ln \lambda_1 - \ln \lambda_2}{\lambda_1 - \lambda_2}. \quad (10)$$

Practically, the optimal value of the maintenance caring out timing (10) can be determined from a diagram in the way of plotting the dependencies of the: probability of the damages, in the between-scheduled period, absence  $P_{\bar{D}}(t)$ ; probability of the damages happening, but not coming forward the failure  $P_{D\bar{F}}(t)$ ; probability of the failure, in the between-scheduled period, coming out  $P_F(t)$  [143, p. 171].

### 3. Determination of the maintenance optimal periodicity via conditional probabilities

In the considered above case (2)-(10) all those three situations together constitute (comprise, make up, form, compose) the complete (total/whole/full/entire) group of non-conjunctive events [143, p. 171]:

$$P_{\bar{D}}(t) + P_{D\bar{F}}(t) + P_F(t) = 1. \quad (11)$$

This is the so-called normalizing condition.

Let us remind that if there is a set of two random events  $A$  and  $B$  possibly happening together, then the total probability for the whole group of non-conjunctive events, [82],

$$P_{AB} + P_{A\bar{B}} + P_{\bar{A}B} + P_{\bar{A}\bar{B}} = 1. \quad (12)$$

For example, the probability of event  $A$  is  $P_A = 0.1$ ; for not  $A$ , i.e.  $\bar{A}$ , it makes  $P_{\bar{A}} = 1 - P_A = 1 - 0.1 = 0.9$ ; conditional probabilities, for

instance: event  $B$  happens on condition of event  $A$  has already happened,  $P_{B|A} = 0.4$ ; then  $P_{\bar{B}|A} = 1 - P_{B|A} = 1 - 0.4 = 0.6$ ;  $P_{B|\bar{A}} = 0.2$ ;  $P_{\bar{B}|\bar{A}} = 1 - P_{B|\bar{A}} = 1 - 0.2 = 0.8$ .

Probabilities for conjunctive (conjoint) events, for Eq. (12):

$$\begin{aligned}
 P_{AB} &= P_A P_{B|A} = 0.1 \cdot 0.4 = 0.04; \\
 P_{A\bar{B}} &= P_A P_{\bar{B}|A} = 0.1 \cdot 0.6 = 0.06; \\
 P_{\bar{A}B} &= P_{\bar{A}} P_{B|\bar{A}} = 0.9 \cdot 0.2 = 0.18; \\
 P_{\bar{A}\bar{B}} &= P_{\bar{A}} P_{\bar{B}|\bar{A}} = 0.9 \cdot 0.8 = 0.72.
 \end{aligned} \tag{13}$$

Substituting results of Eq. (13) for corresponding values of Eq. (12) it yields that result. An illustration of the concept expressed with Eq. (11)-(13) is represented in Fig. 3.

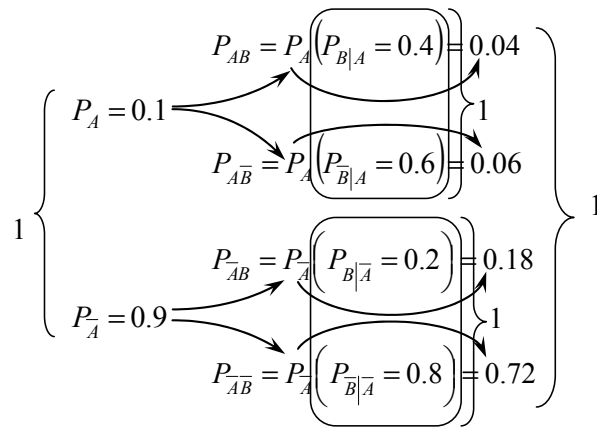


Fig. 3. Illustration of the concept for a set of two conjoint events

In the considered problem setting, Eq. (2)-(11), the failure can occur (arise) only if a damage has already taken place. For probabilities, it may be interpreted in such, for instance, way (Fig. 4).

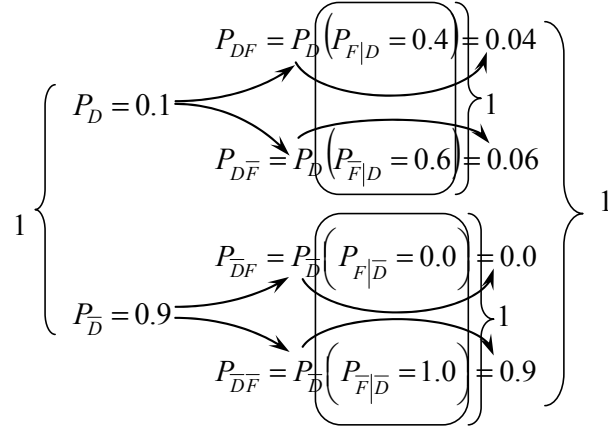


Fig. 4. Illustration of the concept for a set of damage and failure conjoint events

That is why in Eq. (11)  $P_{\bar{D}}(t)$  represents both  $P_{\bar{D}\bar{F}} = P_{\bar{D}}(P_{F|\bar{D}} = 0.0) = 0.0$  and  $P_{\bar{D}F} = P_{\bar{D}}(P_{F|\bar{D}} = 1.0) = P_{\bar{D}}$ ; and  $P_F(t)$  is for  $P_{DF} = P_D(P_{F|D})$ . Therefore all of them, together with  $P_{D\bar{F}} = P_D(P_{F|\bar{D}})$ , see Eq. (7), make the entire group of non-conjoint events. For the probability of  $P_{\bar{D}\bar{F}} = P_{\bar{D}}(P_{F|\bar{D}} = 1.0) = P_{\bar{D}}(t)$  we have

$$P_{\bar{D}}(t) = \int_t^{\infty} \lambda_1 e^{-\lambda_1 x} dx = -e^{-\lambda_1 \cdot \infty} + e^{-\lambda_1 t} = e^{-\lambda_1 t} - 0 = e^{-\lambda_1 t}. \quad (14)$$

For  $P_{DF} = P_D(P_{F|D})$ , see the derivation of Eq. (5) also,

$$P_{F|D}(t) = 1 - P_{\bar{F}|D}(t) = Q(t - \tau) = 1 - e^{-\lambda_2(t-\tau)}. \quad (15)$$

Or on the other hand

$$\begin{aligned} P_{F|D}(t) = Q(t - \tau) &= \int_0^{t-\tau} f_2(x) dx = \int_0^{t-\tau} \lambda_2 e^{-\lambda_2 x} dx = \left[ \lambda_2 \left( -\frac{1}{\lambda_2} \right) e^{-\lambda_2 x} \right]_0^{t-\tau} = \\ &= -e^{-\lambda_2(t-\tau)} + e^{-\lambda_2 \cdot 0} = -e^{-\lambda_2(t-\tau)} + 1. \end{aligned} \quad (16)$$

The yielded result is the same as Eq. (15).

At last for  $P_{DF} = P_D(P_{F|D})$  it will be (see the derivations of Eq. (6), (7) with the only exception, we use  $P_{F|D}(t)$ , Eq. (15) or (16) instead of  $P_{\bar{F}|D}(t)$ , Eq. (5) because here we consider the conjoint event of emergence (appearance, coming out, materialization) of both damage and failure), that is

$$\begin{aligned}
P_{DF}(t) &= \int_0^t [1 - e^{-\lambda_2(t-\tau)}] \lambda_1 e^{-\lambda_1 \tau} d\tau = \int_0^t \lambda_1 e^{-\lambda_1 \tau} d\tau - \int_0^t e^{-\lambda_2(t-\tau)} \lambda_1 e^{-\lambda_1 \tau} d\tau = \\
&= \left[ \lambda_1 \left( -\frac{1}{\lambda_1} \right) e^{-\lambda_1 \tau} \right]_0^t - \lambda_1 e^{-\lambda_2 t} \int_0^t e^{(\lambda_2 - \lambda_1) \tau} d\tau = -e^{-\lambda_1 t} + e^{-\lambda_1 \cdot 0} - \\
&= -\frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} [e^{(\lambda_2 - \lambda_1) \tau}]_0^t = 1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} [e^{(\lambda_2 - \lambda_1) t} - e^{(\lambda_2 - \lambda_1) 0}] = \\
&= 1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} [e^{(\lambda_2 - \lambda_1) t} - 1]. \tag{17}
\end{aligned}$$

Finally

$$P_{DF}(t) = 1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t} - e^{-\lambda_2 t}]. \tag{18}$$

Thus, see Eq. (7), (11), (14), (18), the normalizing condition (11) will be satisfied:

$$\begin{aligned}
&P_{DF}(t) + P_{D\bar{F}}(t) + P_{\bar{D}F}(t) + P_{\bar{D}\bar{F}}(t) = \\
&= 1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t} - e^{-\lambda_2 t}] + \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + 0 + e^{-\lambda_1 t} = 1. \tag{19}
\end{aligned}$$

The diagrams plotted by the numerical simulation with the formulae Eq. (2)-(19) are shown in Fig. 5. Mathematical modeling has been realized for such data:  $\lambda_1 = 5 \cdot 10^{-3} \text{ h}^{-1}$ ;  $\lambda_2 = 1 \cdot 10^{-3} \text{ h}^{-1}$ ;  $t = 0 \dots 1.5 \cdot 10^3 \text{ h}$ .  $t_{\text{opt}} \approx 402 \text{ h}$ .  $P_{\text{DnF}}(t)$  is for  $P_{D\bar{F}}(t)$ ;  $P_{\text{nd}}(t) - P_{\bar{D}}(t)$ ;  $P_{\text{DF}}(t) - P_{DF}(t)$ .

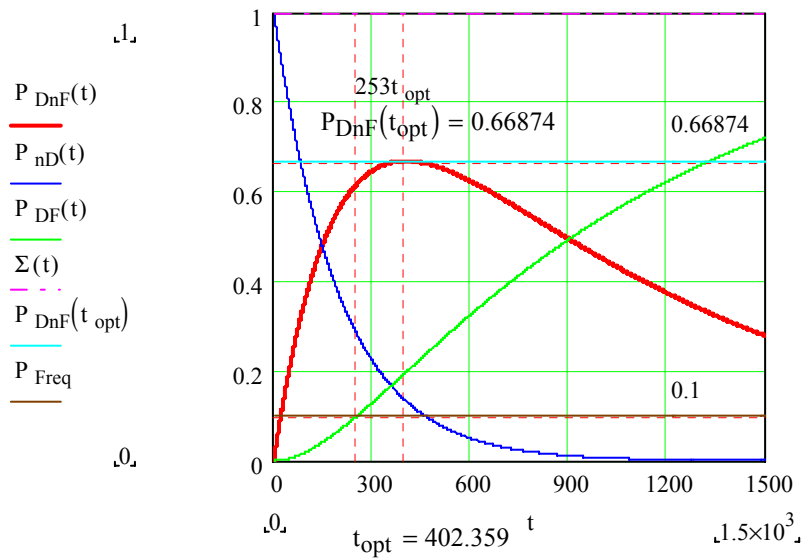


Fig. 5. Results of computer calculation experiments on the MathCad platform

It is obvious that the function of  $P_{\bar{D}}(t)$  is a monotonously decreasing one, and the function of  $P_{DF}(t)$  – non-decreasing [143, p. 171]. The character of the all three situations probability changes is shown in the Fig. 5, [143, p. 172, Fig. 15.2]. at the optimal periodicity  $t_{opt}$  determination one should follow the requirement that the probability of failure  $P_{DF}(t)$  is not higher than the predetermined (prescribed) one  $P_{DF}(t)_{req}$ , [143, p. 171].

At the required level of the non-failure state probability, for example,  $P_{DF}(t)_{req} \leq 0.1$ , designated as  $P_{Freq}$  in Fig. 5, the maintenance periodicity must not exceed 253 hours (also see Fig. 5).

#### 4. Determination of the maintenance optimal periodicity via probabilities density of distributions

Also, the result of Eq. (6), (7) can be obtained on the basis of the concept of separate random variables distribution laws, especially, *conditional laws of distributions*, i.e. given with either *conditional function of distribution*, designated as  $F(x|y)$ , or *conditional density of distribution*, designated as  $f(x|y)$ , [82], or in the 4th stereotype edition's of 1969: Chapter 8, especially Sub-Chapter 8.4, pp. 168-171. This method (approach in the *scheme of random variables*) is analogous to the method of product (multiplication) of probabilities in the *scheme of events*.

For the common density of distribution, the element of probability is, [82], in the 1969 4<sup>th</sup> stereotype edition p. 170, (8.4.4):

$$f(x, y)dxdy = f_1(x)dx f(y|x)dy, \text{ from where } f(x, y) = f_1(x)f(y|x), \quad (20)$$

where  $x$  and  $y$  – specific values of random variables of  $X$  and  $Y$ ;  $f_1(x)$  – distribution law of random variable  $T_1 = \tau - 0$  given with the density of its distribution, the first of the equations of (4);  $f(y|x)$  – conditional density of distribution of random variable  $T_2 = t - \tau$  given with the density of its distribution on condition that the first random variable  $T_1$  has got the specified value of  $\tau$ , the second Eq. (4).

Thus, we have the system of the two random variables  $T_1$  and  $T_2$ . Therefore, integrating the element of probability, it yields

$$\begin{aligned} P_{DF} = P(t, \tau) = P(t) &= \int_0^t \lambda_1 e^{-\lambda_1 x} dx \left( \int_{t-\tau}^{\infty} \lambda_2 e^{-\lambda_2 y} dy \right) = \\ &= \int_0^t \lambda_1 e^{-\lambda_1 x} \left( \left[ -e^{-\lambda_2 y} \right]_{t-\tau}^{\infty} \right) dx = \int_0^t \lambda_1 e^{-\lambda_1 x} \left[ -e^{-\lambda_2 \cdot \infty} + e^{-\lambda_2(t-\tau)} \right] dx = \\ &= \int_0^t \lambda_1 e^{-\lambda_1 x} \left[ -0 + e^{-\lambda_2(t-\tau)} \right] dx. \end{aligned} \quad (21)$$

And we have come to Eq. (6), (7) again.

At the next section, it is going to be considered a few aspects of the modeling, described above with the formulae of (1)-(21), with taking into account provisions of the **MASS SERVICE THEORY**.

This will allow solving the reliability problems [143], relating with the objective measures evaluations (assessments, estimations) of the improvements of the A/C functional system M/T process considered in reference [112], dealing with the **CONTINUING AIRCRAFT AIRWORTHINESS (ICAO DOC 9760)** [1-144], at the higher level of study.

### **MODELLING ON THE BASIS OF THE *MASS SERVICE THEORY***

*The principal theoretical provisions can be found out in references [136, 76-79, 81-84, 86, 113, 139, 140].*

More elite (exclusive) modelling can be performed with the use of *Theory of Mass Service*, [136].

#### **1. The simplest case of the optimal maintenance periodicity determination**

First, let us consider a graph which corresponds to the previous problem. It is represented in Fig. 6.

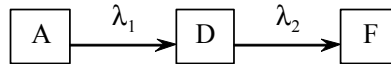


Fig. 6. Graph of three states of an aircraft functional system

Here, in Fig. 6, “A” designates the up state of the system; “D” – damage; “F” – failure. The corresponding system of differential equations by Erlang will have the view of

$$\left. \begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - \lambda_2 P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D. \end{aligned} \right\} \quad (22)$$

Here,  $P_A$ ,  $P_D$ , and  $P_F$  – probabilities of the corresponding states (see Fig. 6);  $t$  – time.

From the first equation of the system (22) we find

$$\frac{dP_A}{dt} = -\lambda_1 P_A; \quad \frac{dP_A}{P_A} = -\lambda_1 dt; \quad \int \frac{dP_A}{P_A} = -\int \lambda_1 dt + C, \quad (23)$$

where  $C$  – constant of integration, it will be found from the initial conditions:

$$t_0 = 0; \quad P_A|_{t=t_0} = 1; \quad P_D|_{t=t_0} = 0; \quad P_F|_{t=t_0} = 0, \quad (24)$$

Eq. (23) yields

$$\ln P_A = -\lambda_1 t + C, \quad (25)$$

from where

$$\ln 1 = -\lambda_1 \cdot 0 + C, \quad 0 = -0 + C, \quad C = 0. \quad (26)$$

Thus,

$$P_A(t) = e^{-\lambda_1 t}. \quad (27)$$

The same result as Eq. (14), which is correct because state “A” means not transition into the state of damage “D” and transition from “A” straight to the state of failure “F” is impossible as well as any of the backward transitions. Therefore

$$P_A(t) = P_D(t) = P_{DF}(t) + P_{DF}(t) = 0 + e^{-\lambda_1 t} = e^{-\lambda_1 t}. \quad (28)$$

Substituting the result of Eq. (27) or (28) for the corresponding value of  $P_A(t)$  in the second equation of the system (22) we get

$$\frac{dP_D}{dt} = \lambda_1 e^{-\lambda_1 t} - \lambda_2 P_D. \quad (29)$$

Thus, we have got a linear not uniform (homogeneous) differential equation of the first order. It can easily be solved with the method represented in [139, 140]. Exactly, accordingly to [140, Chapter XIII, especially § 7, pp. 30-33] the Eq. (29) has the view of [140, Chapter XIII, § 7, p. 30, (1)]

$$\frac{dy}{dx} + P(x)y = Q(x). \quad (30)$$

where  $P(x)$  and  $Q(x)$  – given continuous functions of  $x$  (or constants).

The solution of the linear equation (30). We will be finding the solution of the



equation (30) in the view of a product of two functions of  $x$  [140, Chapter XIII, § 7, p. 30, (2)]:

$$y = u(x)v(x). \quad (31)$$

One of these functions can be taken arbitrary, the other one will be determined on the basis of Eq. (30).

Differentiating both parts of Eq. (31), we find, [140, Chapter XIII, § 7, p. 31],

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}. \quad (32)$$

Substituting the obtained expression of the derivative of  $\frac{dy}{dx}$  into Eq. (30), we will have, [140, Chapter XIII, § 7, p. 31],

$$u \frac{dv}{dx} + v \frac{du}{dx} + Puv = Q, \quad (33)$$

or, [140, Chapter XIII, § 7, p. 31, (3)],

$$u \left( \frac{dv}{dx} + Pv \right) + v \frac{du}{dx} = Q. \quad (34)$$

Let us choose function  $v$  as such as, [140, Chapter XIII, § 7, p. 31, (4)],

$$\frac{dv}{dx} + Pv = 0. \quad (35)$$

Dividing variables in this differential equation with respect to function  $v$ , we find, [140, Chapter XIII, § 7, p. 31],

$$\frac{dv}{v} = -Pdx. \quad (36)$$

Integrating, we get, [140, Chapter XIII, § 7, p. 31],

$$-\ln|C_1| + \ln|v| = -\int Pdx, \quad (37)$$

where  $C_1$  – constant of integration; or, [140, Chapter XIII, § 7, p. 31],

$$v = C_1 e^{-\int Pdx}. \quad (38)$$

Since it is enough to have some different from zero solution of Eq. (35), then we will take [140, Chapter XIII, § 7, p. 31, (5)] as the function of  $v(x)$ , assuming and accepting  $C_1 = 1$ :

$$v(x) = e^{-\int P dx}, \quad (39)$$

where  $\int P dx$  – some antiderivative (counterderivative). It is obviously that  $v(x) \neq 0$ .

Substituting the found value of  $v(x)$  into Eq. (34), [140, Chapter XIII, § 7, p. 31, (3)], we will get (taking into account that  $\frac{dv}{dx} + Pv = 0$ ), [140, Chapter XIII, § 7, p. 31],

$$v(x) \frac{du}{dx} = Q(x), \quad (40)$$

or, [140, Chapter XIII, § 7, p. 31],

$$\frac{du}{dx} = \frac{Q(x)}{v(x)}, \quad (41)$$

from where, [140, Chapter XIII, § 7, p. 31],

$$u = \int \frac{Q(x)}{v(x)} dx + C, \quad (42)$$

where  $C$  – constant of integration.

Substituting  $u$  and  $v$  into the formula of Eq. (31), [140, Chapter XIII, § 7, p. 30, (2)], we finally get

$$y = v(x) \left[ \int \frac{Q(x)}{v(x)} dx + C \right], \quad (43)$$

or, [140, Chapter XIII, § 7, p. 32, (6)],

$$y = v(x) \int \frac{Q(x)}{v(x)} dx + Cv(x). \quad (44)$$

In the considered problem setting (see, compare, and substitute the corresponding functions, expressions, variables, and values of Eq. (29), (30), (44))

$$\frac{dP_D}{dt} + \lambda_2 P_D = \lambda_1 e^{-\lambda_1 t}, \quad \frac{dy}{dx} + P(x)y = Q(x),$$

$$y = P_D, \quad x = t, \quad P(x) = \lambda_2, \quad Q(x) = \lambda_1 e^{-\lambda_1 t}, \quad v(x) = e^{-[\lambda_2 t]} = e^{-\lambda_2 t}. \quad (45)$$

Thus

$$P_D = e^{-\lambda_2 t} \int \frac{\lambda_1 e^{-\lambda_1 t}}{e^{-\lambda_2 t}} dt + C e^{-\lambda_2 t}. \quad (46)$$

Then

$$P_D = e^{-\lambda_2 t} \lambda_1 \int e^{(\lambda_2 - \lambda_1)t} dt + C e^{-\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} [e^{(\lambda_2 - \lambda_1)t}] + C e^{-\lambda_2 t}. \quad (47)$$

From the initial conditions, expressions (24)  $t_0 = 0$ ;  $P_D|_{t=t_0} = 0$ . It yields

$$0 = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 \cdot 0} [e^{(\lambda_2 - \lambda_1) \cdot 0}] + C e^{-\lambda_2 \cdot 0} = \frac{\lambda_1}{\lambda_2 - \lambda_1} \cdot 1 \cdot 1 + C \cdot 1. \quad (48)$$

Therefore

$$C = -\frac{\lambda_1}{\lambda_2 - \lambda_1}. \quad (49)$$

Then

$$P_D = \frac{\lambda_1}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t}] - \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}). \quad (50)$$

Final result Eq. (50) is the same as (6), (7). It is the probability of the damage state “D” of the system (see Fig. 6).

The probability of the failure state “F” will be obtained as the solution from the third equation of the system (22), i.e.

$$\frac{dP_F}{dt} = \lambda_2 P_D = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}). \quad (51)$$

Integration yields

$$P_F = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} [\int e^{-\lambda_1 t} dt - \int e^{-\lambda_2 t} dt] + C, \quad (52)$$

where  $C$  – constant of integration.

Then

$$P_F = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left[ \left( -\frac{1}{\lambda_1} \right) e^{-\lambda_1 t} + \frac{1}{\lambda_2} e^{-\lambda_2 t} \right] + C. \quad (53)$$

From the initial conditions, expressions (24)  $t_0 = 0$ ;  $P_F|_{t=t_0} = 0$ . It yields

$$\begin{aligned}
0 &= \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left[ \left( -\frac{1}{\lambda_1} \right) e^{-\lambda_1 \cdot 0} + \frac{1}{\lambda_2} e^{-\lambda_2 \cdot 0} \right] + C = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) + C = \\
&= \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left( \frac{\lambda_1 - \lambda_2}{\lambda_2 \lambda_1} \right) + C = -1 + C \Rightarrow C = 1. \quad (54)
\end{aligned}$$

Therefore

$$\begin{aligned}
P_F &= \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left[ \frac{e^{-\lambda_2 t}}{\lambda_2} - \frac{e^{-\lambda_1 t}}{\lambda_1} \right] + 1 = \frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} \left[ \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 \lambda_1} \right] + 1 = \\
&= \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} + 1 = \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} + 1 + e^{-\lambda_1 t} - e^{-\lambda_1 t} = \\
&= 1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} + e^{-\lambda_1 t} = \\
&= 1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_1 t} - \lambda_1 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1}.
\end{aligned}$$

Finally

$$P_F = 1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_1 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} = 1 - e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_2 t} - e^{-\lambda_1 t}). \quad (55)$$

The result (55) is the same as (17), (18).

Numerical simulation results as a conducted calculation experiment are shown in Fig. 7. The data for the performed calculations are the same as the mentioned above at fulfilling modeling portrayed in Fig. 5.

In Fig. 7  $y_0$ ,  $y_1$ ,  $y_2$  represent numerical integration of the differential equations system (22) at the initial conditions (24). Symbols “0”, “1”, “2” depict the system states of “A”, “D”, “F” respectively. The curve  $y_1$  plotted for the damage state “D” coincides with  $P_{DnF}(t)$  for  $P_{D\bar{F}}(t)$ ; these are the solutions Eq. (6), (7), and  $P_D$ , Eq. (50) of the second equation of the system (22) correspondingly.

Both analytical research results described with the sequences of the Eq. (1)-(55) and computer simulation numerical solutions for the differential equations system (22) at the initial conditions (24) give the identical results.

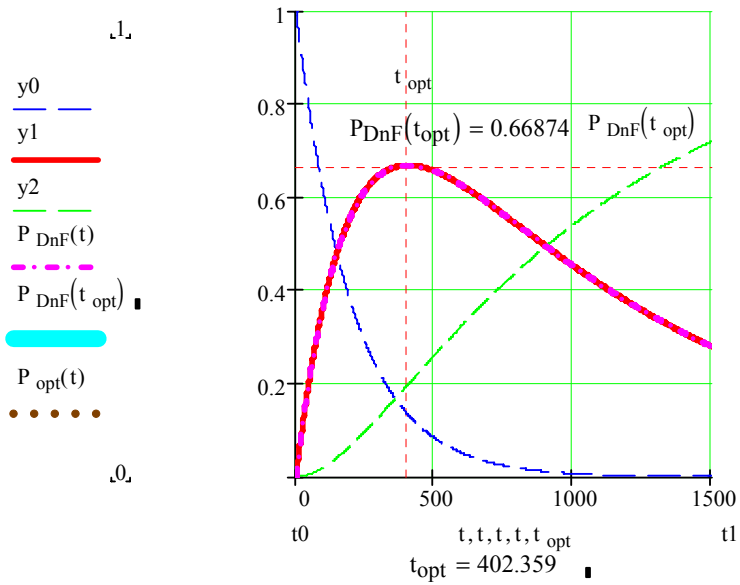


Fig. 7. Results of computer calculation experiments on the MathCad platform

Thus, the detailed solutions for obtaining the reliability objective measures allow successfully assessing the improvements of the A/C functional system M/T process considered in reference [112] on the basis of [136, 140].

## 2. More general modelling implying a possibility of restoration from the damaged into initial state

More general modelling will be done with a step that takes into account a possible return of the system from the damage state “D” into the normal state of “A”. This transition is carried out with the parameter of  $\mu_1$  illustrated on the graph, see Fig. 8.

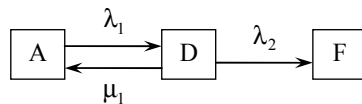


Fig. 8. Graph of three states of an aircraft functional system with a possible transition from the damaged into normal state

The corresponding, to the graph of Fig. 8, system of differential equations by Erlang will have the analogous to the system (22) view of

$$\left. \begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A + \mu_1 P_D; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - (\lambda_2 + \mu_1) P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D. \end{aligned} \right\} \quad (56)$$

**Before solving the system of differential equations (56)** let us solve a simplified problem. The graph is shown in Fig. 9.

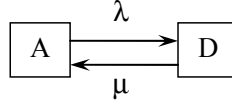


Fig. 9. Graph of two states of an aircraft functional system with a possible transition from the damage into normal state

The corresponding, to the graph of Fig. 9, system of differential equations by Erlang will have the analogous to the system (56) view of

$$\left. \begin{aligned} \frac{dP_A}{dt} &= -\lambda P_A + \mu P_D; \\ \frac{dP_D}{dt} &= \lambda P_A - \mu P_D. \end{aligned} \right\} \quad (57)$$

Let us convert (transform) the system of two ordinary linear differential equations of the first order (57) into one second order ordinary linear differential equation. Differentiate the second equation of the system (57) once more with respect to time.

$$\frac{d^2 P_D}{dt^2} = \frac{d}{dt} \frac{dP_D}{dt} = \frac{d}{dt} (\lambda P_A) - \frac{d}{dt} (\mu P_D) = \lambda \frac{dP_A}{dt} - \mu \frac{dP_D}{dt}. \quad (58)$$

Substitute the derivative  $\frac{dP_A}{dt}$  of the expression (58) with its value from the first equation of the system (57).

$$\frac{d^2 P_D}{dt^2} = \lambda(-\lambda P_A + \mu P_D) - \mu \frac{dP_D}{dt}. \quad (59)$$

But from the second equation of the system (57)

$$P_A = \frac{\frac{dP_D}{dt} + \mu P_D}{\lambda}. \quad (60)$$

Thus, substituting expression (60) for its value into (59) we get

$$\ddot{P}_D = \lambda \left[ -\lambda \left( \frac{\dot{P}_D + \mu P_D}{\lambda} \right) + \mu P_D \right] - \mu \dot{P}_D = \lambda(-\dot{P}_D) - \mu \dot{P}_D = -(\lambda + \mu)\dot{P}_D. \quad (61)$$

Here we have obtained the one second order ordinary linear uniform (homogeneous) differential equation, [140, Chapter XIII, § 21, p. 74, (1)]:

$$\ddot{P}_D + (\lambda + \mu)\dot{P}_D = 0. \quad (62)$$

In order to find the general integral of this equation, it will be sufficient (enough), as that was proven, for instance, in [140, Chapter XIII, § 20, pp. 68-74, Theorem 6, (8)], to find two linearly independent partial solutions, [140, Chapter XIII, § 21, pp. 74-79].

Let the sought partial solutions will be (let us be finding the partial solutions) in the view of [140, Chapter XIII, § 21, p. 74, (2)]:

$$P_D = e^{kt}, \quad \text{where } k = \text{const}; \quad (63)$$

then [140, Chapter XIII, § 21, p. 74]:

$$\dot{P}_D = ke^{kt}, \quad \ddot{P}_D = k^2 e^{kt}. \quad (64)$$

Substituting the obtained expressions of the derivatives into Eq. (62), we are finding [140, Chapter XIII, § 21, p. 74]:

$$k^2 e^{kt} + (\lambda + \mu)ke^{kt} = e^{kt} [k^2 + (\lambda + \mu)k] = 0. \quad (65)$$

Since (in view of the fact that)  $e^{kt} \neq 0$ , hence (for this reason), [140, Chapter XIII, § 21, p. 74, (3)]:

$$k^2 + (\lambda + \mu)k = 0. \quad (66)$$

Therefore, if  $k$  satisfies Eq. (66), then  $e^{kt}$  will be the solution of Eq. (62). Eq. (66) is called the *characteristic equation* in relation with Eq. (62).

The characteristic equation is a quadratic equation, having two roots; let us denote them as  $k_1$  and  $k_2$ . At this, [140, Chapter XIII, § 21, p. 75]:

$$k_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad k_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \quad (67)$$

where  $a$ ,  $b$ ,  $c$  – coefficients of a corresponding quadratic equation likewise

$$ak^2 + bk + c = 0. \quad (68)$$

There can be the following cases, [140, Chapter XIII, § 21, p. 75]:

- I.  $k_1$  and  $k_2$  – real, being unequal one to the other numbers ( $k_1 \neq k_2$ );
- II.  $k_1$  and  $k_2$  – complex numbers;
- III.  $k_1$  and  $k_2$  – real, being equal one to the other numbers ( $k_1 = k_2$ ).

Each case is considered separately at many reference books and literature sources. For example, [140, Chapter XIII, § 21, p. 75-79].

In the stated problem

$$a = 1, \quad b = \lambda + \mu, \quad c = 0. \quad (69)$$

The roots  $k_1$  and  $k_2$  from the equation (66)

$$k(k + \lambda + \mu) = 0, \quad k_1 = 0, \quad k_2 + \lambda + \mu = 0, \quad k_2 = -(\lambda + \mu). \quad (70)$$

This is the condition # I, [140, Chapter XIII, § 21, p. 75]: ( $k_1 \neq k_2$ ). In such case, the partial solutions will be the functions [140, Chapter XIII, § 21, p. 75]:

$$P_{D_1} = e^{k_1 t}, \quad P_{D_2} = e^{k_2 t}. \quad (71)$$

These solutions are linearly independent, since [140, Chapter XIII, § 21, p. 75]:

$$P_{D_1} = e^{k_1 t}, \quad \frac{P_{D_2}}{P_{D_1}} = \frac{e^{k_2 t}}{e^{k_1 t}} = e^{(k_2 - k_1)t} \neq \text{const}. \quad (72)$$

Hence (for this reason), the general integral has the view of [140, Chapter XIII, § 21, p. 75]:



$$P_D = C_1 e^{k_1 t} + C_2 e^{k_2 t} = C_1 e^{0t} + C_2 e^{-(\lambda+\mu)t} = C_1 + C_2 e^{-(\lambda+\mu)t}. \quad (73)$$

From the initial conditions similar to (24)

$$t_0 = 0; \quad P_A|_{t=t_0} = 1; \quad P_D|_{t=t_0} = 0. \quad (74)$$

It yields from Eq. (73)

$$0 = C_1 + C_2 e^{-(\lambda+\mu)0}; \quad 0 = C_1 + C_2; \quad C_1 = -C_2. \quad (75)$$

From where

$$P_D = -C_2 + C_2 e^{-(\lambda+\mu)t} = C_2 [e^{-(\lambda+\mu)t} - 1] = C_1 [1 - e^{-(\lambda+\mu)t}]. \quad (76)$$

Solving Eq. (60) for  $P_A$  with the use of Eq. (76)

$$P_A = \frac{\dot{P}_D + \mu P_D}{\lambda} = \frac{C_1 [(\lambda + \mu) e^{-(\lambda+\mu)t}] + \mu C_1 [1 - e^{-(\lambda+\mu)t}]}{\lambda}. \quad (77)$$

Again, from the initial conditions (74)

$$1 = \frac{C_1 [(\lambda + \mu) e^{-(\lambda+\mu)0}] + \mu C_1 [1 - e^{-(\lambda+\mu)0}]}{\lambda} = \frac{C_1 [(\lambda + \mu)] + \mu C_1 [1 - 1]}{\lambda}.$$

$$1 = \frac{C_1 (\lambda + \mu)}{\lambda}; \quad C_1 = \frac{\lambda}{\lambda + \mu}; \quad C_2 = -\frac{\lambda}{\lambda + \mu}. \quad (78)$$

Then, from Eq. (76) finally we are having

$$P_D = \frac{\lambda}{\lambda + \mu} [1 - e^{-(\lambda+\mu)t}]. \quad (79)$$

On the other hand, let

$$\dot{P}_D = y. \quad (80)$$

Then, for the system (57) converted (transformed) into Eq. (62)

$$\dot{y} + (\lambda + \mu)y = 0. \quad (81)$$

Now we have a differential equation of the first order (81) with splitting variables. Let us split the variables:

$$\frac{dy}{dt} = -(\lambda + \mu)y; \quad \frac{dy}{y} = -(\lambda + \mu)dt. \quad (82)$$

Integration yields

$$\ln|y| = -(\lambda + \mu)t + \ln|C_1|; \quad \ln\left|\frac{y}{C_1}\right| = -(\lambda + \mu)t; \quad \frac{y}{C_1} = e^{-(\lambda+\mu)t}; \quad (83)$$

where  $C_1$  – constant of integration; or

$$y = C_1 e^{-(\lambda+\mu)t}. \quad (84)$$

Next, see Eq. (80)

$$\frac{dP_D}{dt} = C_1 e^{-(\lambda+\mu)t}. \quad (85)$$

Integrating one more time (once more, again), i.e. from Eq. (85) it is obtained

$$P_D = -\frac{C_1}{\lambda+\mu} e^{-(\lambda+\mu)t} + C_2, \quad (86)$$

where  $C_2$  – constant of integration.

From the initial conditions (74)

$$0 = -\frac{C_1}{\lambda+\mu} e^{-(\lambda+\mu)0} + C_2; \quad C_2 = \frac{C_1}{\lambda+\mu}; \quad \frac{C_1}{C_2} = \lambda + \mu. \quad (87)$$

Then Eq. (86) will take the view of

$$-C_2 e^{-(\lambda+\mu)t} + C_2 = C_2 [1 - e^{-(\lambda+\mu)t}]. \quad (88)$$

For the probability of the normal state “A” from the second equation of the system (57), analogously to Eq. (77) with the help of Eq. (86)

$$\begin{aligned} P_A &= \frac{\dot{P}_D + \mu P_D}{\lambda} = \frac{-\frac{C_1}{\lambda+\mu} [-(\lambda+\mu)] e^{-(\lambda+\mu)t} + \mu \left( -\frac{C_1}{\lambda+\mu} e^{-(\lambda+\mu)t} + C_2 \right)}{\lambda} = \\ &= \frac{C_1 e^{-(\lambda+\mu)t} + \mu [-C_2 e^{-(\lambda+\mu)t} + C_2]}{\lambda} = \frac{C_1 e^{-(\lambda+\mu)t} + \mu C_2 [1 - e^{-(\lambda+\mu)t}]}{\lambda}. \end{aligned} \quad (89)$$

Once more from the initial conditions (74) the Eq. (89) yields

$$1 = \frac{C_1 e^{-(\lambda+\mu)0} + \mu C_2 [1 - e^{-(\lambda+\mu)0}]}{\lambda}; \quad \lambda = C_1 + \mu C_2 [1 - 1]; \quad C_1 = \lambda. \quad (90)$$

In accordance to relations (87)

$$C_2 = \frac{\lambda}{\lambda+\mu}. \quad (91)$$

Thus, substituting expression (91) for its value into Eq. (88) we obtain

$$P_D = \frac{\lambda}{\lambda + \mu} [1 - e^{-(\lambda + \mu)t}]. \quad (92)$$

Or for Eq. (86), applying expressions (90) and (91), it yields,

$$P_D = -\frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} + \frac{\lambda}{\lambda + \mu} = \frac{\lambda}{\lambda + \mu} (1 - e^{-(\lambda + \mu)t}), \quad (93)$$

the same result.

Also, using instead of Eq. (86) yielding (89), the result (92) or (93) can be obtained with the help of Eq. (88), which gives

$$P_A = \frac{\dot{P}_D + \mu P_D}{\lambda} = \frac{C_2(\lambda + \mu)[e^{-(\lambda + \mu)t}] + \mu C_2[1 - e^{-(\lambda + \mu)t}]}{\lambda}. \quad (94)$$

Applying the initial conditions (74) once again the Eq. (94) yields

$$1 = \frac{C_2(\lambda + \mu)e^{-(\lambda + \mu)0} + \mu C_2[1 - e^{-(\lambda + \mu)0}]}{\lambda} = \frac{C_2(\lambda + \mu) + \mu C_2[1 - 1]}{\lambda},$$

$$1 = \frac{C_2(\lambda + \mu)}{\lambda}. \quad (95)$$

And finally we come to expression (91):

$$C_2 = \frac{\lambda}{\lambda + \mu}. \quad (96)$$

In such case, for probability of the normal state denoted as ‘‘A’’,  $P_A$ , from Eq. (89) or (94), substituting constants of integration (90) and (91) or (96) for their values into Eq. (89) or (94), we will get

$$P_A = \frac{\lambda[e^{-(\lambda + \mu)t}] + \mu \frac{\lambda}{\lambda + \mu} [1 - e^{-(\lambda + \mu)t}]}{\lambda} = e^{-(\lambda + \mu)t} + \frac{\mu}{\lambda + \mu} [1 - e^{-(\lambda + \mu)t}] =$$

$$= \frac{(\lambda + \mu)e^{-(\lambda + \mu)t} + \mu[1 - e^{-(\lambda + \mu)t}]}{\lambda + \mu} = \frac{(\lambda e^{-(\lambda + \mu)t} + \mu e^{-(\lambda + \mu)t}) + [\mu - \mu e^{-(\lambda + \mu)t}]}{\lambda + \mu}. \quad (97)$$

Finally, cancelling members  $+\mu e^{-(\lambda + \mu)t}$  and  $-\mu e^{-(\lambda + \mu)t}$  in the obtained expression (97), it yields

$$P_A = \frac{\lambda e^{-(\lambda + \mu)t} + \mu}{\lambda + \mu}. \quad (98)$$

It is obvious that normalizing condition for the probabilities of the normal “A” and damaged “D” states,  $P_A$  and  $P_D$  respectively, satisfies:

$$P_A + P_D = \frac{\lambda e^{-(\lambda+\mu)t} + \mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} [1 - e^{-(\lambda+\mu)t}],$$

$$P_A + P_D = \frac{\lambda e^{-(\lambda+\mu)t} + \mu + \lambda - \lambda e^{-(\lambda+\mu)t}}{\lambda + \mu}. \quad (99)$$

After cancelling the similar members  $+\lambda e^{-(\lambda+\mu)t}$  and  $-\lambda e^{-(\lambda+\mu)t}$  and reducing for the member of the denominator  $\lambda + \mu$  in the obtained expression (99), it yields

$$P_A + P_D = 1. \quad (100)$$

**Now we may return to the system for the three states**, namely: normal – “A”, damaged – “D”, failure – “F”, corresponding system of equations is (56).

Here, in accordance with [140, Chapter XIII, § 30, pp. 108-113], the characteristic equation for system (56) will be similarly (likewise) [140, Chapter XIII, § 30, p. 109, (5)]:

$$\begin{vmatrix} -\lambda_1 - k & \mu_1 & 0 \\ \lambda_1 & -(\lambda_2 + \mu_1) - k & 0 \\ 0 & \lambda_2 & 0 - k \end{vmatrix} = 0. \quad (101)$$

Determinant (101) yields

$$\begin{aligned} & (-\lambda_1 - k)[-(\lambda_2 + \mu_1) - k](0 - k) + \lambda_1 \lambda_2 \cdot 0 + \mu_1 \cdot 0 \cdot 0 - \\ & - [-(\lambda_2 + \mu_1) - k] \cdot 0 \cdot 0 - \lambda_1 \mu_1 (0 - k) - (-\lambda_1 - k) \lambda_2 \cdot 0 = 0. \end{aligned} \quad (102)$$

Which means

$$\begin{aligned} & (-\lambda_1 - k)[-(\lambda_2 + \mu_1) - k](-k) - \lambda_1 \mu_1 (-k) = 0, \\ & -(\lambda_1 + k)[\lambda_2 + \mu_1 + k]k + \lambda_1 \mu_1 k = 0. \end{aligned} \quad (103)$$

From where

$$k[\lambda_1 \mu_1 - (\lambda_1 + k)(\lambda_2 + \mu_1 + k)] = 0. \quad (104)$$

Thus, we have already known at least one root:

$$k_1 = 0. \quad (105)$$

Then, for finding two other roots from Eq. (104)

$$\lambda_1 \mu_1 - (\lambda_1 + k)(\lambda_2 + \mu_1 + k) = 0,$$

$$\lambda_1\mu_1 - \lambda_1\lambda_2 - \lambda_1\mu_1 - \lambda_1k - k\lambda_2 - k\mu_1 - k^2 = 0. \quad (106)$$

Reducing Eq. (106) and cancelling the similar members  $\lambda_1\mu_1$  and  $-\lambda_1\mu_1$ , we get the quadratic equation

$$-k^2 - k(\lambda_1 + \lambda_2 + \mu_1) - \lambda_1\lambda_2 = 0. \quad (107)$$

The sought roots are

$$k_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}; \quad k_3 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}; \quad (108)$$

where  $a = -1$ ,  $b = -(\lambda_1 + \lambda_2 + \mu_1)$ ,  $c = -\lambda_1\lambda_2$  – corresponding coefficients of Eq. (107).

For each root  $k_i$  of Eq. (101)-(104), (106), (107), namely  $k_1, k_2, k_3$  Eq. (105) and (108) we will write down the system of linear uniform (homogenous) algebraic equations with respect to their coefficients  $\alpha_1^{(i)}, \alpha_2^{(i)}, \alpha_3^{(i)}$ , [140, Chapter XIII, § 30, p. 108, (3)]:

$$\left. \begin{array}{l} (-\lambda_1 - k)\alpha_1 + \mu_1\alpha_2 + 0 \cdot \alpha_3 = 0; \\ \lambda_1\alpha_1 + [-(\lambda_2 + \mu_1) - k]\alpha_2 + 0 \cdot \alpha_3 = 0; \\ 0 \cdot \alpha_1 + \lambda_2\alpha_2 + (0 - k)\alpha_3 = 0. \end{array} \right\} \quad (109)$$

The system of Eq. (109) derives from an assumption of a partial solution existence in the view of [140, Chapter XIII, § 30, p. 108, (2)]:

$$P_A = \alpha_1 e^{kt}; \quad P_D = \alpha_2 e^{kt}; \quad P_F = \alpha_3 e^{kt}; \quad (110)$$

for the system of Eq. (56).

Since having three roots in the stated problem setting, we obtain, [140, Chapter XIII, § 30, p. 109], the solution of the system of Eq. (56):

for the root of  $k_1$

$$P_A^{(1)} = \alpha_1^{(1)} e^{k_1 t}; \quad P_D^{(1)} = \alpha_2^{(1)} e^{k_1 t}; \quad P_F^{(1)} = \alpha_3^{(1)} e^{k_1 t}; \quad (111)$$

for the root of  $k_2$

$$P_A^{(2)} = \alpha_1^{(2)} e^{k_2 t}; \quad P_D^{(2)} = \alpha_2^{(2)} e^{k_2 t}; \quad P_F^{(2)} = \alpha_3^{(2)} e^{k_2 t}; \quad (112)$$

for the root of  $k_3$

$$P_A^{(3)} = \alpha_1^{(3)} e^{k_3 t}; \quad P_D^{(3)} = \alpha_2^{(3)} e^{k_3 t}; \quad P_F^{(3)} = \alpha_3^{(3)} e^{k_3 t}. \quad (113)$$

In the way (by the method) of direct substitution of partial solutions (111)-(113) into equations, one can be convinced that the system of functions, similarly to [140, Chapter XIII, § 30, p. 109, (6)]:

$$\left. \begin{aligned} P_A &= C_1 P_A^{(1)} + C_2 P_A^{(2)} + C_3 P_A^{(3)} = C_1 \alpha_1^{(1)} e^{k_1 t} + C_2 \alpha_1^{(2)} e^{k_2 t} + C_3 \alpha_1^{(3)} e^{k_3 t}; \\ P_D &= C_1 P_D^{(1)} + C_2 P_D^{(2)} + C_3 P_D^{(3)} = C_1 \alpha_2^{(1)} e^{k_1 t} + C_2 \alpha_2^{(2)} e^{k_2 t} + C_3 \alpha_2^{(3)} e^{k_3 t}; \\ P_F &= C_1 P_F^{(1)} + C_2 P_F^{(2)} + C_3 P_F^{(3)} = C_1 \alpha_3^{(1)} e^{k_1 t} + C_2 \alpha_3^{(2)} e^{k_2 t} + C_3 \alpha_3^{(3)} e^{k_3 t}; \end{aligned} \right\} \quad (114)$$

where  $C_1; C_2; C_3$ ; – arbitrary constants; also is the solution of the differential equations system (56). This is the general solution of the differential equations system (56), [140, Chapter XIII, § 30, p. 110].

Satisfying the condition of Eq. (105) for root  $k_1 = 0$  from the system of Eq. (109) we have

$$\left. \begin{aligned} (-\lambda_1 - k_1) \alpha_1^{(1)} &+ \mu_1 \alpha_2^{(1)} &+ 0 \cdot \alpha_3^{(1)} &= 0; \\ \lambda_1 \alpha_1^{(1)} &+ [-(\lambda_2 + \mu_1) - k_1] \alpha_2^{(1)} &+ 0 \cdot \alpha_3^{(1)} &= 0; \\ 0 \cdot \alpha_1^{(1)} &+ \lambda_2 \alpha_2^{(1)} &+ (0 - k_1) \alpha_3^{(1)} &= 0. \end{aligned} \right\} \quad (115)$$

Thus

$$\left. \begin{aligned} -\lambda_1 \alpha_1^{(1)} &+ \mu_1 \alpha_2^{(1)} &+ 0 \cdot \alpha_3^{(1)} &= 0; \\ \lambda_1 \alpha_1^{(1)} &- (\lambda_2 + \mu_1) \alpha_2^{(1)} &+ 0 \cdot \alpha_3^{(1)} &= 0; \\ 0 &+ \lambda_2 \alpha_2^{(1)} &- 0 \cdot \alpha_3^{(1)} &= 0. \end{aligned} \right\} \quad (116)$$

From where, immediately the coefficients are

$$\alpha_2^{(1)} = 0; \quad \alpha_1^{(1)} = 0; \quad \alpha_3^{(1)} = 1; \quad (117)$$

since  $\alpha_3^{(1)}$  is an arbitrary number, supposedly (by an assumption)  $\alpha_3^{(1)} = 1$ , [140, Chapter XIII, § 30, p. 109].

For the Eq. (107) roots of  $k_2$  and  $k_3$ , Eq. (108), the system of Eq. (109) analogous to the system of Eq. (115) it yields

$$\left. \begin{array}{l} (-\lambda_1 - k_{2,3})\alpha_1^{(2,3)} + \mu_1\alpha_2^{(2,3)} + 0 \cdot \alpha_3^{(2,3)} = 0; \\ \lambda_1\alpha_1^{(2,3)} + [-(\lambda_2 + \mu_1) - k_{2,3}]\alpha_2^{(2,3)} + 0 \cdot \alpha_3^{(2,3)} = 0; \\ 0 \cdot \alpha_1^{(2,3)} + \lambda_2\alpha_2^{(2,3)} + (0 - k_{2,3})\alpha_3^{(2,3)} = 0. \end{array} \right\} \quad (118)$$

The system of Eq. (118) may be solved for unknown sought (wanted/needed) coefficients in a few different ways. One of them is the next.

Since one of the alpha coefficients can be chosen arbitrary, accordingly with [140, Chapter XIII, § 30, p. 109], let us assume

$$\alpha_2^{(2,3)} = 1. \quad (119)$$

Then, from the first equation of system (118)

$$(-\lambda_1 - k_{2,3})\alpha_1^{(2,3)} + \mu_1 = 0; \quad \alpha_1^{(2,3)} = \frac{\mu_1}{\lambda_1 + k_{2,3}}. \quad (120)$$

Or from the second equation

$$\lambda_1\alpha_1^{(2,3)} - (\lambda_2 + \mu_1) - k_{2,3} = 0; \quad \alpha_1^{(2,3)} = \frac{\lambda_2 + \mu_1 + k_{2,3}}{\lambda_1}. \quad (121)$$

Or summing the first and second equations, reducing, and cancelling the similar members

$$\begin{aligned} (-\lambda_1 - k_{2,3})\alpha_1^{(2,3)} + \lambda_1\alpha_1^{(2,3)} + \mu_1\alpha_2^{(2,3)} + [-(\lambda_2 + \mu_1) - k_{2,3}]\alpha_2^{(2,3)} &= 0; \\ -k_{2,3}\alpha_1^{(2,3)} - [\lambda_2 + k_{2,3}]\alpha_2^{(2,3)} &= 0; \quad \alpha_1^{(2,3)} = -\frac{\lambda_2 + k_{2,3}}{k_{2,3}}. \end{aligned} \quad (122)$$

All three expressions for  $\alpha_1^{(2,3)}$ , i.e. Eq. (120)-(122) are equivalent because all of them use the roots  $k_2$  and  $k_3$ , Eq. (108) of the initial quadratic equation Eq. (107).

Indeed.

Equalizing Eq. (120) and (121) we get

$$\begin{aligned} \frac{\mu_1}{\lambda_1 + k_{2,3}} &= \frac{\lambda_2 + \mu_1 + k_{2,3}}{\lambda_1}; \quad \lambda_1\mu_1 = (\lambda_2 + \mu_1 + k_{2,3})(\lambda_1 + k_{2,3}); \\ \lambda_1\mu_1 &= \lambda_1\lambda_2 + \lambda_1\mu_1 + \lambda_1k_{2,3} + \lambda_2k_{2,3} + \mu_1k_{2,3} + k_{2,3}^2. \end{aligned} \quad (123)$$

And cancelling for  $\lambda_1\mu_1$  in both parts of Eq. (123) it yields Eq. (107):

$$\lambda_1\lambda_2 + (\lambda_1 + \lambda_2 + \mu_1)k_{2,3} + k_{2,3}^2 = 0. \quad (124)$$

The same result is obtained if make equal Eq. (120) and (122):

$$\begin{aligned} \frac{\mu_1}{\lambda_1 + k_{2,3}} &= -\frac{\lambda_2 + k_{2,3}}{k_{2,3}}; & \mu_1 k_{2,3} &= -(\lambda_2 + k_{2,3})(\lambda_1 + k_{2,3}); \\ \mu_1 k_{2,3} &= -\lambda_1\lambda_2 - \lambda_1 k_{2,3} - \lambda_2 k_{2,3} - k_{2,3}^2; \\ k_{2,3}^2 + (\lambda_1 + \lambda_2 + \mu_1)k_{2,3} + \lambda_1\lambda_2 &= 0. \end{aligned} \quad (125)$$

When equalling Eq. (121) and (122) it gives the same. Indeed:

$$\begin{aligned} \frac{\lambda_2 + \mu_1 + k_{2,3}}{\lambda_1} &= -\frac{\lambda_2 + k_{2,3}}{k_{2,3}}; & (\lambda_2 + \mu_1 + k_{2,3})k_{2,3} &= -(\lambda_2 + k_{2,3})\lambda_1; \\ (\lambda_2 + \mu_1)k_{2,3} + k_{2,3}^2 &= -\lambda_1\lambda_2 - \lambda_1 k_{2,3}; \\ k_{2,3}^2 + (\lambda_1 + \lambda_2 + \mu_1)k_{2,3} + \lambda_1\lambda_2 &= 0. \end{aligned} \quad (126)$$

For the values of coefficient  $\alpha_3^{(2,3)}$ , we have, from the third equation of the system of Eq. (118) and condition (119),

$$\lambda_2\alpha_2^{(2,3)} - k_{2,3}\alpha_3^{(2,3)} = 0; \quad \alpha_3^{(2,3)} = \frac{\lambda_2}{k_{2,3}}. \quad (127)$$

Thus, turning back to the system of Eq. (114), we determine the unknown coefficients of the general solution of the differential equations system (56), [140, Chapter XIII, § 30, p. 110], satisfying the initial conditions (24):  $t_0 = 0$ ;  $P_A|_{t=t_0} = 1$ ;  $P_D|_{t=t_0} = 0$ ;  $P_F|_{t=t_0} = 0$ ; and

have already known the coefficients of alpha; i.e. Eq. (117):  $\alpha_2^{(1)} = 0$ ;  $\alpha_1^{(1)} = 0$ ;  $\alpha_3^{(1)} = 1$ ; Eq. (119):  $\alpha_2^{(2,3)} = 1$ ; Eq. (120):  $\alpha_1^{(2,3)} = \frac{\mu_1}{\lambda_1 + k_{2,3}}$ ;

Eq. (127):  $\alpha_3^{(2,3)} = \frac{\lambda_2}{k_{2,3}}$ :

$$\left. \begin{aligned} P_A &= C_1\alpha_1^{(1)}e^{k_1t} + C_2\alpha_1^{(2)}e^{k_2t} + C_3\alpha_1^{(3)}e^{k_3t}; \\ P_D &= C_1\alpha_2^{(1)}e^{k_1t} + C_2\alpha_2^{(2)}e^{k_2t} + C_3\alpha_2^{(3)}e^{k_3t}; \\ P_F &= C_1\alpha_3^{(1)}e^{k_1t} + C_2\alpha_3^{(2)}e^{k_2t} + C_3\alpha_3^{(3)}e^{k_3t}; \end{aligned} \right\}_{t_0=0} =$$



$$\begin{aligned}
&= \left\{ \begin{array}{l} 1 = C_1 \cdot 0 \cdot e^{k_1 \cdot 0} + C_2 \alpha_1^{(2)} e^{k_2 \cdot 0} + C_3 \alpha_1^{(3)} e^{k_3 \cdot 0}; \\ 0 = C_1 \cdot 0 \cdot e^{k_1 \cdot 0} + C_2 \cdot 1 \cdot e^{k_2 \cdot 0} + C_3 \cdot 1 \cdot e^{k_3 \cdot 0}; \\ 0 = C_1 \cdot 1 \cdot e^{k_1 \cdot 0} + C_2 \alpha_3^{(2)} e^{k_2 \cdot 0} + C_3 \alpha_3^{(3)} e^{k_3 \cdot 0}; \end{array} \right\} = \\
&= \left\{ \begin{array}{l} 1 = 0 + C_2 \alpha_1^{(2)} + C_3 \alpha_1^{(3)}; \\ 0 = 0 + C_2 + C_3; \\ 0 = C_1 + C_2 \alpha_3^{(2)} + C_3 \alpha_3^{(3)}; \end{array} \right\} = \left\{ \begin{array}{l} 1 = 0 + C_2 \frac{\mu_1}{\lambda_1 + k_2} + C_3 \frac{\mu_1}{\lambda_1 + k_3}; \\ 0 = 0 + C_2 + C_3; \\ 0 = C_1 + C_2 \frac{\lambda_2}{k_2} + C_3 \frac{\lambda_2}{k_3}. \end{array} \right\} \quad (128)
\end{aligned}$$

From the second equation of the system of Eq. (128) it yields

$$C_2 = -C_3. \quad (129)$$

Substituting the values of Eq. (129) for the corresponding members into the first equation of the system of Eq. (128) we get

$$1 = C_3 \left( \frac{\mu_1}{\lambda_1 + k_3} - \frac{\mu_1}{\lambda_1 + k_2} \right); \quad C_3 = \frac{1}{\frac{\mu_1}{\lambda_1 + k_3} - \frac{\mu_1}{\lambda_1 + k_2}}. \quad (130)$$

In order to make the notations shorter let us put down the indications with the alpha symbolizations:

$$1 = -C_3 \alpha_1^{(2)} + C_3 \alpha_1^{(3)} = C_3 [\alpha_1^{(3)} - \alpha_1^{(2)}]; \quad C_3 = \frac{1}{\alpha_1^{(3)} - \alpha_1^{(2)}}. \quad (131)$$

Therefore

$$C_2 = -\frac{1}{\alpha_1^{(3)} - \alpha_1^{(2)}}. \quad (132)$$

From the third equation of the system of Eq. (128) we obtain

$$C_1 = -C_2 \alpha_3^{(2)} - C_3 \alpha_3^{(3)}. \quad (133)$$

Now, all coefficients are expressed through the given values, hence, the system of Eq. (56) is successfully solved.

It may be concluded. The obtained solutions results described with the mathematical expressions of (1)-(133) create a possibility for the students to model the situations relating with the reliability objective measures allowing successfully assessing the improvements of the A/C

functional system M/T process considered in reference [112] on the basis of the methods discussed in [136, 140] in application to [1-144].

Further **PARTS** of the **METHOD GUIDE ON THE SS** problems are going to be intended for several other scientific components of the SS work on the academic subject. Background readings of the new theories [4-60, 64, 65, 89-109, 117, 118, 138] are welcome here in respect with the **CONTINUING AIRCRAFT AIRWORTHINESS (ICAO DOC 9760)** [1-144].

It is very attractive to use the calculus of variations theories [64, 65, 80, 85, 88, 95, 96, 117, 118] emphasizing the uncertainties of different kinds [4-60, 64, 65, 89-109, 117, 118, 138] for optimization of the problems stated in all references of [1-144].

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ПРИДАТНОСТІ  
ПОВІТРЯНИХ СУДЕН  
(ICAO Doc 9760)**

Частина I

**ПОКАЗНИКИ НАДІЙНОСТІ ДЛЯ ОЦІНКИ  
УДОСКОНАЛЕННЯ ПРОЦЕСУ ТЕХНІЧНОГО  
ОБСЛУГОВУВАННЯ ПОВІТРЯНОГО СУДНА**

Методичні рекомендації  
до виконання самостійної роботи  
для студентів 1-го курсу галузі знань  
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(Англійською мовою)

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